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INTERVAL SHEFFER STROKE BASIC ALGEBRAS

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ABSTRACT. In this paper we deal with Sheffer stroke basic algebras $\mathcal{A} = (A; |)$, and we define the operations $|_a$, $|^b$, $|_a^b$ for any elements $a, b \in A$ in such a way that $([a, 1]; |_a)$, $([0, b]; |^b)$, $([a, b]; |_a^b)$ become also Sheffer Stroke basic algebras, respectively. Subsequeutly, we show that these interval Sheffer Stroke basic algebras on a given Sheffer Stroke basic algebra $\mathcal{A} = (A; |)$ verify the patchwork condition.

Keywords: Basic algrebras, interval basic algebra, patchwork condition.

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1. Introduction

The Sheffer stroke operation (|) was primarily given by H. M. Sheffer [10]. This operation attracted many researchers' attention because any axiom in Boolean algebras is expressed via only this operation [12].

Thus, many rearchers wish to apply such a reduction to several algebraic structures such as orthoimplication algebras [13] and sheffer stroke basic algebras [11], etc.

We give definitions and notions about sheffer stroke operation and basic algebras in second section, and we search for an answer to question that how the new operations can be defined on a given interval to obtain a Sheffer Stroke basic algebra in third section.

2. Preliminaries

We give the following fundamental notions.

Definition 2.1. [2] *Let $\mathcal{A} = (A; |)$ be a structure. The operation $|$ is called a Sheffer stroke operation if it satisfies the following conditions:*

$(S1)$ $x|y = y|x$,
$(S2)$ $(x|x)|(x|y) = x$,

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$(S3)$ $x|((y|z)|(y|z)) = ((x|y)|(x|y))|z$,
$(S4)$ $(x|((x|x)|(y|y)))|(x|((x|x)|(y|y))) = x$.
If additionally it satisfies the identity
$(S5)$ $y|(x|(x|x)) = y|y$,
it is called an ortho Sheffer stroke operation.

Lemma 2.1. [2] *Let $|$ be a Sheffer stroke operation on A and $\leq$ be the induced order of $\mathcal{A} = (A, |)$. Then*
(i) $x \leq y$ *if and only if* $y|y \leq x|x$,
(ii) $x|(y|(x|x)) = x|x$ *is the identity of* $\mathcal{A}$,
(iii) $x \leq y$ *implies* $y|z \leq x|z$ *for all* $z \in A$,
(iv) $a \leq x$ *and* $a \leq y$ *imply* $x|y \leq a|a$.

The Sheffer Stroke basic algebras were introduced in [11]. They redefine basic algebras by using only the Sheffer Stroke operation.

Definition 2.2. [11] *An algebra $\mathcal{A} = (A; |)$ of type (2) is called a Sheffer stroke basic algebra if it satisfies the following identities:*
$(SH1)$ $(x|(x|x))|(x|x) = x$,
$(SH2)$ $(x|(y|y))|(y|y) = (y|(x|x))|(x|x)$,
$(SH3)$ $(((x|(y|y))|(y|y))|(z|z))|((x|(z|z))|(x|(z|z))) = x|(x|x)$.

Lemma 2.2. [11] *Let $\mathcal{A} = (A; |)$ be a Sheffer Stroke basic algebra. Then there exists an algebraic constant element $1 \in A$ and $\mathcal{A} = (A, |)$ provides the following identities:*
(i) $x|(x|x) = 1$,
(ii) $x|(1|1) = 1$,
(iii) $1|(x|x) = x$,
(iv) $((x|(y|y))|(y|y))|(y|y) = x|(y|y)$,
(v) $(y|(x|(y|y)))|(x|(y|y)) = 1$.

Theorem 2.1. [11] *Let $\mathcal{A} = (A; \oplus, \neg, 0)$ be a basic algebra. We define $x|y := \neg x \oplus \neg y$. Then $(A; |)$ is a Sheffer Stroke basic algebra.*

Corollary 2.1. [11] *Let $\mathcal{A} = (A; |)$ be a Sheffer Stroke basic algebra with the least element 0 and the greatest element 1. Then $(A; \vee, \wedge_0, ^0, 0, 1)$ is a lattice with an antitone involution.*

3. The Interval Sheffer Stroke Basic Algebras

The first systematic concept for the interval basic algebras is introduced in [5]. Since the Sheffer Stroke basic algebras are obtained by means of basic algebras, we put forward a question how the new operations can be defined on a given interval to get a Sheffer Stroke basic algebra again. In this section, we answer this question.

Lemma 3.1. *Let $\mathcal{A} = (A; |)$ be a Sheffer Stroke basic algebra. Then the relation $\leq$, called an induced order of A, satisfies the following properties:*
(1) *the relation $\leq$ defined by*
$$x \leq y \text{ if and only if } x|(y|y) = 1$$
is a partial order on A such that 0 is the least element of A and 1 is the greatest element of A,
(2) $x \leq y$ *implies that* $(x|x)|(z|z) \leq (y|y)|(z|z)$,
(3) $x \leq y$ *if and only if* $y|y \leq x|x$,
(4) $y \leq (x|x)|(y|y)$,

(5) $1|(x|x) = x$.

Proof. (1) The relation $\leq$ is reflexive by Lemma 2.2(i). Let $x \leq y$ and $y \leq x$. Then $x|(y|y) = 1$ and $y|(x|x) = 1$. By Definition 2.2(SH2) and Lemma 2.2 (iii), we obtain

$$x = 1|(x|x) = (y|(x|x))|(x|x) = (x|(y|y))|(y|y) = 1|(y|y) = y.$$

Hence, the relation $\leq$ is anti-symmetric. Let $x \leq y$ and $y \leq z$. Then $x|(y|y) = 1$ and $y|(z|z) = 1$. By Definition 2.2(SH3) and Lemma 2.2 (iii), we get

$$\begin{aligned} 1 &= (((x|(y|y))|(y|y))|(z|z))|((x|(z|z))|(x|(z|z))) \\ &= ((1|(y|y))|(z|z))|((x|(z|z))|(x|(z|z))) \\ &= (y|(z|z))|((x|(z|z))|(x|(z|z))) \\ &= 1|((x|(z|z))|(x|(z|z))), \\ &= x|(z|z), \end{aligned}$$

that is, $x \leq z$. Thus the relation $\leq$ is transitive. As a result, the relation $\leq$ is a partial order on A, and we get $x \leq 1$ for each $x \in A$ by Lemma 2.2(ii) and $0 \leq x$ for each $x \in A$ by Theorem 2.1 and Corollary 2.1.

(2) If $x \leq y$, i.e. $x|(y|y) = 1$, then by Definition 2.2(SH3) and Lemma 2.2 (iii) we have

$$\begin{aligned} 1 &= (((x|(y|y))|(y|y))|(z|z))|((x|(z|z))|(x|(z|z))) \\ &= ((1|(y|y))|(z|z))|((x|(z|z))|(x|(z|z))) \\ &= (y|(z|z))|((x|(z|z))|(x|(z|z))). \end{aligned}$$

Hence, we have $y|(z|z) \leq x|(z|z)$ by (1). Substituting simultaneously $(x|x)$ instead of y and $(y|y)$ instead of x in the last inequality, we get $(x|x)|(z|z) \leq (y|y)|(z|z)$.

(3) $(\Rightarrow)$: Assume that $x \leq y$. It implies that $(x|x)|(z|z) \leq (y|y)|(z|z)$. We have $(z|z)|(x|x) \leq (z|z)|(y|y)$ by Definition 2.1(S1). Substituting simultaneously 1 instead of $(z|z)$, $(y|y)$ instead of x, and $(x|x)$ instead of y in the last inequality, we have $1|((y|y)|(y|y)) \leq 1|((x|x)|(x|x))$. Hence, $(y|y) \leq (x|x)$.

$(\Leftarrow)$: Suppose that $(y|y) \leq (x|x)$, that is, $(y|y)|((x|x)|(x|x)) = 1$. Then we write $((x|x)|(x|x))|(y|y) = 1$ by Definition 2.1(S1). Since $(x|x)|(x|x) = x$ by Definition 2.1(S2), we get $x|(y|y) = 1$, i.e. $x \leq y$.

(4) We get

$$y = 1|(y|y) \leq (x|x)|(y|y)$$

by substituting simultaneously 1 instead of y, $(x|x)$ instead of x, and y instead of z in $y|(z|z) \leq x|(z|z)$.

(5) Since $(A;|)$ is a Sheffer Stroke basic algebra, we have

$$1|(x|x) = x$$

by Lemma 2.2(iii).

□

Lemma 3.2. *Let $\mathcal{A} = (A;|)$ be a Sheffer Stroke basic algebra, and $\leq$ be the induced order of A. Then $(A, \leq)$ is a lattice where*

$$x \vee y = (x|(y|y))|(y|y)$$

and

$$x \wedge y = ((x|x) \vee (y|y))|((x|x) \vee (y|y)).$$

Proof. By Lemma 3.1(4) and Definition 2.2(SH2), we obtain $x \leq (y|(x|x))|(x|x) = (x|(y|y))|(y|y)$ and $y \leq (x|(y|y))|(y|y)$. Thus, $(x|(y|y))|(y|y)$ is an upper bound for x and y. Let $x, y \leq z$. Then by Lemma 3.1(2), Definition 2.2(SH2), and Lemma 2.2(iii), we get

$$(x|(y|y))|(y|y) \leq (z|(y|y))|(y|y) = (y|(z|z))|(z|z) = 1|(z|z) = z.$$

Therefore, $(x|(y|y))|(y|y)$ is the least upper bound for x and y, that is, $x \vee y = (x|(y|y))|(y|y)$ is the supremum of x and y.

As a consequence of Lemma 3.1(2) we have that $x \wedge y = ((x|x) \vee (y|y))|((x|x) \vee (y|y))$ is the greatest lower bound for x and y, that is, $x \wedge y = ((x|x) \vee (y|y))|((x|x) \vee (y|y))$ is the infimum of x and y. □

Theorem 3.1. *Let $\mathcal{A} = (A; |)$ be a Sheffer Stroke basic algebra, and $\leq$ be the induced order of A and $a \in A$. We define an operation $|_a$ on the interval $[a, 1]$ by*

$$x|_a y := x|((y|(a|a))|(y|(a|a)))$$

for all $x, y \in [a, 1]$. Then $([a, 1]); |_a)$ is a Sheffer Stroke basic algebra.

Proof. Let $x, y \in [a, 1]$. Then $a \leq y \leq x|((y|(a|a))|(y|(a|a))) = x|_a y$ by Lemma 3.1(4). Thus, $|_a$ is a binary operation on $[a, 1]$. We call that

$$x|_a x = x|((x|(a|a))|(x|(a|a))) = ((x|x)|(x|x))|(a|a) = x|(a|a)$$

by the definition of $|_a$, Definition 2.1(S2) and (S3). We show that axioms (SH1)-(SH3) hold:

$(SH1)$: By Lemma 3.2 we get

$$\begin{aligned}
(x|_a(x|_a x))|_a(x|_a x) &= (x|(((x|(a|a))|(a|a))|((x|(a|a))|(a|a))))| \\
&\quad (((x|(a|a))|(a|a))|((x|(a|a))|(a|a))) \\
&= (x|((x \vee a)|(x \vee a)))|((x \vee a)|(x \vee a)) \\
&= (x|(x|x))|(x|x) \\
&= x.
\end{aligned}$$

$(SH2)$: By Lemma 3.2, we have

$$\begin{aligned}
(x|_a(y|_a y))|_a(y|_a y) &= (x|(((y|(a|a))|(a|a))|((y|(a|a))|(a|a))))| \\
&\quad (((y|(a|a))|(a|a))|((y|(a|a))|(a|a))) \\
&= (x|((y \vee a)|(y \vee a)))|((y \vee a)|(y \vee a)) \\
&= (x|(y|y))|(y|y) \\
&= (y|(x|x))|(x|x) \\
&= (y|_a(x|_a x))|_a(x|_a x).
\end{aligned}$$

$(SH3)$: By Lemma 3.2, we obtain

$$\begin{aligned}
((x|_a(y|_a y))|_a(y|_a y))|_a(z|_a z) &= ((x|(((y|(a|a))|(a|a))|((y|(a|a))|(a|a))))| \\
&\quad (((y|(a|a))|(a|a))|((y|(a|a))|(a|a))))| \\
&\quad (((z|(a|a))|(a|a))|((z|(a|a))|(a|a))) \\
&= ((x|((y \vee a)|(y \vee a)))|((y \vee a)|(y \vee a)))| \\
&\quad ((z \vee a)|(z \vee a)) \\
&= ((x|(y|y))|(y|y))|(z|z). \qquad (a)
\end{aligned}$$

By Lemma 3.2, and $x|_a x = x|(a|a)$, we get

$$\begin{aligned}
(x|_a(z|_a z))|_a(x|_a(z|_a z)) &= (x|_a(z|_a z))|(a|a) \\
&= (x|(((z|(a|a))|(a|a))|((z|(a|a)) \\
&\quad |(a|a))))|(a|a) \\
&= (x|((z \vee a)|(z \vee a)))|(a|a) \\
&= (x|(z|z))|(a|a). \qquad (b)
\end{aligned}$$

By Lemma 3.2, we have

$$\begin{aligned} x|_a(x|_a x) &= x|(((x|(a|a))|(a|a))| \\ &\quad ((x|(a|a))|(a|a))) \\ &= x|((x\vee a)|(x\vee a)) \\ &= x|(x|x). \qquad\qquad (c) \end{aligned}$$

By using (a), (b) and (c), and Lemma 3.2, we get

$$\begin{aligned} (((x|_a(y|_a y))|_a(y|_a y))|_a(z|_a z))|_a(x|_a(z|_a z))|_a(x|_a(z|_a z)) &= (((x|(y|y))|(y|y))|(z|z))| \\ &\quad ((((x|(z|z))|(a|a))|(a|a))| \\ &\quad (((x|(z|z))|(a|a))|(a|a))) \\ &= (((x|(y|y))|(y|y))|(z|z))|(((x| \\ &\quad (z|z))\vee a)|((x|(z|z))\vee a)) \\ &= (((x|(y|y))|(y|y))|(z|z))| \\ &\quad ((x|(z|z))|(x|(z|z))) \\ &= x|(x|x) \\ &= x|_a(x|_a x). \end{aligned}$$

□

Corollary 3.1. *Let $\mathcal{A}=(A;|)$ be a Sheffer Stroke basic algebra, $a,b\in A$ and $a\le b$. Then we have*

$$x|_b y = x|_a((y|_a(b|_a b))|_a(y|_a(b|_a b))) \qquad (PC)$$

for all $x,y\in[b,1]$.

Since the induced lattice of $\mathcal{A}=(A;|)$ is patched up from its intervals, (PC) is called the *patchwork condition.*

Theorem 3.2. *Let $\mathcal{A}=(A;|)$ be a Sheffer Stroke basic algebra, and $\le$ be the induced order of A and $b\in A$. We define an operation $|^b$ on the interval $[0,b]$ by*

$$x|^b y := ((x|x)|b)|((y|y)|b)$$

for all $x,y\in[0,b]$. Then $([0,b]);|^b)$ is a Sheffer Stroke basic algebra.

Proof. We have known $b|b \le (x|x)|((b|b)|(b|b)) = (x|x)|b$ and $b|b\le (y|y)|((b|b)|(b|b)) = (y|y)|b$ by Lemma 3.1(4) and Definition 2.1 (S2). Then $x|^b y = ((x|x)|b)|((y|y)|b) \le ((b|b)|(b|b)) = b$ by Lemma 2.1(iv) and Definition 2.1 (S2), that is, for all $x,y\in[0,b]$ we obtain $x|^b y\in[0,b]$. Thus $|^b$ is a binary operation on $[0,b]$.

We show that axioms (SH1)-(SH3) hold:

$(SH1):$ We obtain

$$\begin{aligned} (x|^b(x|^b x))|^b(x|^b x) &= ((((x|x)|b)|b)|((((x|x)|b)|b)|(((x|x)|b)|b)))| \\ &\quad (((x|x)|b)|b) && (\textit{by } (S1),(S3), \textit{ and } (S2), \textit{ respectively}) \\ &= 1|(((x|x)|b)|b) && (\textit{by Lemma } 2.2(i)) \\ &= 1|(((x|x)|((b|b)|(b|b)))|((b|b)|(b|b))) && (\textit{by } (S2)) \\ &= 1|((x|x)\vee(b|b)) && (\textit{by Lemma } 3.2) \\ &= 1|(x|x) && (\textit{by Lemma } 3.1(3)) \\ &= x && (\textit{by Lemma } 2.2(iii)). \end{aligned}$$

$(SH2)$: We obtain

$$
\begin{aligned}
(x|^b(y|^by))|^b(y|^by) &= (x|(y|y))|(y|y) && (\text{by } (S1),(S2),(S3), \text{ Lemma } 3.2, \text{ and Lemma } 3.1(3)) \\
&= (y|(x|x))|(x|x) \\
&= (y|^b(x|^bx))|^b(x|^bx).
\end{aligned}
$$

$(SH3)$: To prove (SH3), we will give the following claims and their proofs:
Claim 1 Let $x \in [0,b]$. Then $((x|x)|b)|b = x|x$.

Proof. By Lemma 3.2, Lemma 3.1(3), and (S2), we obtain

$$
\begin{aligned}
((x|x)|b)|b &= ((x|x)|((b|b)|(b|b)))|((b|b)|(b|b)) \\
&= (x|x) \vee (b|b) \\
&= x|x.
\end{aligned}
$$

□

Claim 2 Let $x \in [0,b]$. Then $x|^b(x|^bx) = b$.

Proof. By using Lemma 3.2, (S1) and (S2), and Claim 1, we obtain

$$
\begin{aligned}
x|^b(x|^bx) &= ((x|x)|b)|(((x|x)|b)|b) \\
&= ((x|x)|b)|(x|x) \\
&= (b|(x|x))|(x|x) \\
&= b \vee x \\
&= b.
\end{aligned}
$$

□

Claim 3 Let $x, y \in [0,b]$. If $x \leq y$, then $(b|(x|x))|(y|y) = b$.

Proof. By Lemma 3.1, if $x \leq y$, then $(x|x)|b \leq (y|y)|b$, that is, $((x|x)|b)|(((y|y)|b)|((y|y)|b)) = 1$. Thus, we have

$$
\begin{aligned}
(b|(x|x))|(y|y) &= ((x|x)|b)|(y|y) && (\text{by } (S1)) \\
&= (((((x|x)|b)|(y|y))|(((x|x)|b)|(y|y)))|b)|b)| \\
&\quad (((((x|x)|b)|(y|y))|(((x|x)|b)|(y|y)))|b)|b) && (\text{by } (S2) \text{ and Claim } 1) \\
&= ((((x|x)|b)|(((y|y)|b)|((y|y)|b)))|b)| \\
&\quad ((((x|x)|b)|(((y|y)|b)|((y|y)|b)))|b) && (\text{by } (S3)) \\
&= (1|b)|(1|b) && (\text{by hypothesis}) \\
&= (1|((b|b)|(b|b)))|(1|((b|b)|(b|b))) && (\text{by } (S2)) \\
&= (b|b)|(b|b) && (\text{by Lemma } 2.2(iii)) \\
&= b && (\text{by } (S2)).
\end{aligned}
$$

□

From (SH2), we have alreeady known $(x|^b(y|^by))|^b(y|^by) = (x|(y|y))|(y|y)$. Then, by using (S2), and Claim 1, we get

$$
\begin{aligned}
((x|^b(y|^by))|^b(y|^by))|^b(z|^bz) &= ((((x|(y|y))|(y|y))|((x|(y|y))|(y|y)))|b)| \\
&\quad (((((z|z)|b)|((z|z)|b))|(((z|z)|b)|((z|z)|b)))|b) \\
&= ((((x|(y|y))|(y|y))|((x|(y|y))|(y|y)))|b)|(((z|z)|b)|b) \\
&= ((((x|(y|y))|(y|y))|((x|(y|y))|(y|y)))|b)|(z|z). \qquad (*)
\end{aligned}
$$

We get

$$\begin{aligned}(x|^b(z|^bz))|^b(x|^b(z|^bz)) &= (((((x|x)|b)|(((z|z)|b)|b))|(((x|x)|b)| \\ &\quad (((z|z)|b)|b)))|b)|(((((x|x)|b)|(((z|z)| \\ &\quad b)|b))|(((x|x)|b)|(((z|z)|b)|b)))|b) && (by\ (S2)) \\ &= ((((z|z)|b)|b)|((((x|x)|b)|b)|(((x|x)|b)|b)))| \\ &\quad ((((z|z)|b)|b)|((((x|x)|b)|b)|(((x|x)|b)|b))) && (by\ (S1) \\ &&& and\ (S3)) \\ &= ((z|z)|((x|x)|(x|x)))|((z|z)|((x|x)|(x|x))) && (by\ Claim\ 1) \\ &= ((z|z)|x)|((z|z)|x) && (by\ (S2)) \\ &= (x|(z|z))|(x|(z|z)) && (by\ (S1)).\ (**)\end{aligned}$$

By applying (*) and (**), we obtain

$$\begin{aligned}(((x|^b(y|^by))|^b(y|^by))|^b(z|^bz))|^b((x|^b(z|^bz))|^b(x|^b(z|^bz))) &= (((((((x|(y|y))|(y|y))| \\ &\quad ((x|(y|y))|(y|y)))|b)| \\ &\quad (z|z))|(((((x|(y|y))| \\ &\quad (y|y))|((x|(y|y))| \\ &\quad (y|y)))|b)|(z|z)))|b)| \\ &\quad ((x|(z|z))|b) && (by\ (S2)) \\ &= b && (by\ (S1), \\ &&& (S2),(S3), \\ &&& Lemma\ 3.2, \\ &&& Claim\ 1, \\ &&& Claim\ 3) \\ &= x|^b(x|^bx) && (by\ Claim\ 2).\end{aligned}$$

□

Theorem 3.3. *Let $\mathcal{A} = (A;|)$ be a Sheffer Stroke basic algebra, and $\leq$ be the induced order of A and $a, b \in A$. We define an operation $|_a^b$ on the interval $[a, b]$ such that*

$$x|_a^b y := (b|(x|x))|(((b|(y|y))|(a|a))|((b|(y|y))|(a|a)))$$

for all $x, y \in [a, b]$. Then $\mathcal{A}(a, b) = ([a, b]; |_a^b)$ is a Sheffer Stroke basic algebra.

Proof. $([0, b], |^b)$ is a Sheffer Stroke basic algebra by Theorem 3.2 and $a \in [0, b]$. Then, by Theorem 3.1 to the operation $|^b$, we obtain

$$x|_a^b y = x|^b((y|^b(a|^ba))|^b(y|^b(a|^ba))).$$

Thus, $\mathcal{A}(a, b) = ([a, b]; |_a^b)$ is a Sheffer Stroke basic algebra. □

Similarly, in the proof of Theorem 3.3, by applying Theorem 3.2 to the operation $|_a$, we get

$$x|_a^b y = ((x|_a x)|_a b)|_a((y|_a y)|_a b),$$

since $([a, 1]; |_a)$ is the Sheffer Stroke basic algebra by Theorem 3.1.

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References

[1] I. Chajda, (2015), Basic algebras, logics, trends and applications, *Asian-European Journal of Mathematics* 8.03, 1550040.

[2] I. Chajda, (2005), Sheffer operation in ortholattices, *Acta Universitatis Palackianae Olomucensis, Facultas Rerum Naturalium, Mathematica,* 44.1, 19-23.

[3] I. Chajda, R. Halaš, and J. Kühr, (2007), *Semilattice structures,* Vol. 30, Heldermann Verlag, Lemgo, Germany.

[4] I. Chajda, and M. Kolařík, (2009), Independence of the axiomatic system of basic algebras, *Soft Computing* 13, 41-43.

[5] I. Chajda, and M. Kolařík, (2009), Interval Basic Algebras, *NOVI SAD J. MATH.,* Vol. 39-2.

[6] I. Chajda, and J. Kühr, (2006), A note on interval MV-algebras, *Math. Slovaca,* 56, 47-52.

[7] I. Chajda, and J. Kühr, (2006), GMV-algebras and meet-semilattices with sectionally antitone permutations, *Math. Slovaca,* 56, 275-288

[8] A. Dvurečenskij, and M. Hyč ko, (2006), Algebras on subintervals of BL-algebras, pseudo BL-algebras and bounded residuated $\updownarrow$-monoids, *Math. Slovaca,* 56, 125-144.

[9] J. Jakubik, (2006), On intervals and the dual of pseudo MV-algebras, *Math. Slovaca,* 56, 213-221.

[10] H. M. Sheffer, (1913), A set of five independent postulates for Boolean algebras, with application to logical constants, *Transactions of the American Mathematical Society,* 14(4), 481-488.

[11] Oner T., Senturk I., (2017), The Sheffer Stroke Operation Reducts of Basic Algebras, *Open Math.,* 15, 926-935.

[12] McCune, William, et al., (2002), Short single axioms for Boolean algebra, *Journal of Automated Reasoning,* 29.1, 1-16.

[13] Abbott J. C., (1967), Implicational algebras, *Bulletin mathématique de la Société des Sciences Mathématiques de la République Socialiste de Roumanie,* 11.1, 3-23.

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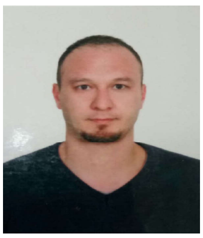

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