TWMS J. App. and Eng. Math. V.9, No.1, Special Issue, 2019, pp. 49-57

MATHEMATICAL MODELING OF THE DEPENDENCE OF HEAT DISTRIBUTION ON DIRECTION IN SURFACE TEXTURING OF CONTINUOUS CARBON FIBRE REINFORCED EPOXY COMPOSITES BY ND.YAG LASER

TIMUR CANEL[1], İREM BAĞLAN[2], §

Abstract. Major developments, especially in the aircraft, automobile and energy sectors, have led to the need to use highly featured new materials used in these fields . Carbon fiber reinforced plastics (CFRP) have been used frequently in the transportation and energy sectors because of their high strength-to-weight ratio. Mathematical Modeling of laser beam ablation of carbon fiber reinforced plastic (CFRP) is performed.Heat transfer equation based on Fourier's law of conduction is used for calculation of the temperature distribution in the target material. Under some natural regularity and consistency conditions on the input data the existence, uniqueness of solution are shown by using the generalized Fourier method.

Keywords: Laser ablation, Mathematical modeling, Fourier method, Carbon fibre Epoxy, Surface texturing.

1. Introduction

Major developments, especially in the aircraft, automobile and energy sectors, required the use of high-performance new materials used in these fields [1].Polymers and their composites are the candidate materials for future applications in numerous industrial areas. Since thermoplastic based materials are easy to weld, shape, store and recycle, many field uses have been increased by incorporating composite materials into production processes [1]. Carbon fiber reinforced plastics (CFRP) have been used frequently in the transportation and energy sectors because of their high strength-to-weight ratio [2]. Lasers are frequently used in cutting, drilling and surface processing as they can carry out high energy transfer in various forms [6]. With the growing commercial application of these composite structures there is an interest in the use of lasers to cut and drill such materials [9]. The main problem in the processing of CFRPs is that the matrix material and carbon fibers have different thermal conductivities and melting temperatures [11]. This difference causes thermal damage. In many applications, it is aimed to reduce thermal damage in

---

[1] Department of Mathematics, Kocaeli University, Kocaeli 41380, Turkey.
e-mail: tcanel@kocaeli.edu.tr; ORCID:https//orcid.org/0000-0002-4282-1806;

[2] Department of Mathematics, Kocaeli University, Kocaeli 41380, Turkey.
e-mail: isakinc@kocaeli.edu.tr; ORCID:https//orcid.org/0000-0002-1877-9791;

§ Selected papers of International Conference on Life and Engineering Sciences (ICOLES 2018), Kyrenia, Cyprus, 2-6 September, 2018.

TWMS Journal of Applied and Engineering Mathematics, Vol.9, No.1, Special Issue, 2019; © Işık University, Department of Mathematics; all rights reserved.

the polymer matrix in the majority of CFRP laser drilling or ablation studies. One of the most important of these problems is the control of the Heat Affected Zone (HAZ). Numerous studies have been carried out on the effect of laser parameters on the HAZ.

Because of the quality degradation in the laser process, especially in the efficient processing of CFRPs, high energy and other laser parameters compatible with these energy levels are required. Thermal damage is the most important quality reducing effect in laser processing of CFRP [11]. Therefore, it is important to select the appropriate parameters when laser machining of materials. By selecting appropriate laser parameters and process strategies, CFRP can be processed very efficiently by reducing the thermal damage caused by laser radiation.

In this study, it was examined the dependence of heat distribution on direction for CFRP and also influnce of energy on aspect ratio.

When a laser beam of intensity is irradiated on the surface of insulators, it results in vibrations. This excitation energy is rapidly converted into heat (time duration in the range $10-12to10-6s$for nonmetals). This is followed by various heat transfer processes such as conduction into the materials, and convection and radiation from the surface. The most significant heat transfer process being the heat conduction into the material. The generation of heat at the surface and its conduction into the material establishes the temperature distributions in the material depending on the thermo-physical properties of the material and laser parameters [4]. If the incident laser intensity is sufficiently high, the absorption of laser energy can result in the phase transformations such as surface melting and evaporation. Generally, these phase transformations are associated with threshold (minimum) laser intensities referred to as melting and evaporation thresholds (Im and Iv). Melting and evaporation are the efficient material removal mechanisms during many machining processes.

2. Heat Transfer Model

To understand the effects of laser irradiation on the material, it is necessary to evaluate the temporal and spatial variation of temperature distribution. The most simplified thermal analysis is based on the solution of one-dimensional heat conduction equation with simplified assumptions such as [10];

1. Material is homogeneous. The thermo-physical properties are independent of temperature.
2. The initial temperature of the material is constant.
3. Heat input is uniform during the irradiation time.
4. The convection and radiation losses from the surface are negligible.

The governing equation for the one dimensional heat transfer can be written as:

$$\frac{\partial T(z,t)}{\partial t} = \alpha^2 \frac{\partial^2 T(z,t)}{\partial z^2} \tag{1}$$

where T is the temperature at location z after time t, $3b1$ is the thermal diffusivity.

$$\alpha^2 = \frac{k}{c\rho}$$

where,
k denotes the heat conduction coefficient
c specific heat $(1500J/(kg.K))$.
ρ density $(1650k/m3)$

Let $t_p > 0$ be a fixed number and denote by $D = \{(z.t) : 0 < z < l, 0 < t < t_p\}$, where t_p is the pulse duration.

The initial condition can be written as;

$$T(z,0) = T_0, 0 < z < l \tag{2}$$

where T_0 is the initial constant temperature of the material (room temperature $20C$).

The simple boundary condition at the surface ($z = 0$) assuming that laser energy absorbed at the surface equals the energy conducted can be written as:

$$\frac{\partial T(0,t)}{\partial t} = \frac{\partial T(l,t)}{\partial t} = 0, (t > 0) \tag{3}$$

This problem will be called an parabolic problem where $T(z,t) \in C^{2,1}(D) \cap C^{1,0}(D)$ is called classical solution of the problem (1)-(3). The problem of finding the heat source in a parabolic equation has been investigated in many studies [3, 5, 6, 7].

3. Volumetric Heat Generation

When the thin film absorbs the laser beam energy, it generates heat [9]. The heat generation rate is defined as ,

$$S = \frac{-dI}{dz} \tag{4}$$

where

$$I = I_0 e^{-\int\limits_b^z a dz} \tag{5}$$

, $I = I(z)$is the laser intensity at a depth z in units of $W/m2$, z is the depth along the thin film in units of m and a is the absorptivity or the absorption coefficient of the material in units of $m1$.Substituting eq. (4) in (5) and using Leibniz rule yields

$$S = \frac{-d}{dz}\left\{ I_0 e^{-\int\limits_b^z a dz} \right\}$$

Hence, the volumetric heat generation becomes:

$$S = I_0 e^{-\int\limits_b^z a dz} \tag{6}$$

4. Mathematical Modeling of Ablation

Obtained Mathematical model for ablation is given as:

$$\frac{\partial T(z,t)}{\partial t} = \alpha^2 \frac{\partial^2 T(z,t)}{\partial z^2} + S(z,t) \tag{7}$$

with initial condition

$$T(z,0) = T_0, 0 < z < l \tag{8}$$

the boundary condition is

$$\frac{\partial T(0,t)}{\partial t} = \frac{\partial T(l,t)}{\partial t} = 0, (t > 0) \tag{9}$$

We look for the following representation for the solution of (7)-(9)

$$T(z,t)=\sum_{k=0}^{\infty}\left(T_{ck}\cos\frac{2\pi k}{l}z+T_{sk}\sin\frac{2\pi k}{l}z\right), \tag{10}$$

we obtain the solution of (7)-(9)

$$\begin{aligned} T(z,t) \;=\; & \sum_{k=0}^{\infty}\varphi_{ck}e^{-\left(\frac{2\pi k}{l}\right)^2 t}+\int_0^t\int_0^l\left(S(z,t)\cos\frac{2\pi k}{l}\tau e^{-\left(\frac{2\pi k}{l}\right)^2(t-\tau)}\right)\cos\frac{2\pi k}{l}z \qquad (11)\\ & \sum_{k=0}^{\infty}\varphi_{sk}e^{-\left(\frac{2\pi k}{l}\right)^2 t}+\int_0^t\int_0^l\left(S(z,t)\sin\frac{2\pi k}{l}\tau e^{-\left(\frac{2\pi k}{l}\right)^2(t-\tau)}\right)\sin\frac{2\pi k}{l}z. \end{aligned}$$

In our study, the simple boundary condition at the surface ($z = 0$) assuming that laser energy absorbed at the surface equals the energy conducted can be written as [8]:

$$-k\frac{\partial T(0,t)}{\partial t}=-\delta H$$

where k is the thermal conductivity and H is the absorbed laser energy . The absorbed laser energy H can be given by the product of absorptivity A and incident laser power density $I_0(i.e., H = AI_0)$. If tp is the irradiation time (pulse on time) then the parameter f064 equals unity when the laser is on, i.e., $0 \leq t \leq tp$. It can be taken as zero when the laser is off, i.e., $t > tp$.

$$\frac{\partial T(z,t)}{\partial t}=\alpha^2\frac{\partial^2 T(z,t)}{\partial z^2}+S(z,t) \tag{12}$$

with initial condition

$$T(z,0)=T_0, 0<z<l \tag{13}$$

the boundary condition is

$$\frac{\partial T(0,t)}{\partial t}=0, \frac{\partial T(l,t)}{\partial t}=\frac{-H}{k}, (t>0) \tag{14}$$

The solutions of these equations can be obtained as follows:
During heating $(0 < t < tp)$:

$$\begin{aligned} T(z,t) \;=\; & \sum_{k=0}^{\infty}\varphi_{ck}e^{-\left(\frac{2\pi k}{l}\right)^2 t}+\int_0^t\int_0^l\left(S(z,t)\cos\frac{2\pi k}{l}\tau e^{-\left(\frac{2\pi k}{l}\right)^2(t-\tau)}\right)\cos\frac{2\pi k}{l}z \qquad (15)\\ & \sum_{k=0}^{\infty}\varphi_{sk}e^{-\left(\frac{2\pi k}{l}\right)^2 t}+\int_0^t\int_0^l\left(S(z,t)\sin\frac{2\pi k}{l}\tau e^{-\left(\frac{2\pi k}{l}\right)^2(t-\tau)}\right)\sin\frac{2\pi k}{l}z-\frac{zH}{lk}. \end{aligned}$$

5. EXPERIMENTAL RESULTS AND MEASUREMENTS

The2.9 mm thick multi-layer carbon fibre composites plaques to be used were cleaned with chloroform. In the ablation process GSI lumonics JK760TR Series Laser (Class 4) system in a CNC cabin was used. The process parameters are selected as: Focus positions are +2, mm (upward from the surface). Spot size is 650 m, pulse durations are2 ms. Pulse energies of laser are varied from $0.5J to 2.5$J with 0.5 J step. $(0.5, 1, 1.5, 2, 2.5J)$ for single pulse. The JK760TR Series of laser is an Nd:YAG laser that has 1064 nm wavelength, 0.3–50ms. pulse length and 500 Hz maximum repetition rate. The average power that can be obtained is 600Watt. And also JK760 TR series laser has a pulse shaping ability. Laser output power is delivered via a 600 m radius fiber optic cable to the focus head at the workstation for process. The laser beam is focused on sample using 160mm plano convex lens. The minimum spot size on the plates has been 0.4mm. There was no shielding gas used during the experiments. The surface topography and surface roughness parameters of the test coupons were analysed in detail by using a laser non-contact optical profilometer (Nanovea PS50, USA). Formed crater surface was scanned with 0.1μm precision. The analyses were performed to investigate the surface roughness and 3D topography characterization. The areal surface texture were analysed in accordance with $\text{ISO}25178 - 2 : 2012$ standard. In order to visualize 3D topography of the spcimens, the measurement software named Mountains ® surface imaging & metrology software, FR was used.

6. Results and discussion

Top view sample image of dimples scanned by profilometer is given in Fig. 1 (a) and (c). The sample image taken from stereo microscope is given in Fig 1 (b) and (d).

Top view image taken from stereo microscope and profilometer of dimples for 2.5 joule pulse energy are given Fig 1a and 1 b respectively. Top view image taken from stereo microscope and profilometer of dimples for 2.0 joule pulse energy are given Fig 1c and 1 d respectively. Top view image taken from stereo microscope and profilometer of dimples for 1.5 joule pulse energy are given Fig 1e and 1 f respectively. Top view image taken from stereo microscope and profilometer of dimples for 1.0 joule pulse energy are given Fig 1g and 1 h respectively. Top view image taken from stereo microscope and profilometer of dimples for 0.5 joule pulse energy are given Fig 1i and 1 j respectively.

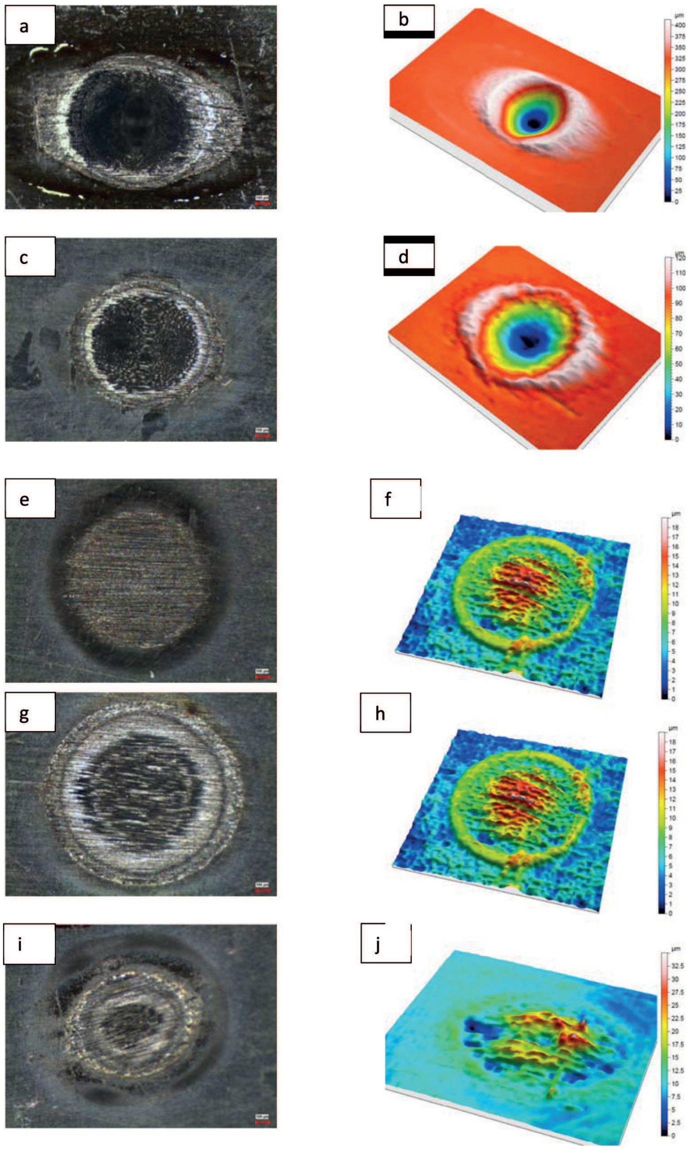
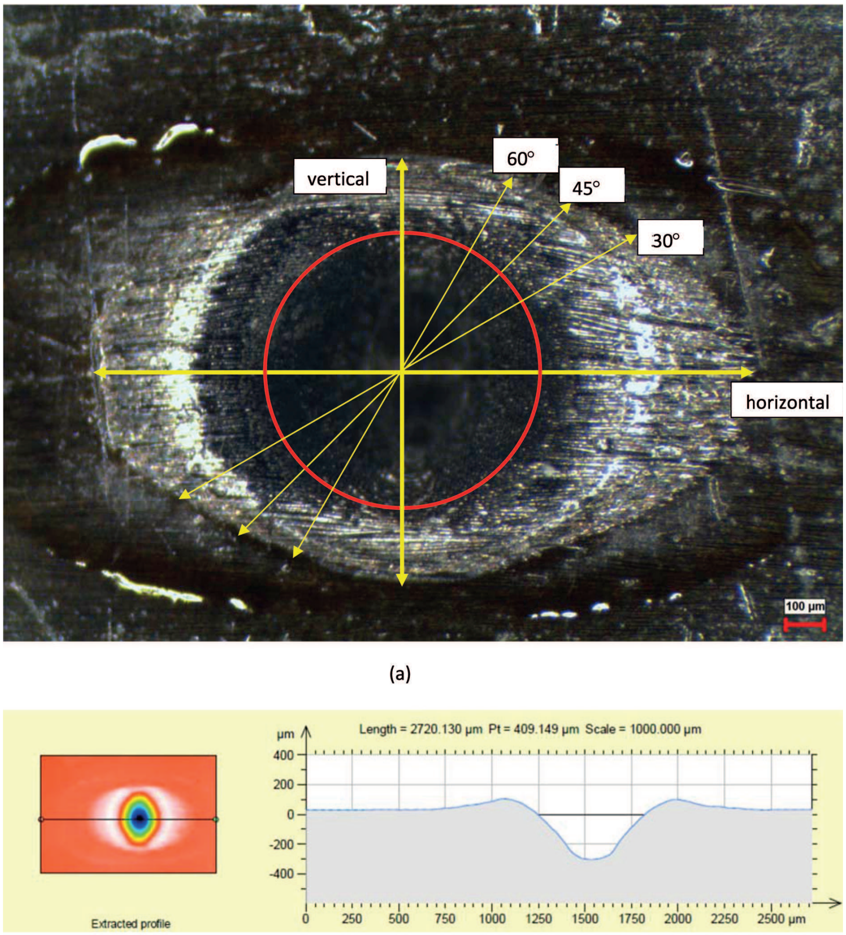


Figure 1. Optical microscope images for 2.5, 2, 1.5, 1, 0.5 joule represents 1a,1c,1e,1g and 1i respectively. Profilometer images for 2.5, 2, 1.5, 1, 0.5 joule represents 1b, 1d, 1f, 1h and 1j respectively.

7. Heat Affected Zone (HAZ)

Figure 2.a and 2.b shows the sample investigation of heat distribution as a function of angle and extracted profile calculated by profilometer respectively for 2.5 joule pulse energy. Same measurement method was applied for the others pulse energies.



(a)

(b)

Figure 2. (a) Heat distribution as a function of angle for 2.5 joule pulse energy. (b) Extracted profile calculated by profilometer for 2.5 joule pulse energy.

Table1. Measurement of heat distribution from optical microscope and profilometer for 2.5 joule pulse energy.

Enerji	0 deg(Horizontal)	30 deg	45 deg	60 deg	90deg(Vertical)
2.5	1560	1251	1114	1039	1013
2.0	1195	1074	1019	990	986
1.5	1098	1070	1061	1047	1034
1.0	1486	1427	1421	1375	1261
0.5	1098	925	884	851	836

As shown in Table1, as the angle increases, the HAZ dimensions decrease. This indicates that the fibers transmit heat faster than the matrix. Max HAZ occurs on fibers direction. As seen in figures for all energies, the max. cavity diameter has remained almost unchanged up to 1.5 joules pulse energy. After this energy value, the size of the HAZ has increased rapidly. If we accept the heat distribution perpendicular to the direction of the fibers. We can say that the HAZ dimensions increase exponentially as we approach the direction of the fibers.

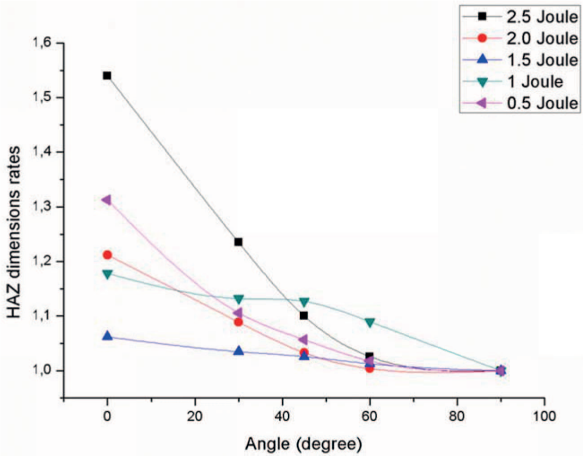


FIGURE 3. HAZ dimension rates as a function of angle.

REFERENCES

[1] Jaeschke, P., Kern, M., Stute, U., (2014), D. Kracht and Heinz Haferkamp, Laser processing of continuous carbon fibre reinforced polyphenylene sulphide organic sheets—Correlation of process parameters and reduction in static tensile strength properties. Journal of Thermoplastic Composite Materials, 27(3), 324–337.

[2] Staehr, R., Bluemel, S., Hansen, P., Jaeschke, P., (2015), O. Suttmann and L. Overmeyer, The influence of moisture content on the heat affected zone and the resulting in-plane shear strength of laser cut thermoplastic CFRP, Plastics, Rubber and Composites, 44, 111-116.

[3] Geersa, M. D., de Borstb, R., Brekelmansc, W. A. M. and Peerlingsc, R. H. J., (1999), Validation and internal length scale determination for a gradient damage model] application to short glass fibre reinforced polypropylene, International Journal of Solids and Structures 25, 1446-1472.

[4] Thomason, J. L. and Vlug, M. A., (1996), Influence of fibre length and concentration on the properties of glass fibre-reinforced polypropylene: 1. Tensile and flexural modulus, Conlposites: Part A 27A, 417-484.

[5] Jin, F. L., Lee, S. Y. and Park, S. J., (2013), Polymer matrices for carbon fiber-reinforced polymer-composites, Carbon Letters, 14, 76-88.

[6] Canel, T., Kaya, A.U and Celik, B., (2012), Parameter optimization of nanosecond laser for microdrilling on PVC by Taguchi method, Optics & Laser Technology 44, 2347–2353.

[7] Baglan, I., (2015), Determination of a coefficient in a quasilinear parabolic equation with periodic boundary condition, Inverse Problems in Science and Engineering, 10.1080/17415977.2014.947479,23,5.

[8] Dahotre, B.N., Harimkar, P. S., (2008), Laser Fabrication and Machining of Materials,2008 Springer Science Business Media, LLC.

[9] Mansoor, S. B. and Yilbas, B. S., (2012), Laser Short-Pulse Heating: Influence of Spatial Distribution of Absorption Coefficient on Temperature Field in Silicon Film, JLMN-Journal of Laser Micro/Nanoengineering Vol. 7, No. 2.

[10] Carslaw , HS., Jaeger, JC., (1959), Conduction of Heat in Solids. Oxford University Press, Oxford.

[11] Weber, R., Hafner, M., (2011), A. Michalowski and T. Graf, Minimum Damage in CFRP Laser Processing, Physics Procedia 12 302–307.

Timur Canel is a Associate Professor. He graduated from Department of physics, ODTÜ, Gaziantep, Turkey in 1993. He received MS - PhD degree in physics from Kocaeli University. He is a member of Department of physics in Kocaeli University by associate professor since 2014. His research interests focus mainly in Laser ablation, Carbon fibre Epoxy, Surface texturing.

İrem Bağlan for the photography and short authobiography, see page 48.