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RESULTS OF SOME SEPARATION AXIOMS IN SUPRA SOFT TOPOLOGICAL SPACES

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Abstract. In this paper, we define separability axioms in supra soft topological spaces by using the soft point given in [1, 3] and investigate some of their characterizations.

Keywords: soft set, soft point, supra soft topology, supra soft separation axioms.

AMS Subject Classification: 54A40, 54E55

1. Introduction

Many practical problems in economics, engineering, environment, social science, medical science etc. cannot be dealt with by classical methods, because classical methods have inherent difficulties. Molodtsov [7] initiated completely a new approach for modeling uncertainties. Shabir and Naz [9] firstly introduced the notion of soft topological space which are defined over an initial universe with a fixed set of parameters and showed that a soft topological space gives a parameterized family of topological spaces. Theoretical studies of soft topological spaces have also been by some authors in [2, 6, 8, 10]. In these studies the concept of soft point is given almost samely. Consequently, some results of classical topology are not valid in soft topological spaces. Mashhour et al. [5] introduced the concepts of supra topological spaces, supra closed sets, supra open sets and supra continuous mapping. Many results of topological spaces remain valid in supratopological spaces, but some become false. As a generalized of soft topological spaces, the notion of supra soft topological spaces by dropping only the soft intersection condition was introduced by El-Sheikh and Abd El-latif [4].

In the present study, we give separability axioms in supra soft topological spaces and study some of their characterizations.

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2. Separation axioms in supra soft topological spaces

Definition 2.1. [7] *A pair (F,E) is called a soft set over X, where F is a mapping given by $F : E \to P(X)$, where $P(X)$ denotes the power set of X. $SS(X)_E$ denotes the family of all soft sets over X with a fixed set of parameters E.*

Definition 2.2. [1, 3] *Let (F,E) be a soft set over X. The soft set (F,E) is called a soft point, denoted by (x_e,E) , if for the element $e \in E$, $F(e) = \{x\}$ and $F(e') = \phi$ for all $e' \in E - \{e\}$.*

Definition 2.3. [1] *Two soft points (x_e,E) and $(y_{e'},E)$ over a common universe X, we say that the soft points are different if $x \neq y$ or $e \neq e'$.*

Definition 2.4. [1] *Let μ be the collection of soft sets over a universe X with a fixed set of parameters E. Then μ is called a supra soft topology on X with a fixed set E if*

(1) Φ ,$\widetilde{X} \in \mu$
(2) *the union of any number of soft sets in μ belongs to μ.*

(X,μ,E) is called a supra soft topological space and the members of μ are called a supra soft open sets. A soft set (F,E) is a supra soft closed set if and only if its complement $(F,E)^c$ is a supra soft open set. It is clear that supra soft topological spaces are very natural generalization of supra topological spaces.

Proposition 2.1. *The collection τ^* of all supra soft closed sets in a supra soft topological space (X,μ,E) satisfies the following conditions:*

(1) Φ ,$\widetilde{X} \in \tau^*$
(2) *the intersection of any number of supra soft closed sets in τ^* belongs to τ^*.*

Proof. It is obvious. □

Remark 2.1. [4] *Every soft topological space is a supra soft topological space, but the converse is not true in general as shown in the following example.*

Example 2.1. *Let $X = \{h_1,h_2,h_3,h_4,h_5\}$, $E = \{e_1,e_2\}$ and*
$\mu = \left\{\Phi, \widetilde{X}, (F_1,E), (F_2,E), (F_3,E)\right\}$ *where (F_i,E) are soft sets over X, $1 \leq i \leq 3$, defined as follows:*
$F_1(e_1) = \{h_1,h_4\}$, $F_1(e_2) = \{h_1,h_3\}$
$F_2(e_1) = \{h_2,h_4\}$, $F_2(e_2) = \{h_2,h_3\}$
$F_3(e_1) = \{h_1,h_2,h_4\}$, $F_3(e_2) = \{h_1,h_2,h_3\}$
Then (X,μ,E) is a supra soft topological space over X, but it is not a soft topological space.

Remark 2.2. *The intersection of two supra soft open sets need not supra soft open set. Also, the intersection of a soft open set and a supra soft open set need not be supra soft open set. But if $\mu = SS(X)_E$, $(F,E) \in \tau$ and $(G,E) \in \mu$, then $(F,E) \cap (G,E) \in \mu$.*

Definition 2.5. [4] *Let (X,τ,E) be a supra soft topological space over X . A soft set (F,E) in (X,τ,E) is called a supra soft neighborhood of the soft point $(x_e,E) \in (F,E)$ if there exists a supra soft open set (G,E) such that $(x_e,E) \in (G,E) \subset (F,E)$.*

The supra soft neighborhood system of a soft point (x_e,E), denoted by $U(x_e,E)$, is the family of all its neighborhoods.

Theorem 2.1. *The supra soft neighborhood system $U(x_e,E)$ at (x_e,E) in a supra soft topological space (X,τ,E) has the following properties:*

(1) *If $(F,E) \in U(x_e,E)$, then $(x_e,E) \in (F,E)$,*

(2) *If* $(F,E) \in U(x_e,E)$ *and* $(F,E) \subset (G,E)$*, then* $(G,E) \in U(x_e,E)$*,*
(3) *If* $(F,E) \in U(x_e,E)$*, then there is a* $(G,E) \in U(x_e,E)$ *such that* $(F,E) \in U(y_{e'},E)$ *for each* $(y_{e'},E) \in (G,E)$*.*

Definition 2.6. *Let* (X,τ,E) *be a supra soft topological space and* (F,E) *be a supra soft set over* X *. The soft point* $(x_e,E) \in (F,E)$ *is called a soft interior point of a soft set* (F,E) *if there exists a supra soft open set* $(G,E) \in U(x_e,E)$ *such that* $(x_e,E) \in (G,E) \subset (F,E)$*.*

Proposition 2.2. *Let* (X,τ,E) *be a supra soft topological space and* (F,E) *be a soft set over* X *. Then* (F,E) *is a supra soft open set if and only if* (F,E) *is a supra soft neighborhood of its soft points.*

Proof. The proof is clear. □

Definition 2.7. *Let* (X,τ,E) *be a supra soft topological space,* (F,E) *be a soft set over* X *and* (x_e,E) *be a soft point. Then* (x_e,E) *is said to be a soft closure point of* (F,E) *if* $(F,E) \cap (G,E) \neq \Phi$*, for arbitrary* $(G,E) \in U(x_e,E)$*.*

Theorem 2.2. *Let* (X,τ,E) *be a supra soft topological space and* (F,E) *be a soft set over* X *. Then* (F,E) *is a supra soft closed set in* X *if and only if every soft closure point of* (F,E) *belongs to it.*

Proof. Let (F,E) be a supra soft closed set, (x_e,E) be a soft closure point and $(x_e,E) \notin (F,E)$. Then $(x_e,E) \in (F,E)^c$. Since $(F,E)^c$ is a supra soft open set in τ, it is a supra soft neighborhood of (x_e,E). Then $(F,E)^c \cap (F,E) = \Phi$. It follows that $(x_e,E) \in (F,E)$.

Conversely, $(x_e,E) \in (F,E)^c$ be any soft point. Then $(x_e,E) \notin (F,E)$. Since (x_e,E) is not a soft closure point of (F,E), there exists a supra soft neighborhood (G,E) of (x_e,E) such that $(F,E) \cap (G,E) = \Phi$. Since $(x_e,E) \in (G,E) \subset (F,E)^c$, we have that $(F,E)^c$ is a supra soft open set, i.e. (F,E) is a supra soft closed set. □

Proposition 2.3. *Let* (X,τ,E) *be a supra soft topological space,* (F,E) *be a supra soft set over* X *and* $x \in X$*. If* (x_e,E) *is a soft interior point of* (F,E)*, then* x *is an interior point of* $F(e)$ *in* (X,τ_e)*.*

Proof. For any $e \in E$, $F(e) \subset X$. If (x_e,E) is a soft interior point of (F,E), then there exists $(G,E) \in \tau$ such that $(x_e,E) \in (G,E) \subset (F,E)$. This means that, $x \in G(e) \subset F(e)$ and $G(e) \in \tau_e$. x is an interior point of $F(e)$ in τ_e. □

Proposition 2.4. *Let* (X,τ,E) *be a supra soft topological space,* (F,E) *be a supra soft set over* X *and* $x \in X$*. If* x *is a closure point of* $F(e)$ *in* (X,τ_e)*, then* (x_e,E) *is a soft closure point of* (F,E)*.*

Proof. The proof is clear. □

Example 2.2. *Let* $X = \{x_1,x_2,x_3\}$*,* $E = \{e_1,e_2\}$ *and*
$\tau = \left\{\Phi, \widetilde{X}, (F_1,E), (F_2,E), (F_3,E)\right\}$ *where*

$$\begin{aligned} F_1(e_1) &= \{x_1,x_2\},\ F_1(e_2) = \{x_1,x_3\},\ F_2(e_1) = \{x_2\},\ F_2(e_2) = \{x_2,x_3\}, \\ F_3(e_1) &= \{x_1,x_2\},\ F_3(e_2) = X. \end{aligned}$$

Then (X,τ,E) *is a supra soft topological space over* X*. Thus* (F,E) *is defined as follows:*

$$F(e_1) = \{x_1,x_3\},\ F(e_2) = \{x_1,x_3\}$$

Then there is not soft interior point of (F,E)*. But* x_1 *and* x_3 *are interior points of* $F(e_2)$ *in* τ_{e_2} *. Here* $\tau_{e_2} = \{\phi, X, \{x_1,x_3\}, \{x_2,x_3\}\}$*.*

Definition 2.8. *Let (X,τ,E) be a supra soft topological space and $(x_e,E)\neq(y_{e'},E)$.*

(1) *If there exist two supra soft open sets (F,E) and (G,E) such that*
$$(x_e,E)\in(F,E) \text{ and } (y_{e'},E)\notin(F,E) \text{ or } (y_{e'},E)\in(G,E) \text{ and } (x_e,E)\notin(G,E),$$
then (X,τ,E) is called a supra soft T_0-space.

(2) *If there exist two supra soft open sets (F,E) and (G,E) such that*
$$(x_e,E)\in(F,E),\ (y_{e'},E)\notin(F,E) \text{ and } (y_{e'},E)\in(G,E),\ (x_e,E)\notin(G,E),$$
then (X,τ,E) is called a supra soft T_1-space.

(3) *If there exist two supra soft open sets (F,E) and (G,E) such that $(x_e,E)\in(F,E)$, $(y_{e'},E)\in(G,E)$ and $(F,E)\cap(G,E)=\Phi$, then (X,τ,E) is called a supra soft T_2-space.*

Theorem 2.3. *Let (X,τ,E) be a supra soft topological space over X . Then (X,τ,E) is a supra soft T_1-space if and only if each soft point is a supra soft closed set.*

Proof. Let (X,τ,E) be a supra soft T_1-space and (x_e,E) be an arbitrary soft point. We show that $(x_e,E)^c$ is a supra soft open set. Let $(y_{e'},E)\in(x_e,E)^c$, $(x_e,E)\neq(y_{e'},E)$. Since (X,τ,E) is a supra soft T_1- space, there exists a supra soft open set (G,E) such that $(y_{e'},E)\in(G,E)$, $(x_e,E)\notin(G,E)$. Then $(y_{e'},E)\in(G,E)\subset(x_e,E)^c$. This implies that $(x_e,E)^c$ is a supra soft open set, i.e. (x_e,E) is a supra soft closed set.

Suppose that for each (x_e,E) is a supra soft closed set in τ. Then $(x_e,E)^c$ is a supra soft open set in τ. Let $(x_e,E)\neq(y_{e'},E)$. Thus $(y_{e'},E)\in(x_e,E)^c$ and $(x_e,E)\notin(x_e,E)^c$. Similarly $(y_{e'},E)^c$ is a supra soft open set in τ such that $(x_e,E)\in(y_{e'},E)^c$ and $(y_{e'},E)\notin(y_{e'},E)^c$. Therefore (X,τ,E) is a supra soft T_1-space over X. □

Proposition 2.5. *Let (X,τ,E) be a supra soft topological space. If (X,τ,E) is a supra soft T_i-space, then (X,τ_e) is a supra T_i-space for each $e\in E$. $(i=0,1,2)$*

Proof. The proof is clear. □

Remark 2.3. *Every supra soft T_1-space is a supra soft T_0-space, every supra soft T_2-space is a supra soft T_1-space.*

Theorem 2.4. *Let (X,τ,E) be a supra soft T_1-space, for every soft point (x_e,E), $(x_e,E)\in(G,E)$ and $(G,E)\in\tau$. If there exists a supra soft open set (F,E) such that $(x_e,E)\in(F,E)\subset cl^s(F,E)\subset(G,E)$, then (X,τ,E) is a supra soft T_2-space.*

Proof. Suppose that $(x_e,E)\neq(y_{e\prime},E)$. Since (X,τ,E) is a supra soft T_1-space, (x_e,E) and $(y_{e\prime},E)$ are supra soft closed sets in τ. Thus $(x_e,E)\in(y_{e\prime},E)^c$ and $(y_{e\prime},E)^c$ is a supra soft open set in τ. Then there exists a supra soft open set (F,E) in τ such that
$$(x_e,E)\in(F,E)\subset cl^s(F,E)\subset(y_{e\prime},E)^c.$$
Hence we have $(y_{e\prime},E)\in cl^s(F,E)^c$, $(x_e,E)\in(F,E)$ and $(F,E)\cap cl^s(F,E)^c=\Phi$, i.e, (X,τ,E) is a supra soft T_2-space. □

Definition 2.9. *Let (X,τ,E) be a supra soft topological space over X.*

(1) *(F,E) be a supra soft closed set in X and $(x_e,E)\notin(F,E)$. If there exist supra soft open sets (G_1,E) and (G_2,E) such that $(x_e,E)\in(G_1,E)$, $(F,E)\subset(G_2,E)$ and $(G_1,E)\cap(G_2,E)=\Phi$, then (X,τ,E) is called a supra soft regular space.*

(2) *(X,τ,E) is said to be a supra soft T_3-space if it is supra soft regular and supra soft T_1-space.*

Theorem 2.5. *Let (X, τ, E) be a supra soft topological space over X. (X, τ, E) is a supra soft T_3-space if and only if for every $(x_e, E) \in (F, E) \in \tau$ there exists $(G, E) \in \tau$ such that $(x_e, E) \in (G, E) \subset cl^s(G, E) \subset (F, E)$.*

Proof. Let (X, τ, E) be a supra soft T_3-space and $(x_e, E) \in (F, E) \in \tau$. Since (X, τ, E) is a supra soft T_3-space for the soft point (x_e, E) and supra soft closed set $(F, E)^c$, there exists $(G_1, E), (G_2, E) \in \tau$ such that $(x_e, E) \in (G_1, E)$, $(F, E)^c \subset (G_2, E)$ and $(G_1, E) \cap (G_2, E) = \Phi$. Thus, we have $(x_e, E) \in (G_1, E) \subset (G_2, E)^c \subset (F, E)$. Since $(G_2, E)^c$ is supra soft closed, $cl^s(G_1, E) \subset (G_2, E)^c$.

Conversely, let $(x_e, E) \notin (H, E)$ and (H, E) be a supra soft closed set. Thus, $(x_e, E) \in (H, E)^c$ and from the condition of the theorem , we have $(x_e, E) \in (G, E) \subset cl^s(G, E) \subset (H, E)^c$. Then $(x_e, E) \in (G, E)$, $(H, E) \subset (cl^s(G, E))^c$ and $(G, E) \cap (cl^s(G, E))^c = \Phi$ are satisfied, i.e., (X, τ, E) is a supra soft T_3-space. □

Theorem 2.6. *Let (X, τ, E) be a supra soft topological space over X. If (X, τ, E) is a supra soft T_3-space, then (X, τ_e) is a supra T_3-space, for each $e \in E$.*

Proof. Let (X, τ, E) be a supra soft topological space over X. By Proposition 2.5, (X, τ_e) is a supra T_1-space. Let $B \subset X$ be a closed set in τ_e and $x \notin B$. From the definition of τ_e, there exists a soft closed set (F, E) and $(x_e, E) \notin (F, E)$ such that $F(e) = B$. Since (X, τ, E) is a supra soft regular space, there exist supra soft open sets (G_1, E) and (G_2, E) such that $(x_e, E) \in (G_1, E)$, $(F, E) \subset (G_2, E)$ and $(G_1, E) \cap (G_2, E) = \Phi$. Thus we have $x \in G_1(e)$, $B \subset G_2(e)$ and $G_1(e) \cap G_2(e) = \phi$, i.e. (X, τ_e) is a supra T_3-space for each $e \in E$. □

Definition 2.10. (1) *Let (X, τ, E) be a supra soft topological space (F, E) and (G, E) supra soft closed sets such that $(F, E) \cap (G, E) = \Phi$. If there exist supra soft open sets $(F_1, E), (F_2, E)$ such that $(F, E) \subset (F_1, E)$, $(G, E) \subset (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \Phi$, then (X, τ, E) is called a supra soft normal space.*

(2) *(X, τ, E) is said to be a supra soft T_4-space if it is supra soft normal and supra soft T_1-space.*

Remark 2.4. *A supra soft T_3-space is a supra soft T_2-space, a supra soft T_4-space is a supra soft T_3-space.*

Theorem 2.7. *Let (X, τ, E) be a supra soft topological space over X. Then (X, τ, E) is a supra soft T_4-space if and only if, for each supra soft closed set (F, E) and supra soft open set (G, E) with $(F, E) \subset (G, E)$, there exists supra soft open set (D, E) such that*

$$(F, E) \subset (D, E) \subset cl^s(D, E) \subset (G, E).$$

Proof. Let (X, τ, E) be a supra soft T_4-space, (F, E) be a supra soft closed set and $(F, E) \subset (G, E)$, $(G, E) \in \tau$. Then $(G, E)^c$ is a supra soft closed set and $(F, E) \cap (G, E)^c = \Phi$. Since (X, τ, E) is a supra soft T_4-space, there exist supra soft open sets (D_1, E) and (D_2, E) such that $(F, E) \subset (D_1, E)$, $(G, E)^c \subset (D_2, E)$ and $(D_1, E) \cap (D_2, E) = \Phi$. This implies that

$$(F, E) \subset (D_1, E) \subset (D_2, E)^c \subset (G, E).$$

$(D_2, E)^c$ is a supra soft closed set and $cl^s(D_1, E) \subset (D_2, E)^c$ is satisfied. Thus

$$(F, E) \subset (D_1, E) \subset cl^s(D_1, E) \subset (G, E)$$

is obtained.

Conversely, let (F_1, E), (F_2, E) be two supra soft closed sets and $(F_1, E) \cap (F_2, E) = \Phi$. Then $(F_1, E) \subset (F_2, E)^c$. From the condition of theorem, there exists a supra soft open set (D, E) such that

$$(F_1, E) \subset (D, E) \subset cl^s(D, E) \subset (F_2, E)^c.$$

So, (D, E), $(cl^s(D, E))^c$ are supra soft open sets and $(F_1, E) \subset (D, E)$, $(F_2, E) \subset (cl^s(D, E))^c$ and $(D, E) \cap (cl^s(D, E))^c = \Phi$ are obtained. Hence (X, τ, E) is a supra soft T_4-space. $\square$

3. Conclusion

We have introduced separation axioms in supra soft topological spaces which are defined over an initial universe with a fixed set of parameters. Later their important properties are investigated.

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