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# ZAGREB INDICES AND MULTIPLICATIVE ZAGREB INDICES OF DOUBLE GRAPHS OF SUBDIVISION GRAPHS

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Abstract. Let $G$ be a simple graph. The subdivision graph and the double graph are the graphs obtained from a given graph $G$ which have several properties related to the properties of $G$. In this paper, the first and second Zagreb and multiplicative Zagreb indices of double graphs, subdivision graphs, double graphs of the subdivision graphs and subdivision graphs of the double graphs of $G$ are obtained. In particular, these numbers are calculated for the frequently used null, path, cycle, star, complete, complete bipartite or tadpole graph.

Keywords: Zagreb indices, multiplicative Zagreb indices, double graphs, subdivision graphs

AMS Subject Classification: 05C07, 05C10, 05C30

## 1. Introduction

Let $G = (V, E)$ be a simple graph with $V(G) = n$ vertices and $E(G) = m$ edges. For a vertex $v \in V(G)$, we denote the degree of $v$ by $d_G(v)$. A vertex with degree one is specially called a pendant vertex. As usual, we denote by $N_n, P_n, C_n, S_n, K_n, K_{t,s}$ and $T_{t,s}$ the null, path, cycle, star, complete, complete bipartite and tadpole graphs, respectively.

Tens of topological graph indices have been defined and studied by many mathematicians and also by chemists as graphs are used to model molecules by replacing atoms with vertices and bonds with edges. Two of the most important topological graph indices are called first and second Zagreb indices denoted by $M_1(G)$ and $M_2(G)$, respectively:

$$M_1(G) = \sum_{u \in V(G)} d_G^2(u) \quad \text{and} \quad M_2(G) = \sum_{u,v \in E(G)} d_G(u) d_G(v). \tag{1}$$

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They were first defined 41 years ago by Gutman and Trinajstic [11], and are referred to due to their uses in QSAR and QSPR. There is very large number of papers dealing with the first and second Zagreb indices, see e.g. [1] - [9], [14] - [16], [18] - [29]. Recently, Todeschini and Consonni [21] have introduced the multiplicative variants of these additive graph invariants by

$$\Pi_1(G) = \prod_{u\in V(G)} d_G^2(u) \qquad \text{and} \qquad \Pi_2(G) = \prod_{u,v\in E(G)} d_G(u)d_G(v) \tag{2}$$

and called them multiplicative Zagreb indices.

For a graph $G$ with vertex set $V(G) = \{v_1,\ v_2,\ \cdots,\ v_n\}$, we take another copy of $G$ with vertices labelled by $\{v_1,\ v_2,\ \cdots,\ v_n\}$, this time, where $v_i$ corresponds to $v_i$ for each $i$. If we connect $v_i$ to the neighbours of $v_i$ for each $i$, we obtain a new graph called the double graph of $G$. It is denoted by $D(G)$. In Figure 1 and 2, the double graphs of two graphs $P_6$ and $C_4$ are shown. Naturally, the more complex is the graph, more complex is its double graph.

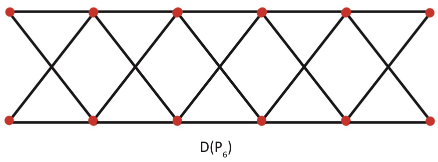


FIGURE 1. Double graph of $P_6$

Double graphs were first introduced by Indulal and Vijayakumari [13] in the study of equienergetic graphs. Later Munarini [17] et al. calculates the double graphs of $N_n$ and $K_{t,s}$ as $N_{2n}$ and $K_{2t,2s}$, respectively.

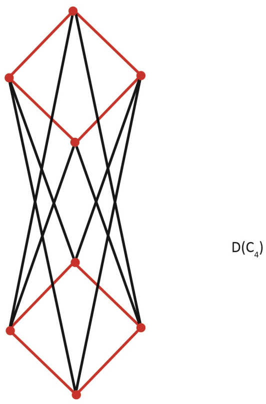


FIGURE 2. Double graph of $C_4$

Let now $G$ be a simple graph. The degree sequence of the double graph of $G$ can be given in terms of the degree sequence of $G$ as follows:

**Lemma 1.1.** [28] *Let the DS of a graph $G$ be $DS(G)$=$\{d_1,\ d_2,\ \cdots,\ d_n\}$. Then the DS $DS(D(G))$ of the double graph is given by*

$$DS(D(G)) = \left\{2d_1^{(2)},\ 2d_2^{(2)},\ \cdots,\ 2d_n^{(2)}\right\}.$$

Hence we have

**Theorem 1.1.** *Let $G$ be a simple graph with $m$ edges. Then the first and second Zagreb indices of the double graph $D(G)$ of $G$ are*

$$M_1(D(G)) = 8M_1(G)$$

*and*

$$M_2(D(G)) = 4M_2(G).$$

*Proof.* By Lemma 1.1, we get

$$\begin{aligned} M_1(D(G)) &= \sum_{u\in V(D(G))} d^2(u) \\ &= 2\left(4d_1^2 + 4d_2^2 + \cdots + 4d_n^2\right) \\ &= 8M_1(G) \end{aligned}$$

and as $2M_2(G)$ is obtained for two copies of $G$ in $D(G)$ and $2M_2(G)$ is obtained for the new edges between the two copies of $G$, we obtain

$$M_2(D(G)) = 4M_2(G).$$

□

Let now $G$ be a simple graph. The degree sequence of the double graph of $G$ can be given in terms of the degree sequence of $G$ as follows:

Here we first calculate subgraphs of other simple graph types such as cycle graph $C_n$, path graph $P_n$, star graph $S_n$, complete graph $K_n$, complete bipartite graph $K_{t,s}$ and tadpole graph $T_{t,s}$.

The subdivision graph $S(G)$ of a graph $G$ is the graph obtained from $G$ by replacing each of its edges by a path of length 2, or equivalently by inserting an additional vertex into each edge of $G$. Subdivision graphs are used to obtain several mathematical and chemical properties of more complex graphs from more basic graphs and there are many results on these graphs. Similarly the r-subdivision graph of $G$ denoted by $S^r(G)$ is defined by adding $r$ vertices to each edge, [23], [25]. Then, we obtain the double graphs of these subdivision graphs. These subdivision and $r$-subdivision graphs were recently studied by several authors, [12, 18, 22, 23, 24, 25, 28, 29]. In that paper, ten types of Zagreb indices including first and second Zagreb indices and multiplicative Zagreb indices that we shall be concentrating in this paper on were calculated.

**Lemma 1.2.** [28] *Let the DS of a graph $G$ be $DS(G)$=$\{d_1,\ d_2,\ \cdots,\ d_n\}$. Then the DS $DS(S(G))$ of the subdivision graph is given by*

$$DS(S(G)) = \left\{2^{(m)},\ d_1,\ d_2,\ \cdots,\ d_n\right\}.$$

The following result gives the first and second Zagreb indices of the subdivision graph $S(G)$ of any simple graph $G$, [14]:

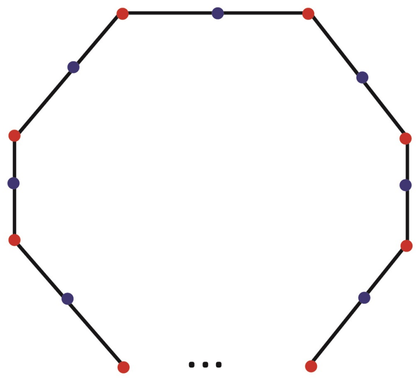


FIGURE 3. Subdivision graph of $C_n$

**Theorem 1.2.** *Let $G$ be a simple graph with $m$ edges. Then the first and second Zagreb indices of the subdivision graph $S(G)$ of $G$ are*

$$M_1(S(G)) = 4m + M_1(G)$$

*and*

$$M_2(S(G)) = 2M_1(G).$$

*Proof.* By Lemma 1.2, we have each degree of $G$ twice in the double graph $D(G)$ and therefore we get

$$\begin{aligned} M_1(S(G)) &= \sum_{u \in V(S(G))} d^2(u) \\ &= m \cdot 2^2 + \sum_{u \in V(G)} d^2(u) \\ &= 4m + M_1(G) \end{aligned}$$

and

$$\begin{aligned} M_2(S(G)) &= d_1(2d_1) + d_2(2d_2) + \cdots + d_n(2d_n) \\ &= 2\sum_{i=1}^{n} d_i^2 \\ &= 2M_1(G). \end{aligned}$$

□

## 2. Double graphs of subdivision graphs

For a null graph $N_n$, one can not obtain a subdivision graph by adding a new vertex so our result will be given for other graph types.

The following result gives the DS of the double graph of the subdivision graph of any simple graph $G$:

**Lemma 2.1.** *Let the DS of a simple graph G be* $DS(G)=\{d_1,\ d_2,\ \cdots,\ d_n\}$*. Then the* $DS(D(S(G)))$ *of the double graph of the subdivision graph is given by*

$$DS(D(S(G))) = \left\{2d_1^{(2)},\ 2d_2^{(2)},\ \cdots,\ 2d_n^{(2)}, 4^{(2m)}\right\}.$$

The following result gives the first and second Zagreb indices of the double graph $D(S(G))$ of the subdivision graph of any simple graph $G$:

**Theorem 2.1.** *Let G be a simple graph with m edges. Then the first and second Zagreb indices of the subdivision graph $S(G)$ of G are*

$$M_1(D(S(G))) = 32m + 8M_1(G)$$

*and*

$$M_2(D(S(G))) = 32M_1(G).$$

*Proof.* By Lemma 2.1, we get

$$\begin{aligned} M_1(D(S(G))) &= \sum_{u\in V(D(S(G)))} d^2(u) \\ &= 2m\cdot 4^2 + 2\left(4\sum_{u\in V(G)} d^2(u)\right) \\ &= 32m + 8M_1(G) \end{aligned}$$

and

$$\begin{aligned} M_2(D(S(G))) &= 2\left[2d_1(4\cdot 2d_1) + 2d_2(4\cdot 2d_2) + \cdots + 2d_n(4\cdot 2d_n)\right] \\ &= 32\sum_{i=1}^{n} d_i^2 \\ &= 32M_1(G). \end{aligned}$$

□

The following result gives these two indices for some very frequently used graph classes:

**Corollary 2.1.** *Let G be one of the path, cycle, star, complete, complete bipartite or tadpole graphs. Then the first and second Zagreb indices of the double graph of the subdivision of G are given by*

$$M_1((D(S(G))) = \begin{cases} 16 + 32(n-2) & \text{if } G = P_n, n \geq 2 \\ 64n & \text{if } G = C_n, n > 2 \\ 8(n-1)(n+4) & \text{if } G = S_n, n \geq 2 \\ 8(n^3-n) & \text{if } G = K_n, n \geq 2 \\ 8ts(t+s+4) & \text{if } G = K_{t,s},\ t,s \geq 1 \\ 16(4t+4s+1) & \text{if } G = T_{t,s}, t \geq 3, s \geq 1 \end{cases}$$

*and*

$$M_2((D(S(G))) = \begin{cases} 64(2n-3) & \text{if } G = P_n, n \geq 2 \\ 128n & \text{if } G = C_n, n > 2 \\ 64(n-1) & \text{if } G = S_n, n \geq 2 \\ 32n(n-1)^2 & \text{if } G = K_n, n \geq 2 \\ 128ts & \text{if } G = K_{t,s},\ t,s \geq 1 \\ 64(2(t+s)+1) & \text{if } G = T_{t,s}, t \geq 3, s \geq 1 \end{cases}$$

*Proof.* It is a direct consequence of Theorem 2.1. □

## 3. Multiplicative Zagreb indices

In this section, similarly to the above section, we shall calculate the multiplicative Zagreb indices of double graphs, subdivision graphs, subdivision of double graphs and double graphs of subdivision graphs. Our main tool will be Lemmas 1.1, 2.1 and 1.2 giving the degree sequences of those graphs.

**Theorem 3.1.** *Let $G$ be a simple graph. Then the first and second multiplicative Zagreb indices of the double graph $D(G)$ of $G$ are*

$$\Pi_1(D(G)) = 16\,(\Pi_1(G))^4$$

*and*

$$\Pi_2(D(G)) = \prod_{i=1}^{n} d_i^{4d_i}.$$

*Proof.* By Lemma 1.1, we get the required results similarly to the above proofs. □

**Theorem 3.2.** *Let $G$ be a simple graph with $n$ vertices and $m$ edges. Then the first and second multiplicative Zagreb indices of the subdivision graph $S(G)$ of $G$ are*

$$\Pi_1(S(G)) = 2^{2m}\Pi_1(G)$$

*and*

$$\Pi_2(S(G)) = 2^n\Pi_1(G).$$

*Proof.* By Lemma 1.2, we get the proof of the first assertion. For the second part, note that each vertex degree $d_i$ is always multiplied with 2 and this happens $d_i$ times giving the proof. □

## 4. Subdivision graphs of double graphs

Finally, we calculate the first and second Zagreb indices of the subdivision graph of the double graph of a simple graph $G$. The DSs of the double graph $D(G)$ and the subdivision graph $S(D(G))$ of the double graph of $G$ are given by the following result:

**Lemma 4.1.** *Let the DS of a graph $G$ be $DS(G)=\{d_1,\ d_2,\ \cdots,\ d_n\}$. Then the DS of the double graph $DS(D(G))$ and the subdivision graph $DS(S(D(G)))$ of the double graph are given by*

$$DS(D(G)) = \left\{2d_1^{(2)},\ 2d_2^{(2)},\ \cdots,\ 2d_n^{(2)},\right\}$$

*and*

$$DS(S(D(G))) = \left\{2d_1^{(2)},\ 2d_2^{(2)},\ \cdots,\ 2d_n^{(2)}, 2^{(|E(D(G))|)}\right\}.$$

Here it is not difficult to see that $\mid E(D(G)) \mid= 4m$. Therefore we have

**Theorem 4.1.** *Let $G$ be a simple graph with $m$ edges. Then the first and second Zagreb indices of the subdivision graph $S(D(G))$ of the double graph of $G$ are*

$$M_1(S(D(G))) = 16m + 8M_1(G)$$

*and*

$$M_2(S(D(G))) = 8M_1(G).$$

*Proof.* By Lemma 4.1, we get

$$\begin{aligned} M_1(S(D(G))) &= 2\left(4d_1^2 + 4d_2^2 + \cdots + 4d_n^2\right) + 4|E(D(G))| \\ &= 16m + 8M_1(G) \end{aligned}$$

and

$$\begin{aligned} M_2(S(D(G))) &= 4\left(2d_1 \cdot d_1 + 2d_2 \cdot d_2 + \cdots + 2d_n \cdot d_n\right) \\ &= 8M_1(G). \end{aligned}$$

□

Finally, we give the first multiplicative Zagreb indices of the subdivision of double and double of subdivision graphs:

**Theorem 4.2.** *Let $G$ be a simple graph with $n$ vertices and $m$ edges. Then the first multiplicative Zagreb indices of the subdivision graph $S(D(G))$ of the double graph of $G$ and the double graph $D(S(G))$ of the subdivision graph of $G$ are*

$$\Pi_1(S(D(G))) = 2^{4(n+2m)}\Pi_1^2(G)$$

*and*

$$\Pi_1(D(S(G))) = 2^{4(n+2m)}\Pi_1^2(G).$$

*Note that these two indices are equal for any simple graph.*

*Proof.* Both results follow by Lemmas 2.1 and 4.1, respectively. □

**Corollary 4.1.** *Let $G$ be one of the path, cycle, star, complete, complete bipartite or tadpole graphs. Then the first and second multiplicative Zagreb indices of the double graph of the subdivision of $G$ are given by*

$$\Pi_1((D(S(G))) = \begin{cases} 2^{16(n-1)} & \text{if } G = P_n, n \geq 2 \\ 2^{16n} & \text{if } G = C_n, n > 2 \\ 2^{4(3n-2)} \cdot (n-1)^4 & \text{if } G = S_n, n \geq 2 \\ 2^{4n^2} \cdot (n-1)^{4n} & \text{if } G = K_n, n \geq 2 \\ 2^{4(t+s+2ts)} \cdot s^{4t} \cdot t^{4s} & \text{if } G = K_{t,s},\ t, s \geq 1 \\ 2^{16(t+s)-8} \cdot 3^4 & \text{if } G = T_{t,s}, t \geq 3, s \geq 1 \end{cases}$$

*and*

$$\Pi_2((D(S(G))) = \begin{cases} 2^{8(4n-5)} & \text{if } G = P_n, n \geq 2 \\ 2^{32n} & \text{if } G = C_n, n > 2 \\ 2^{24(n-1)} \cdot (n-1)^{4(n-1)} & \text{if } G = S_n, n \geq 2 \\ 2^{12n(n-1)} \cdot (n-1)^{4n(n-1)} & \text{if } G = K_n, n \geq 2 \\ (8s)^{8t} \cdot (8t)^{8s} & \text{if } G = K_{t,s},\ t, s \geq 1 \\ 2^{32(t+s)-16} \cdot 3^{12} & \text{if } G = T_{t,s}, t \geq 3, s \geq 1 \end{cases}$$

*Proof.* The proof of the first part directly follows from the second formula in Theorem 4.2. So we prove the second part for tadpole graphs. Similar methods can be used for others. Let $G$ be the tadpole graph $T_{t,s}$. There are four types of entries in $M_2(D(S(T_{t,s})))$:

**i)** Let $u$ be a pendant vertex in $G$ with degree 2 and $v$ is a vertex forms an edge with $u$ of degree 4. So in $D(S(T_{t,s}))$, for each $u$ and $v$ there are 4 edges so each vertex pair adds $2 \cdot 4 \cdot 4$ is added to $\Pi_2(D(S(T_{t,s})))$.

**ii)** Both $u$ and $v$ are the vertices (of degree 4) belong the cycle part which form edges. So each vertex pair adds $2 \cdot 4(t-1) \cdot 4 \cdot 4$ is added to $\Pi_2(D(S(T_{t,s})))$.

**iii)** Both $u$ and $v$ are the vertices (of degree 4) belong the path part which form edges. So each vertex pair adds $2 \cdot 4(s-1) \cdot 4 \cdot 4$ is added to $\Pi_2(D(S(T_{t,s})))$.

**iv)** Let $u$ be a common vertex of path and cycle parts of tadpole graph with degree 6 and $v$ is a vertex which forms an edge with $u$ with degree 4 so each vertex pair adds $4 \cdot 6 \cdot 2 \cdot 6$ is added to $\Pi_2(D(S(T_{t,s})))$.

$$\begin{aligned} \Pi_2(S(D(K_n))) &= (2 \cdot 4)^4 \cdot (4 \cdot 4)^{8(t-1)} \cdot (4 \cdot 4)^{8(s-1)} \cdot (4 \cdot 6)^{12} \\ &= 2^{32(t+s)-16} \cdot 3^{12}. \end{aligned}$$

□

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