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PRODUCT OF BIPOLAR INTUITIONISTIC FUZZY GRAPHS AND THEIR DEGREE

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Abstract. In this paper, bipolar intuitionistic fuzzy graphs with four operations namely Cartesian product, composition, tensor product, normal product are defined. Also, the degrees of the vertices of the resultant graphs which are obtained from two given bipolar intuitionistic fuzzy graphs G_1 and G_2 using the operations Cartesian product, composition, tensor product, normal product are determined.

Keywords: Bipolar intuitionistic fuzzy graph, Cartesian product, composition of graph, tensor product, normal product.

AMS Subject Classification: 05C72

1. Introduction

In 1965, Zadeh [30] represented the uncertainty as fuzzy subset of sets. Since then, the theory of fuzzy sets has become a vigorous area of research in different disciplines including medical and life sciences, management sciences, social sciences, engineering, statistics, graph theory, artificial intelligence, signal processing, multiagent systems, pattern recognition, robotics, computer networks, expert systems, decision-making, automata theory, etc. Graph theory has numerous applications to problems in computer science, networking routing, system analysis, electrical engineering, operations research, economics, transportation and many others. In many cases, some aspects of a graph-theoretic problem may be uncertain. The bipolar fuzzy sets have been explained by Zhang [31] in 1994. Zhang extended the fuzzy sets as bipolar fuzzy sets by assigning the membership value in the range $[-1, 1]$. In a bipolar fuzzy set, the membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree $(0, 1]$ of an element indicates that the element somewhat satisfies the property, and the membership degree $[-1, 0]$ of an element indicates that the element somewhat satisfies the implicit counter property.

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In 2001, Mordeson and Nair [7] discussed about the properties of fuzzy graphs and hypergraphs. After that, the operation of union, join, Cartesian product and composition on two fuzzy graphs was defined by Mordeson and Peng [8]. Bhattacharya in 1987 developed some remarks on fuzzy graphs. Hai Long Yang et al.[6] gave the generalized bipolar fuzzy graphs. Atanassov [5] introduced the concept of intuitionistic fuzzy set as a generalization of fuzzy sets. Atanassov added a new components which determines the degree of non-membership in the definition of fuzzy set. In 1975, Rosenfeld [16] discussed the concept of fuzzy graph. The fuzzy relations between fuzzy sets were also considered by Rosenfeld and he developed the structure of fuzzy graphs, obtained analogs of several graphs theoretical concepts. After Rosenfeld [16] the fuzzy graph theory increases with its various types of branches, such as - fuzzy tolerance graph [24], fuzzy threshold graph [23], bipolar fuzzy graphs [14, 15, 28], balanced interval-valued fuzzy graphs [10, 12], fuzzy k-competition graphs and p-competition fuzzy graphs [26], fuzzy planar graphs [22, 29], bipolar fuzzy hypergraphs [25], etc. Also several works have been done on fuzzy graphs by Samanta and Pal[28]. Sahoo and Pal [19] discussed the concept of intuitionistic fuzzy competition graph. They also discussed intuitionistic fuzzy tolerance graph with application [20], different types of products on intuitionistic fuzzy graphs [18] and product of intuitionistic fuzzy graphs and their degrees [21].

2. Preliminaries

Let V be a universe of discourse. It may be taken as the set of vertices of a graph G. If the membership value of $u \in V$ is non-zero, then u is considered as a vertex of G.

Definition 2.1. *A fuzzy set of a set V is a mapping σ from V to $[0,1]$. A fuzzy graph $G = (V, \sigma, \mu)$, where V is a set of vertices, σ and μ are two functions defined as $\mu : V \times V \longrightarrow [0,1]$ is a symmetric fuzzy relation and $\sigma : V \to [0,1]$, such that $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$, where $\sigma(u)$ and $\mu(u,v)$ represent the membership values of the vertex u and the edge (u,v) or uv respectively. The underlying crisp graph of $G = (V, \sigma, \mu)$ is denoted by $G^* = (V, E)$ where $E \subseteq V \times V$.*

Definition 2.2. *Let $G = (V, \sigma, \mu)$ be a fuzzy graph, the degree of a vertex u in G is defined by*
$d_u = \sum_{u \neq v} \mu(u,v) = \sum_{uv \in E} \mu(u,v)$.

Definition 2.3. *A bipolar fuzzy graph with an underlying set $V$is defined to be a pair $G = (V, A, B)$ where $A = (\mu_A^P, \mu_A^N)$ is a bipolar fuzzy set in V and $B = (\mu_B^P, \mu_B^N)$ is a bipolar fuzzy set in $E \subseteq V \times V$ such that*
$\mu_B^P(x,y) \leq min(\mu_A^P(x), \mu_A^P(y))$ *and* $\mu_B^N(x,y) \geq max(\mu_A^N(x), \mu_A^N(y))$ *for all* $(x,y) \in E$.

Definition 2.4. *An intuitionistic fuzzy graph is of the form $G = (V, \mu, \lambda)$ where*
(i)The vertex set $V = \{v_0, v_1, \ldots, v_n\}$ such that $\mu_1 : V \to [0,1]$ and $\lambda_1 : V \to [0,1]$, denote the degree of membership and non-membership of the vertex $v_i \in V$ respectively and $0 \leq \mu_1(v_i) + \lambda_1(v_i) \leq 1$ for every $v_i \in V$ $(i = 1, 2, \ldots, n)$, and
(ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \to [0,1]$ and $\lambda_2 : V \times V \to [0,1]$, where $\mu_2(v_i, v_j)$ and $\lambda_2(v_i, v_j)$ denote the the degree of membership and non-membership value of the edge (v_i, v_j) respectively such that $\mu_2(v_i, v_j) \leq \min\{\mu_1(v_i), \mu_1(v_j)\}$ and $\lambda_2(v_i, v_j) \leq \max\{\lambda_1(v_i), \lambda_1(v_j)\}$, $0 \leq \mu_2(v_i, v_j) + \lambda_2(v_i, v_j) \leq 1$ for every edge (v_i, v_j).

The main objective of this paper is to study the bipolar intuitionistic fuzzy graph and this graph is based on the bipolar intuitionistic fuzzy set defined below.

3. Bipolar intuitionistic fuzzy graphs

Definition 3.1. *Let X be a non empty set. A bipolar intuitionistic fuzzy set $B = \{(x, \mu^P(x), \mu^N(x)$*
$, \lambda^P(x), \lambda^N(x)\}$ where $\mu^P : X \longrightarrow [0,1]$, $\mu^N : X \longrightarrow [-1,0]$, $\lambda^P : X \longrightarrow [0,1]$, $\lambda^N : X \longrightarrow [-1,0]$ are the mappings such that $0 \leq \mu^P(x) + \lambda^P(x) \leq 1$, $-1 \leq \mu^N(x) + \lambda^N(x) \leq 0$.

Definition 3.2. *A bipolar intuitionistic fuzzy graph with an underlying set V is defined to be a pair $G = (A, B)$, where $A = (\mu^P_A, \mu^N_A, \lambda^P_A, \lambda^N_A)$ is a bipolar intuitionistic fuzzy set on $V$and $B = (\mu^P_B, \mu^N_B, \lambda^P_B, \lambda^N_B)$ is a bipolar intuitionistic fuzzy set on $E \subseteq V \times V$ such that*
$\mu^P_B(x, y) \leq \min(\mu^P_A(x), \mu^P_A(y)$
$\mu^N_B(x, y) \geq \max(\mu^N_A(x), \mu^N_A(y)$
$\lambda^P_B(x, y) \leq \max(\lambda^P_A(x), \lambda^P_A(y)$
$\lambda^N_B(x, y) \geq \min(\lambda^N_A(x), \lambda^N_A(y)$ *for all* $(x, y) \in E$
we call A the bipolar intuitionistic fuzzy vertex set of V, B the bipolar intuitionistic fuzzy edge set of E, respectively. A bipolar intuitionistic fuzzy relation A on X is called symmetric if $\mu^P_B(x, y) = \mu^P_B(y, x)$, $\mu^N_B(x, y) = \mu^N_B(y, x)$,$\lambda^P_B(x, y) = \lambda^P_B(\{y, x\})$, $\lambda^N_B(x, y) = \lambda^P_N(y, x)$, for all $(x, y) \in X$.
We use the notation xy for an elamant of E.

Now, we give an example of bipolar intuitionistic fuzzy graph:

Example 3.1. *Consider a graph $G^* = (V, E)$ such that $V = \{a, b, c\}$, $E = \{ab, bc, ca\}$(figure 1). Let $A = (\mu^P_A, \mu^N_A, \lambda^P_A, \lambda^N_A)$ be a bipolar intuitionistic fuzzy subset of V and let $B = (\mu^P_B, \mu^N_B, \lambda^P_B, \lambda^N_B)$ be a bipolar intuitionistic fuzzy subset of $E \subseteq V \times V$ defined by*

	a	*b*	*c*
μ^P_A	*0.4*	*0.5*	*0.3*
μ^N_A	*−0.5*	*−0.4*	*−0.2*
λ^P_A	*0.5*	*0.3*	*0.7*
λ^N_A	*−0.3*	*−0.4*	*−0.5*

	ab	*bc*	*ca*
μ^P_B	*0.3*	*0.2*	*0.3*
μ^N_B	*−0.4*	*−0.1*	*−0.2*
λ^P_B	*0.4*	*0.6*	*0.6*
λ^N_B	*−0.3*	*−0.5*	*−0.4*

The corresponding intuitionistic fuzzy graph is shown in Figure 1.

Definition 3.3. *Let $G = (V, A, B)$ be a bipolar intuitionistic fuzzy graph. The open degree of a vertex u is defined as $deg(u) = (deg^P_\mu(u), deg^N_\mu(u), deg^P_\lambda(u), deg^N_\lambda(u))$ where,*
$deg^P_\mu(u) = \sum\limits_{u \neq v, v \in V} \mu^P_B(u, v), deg^N_\mu(u) = \sum\limits_{u \neq v, v \in V} \mu^N_B(u, v)$ *and*
$deg^P_\lambda(u) = \sum\limits_{u \neq v, v \in V} \lambda^P_B(u, v), deg^N_\lambda(u) = \sum\limits_{u \neq v, v \in V} \lambda^N_B(u, v).$

4. Product and degree of bipolar intuitionistic fuzzy graphs

In this section, we are consider different types of product of bipolar intuitionistic fuzzy graph(BIFG). Then we determine the degree of the resultant graphs.

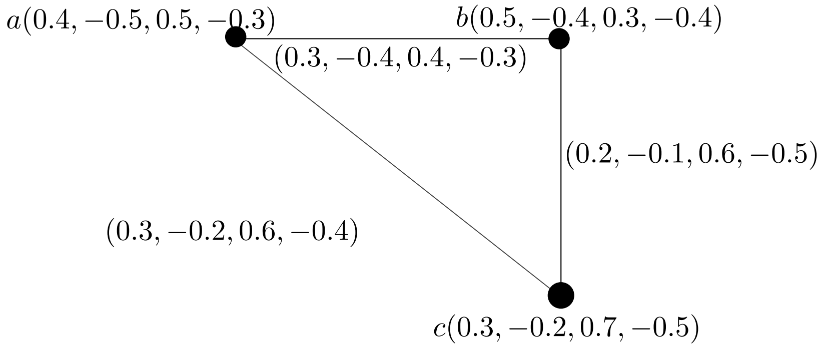

FIGURE 1. An bipolar intuitionistic fuzzy graph

4.1. **Cartesian product of two bipolar intuitionistic fuzzy graphs.**

Definition 4.1. *Let us consider two BIFG G_1 and G_2. The Cartesian product of two BIFGs $G_1 = (V_1, A_1, B_1)$ and $G_2 = (V_2, A_2, B_2)$ with underline crisp graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ respectively is defined as a bipolar intuitionistic fuzzy graph $G = G_1 \times G_2 = (A_1 \times A_2, B_1 \times B_2)$ where $V = V_1 \times V_2$ and $E = \{(u_1, u_2)(v_1, v_2) | u_1 = v_1, u_2v_2 \in E_2$ or $u_2 = v_2, u_1v_1 \in E_1\}$ with*
(i)

$$\begin{aligned}
\mu^P_{A_1\times A_2}(u_1,u_2) &= \mu^P_{A_1}(u_1) \wedge \mu^P_{A_2}(u_2) \\
\mu^N_{A_1\times A_2}(u_1,u_2) &= \mu^N_{A_1}(u_1) \vee \mu^N_{A_2}(u_2) \\
\lambda^P_{A_1\times A_2}(u_1,u_2) &= \lambda^P_{A_1}(u_1) \vee \lambda^P_{A_2}(u_2) \\
\lambda^N_{A_1\times A_2}(u_1,u_2) &= \lambda^N_{A_1}(u_1) \wedge \lambda^N_{A_2}(u_2),
\end{aligned}$$

for all $(u_1, u_2) \in V$
(ii)

$$\begin{aligned}
\mu^P_{B_1\times B_2}((u,u_2),(u,v_2)) &= \mu^P_{A_1}(u) \wedge \mu^P_{B_2}(u_2,v_2) \\
\mu^N_{B_1\times B_2}((u,u_2),(u,v_2)) &= \mu^N_{A_1}(u) \vee \mu^N_{B_2}(u_2,v_2) \\
\lambda^P_{B_1\times B_2}((u,u_2),(u,v_2)) &= \lambda^P_{A_1}(u) \vee \lambda^P_{B_2}(u_2,v_2) \\
\lambda^N_{B_1\times B_2}((u,u_2),(u,v_2)) &= \lambda^N_{A_1}(u) \wedge \lambda^N_{B_2}(u_2,v_2),
\end{aligned}$$

for all $u \in V_1$ and $u_2v_2 \in E_2$
(iii)

$$\begin{aligned}
\mu^P_{B_1\times B_2}((u_1,v),(v_1,v)) &= \mu^P_{B_1}(u_1,v_1) \wedge \mu^P_{A_2}(v) \\
\mu^N_{B_1\times B_2}((u_1,v),(v_1,v)) &= \mu^N_{B_1}(u_1,v_1) \vee \mu^N_{A_2}(v) \\
\lambda^P_{B_1\times B_2}((u_1,v),(v_1,v)) &= \lambda^P_{B_1}(u_1,v_1) \vee \lambda^P_{A_2}(v) \\
\lambda^N_{B_1\times B_2}((u_1,v),(v_1,v)) &= \lambda^N_{B_1}(u_1,v_1) \wedge \lambda^N_{A_2}(v),
\end{aligned}$$

for all $v \in V_2$ and $u_1v_1 \in E_1$

Definition 4.2. *Let $G = (V, E)$ where $V = V_1 \times V_2$, $E = E_1 \times E_2$ be the Cartesian product of two bipolar intuitionistic fuzzy graphs $G_1 = (V_1, A_1, B_1)$ and $G_2 = (V_2, A_2, B_2)$ with underline crisp graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ respectively. Then the degree of the vertex (u_1, u_2) in V is denoted by*

$d_{G_1\times G_2}(u_1,u_2)=(d^P_{\mu G_1\times G_2}(u_1,u_2),d^N_{\mu G_1\times G_2}(u_1,u_2),d^P_{\lambda G_1\times G_2}(u_1,u_2),d^N_{\lambda G_1\times G_2}(u_1,u_2))$ *where*

$$d^P_{\mu G_1\times G_2}(u_1,u_2) = \sum_{u_1=v_1,u_2v_2\in E_2}\mu^P_{A_1}(u_1)\wedge\mu^P_{B_2}(u_2,v_2)+\sum_{u_2=v_2,u_1v_1\in E_1}\mu^P_{B_1}(u_1,v_1)\wedge\mu^P_{A_2}(v_2)$$

$$d^N_{\mu G_1\times G_2}(u_1,u_2) = \sum_{u_1=v_1,u_2v_2\in E_2}\mu^N_{A_1}(u_1)\vee\mu^N_{B_2}(u_2,v_2)+\sum_{u_2=v_2,u_1v_1\in E_1}\mu^N_{B_1}(u_1,v_1)\vee\mu^N_{A_2}(v_2)$$

$$d^P_{\lambda G_1\times G_2}(u_1,u_2) = \sum_{u_1=v_1,u_2v_2\in E_2}\lambda^P_{A_1}(u_1)\vee\lambda^P_{B_2}(u_2,v_2)+\sum_{u_2=v_2,u_1v_1\in E_1}\lambda^P_{B_1}(u_1,v_1)\vee\lambda^P_{A_2}(v_2)$$

$$d^N_{\lambda G_1\times G_2}(u_1,u_2) = \sum_{u_1=v_1,u_2v_2\in E_2}\lambda^N_{A_1}(u_1)\wedge\lambda^N_{B_2}(u_2,v_2)+\sum_{u_2=v_2,u_1v_1\in E_1}\lambda^N_{B_1}(u_1,v_1)\wedge\lambda^N_{A_2}(v_2)$$

Theorem 4.1. *Let $G_1=(A_1,B_1)$ and $G_2=(A_2,B_2)$ be two bipolar intuitionistic fuzzy graphs. If $\mu^P_{A_1}\geq\mu^P_{B_2}$, $\mu^N_{A_1}\leq\mu^N_{B_2}$, $\lambda^P_{A_1}\leq\lambda^P_{B_2}$, $\lambda^N_{A_1}\geq\lambda^N_{B_2}$ and $\mu^P_{A_2}\geq\mu^P_{B_1}$, $\mu^N_{A_2}\leq\mu^N_{B_1}$, $\lambda^P_{A_2}\leq\lambda^P_{B_1}$, $\lambda^N_{A_2}\geq\lambda^N_{B_1}$, then $d_{G_1\times G_2}(u_1,u_2)=d_{G_1}(u_1)+d_{G_2}(u_2)$.*

Proof. From the defination of vertex in Cartesian product, we have,

$$\begin{aligned}
d^P_{\mu G_1\times G_2}(u_1,u_2) &= \sum_{((u_1,u_2)(v_1,v_2))\in E}\mu^P_{B_1\times B_2}((u_1,u_2),(v_1,v_2))\\
&= \sum_{u_1=v_1,(u_2,v_2)\in E_2}\mu^P_{A_1}(u_1)\wedge\mu^P_{B_2}(u_2,v_2)+\sum_{(u_2=v_2,(u_1,v_1)\in E_1)}\mu^P_{B_1}(u_1,v_1)\wedge\mu^P_{A_2}(v_2)\\
&= \sum_{(u_2,v_2)\in E_2}\mu^P_{B_2}(u_2,v_2)+\sum_{(u_1,v_1)\in E_1}\mu^P_{B_1}(u_1,v_1) \qquad [\textit{since } \mu^P_{A_1}\geq\mu^P_{B_2} \textit{ and } \mu^P_{A_2}\geq\mu^P_{B_1}]\\
&= d^P_{\mu G_2}(u_2)+d^P_{\mu G_1}(u_1)\\
&= d^P_{\mu G_1}(u_1)+d^P_{\mu G_2}(u_2)
\end{aligned}$$

similarly, we proved $d^N_{\mu G_1\times G_2}(u_1,u_2)=d^N_{\mu G_1}(u_1)+d^N_{\mu G_2}(u_2)$

$d^P_{\lambda G_1\times G_2}(u_1,u_2)=d^P_{\lambda G_1}(u_1)+d^P_{\lambda G_2}(u_2)$

$d^N_{\lambda G_1\times G_2}(u_1,u_2)=d^N_{\lambda G_1}(u_1)+d^N_{\lambda G_2}(u_2)$.

Hence, $d_{G_1\times G_2}(u_1,u_2)=d_{G_1}(u_1)+d_{G_2}(u_2)$.

Example 4.1. *Here, $\mu^P_{A_1}\geq\mu^P_{B_2}$, $\mu^N_{A_1}\leq\mu^N_{B_2}$, $\lambda^P_{A_1}\leq\lambda^P_{B_2}$,$\lambda^N_{A_1}\geq\mu^N_{B_2}$ and $\mu^P_{A_2}\geq\mu^P_{B_1}$, $\mu^N_{A_2}\leq\mu^N_{B_1}$, $\lambda^P_{A_2}\leq\lambda^P_{B_1}$,$\lambda^N_{A_2}\geq\mu^N_{B_1}$. Then by Theorem 1, we have*
$d^P_{\mu G_1\times G_2}(u_1,u_2)=d^P_{\mu G_1}(u_1)+d^P_{\mu G_2}(u_2)=0.1+0.1=0.2$
$d^N_{\mu G_1\times G_2}(u_1,u_2)=d^N_{\mu G_1}(u_1)+d^N_{\mu G_2}(u_2)=-0.2-0.3=-0.5$
$d^P_{\lambda G_1\times G_2}(u_1,u_2)=d^P_{\lambda G_1}(u_1)+d^P_{\lambda G_2}(u_2)=0.4+0.4=0.8$
$d^N_{\lambda G_1\times G_2}(u_1,u_2)=d^N_{\lambda G_1}(u_1)+d^N_{\lambda G_2}(u_2)=-0.3-0.3=-0.6$ *So, $d_{G_1\times G_2}(u_1,u_2)=(0.2,-0.5,0.8,-0.6)$. Similarly, we can find the degree of all vertices in $G_1\times G_2$.This is verfied in Figure 2.*

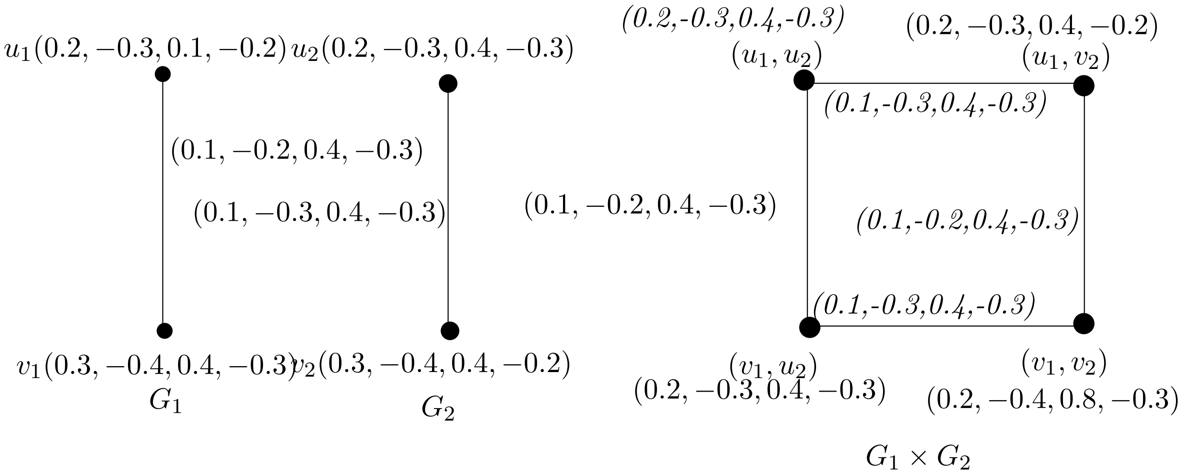

FIGURE 2. Cartesian product of two bipolar intuitionistic fuzzy graphs

4.2. **Composition of BIFG.** Here, we define the Composition of BIFG and also we determined the degree of each vertex of the resultant graph.

Definition 4.3. *The Composition of two bipolar intuitionistic fuzzy graphs* $G_1 = (V_1, A_1, B_1)$ *and* $G_2 = (v_2, A_2, B_2)$ *with underline crisp graphs* $G_1^* = (V_1, E_1)$ *and* $G_2^* = (V_2, E_2)$ *respectively is defined as a bipolar intuitionistic fuzzy graph* $G = G_1[G_2] = (A_1 \circ A_2, B_1 \circ B_2)$ *with underline crisp graph* $G^* = (V, E)$ *where* $V = V_1 \times V_2$ *and* $E = \{(u_1, u_2)(v_1, v_2) | u_1 = v_1, u_2v_2 \in E_2$ *or* $u_2 = v_2, u_1v_1 \in E_1\}$ *and* $E^* = E \cup \{(u_1, u_2)(v_1, v_2) | u_1v_1 \in E_1, u_2 \neq v_2\}$ *with*

(i)

$$
\begin{aligned}
\mu^P_{A_1 \circ A_2}(u_1, u_2) &= \mu^P_{A_1}(u_1) \wedge \mu^P_{A_2}(u_2) \\
\mu^N_{A_1 \circ A_2}(u_1, u_2) &= \mu^N_{A_1}(u_1) \vee \mu^N_{A_2}(u_2) \\
\lambda^P_{A_1 \circ A_2}(u_1, u_2) &= \lambda^P_{A_1}(u_1) \vee \lambda^P_{A_2}(u_2) \\
\lambda^N_{A_1 \circ A_2}(u_1, u_2) &= \lambda^N_{A_1}(u_1) \wedge \lambda^N_{A_2}(u_2),
\end{aligned}
$$

for all $(u_1, u_2) \in V$

(ii)

$$
\begin{aligned}
\mu^P_{B_1 \circ B_2}((u, u_2), (u, v_2)) &= \mu^P_{A_1}(u) \wedge \mu^P_{B_2}(u_2, v_2) \\
\mu^N_{B_1 \circ B_2}((u, u_2), (u, v_2)) &= \mu^N_{A_1}(u) \vee \mu^N_{B_2}(u_2, v_2) \\
\lambda^P_{B_1 \circ B_2}((u, u_2), (u, v_2)) &= \lambda^P_{A_1}(u) \vee \lambda^P_{B_2}(u_2, v_2) \\
\lambda^N_{B_1 \circ B_2}((u, u_2), (u, v_2)) &= \lambda^N_{A_1}(u) \wedge \lambda^N_{B_2}(u_2, v_2),
\end{aligned}
$$

for all $u \in V_1$ *and* $u_2v_2 \in E_2$

(iii)

$$\begin{aligned}
\mu^P_{B_1\circ B_2}((u_1,v),(v_1,v)) &= \mu^P_{A_2}(v) \wedge \mu^P_{B_1}(u_1,v_1)\\
\mu^N_{B_1\circ B_2}((u_1,v),(v_1,v)) &= \mu^N_{B_1}(u) \vee \mu^N_{A_2}(u_2,v_2)\\
\lambda^P_{B_1\circ B_2}((u_1,v),(v_1,v)) &= \lambda^P_{A_2}(v) \vee \lambda^P_{B_1}(u_1,v_1)\\
\lambda^N_{B_1\circ B_2}((u_1,v),(v_1,v)) &= \lambda^N_{A_2}(v) \wedge \lambda^N_{B_1}(u_1,v_1),
\end{aligned}$$

for all $v \in V_2$ *and* $u_1v_1 \in E_1$

(iv)

$$\begin{aligned}
\mu^P_{B_1\circ B_2}((u_1,u_2),(v_1,v_2)) &= \mu^P_{A_2}(u_2) \wedge \mu^P_{A_2}(v_2) \wedge \mu^P_{B_1}(u_1,v_1)\\
\mu^N_{B_1\circ B_2}((u_1,u_2),(v_1,v_2)) &= \mu^N_{A_2}(u_2) \vee \mu^N_{A_2}(v_2) \vee \mu^N_{B_1}(u_1,v_1)\\
\lambda^P_{B_1\circ B_2}((u_1,u_2),(v_1,v_2)) &= \lambda^P_{A_2}(u_2) \vee \lambda^P_{A_2}(v_2) \vee \lambda^P_{B_1}(u_1,v_1)\\
\lambda^N_{B_1\circ B_2}((u_1,u_2),(v_1,v_2)) &= \lambda^N_{A_2}(u_2) \wedge \mu^N_{A_2}(v_2) \wedge \lambda^N_{B_1}(u_1,v_1),
\end{aligned}$$

for all $(u_1,u_2)(v_1,v_2) \in E^* - E$

Definition 4.4. *Let* $G = G_1[G_2] = (A_1 \circ A_2, B_1 \circ B_2)$ *with underline crisp graph* $G^* = (V,E)$ *where* $V = V_1 \times V_2$, $E = E_1 \times E_2$ *be the composition of two bipolar intuitionistic fuzzy graphs* $G_1 = (V_1, A_1, B_1)$ *and* $G_2 = (V_2, A_2, B_2)$ *with crisp graph* $G_1^* = (V_1, E_1)$ *and* $G_2^* = (V_2, E_2)$ *respectively.Then the degree of the vertex* (u_1,u_2) *in* V *is denoted by* $d_{G_1[G_2]}(u_1,u_2) = (d^P_{\mu G_1[G_2]}(u_1,u_2), d^N_{\mu G_1[G_2]}(u_1,u_2), d^P_{\lambda G_1[G_2]}(u_1,u_2), d^N_{\lambda G_1[G_2]}(u_1,u_2))$, *where*

$$d^P_{\mu G_1[G_2]}(u_1,u_2) = \sum_{u_1=v_1,u_2v_2\in E_2} \mu^P_{A_1}(u_1) \wedge \mu^P_{B_2}(u_2,v_2) + \sum_{u_2=v_2,u_1v_1\in E_1} \mu^P_{B_1}(u_1,v_1) \wedge \mu^P_{A_2}(v_2)$$
$$+ \sum_{u_2\neq v_2,u_1v_1\in E_1} \mu^P_{A_2}(u_2) \wedge \mu^P_{A_2}(v_2) \wedge \mu^P_{B_1}(u_1,v_1).$$

$$d^N_{\mu G_1[G_2]}(u_1,u_2) = \sum_{u_1=v_1,u_2v_2\in E_2} \mu^N_{A_1}(u_1) \vee \mu^N_{B_2}(u_2,v_2) + \sum_{u_2=v_2,u_1v_1\in E_1} \mu^N_{B_1}(u_1,v_1) \vee \mu^N_{A_2}(v_2)$$
$$+ \sum_{u_2\neq v_2,u_1v_1\in E_1} \mu^N_{A_2}(u_2) \vee \mu^N_{A_2}(v_2) \vee \mu^N_{B_1}(u_1,v_1).$$

$$d^P_{\lambda G_1[G_2]}(u_1,u_2) = \sum_{u_1=v_1,u_2v_2\in E_2} \lambda^P_{A_1}(u_1) \vee \lambda^P_{B_2}(u_2,v_2) + \sum_{u_2=v_2,u_1v_1\in E_1} \lambda^P_{B_1}(u_1,v_1) \vee \lambda^P_{A_2}(v_2)$$
$$+ \sum_{u_2\neq v_2,u_1v_1\in E_1} \lambda^P_{A_2}(u_2) \vee \lambda^P_{A_2}(v_2) \vee \lambda^P_{B_1}(u_1,v_1).$$

$$d^N_{\lambda G_1[G_2]}(u_1,u_2) = \sum_{u_1=v_1,u_2v_2\in E_2} \lambda^N_{A_1}(u_1) \wedge \lambda^N_{B_2}(u_2,v_2) + \sum_{u_2=v_2,u_1v_1\in E_1} \lambda^N_{B_1}(u_1,v_1) \wedge \lambda^N_{A_2}(v_2)$$
$$+ \sum_{u_2\neq v_2,u_1v_1\in E_1} \lambda^N_{A_2}(u_2) \wedge \mu^N_{A_2}(v_2) \wedge \lambda^N_{B_1}(u_1,v_1).$$

Theorem 4.2. *Let* $G_1 = (V_1, A_1, B_1)$ *and* $G_2 = (V_2, A_2, B_2)$ *be two bipolar intuitionistic fuzzy graphs. If* $\mu^P_{A_1} \geq \mu^P_{B_2}$, $\mu^N_{A_1} \leq \mu^N_{B_2}$, $\lambda^P_{A_1} \leq \lambda^P_{B_2}$,$\lambda^N_{A_1} \geq \lambda^N_{B_2}$ *and* $\mu^P_{A_2} \geq \mu^P_{B_1}$, $\mu^N_{A_2} \leq \mu^N_{B_1}$, $\lambda^P_{A_2} \leq \lambda^P_{B_1}$,$\lambda^N_{A_2} \geq \lambda^N_{B_1}$, *then* $d_{G_1[G_2]}(u_1,u_2) = |V_2| d_{G_1}(u_1) + d_{G_2}(u_2)$.

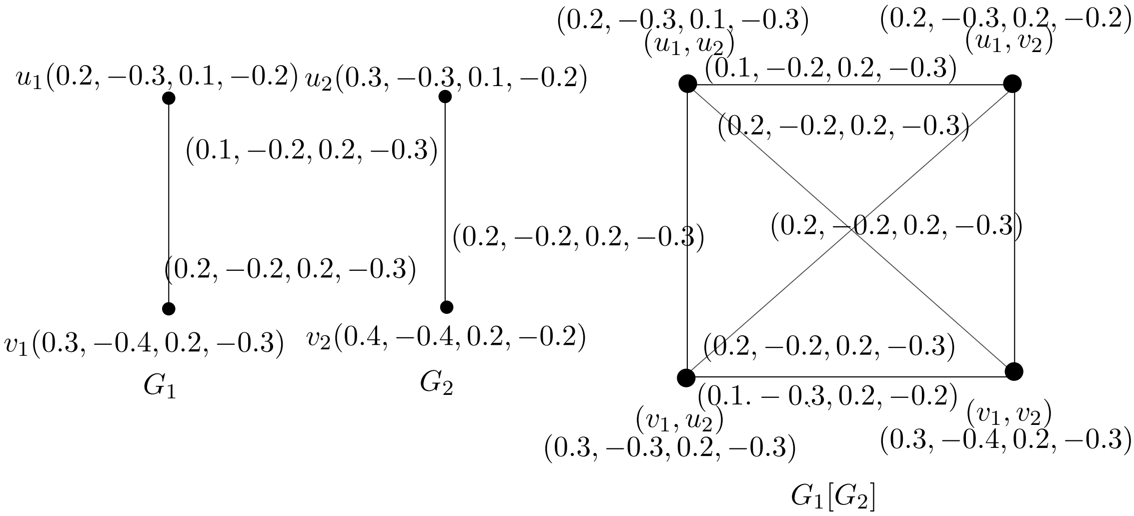

FIGURE 3. Composition of two bipolar intuitionistic fuzzy graphs

Example 4.2. *Here,* $\mu^P_{A_1} \geq \mu^P_{B_2}$, $\mu^N_{A_1} \leq \mu^N_{B_2}$, $\lambda^P_{A_1} \leq \lambda^P_{B_2}$,$\lambda^N_{A_1} \geq \mu^N_{B_2}$ *and* $\mu^P_{A_2} \geq \mu^P_{B_1}$, $\mu^N_{A_2} \leq \mu^N_{B_1}$, $\lambda^P_{A_2} \leq \lambda^P_{B_1}$,$\lambda^N_{A_2} \geq \mu^N_{B_1}$. *Then by Theorem 2, we have*

$d^P_{\mu G_1[G_2]}(u_1, u_2) = |V_2| d^P_{\mu G_1}(u_1) + d^P_{\mu G_2}(u_2) = 2 \times (0.2) + 0.1 = 0.5$

$d^N_{\mu G_1[G_2]}(u_1, u_2) = |V_2| d^N_{\mu G_1}(u_1) + d^N_{\mu G_2}(u_2) = 2 \times (-0.2) + (-0.2) = -0.5$

$d^P_{\lambda G_1[G_2]}(u_1, u_2) = |V_2| d^P_{\lambda G_1}(u_1) + d^P_{\lambda G_2}(u_2) = 2 \times (0.2) + 0.2 = 0.6$

$d^N_{\lambda G_1[G_2]}(u_1, u_2) = |V_2| d^N_{\lambda G_1}(u_1) + d^N_{\lambda G_2}(u_2) = 2 \times (-0.3) - 0.3 = -0.8$

So, $d_{G_1[G_2]}(u_1, u_2) = (0.5, -0.6, 0.6, -0.8)$. *Similarly, we can find the degree of all vertices in* $G_1[G_2]$.*This is verfied in Figure 3.*

4.3. **Tensor product of two BIFGs.** In this section, we described tensor product of two BIFGs. The degree of each vertex of the resultant graph.

Definition 4.5. *The tensor product of two bipolar intuitionistic fuzzy graphs* $G_1 = (V_1, A_1, B_1)$ *and* $G_2 = (V_2, A_2, B_2)$ *with underline crisp graphs* $G_1^* = (V_1, E_1)$ *and* $G_2^* = (V_2, E_2)$ *respectively is defined as a bipolar intuitionistic fuzzy graph* $G = G_1 \otimes G_2 = (A_1 \otimes A_2, B_1 \otimes B_2)$ *with underline crisp graph* $G^* = (V, E)$ *where* $V = V_1 \times V_2$ *and* $E = \{(u_1, u_2)(v_1, v_2) | u_1 v_1 \in E_1, u_2 v_2 \in E_2\}$ *with*
(i)

$$\begin{aligned}
\mu^P_{A_1 \otimes A_2}(u_1, u_2) &= \mu^P_{A_1}(u_1) \wedge \mu^P_{A_2}(u_2) \\
\mu^N_{A_1 \otimes A_2}(u_1, u_2) &= \mu^N_{A_1}(u_1) \vee \mu^N_{A_2}(u_2) \\
\lambda^P_{A_1 \otimes A_2}(u_1, u_2) &= \lambda^P_{A_1}(u_1) \vee \lambda^P_{A_2}(u_2) \\
\lambda^N_{A_1 \otimes A_2}(u_1, u_2) &= \lambda^N_{A_1}(u_1) \wedge \lambda^N_{A_2}(u_2),
\end{aligned}$$

for all $(u_1, u_2) \in V$ *(ii)*

$$\begin{aligned}
\mu^P_{B_1 \otimes B_2}((u_1, u_2), (v_1, v_2)) &= \mu^P_{B_1}(u_1, v_1) \wedge \mu^P_{B_2}(u_2, v_2) \\
\mu^N_{B_1 \otimes B_2}((u_1, u_2), (v_1, v_2)) &= \mu^N_{B_1}(u_1, v_1) \vee \mu^N_{B_2}(u_2, v_2) \\
\lambda^P_{B_1 \otimes B_2}((u_1, u_2), (v_1, v_2)) &= \lambda^P_{B_1}(u_1, v_1) \vee \lambda^P_{B_2}(u_2, v_2) \\
\lambda^N_{B_1 \otimes B_2}((u_1, u_2), (v_1, v_2)) &= \lambda^N_{B_1}(u_1, v_1) \wedge \lambda^N_{B_2}(u_2, v_2),
\end{aligned}$$

for all $u_1v_1 \in E_1$ *and* $u_2v_2 \in E_2$

Definition 4.6. *Let* $G = G_1 \otimes G_2$ *with underline crisp graph* (V, E) *where* $V = V_1 \times V_2$, $E = E_1 \times E_2$ *be the Cartesian product of two bipolar intuitionistic fuzzy graphs* $G_1 = (V_1, A_1, B_1)$ *and* $G_2 = (V_2, A_2, B_2)$*. Then the degree of the vertex* (u_1, u_2) *in* V *is denoted by*
$d_{G_1\otimes G_2}(u_1, u_2) = (d^P_{\mu G_1\otimes G_2}(u_1, u_2), d^N_{\mu G_1\otimes G_2}(u_1, u_2), d^P_{\lambda G_1\otimes G_2}(u_1, u_2), d^N_{\lambda G_1\otimes G_2}(u_1, u_2))$ *and defined by*

$$\begin{aligned}
d^P_{\mu G_1\otimes G_2}(u_1, u_2) &= \sum_{u_1v_1\in E_1} \mu^P_{B_1}(u_1, v_1) \wedge \mu^P_{B_2}(u_2, v_2). \\
d^N_{\mu G_1\otimes G_2}(u_1, u_2) &= \sum_{u_1v_1\in E_1} \mu^N_{B_1}(u_1, v_1) \vee \mu^N_{B_2}(u_2, v_2). \\
d^P_{\lambda G_1\otimes G_2}(u_1, u_2) &= \sum_{u_1v_1\in E_1} \lambda^P_{B_1}(u_1, v_1) \vee \lambda^P_{B_2}(u_2, v_2). \\
d^N_{\lambda G_1\otimes G_2}(u_1, u_2) &= \sum_{u_1v_1\in E_1} \lambda^N_{B_1}(u_1, v_1) \wedge \lambda^N_{B_2}(u_2, v_2).
\end{aligned}$$

Theorem 4.3. *Let* $G_1 = (V_1, A_1, B_1)$ *and* $G_2 = (V_2, A_2, B_2)$ *be two bipolar intuitionistic fuzzy graphs. If* $\mu^P_{B_2} \geq \mu^P_{B_1}$, $\mu^N_{B_2} \leq \mu^N_{B_1}$, $\lambda^P_{B_2} \leq \lambda^P_{B_1}$,$\lambda^N_{B_2} \geq \mu^N_{B_1}$, *then* $d_{G_1\otimes G_2}(u_1, u_2) = d_{G_1}(u_1)$ *and if* $\mu^P_{B_1} \geq \mu^P_{B_2}$, $\mu^N_{B_1} \leq \mu^N_{B_2}$, $\lambda^P_{B_1} \leq \lambda^P_{B_2}$,$\lambda^N_{B_1} \geq \mu^N_{B_2}$, *then* $d_{G_1\otimes G_2}(u_1, u_2) = d_{G_2}(u_2)$.

4.4. **Normal product of two BIFGs.** In this section, we consider the normal product of two BIFGs. The degree of each vertex of the resultant graph in terms of the original graphs in calculated.

Definition 4.7. *The normal product of two bipolar intuitionistic fuzzy graphs* $G_1 = (V_1, A_1, B_1)$ *and* $G_2 = (V_2, A_2, B_2)$ *with underline crisp graph* $G_1^* = (V_1, E_1)$ *and* $G_2^* = (V_2, E_2)$ *respectively is defined as a bipolar intuitionistic fuzzy graph* $G = G_1 \bullet G_2 = (A_1 \bullet A_2, B_1 \bullet B_2)$ *with underline crisp graph* $G^* = (V, E)$ *where* $V = V_1 \times V_2$ *and* $E = \{(u_1, u_2)(v_1, v_2)|u_1 = v_1, u_2v_2 \in E_2 \text{ or } u_2 = v_2, u_1v_1 \in E_1\} \cup E = \{(u_1, u_2)(v_1, v_2)|u_1v_1 \in E_1, u_2v_2 \in E_2\}$ *with*
(i)

$$\begin{aligned}
\mu^P_{A_1\bullet A_2}(u_1, u_2) &= \mu^P_{A_1}(u_1) \wedge \mu^P_{A_2}(u_2) \\
\mu^N_{A_1\bullet A_2}(u_1, u_2) &= \mu^N_{A_1}(u_1) \vee \mu^N_{A_2}(u_2) \\
\lambda^P_{A_1\bullet A_2}(u_1, u_2) &= \lambda^P_{A_1}(u_1) \vee \lambda^P_{A_2}(u_2) \\
\lambda^N_{A_1\bullet A_2}(u_1, u_2) &= \lambda^N_{A_1}(u_1) \wedge \lambda^N_{A_2}(u_2),
\end{aligned}$$

for all $(u_1, u_2) \in V$
(ii)

$$\begin{aligned}
\mu^P_{B_1\bullet B_2}((u, u_2), (u, v_2)) &= \mu^P_{A_1}(u) \wedge \mu^P_{B_2}(u_2, v_2) \\
\mu^N_{B_1\bullet B_2}((u, u_2), (u, v_2)) &= \mu^N_{A_1}(u) \vee \mu^N_{B_2}(u_2, v_2) \\
\lambda^P_{B_1\bullet B_2}((u, u_2), (u, v_2)) &= \lambda^P_{A_1}(u) \vee \lambda^P_{B_2}(u_2, v_2) \\
\lambda^N_{B_1\bullet B_2}((u, u_2), (u, v_2)) &= \lambda^N_{A_1}(u) \wedge \lambda^N_{B_2}(u_2, v_2),
\end{aligned}$$

for all $u \in V_1$ *and* $u_2v_2 \in E_2$

(iii)

$$\begin{aligned}
\mu^P_{B_1\bullet B_2}((u_1,v),(v_1,v)) &= \mu^P_{A_2}(v) \wedge \mu^P_{B_1}(u_1,v_1)\\
\mu^N_{B_1\bullet B_2}((u_1,v),(v_1,v)) &= \mu^N_{B_1}(u) \vee \mu^N_{A_2}(u_2,v_2)\\
\lambda^P_{B_1\bullet B_2}((u_1,v),(v_1,v)) &= \lambda^P_{A_2}(v) \vee \lambda^P_{B_1}(u_1,v_1)\\
\lambda^N_{B_1\bullet B_2}((u_1,v),(v_1,v)) &= \lambda^N_{A_2}(v) \wedge \lambda^N_{B_1}(u_1,v_1),
\end{aligned}$$

for all $v \in V_2$ *and* $u_1v_1 \in E_1$

(iv)

$$\begin{aligned}
\mu^P_{B_1\bullet B_2}((u_1,u_2),(v_1,v_2)) &= \mu^P_{B_1}(u_1,v_1) \wedge \mu^P_{B_2}(u_2,v_2)\\
\mu^N_{B_1\bullet B_2}((u_1,u_2),(v_1,v_2)) &= \mu^N_{B_1}(u_1,v_1) \vee \mu^N_{B_2}(u_2,v_2)\\
\lambda^P_{B_1\bullet B_2}((u_1,u_2),(v_1,v_2)) &= \lambda^P_{B_1}(u_1,v_1) \vee \lambda^P_{B_2}(u_2,v_2)\\
\lambda^N_{B_1\bullet B_2}((u_1,u_2),(v_1,v_2)) &= \lambda^N_{B_1}(u_1,v_1) \wedge \lambda^N_{B_2}(u_2,v_2),
\end{aligned}$$

for all $u_1v_1 \in E_1$ *and* $u_2v_2 \in E_2$

Definition 4.8. *Let* $G = G_1 \bullet G_2$ *with underline crisp graph* $G^* = (V,E)$ *where* $V = V_1 \times V_2$, $E = E_1 \times E_2$ *be the normal product of two bipolar intuitionistic fuzzy graphs* $G_1 = (A_1,B_1)$ *and* $G_2 = (A_2,B_2)$ *with crisp graph* $G_1^* = (V_1,E_1)$ *and* $G_2^* = (V_2,E_2)$ *respectively.Then the degree of the vertex* (u_1,u_2) *in* V *is denoted by* $d_{G_1\bullet G_2}(u_1,u_2) = (d^P_{\mu G_1\bullet G_2}(u_1,u_2), d^N_{\mu G_1\bullet G_2}(u_1,u_2), d^P_{\lambda G_1\bullet G_2}(u_1,u_2), d^N_{\lambda G_1\bullet G_2}(u_1,u_2))$ *and defined by*

$$d^P_{\mu G_1\bullet G_2}(u_1,u_2) = \sum_{u_1=v_1,(u_2,v_2)\in E_2} \mu^P_{A_1}(u_1) \wedge \mu^P_{B_2}(u_2,v_2) + \sum_{u_2=v_2,(u_1,v_1)\in E_1} \mu^P_{B_1}(u_1,v_1) \wedge \mu^P_{A_2}(v_2)$$
$$+ \sum_{u_1v_1\in E_1} \mu^P_{B_1}(u_1,v_1) \wedge \mu^P_{B_2}(u_2,v_2).$$

$$d^N_{\mu G_1\bullet G_2}(u_1,u_2 = \sum_{u_1=v_1,(u_2,v_2)\in E_2} \mu^N_{A_1}(u_1) \vee \mu^N_{B_2}(u_2,v_2) + \sum_{u_2=v_2,(u_1,v_1)\in E_1} \mu^N_{B_1}(u_1,v_1) \vee \mu^N_{A_2}(v_2)$$
$$+ \sum_{u_1v_1\in E_1} \mu^N_{B_1}(u_1,v_1) \vee \mu^N_{B_2}(u_2,v_2).$$

$$d^P_{\lambda G_1\bullet G_2}(u_1,u_2) = \sum_{u_1=v_1,(u_2,v_2)\in E_2} \lambda^P_{A_1}(u_1) \vee \lambda^P_{B_2}(u_2,v_2) + \sum_{u_2=v_2,(u_1,v_1)\in E_1} \lambda^P_{B_1}(u_1,v_1) \vee \lambda^P_{A_2}(v_2)$$
$$+ \sum_{u_1v_1\in E_1} \lambda^P_{B_1}(u_1,v_1) \vee \lambda^P_{B_2}(u_2,v_2).$$

$$d^N_{\lambda G_1\bullet G_2}(u_1,u_2) = \sum_{u_1=v_1,(u_2,v_2)\in E_2} \lambda^N_{A_1}(u_1) \wedge \lambda^N_{B_2}(u_2,v_2) + \sum_{u_2=v_2,(u_1,v_1)\in E_1} \lambda^N_{B_1}(u_1,v_1) \wedge \lambda^N_{A_2}(v_2)$$
$$+ \sum_{u_1v_1\in E_1} \lambda^N_{B_1}(u_1,v_1) \wedge \lambda^N_{B_2}(u_2,v_2).$$

Theorem 4.4. *Let* $G_1 = (V_1,A_1,B_1)$ *and* $G_2 = (V_2,A_2,B_2)$ *be two bipolar intuitionistic fuzzy graphs. If* $\mu^P_{A_1} \geq \mu^P_{B_2}$, $\mu^N_{A_1} \leq \mu^N_{B_2}$, $\lambda^P_{A_1} \leq \lambda^P_{B_2}$, $\lambda^N_{A_1} \geq \lambda^N_{B_2}$, $\mu^P_{A_2} \geq \mu^P_{B_1}$, $\mu^N_{A_2} \leq \mu^N_{B_1}$,

$\lambda^P_{A_2} \leq \lambda^P_{B_1}$, $\lambda^N_{A_2} \geq \lambda^N_{B_1}$ *and* $\mu^P_{B_2} \geq \mu^P_{B_1}$, $\mu^N_{B_2} \leq \mu^N_{B_1}$, $\lambda^P_{B_2} \leq \lambda^P_{B_1}$, $\lambda^N_{B_2} \geq \mu^N_{B_1}$, *then* $d_{G_1 \bullet G_2}(u_1, u_2) = |V_2| d_{G_1}(u_1) + d_{G_2}(u_2)$.

5. Conclusion

In this paper, we defined bipolar intuitionistic fuzzy sets, bipolar intuitionistic fuzzy graphs and then determined the degree of the vertices of the graphs $G_1 \times G_2$, $G_1[G_2]$, $G_1 \otimes G_2$ and $G_1 \bullet G_2$, in terms of the degree of the vertices of the bipolar intuitionistic fuzzy graphs G_1 and G_2 under some conditions and illustrated them through examples. The vertices and their degree of any graph are very important parameters. This study is very useful to analyse various properties of Cartesian product, composition, tensor product and normal product of two bipolar intuitionistic fuzzy graphs.

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