TWMS J. App. Eng. Math. V.8, N.2, 2018, pp. 438-447

HIGHER ORDER PERTURBATION EXPANSION FOR ION-ACOUSTIC SOLITARY WAVES WITH Q-NONEXTENSIVE NONTHERMAL VELOCITY DISTRIBUTION

H. DEMIRAY[1], §

Abstract. The basic nonlinear equations describing the dynamics of a two component plasma consisting of cold positive ions and electrons obeying hybrid q- nonextensive non-thermal velocity distribution are examined through the use of modified PLK formalism and the reductive perturbation method and obtained the KdV equation for the lowest order term in the perturbation expansion. The method is further extended to include the contribution of higher order terms in the expansion; the evolution equation for the second order term is found to be the degenerate(linearized) KdV equation with non-homogeneous term. Seekink the localized travelling wave solution (solitons) to these evolution equations we obtained the speed correction terms and the wave profiles. Numerical results for the set of suitable parameters(Williams et. al. [23]) are shown inb the form of some graphs. The combined effect of nonextensive parameter (q) and the nonthermal parameter (α) on the soliton dynamics has also been studied.

Keywords: Solitons, Tsallis statistics, Nonthermal distribution.

AMS Subject Classification: 83-02, 35Q51

1. Introduction

Solitons are localized pulse shaped stable nonlinear entities which arise as a manifestation of balance between the nonlinearity and dispersion. Washimi and Taniuti [1] were the first to use the reductive perturbation method to derive the Korteweg-deVries(KdV) equation for ion-acoustic solitons(IASs) in plasma. Early investigations on IASs were based on the particle distribution obeying Maxwellian distribution. Such distributions are believed to be valid universally for the macroscopic ergodic equilibrium systems. However, it is now well established that the space and laboratory plasmas indicate the presence of energetic particles in tailed- particle distribution. Different models proposed to describe this phenomenon manifest themselves by influencing wave dynamics through modification of the particle distribution. Thus, electron and ion distributions play a crucial role in characterizing the physics of nonlinear waves [2]. Observations made by Viking spacecraft [3] and Freja satellite [4] showed the significance of electrostatic solitary structures in magetosphere with density depletion. To explain these observations Cairns et al. [5]

[1] Isik University, Faculty of Arts and Sciences, Department of Mathematics, 34980 Sile, Istanbul, Turkey.
e-mail: hilmi.demiray@isikun.edu.tr; ORCID: http://orcid.org/0000-0001-8590-3396.

§ Manuscript received: Febrary 13, 2017; accepted; September 11, 2018.
TWMS Journal of Applied and Engineering Mathematics, Vol.8, No.2

introduced a distribution to model enhanced high energy tails in terms of a parameter α, which measures the deviation from the Maxwellian distribution function. It was shown that, the nature of ion sound solitary structure may change in the presence of nonthermal electrons.

In another development, there have been growing interest to study nonextensive statistical mechanics based on the deviations of the Boltzmann-Gibbs-Shannon (BGS) entropic measures. It is to be pointed out that the system with long range interactions, long range memory and intrinsic inhomogeneity are intractable within the framework of BGS statistics. As is known, the plasma systems far away from equilibrium with power law distribution, can be studied within the framework of nonextensive statistics initiated by Tsallis [6] as a generalization of BGS statistics. A crucial property of this entroy is pseudo-additivity such that

$$S_q(A+B) = S_q(A) + S_q(B) + (1-q)S_q(A)S_q(B)$$

for the systems A and B. It may be mentioned that nonlocality or long range interactions are introduced by the multiplicative term accounting for correlation between the subsystems. The index q that underpins the generalized Tsallis entropy is connected to the dynamics of the system and measures the amount of its nonextensivity. Tsallis entropy has been used with some success in a number of research work in plasma physics(see, for instance [7, 8]). Recently, Tribeche et al.[9] extended the work Cairn et al.[5] and proposed a hybrid Cairn-Tsallis distribution. This distribution is suitable to provide a better fit over a wide range of the space observations due to two parameter representation as well as the flexibility of the nonextensive q- parameter. Amour et al. [10] applied this hybrid model to study the electron-acoustic solitary waves in plasma and showed that this model did not support the compressive electron-acoustic soliton. Recently, Wang et al. [11] used this model to study solitary waves and rogue waves. From the study of modulational instability, they found that the electron-acoustic solitary waves can exist in such a plasma. It was further shown that the regions of existence and amplitude of solitary waves and rogue waves are effected by the nonextensive parameter q and the nonthermal parameter α. More recently, Bouzit et al.[12] studied ion-acoustic waves in a mixed nonextensive high energy -tail electron distribution. They investigated the effect of an interplay between nonthermality and nonextensivity on ion-acoustic soliton nature and associated rogue waves. They further observed that that, ion-acoustic soliton exhibits compression or rarefaction depending on the nonextensivity and nonthermality of modulational instability of ion-acoustic waves, Bouzit et al. [13] employed Cairns -Tsallis velocity distribution and observed that both nonthermal and nonextensive parameters affect the domains of instabilities.

As is known, in solving the nonlinear evolution equations through the use of reductive perturbation method the lowest order term in the perturbation expansion is characterized by the Korteweg-deVries(KdV) equation. As the amplitude of the wave increases, the width and velocity of a soliton deviates from from the one predicted by the KdV model. Ikezi et al.[14] and Ikezi[15] claimed that the experimental data for ion-acoustic solitary waves (IASWs) do not fit well for the soliton solution obtained from the KdV equations. To improve the accuracy of the solution the modications in the theory have been suggested by Kato et al.[16]. To study the higher order terms in the perturbation expansion, the use of reductive perturbation method (Taniuti[17]) lead to the secularity in the solution which can be removed by introducing some additional slow scale variables (Sugimoto and Kakutani[18] or by the re-normalization procedure by Kodama and Taniuti[19]. The re-normalization method is rather heuristic and criticized by several scientists (see, Malfliet and Wieers [20]; Demiray [21, 22]). To eliminate such secularities in the solution, Demiray [21] presented a method so called "the modified reductive perturbation method", which is based on the idea that the higher order nonlinearities should be balanced by higher order dispersive effects. The detail of the method can be found in the above references.

In the present work, using the basic nonlinear equations of a two component plasma consisting of cold positive ions and electrons obeying hybrid q- nonextensive nonthermal velocity distribution, we studied the propagation of small but finite amplitude waves in such a medium through the use of modified PLK formalism and the reductive perturbation method and obtained the KdV equation for the lowest order term in the perturbation expansion. The method is further extended to include the contribution of higher order terms in the expansion; the evolution equation for the second order term is found to be the degenerate(linearized) KdV equation with non-homogeneous term. Seeking the localized travelling wave solution (solitons) to these evolution equations we obtained the speed correction terms and the wave profiles. Numerical results for the set of suitable parameters(Williams et. al. [23]) are shown inb the form of some graphs. The combined effect of nonextensive parameter (q) and the nonthermal parameter (α) on the soliton dynamics has also been studied.

2. Basic Field Equations

We consider a homogeneous, collisionless, unmagnetized plasma consisting of cold positive ions with electrons obeying q-nonextensive nonthermal velocity distribution. The dynamics of such system may be described by the set of normalized equations[23],

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(u n_i) = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{\partial \phi}{\partial x} = 0, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n_i, \tag{3}$$

where n_i and n_e are the normalized ion and electron number densities, u is the fluid velocity of ions and ϕ is the electrostatic potential, respectively. The normalized q-nonextensive nonthermal electron density profile is given by [9]

$$n_e = [1 + (q-1)\phi]^{\frac{q+1}{2(q-1)}}(1 + G_1\phi + G_2\phi^2), \tag{4}$$

where the parameter q, called the nonextensive parameter, stands for the strength of nonextensive system and the coefficients G_1 and G_2 are defined by

$$G_1 = -16q\alpha/(3 - 14q + 15q^2 + 12\alpha), \quad G_2 = -(2q-1)G_1. \tag{5}$$

Here α is a parameter determining the number of nonthermal electrons in the model. The range and the validity of (q, α) for solitons are discussed by Williams et. al. [23]. In the extensive limiting case ($q \to 1$) and $\alpha = 0$, the above distribution reduces to the well-known Maxwell- Boltzmann velocity distribution. For ($q \to 1$) and $\alpha \neq 0$, the above distribution reduces to Cairn distribution [5].

For small ϕ, by expanding (4) into a power series, the electron number density n_e may be expressed by

$$n_e = 1 + d_1\phi + d_2\phi^2 + d_3\phi^3 + \dots \ , \tag{6}$$

where the coefficients $d_1,\ d_2,\ d_3,$

are defined by

$$d_1 = G_1 + \frac{q+1}{2}, \quad d_2 = G_2 + G_1(\frac{q+1}{2}) + \frac{(q+1)(3-q)}{8},$$
$$d_3 = \frac{(q+1)(3-q)(5-3q)}{48} + G_1\frac{(q+1)(3-q)}{8} + G_2\frac{(q+1)}{2}. \tag{7}$$

The equations (1)-(3) and (6) will be used as we study the solitary waves of small- but finite amplitude.

3. Modified PLK Formalism

When the classical reductive perturbation method (Washimi and Taniuti [1]) is applied to study the higher order perturbation expansion it leads to the secularity in the progressive wave solution (Sugimoto and Kakutani [18]). To remove such secularities Demiray [21] presented the modified reductive perturbation method, which is mainly based on the balance between the higher order nonlinearities and higher order dispersive effects.

In the present work, we shall show that the modified form of extended PLK method can efficiently used to study the effects of higher order perturbation expansion. For that purpose we introduce the following strained coordinates

$$\epsilon^{1/2}(x - \lambda t) = \xi + p(\tau), \quad \tau = \epsilon^{3/2} t, \tag{8}$$

where ϵ is the smallness parameter measuring the weakness of nonlinearity, λ is a parameter to be determined from the solution, and $p(\tau)$ is an unknown function characterizing the higher order dispersive effects. Introducing (8) into (1)-(3) and (6) the following equations are obtained:

$$-(\lambda + \epsilon\frac{dp}{d\tau})\frac{\partial n_i}{\partial \xi} + \epsilon\frac{\partial n_i}{\partial \tau} + \frac{\partial}{\partial \xi}(n_i u) = 0, \tag{9}$$

$$-(\lambda + \epsilon\frac{dp}{d\tau})\frac{\partial u}{\partial \xi} + \epsilon\frac{\partial u}{\partial \tau} + u\frac{\partial u}{\partial \xi} + \frac{\partial \phi}{\partial \xi} = 0, \tag{10}$$

$$\epsilon\frac{\partial^2 \phi}{\partial \xi^2} = 1 + d_1\phi + d_2\phi^2 + d_3\phi^3 + ... - n_i. \tag{11}$$

We shall assume that the field quantities $n_i,\ u,\ \phi$ and the unknown function $p(\tau)$ can be expanded into power series in ϵ as:

$$n_i = 1 + \epsilon n_i^{(1)} + \epsilon^2 n_i^{(2)} + \epsilon^3 n_i^{(3)} + ...,$$
$$u = \epsilon u^{(1)} + \epsilon^2 u^{(2)} + \epsilon^3 u^{(3)} + ...,$$
$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \epsilon^3 \phi^{(3)} + ...,$$
$$p = \epsilon p_1 + \epsilon^2 p_2 + \epsilon^3 p_3 + ... \quad . \tag{12}$$

Introducing the expansion (12) into the field equations (1)-(3) and (6) and setting the coefficients of like powers of ϵ equal to zero, the following sets of differential equations are obtained:

O(ϵ) equations:

$$-\lambda\frac{\partial n_i^{(1)}}{\partial \xi} + \frac{\partial u^{(1)}}{\partial \xi} = 0, \quad -\lambda\frac{\partial u^{(1)}}{\partial \xi} + \frac{\partial \phi^{(1)}}{\partial \xi} = 0, \quad n_i^{(1)} = d_1\phi^{(1)}. \tag{13}$$

O(ϵ^2) equations:

$$-\lambda\frac{\partial n_i^{(2)}}{\partial\xi}+\frac{\partial u^{(2)}}{\partial\xi}+\frac{\partial n_i^{(1)}}{\partial\tau}+\frac{\partial}{\partial\xi}[n_i^{(1)}u^{(1)}]=0,$$
$$-\lambda\frac{\partial u^{(2)}}{\partial\xi}+\frac{\partial\phi^{(2)}}{\partial\xi}+\frac{\partial u^{(1)}}{\partial\tau}+\frac{\partial}{\partial\xi}[\frac{{u^{(1)}}^2}{2}]=0,$$
$$n_i^{(2)}=d_1\phi^{(2)}+d_2(\phi^{(1)})^2-\frac{\partial^2\phi^{(1)}}{\partial\xi^2}.. \tag{14}$$

O(ϵ^3) equations:

$$-\lambda\frac{\partial n_i^{(3)}}{\partial\xi}+\frac{\partial u^{(3)}}{\partial\xi}+\frac{\partial n_i^{(2)}}{\partial\tau}-\frac{dp_1}{d\tau}\frac{\partial n_i^{(1)}}{\partial\xi}+\frac{\partial}{\partial\xi}[n_i^{(1)}u^{(2)}+n_i^{(2)}u^{(1)}]=0,$$
$$-\lambda\frac{\partial u^{(3)}}{\partial\xi}+\frac{\partial\phi^{(3)}}{\partial\xi}+\frac{\partial u^{(2)}}{\partial\tau}-\frac{dp_1}{d\tau}\frac{\partial u^{(1)}}{\partial\xi}+\frac{\partial}{\partial\xi}[u^{(1)}u^{(2)}]=0,$$
$$n_i^{(3)}=d_1\phi^{(3)}+2d_2\phi^{(1)}\phi^{(2)}+d_3[\phi^{(1)}]^3-\frac{\partial^2\phi^{(2)}}{\partial\xi^2}. \tag{15}$$

3.1. **Solution of the field equations.** From the solution of the set (13) one obtains

$$\phi^{(1)}=\Phi_1(\xi,\tau),\quad u^{(1)}=d_1^{1/2}\Phi_1,\quad n_i^{(1)}=d_1\Phi_1,\quad \lambda=(d_1)^{-1/2}, \tag{16}$$

where $\Phi_1(\xi,\tau)$is an unknown function of its arguments and its evolution equation will be obtained later.

Introducing (16) into the set (14) we have

$$-\lambda\frac{\partial n_i^{(2)}}{\partial\xi}+\frac{\partial u^{(2)}}{\partial\xi}+d_1\frac{\partial\Phi_1}{\partial\tau}+d_1^{3/2}\frac{\partial}{\partial\xi}(\Phi_1^2)=0,$$
$$-\lambda\frac{\partial u^{(2)}}{\partial\xi}+\frac{\partial\phi^{(2)}}{\partial\xi}+d_1^{1/2}\frac{\partial\Phi_1}{\partial\tau}+\frac{d_1}{2}\frac{\partial}{\partial\xi}(\Phi_1^2)=0,$$
$$n_i^{(2)}=d_1\Phi_2+d_2{\Phi_1}^2-\frac{\partial^2\Phi_1}{\partial\xi^2}.. \tag{17}$$

The solution of (17) gives

$$n_i^{(2)}=d_1\Phi_2+d_2{\Phi_1}^2-\frac{\partial^2\Phi_1}{\partial\xi^2},$$
$$u^{(2)}={d_1}^{1/2}\Phi_2+(\frac{d_2}{2{d_1}^{1/2}}-\frac{{d_1}^{3/2}}{4})\Phi_1^2-\frac{1}{2{d_1}^{1/2}}\frac{\partial^2\Phi_1}{\partial\xi^2}, \tag{18}$$

and

$$\frac{\partial\Phi_1}{\partial\tau}+\mu_1\Phi_1\frac{\partial\Phi_1}{\partial\xi}+\mu_2\frac{\partial^3\Phi_1}{\partial\xi^3}=0. \tag{19}$$

where the coefficients μ_1 and μ_2 are defined by

$$\mu_1=\frac{3}{2}d_1^{1/2}-\frac{d_2}{d_1^{3/2}},\quad \mu_2=\frac{1}{2d_1^{3/2}}. \tag{20}$$

The evolution equation (20) is known as the conventional Korteweg-deVrie(KdV) equation.

To find the solution for $O(\epsilon^3)$ equations, we first eliminate $u^{(3)}$ between the first two equations of (15) and obtain

$$\frac{\partial}{\partial\xi}[\phi^{(3)} - \lambda^2 n_i^{(3)}] + \frac{\partial}{\partial\tau}[u^{(2)} + \lambda n_i^{(2)}] - \frac{dp_1}{d\tau}\frac{\partial}{\partial\xi}[\lambda n_i^{(1)} + u^{(1)}]$$
$$+\frac{\partial}{\partial\xi}[\lambda n_i^{(1)} u^{(2)} + \lambda n_i^{(2)} u^{(1)} + u^{(1)} u^{(2)}] = 0. \tag{21}$$

Inserting the solutions (16) and (18) into (21) we have

$$\frac{\partial\Phi_2}{\partial\tau} + (\frac{3}{2}d_1^{1/2} - \frac{d_2}{d_1^{3/2}})\frac{\partial}{\partial\xi}(\Phi_1\Phi_2) + \frac{1}{2d_1^{3/2}}\frac{\partial^3\Phi_2}{\partial\xi^3} = R_2(\Phi_1). \tag{22}$$

where $R_2(\Phi_1)$ is defined by

$$R_2(\Phi_1) = (\frac{d_1}{4} - \frac{3}{2}\frac{d_2}{d_1})\Phi_1\frac{\partial\Phi_1}{\partial\tau} + \frac{3}{4d_1}\frac{\partial^3\Phi_1}{\partial\xi^2\partial\tau} + (\frac{3}{2}\frac{d_3}{{d_1}^{3/2}} + \frac{3}{4}{d_1}^{3/2} - \frac{3d_2}{{d_1}^{1/2}})\Phi_1^2\frac{\partial\Phi_1}{\partial\xi}$$
$$+\frac{1}{{d_1}^{1/2}}\frac{\partial}{\partial\xi}(\Phi_1\frac{\partial^2\Phi_1}{\partial\xi^2}) + \frac{dp_1}{\partial\tau}\frac{\partial\Phi_1}{\partial\xi}. \tag{23}$$

Here the evolution equation (22) is the linearized (degenerate) KdV equation with non-homogeneous term $R_2(\Phi_1)$.

3.2. **Solitary waves.** In this sub-section we shall study the propagation of solitary waves in such a medium. For that purpose, we shall seek a progressive wave solution to the evolution equations (19) and (22) of the following form

$$\Phi_i(\xi,\tau) = \Phi_i(\zeta), \quad \zeta = \chi(\xi - c\tau), \quad i = 1, 2, \tag{24}$$

where c is the speed of propagation and χ is a constant to be determined from the solution. Inserting (24) for $i = 1$ into the evolution equation (19), we have

$$-c\Phi_1' + (\frac{3}{2}d_1^{1/2} - \frac{d_2}{d_1^{3/2}})\Phi_1{\Phi_1}' + \frac{\chi^2}{2d_1^{3/2}}{\Phi_1}''' = 0, \tag{25}$$

where the prime denotes the differentiation of the corresponding quantity with respect to ζ. Integrating (25) with respect to ζ and utilizing the localization condition, i.e., Φ_1 and its various order derivatives vanish as $\zeta \to \pm\infty$, we obtain

$$-c\Phi_1 + (\frac{3}{4}d_1^{1/2} - \frac{d_2}{2d_1^{3/2}}){\Phi_1}^2 + \frac{\chi^2}{2d_1^{3/2}}{\Phi_1}'' = 0. \tag{26}$$

This nonlinear ordinary differential equation assumes a solitary wave solution of the form

$$\Phi_1 = a\ \mathrm{sech}^2\zeta, \tag{27}$$

where a is the constant wave amplitude, c and χ are given by

$$c = (\frac{{d_1}^{1/2}}{2} - \frac{d_2}{3{d_1}^{3/2}})a, \quad \chi^2 = (\frac{{d_1}^2}{4} - \frac{d_2}{6})a. \tag{28}$$

To obtain the progressive wave solution to the evolution equation (22), we insert (24) for $i = 2$ into (22), integrate the result with respect to ζ and utilize the localization condition yields the following non-homogeneous differential equation

$$\Phi_2'' + (\frac{12}{a}\Phi_1 - 4)\Phi_2 = (3\frac{d_2}{d_1} - \frac{d_1}{2}){\Phi_1}^2 - 3\frac{\chi^2}{d_1}\Phi_1''$$

$$+\frac{1}{\chi^2}(d_3+\frac{{d_1}^3}{2}-2d_1d_2){\Phi_1}^3+2d_1\Phi_1\Phi_1''+2\frac{{d_1}^{3/2}}{\chi^2}\frac{dp_1}{d\tau}\Phi_1. \quad (29)$$

Noting the relation

$$\Phi_1''=4\Phi_1-\frac{6}{a}{\Phi_1}^2, \quad (30)$$

the equation (29) can be written as follows:

$$\Phi_2''+(\frac{12}{a}\Phi_1-4)\Phi_2=(2\frac{{d_1}^{3/2}}{\chi^2}\frac{dp_1}{d\tau}-12\frac{\chi^2}{d_1})\Phi_1+12d_1{\Phi_1}^2+\frac{1}{\chi^2}(d_3-\frac{5}{2}{d_1}^3){\Phi_1}^3. \quad (31)$$

The term proportional to Φ_1 causes secularity (Demiray [22]). In order to remove the secularity the coefficient of Φ_1 must vanish, which yields

$$\frac{dp_1}{d\tau}=6\frac{\chi^4}{{d_1}^{5/2}}, \quad \text{or} \quad p_1=6\frac{\chi^4}{{d_1}^{5/2}}\tau. \quad (32)$$

Here $\frac{dp_1}{d\tau}=6\chi^4/{d_1}^{5/2}$ corresponds to the speed correction term of order ϵ. The remaining parts of the equation (31) become

$$\Phi_2''+(\frac{12}{a}\Phi_1-4)\Phi_2=12d_1{\Phi_1}^2+\frac{1}{\chi^2}(d_3-\frac{5}{2}{d_1}^3){\Phi_1}^3. \quad (33)$$

The particular solution of this equation may be given as follows:

$$\Phi_2=c_1\Phi_1+c_2{\Phi_1}^2, \quad (34)$$

where the coefficients c_1 and c_2 are defined by

$$c_1=\frac{(6d_3-8d_1d_2-3{d_1}^3)a^2}{24\chi^2}, \quad c_2=\frac{(5{d_1}^3-2d_3)a}{16\chi^2}. \quad (35)$$

The total solution up to $O(\epsilon^3)$ may be given by

$$\phi=\epsilon a\{[1+\epsilon\frac{(6d_3-8d_1d_2-3{d_1}^3)a^2}{24\chi^2}]\text{sech}^2\zeta$$
$$+\epsilon[\frac{(6{d_1}^3-2d_3)}{16\chi^2}a^2]\text{sech}^4\zeta\}+\text{O}(\epsilon^3). \quad (36)$$

with

$$\zeta=\epsilon^{1/2}\chi\{x-[\frac{1}{{d_1}^{1/2}}-\epsilon(\frac{2\chi^2}{{d_1}^{3/2}})-\epsilon^2(\frac{6\chi^4}{{d_1}^{5/2}})-..]t\}, \quad (37)$$

where

$$\chi^2=(\frac{d_1^2}{4}-\frac{d_2}{6})a. \quad (38)$$

3.3. **Results and Discussions.** In the present work, we studied the propagation of weakly nonlinear waves in a two component plasma consisting of cold ions and electrons obeying nonextensive and nonthermal velocity distribution. By applying the modified PLK formalism and the reductive perturbation method to the basic equations, we obtained a KdV equation (19) for ion-acoustic solitons to the lowest order term of perturbation. We analyze numerically the range the range of parameters valid for model to exhibit soliton behavior(Williams et. al. [23]). The sign of μ_1 is important for the characterization of solitons. The region $\mu_1<0$ corresponds to the rarefactive solitons whereas $\mu_1>0$ is for compressive solitons . The regions of validity of these two types of solitons are depicted on Fig.1. The transition from compressive to rarefactive solitons occurs with increasing α for fixed values of q.

The Fig.2, which characterizes the compressive solitons, displays the lowest order (Φ_1), second order (Φ_2) and the dressed solitons $\Phi = \epsilon\Phi_1 + \epsilon^2\Phi_2$ as a function of ζ for the nonextensive parameter $q = 0.8$, the nonthermal parameter $\alpha = 0.0$ and $\epsilon = 0.5,\ a = 1$. It is seen the wave amplitudes are always positive. The Fig.3, which characterizes the rarefactive solitons, again displays the lowest order, the second order and the dressed solitons as a function of ζ for the nonextensive parameter $q = 0.7$, the nonthermal parameter $\alpha = 0.1$ and $\epsilon = 0.5,\ a = -1$. As is seen from the figure, the amplitude of the lowest order and the dressed solitons are negative whereas the amplitude of the second order soliton is positive.

4. Conclusion

In the present work, using the basic nonlinear equations of a two component plasma consisting of cold positive ions and electrons obeying hybrid q- nonextensive nonthermal velocity distribution, we studied the propagation of small but finite amplitude waves in such a medium through the use of modified PLK formalism and the reductive perturbation method and obtained the KdV equation for the lowest order term in the perturbation expansion. The method is further extended to include the contribution of higher order terms in the expansion; the evolution equation for the second order term is found to be the degenerate(linearized) KdV equation with non-homogeneous term. Seeking the localized travelling wave solution (solitons) to these evolution equations we obtained the speed correction terms and the wave profiles. Numerical results for the set of suitable parameters(Williams et. al. [23]) are shown in the form of some graphs. The combined effect of nonextensive parameter (q) and the nonthermal parameter (α) on the soliton dynamics has also been studied.

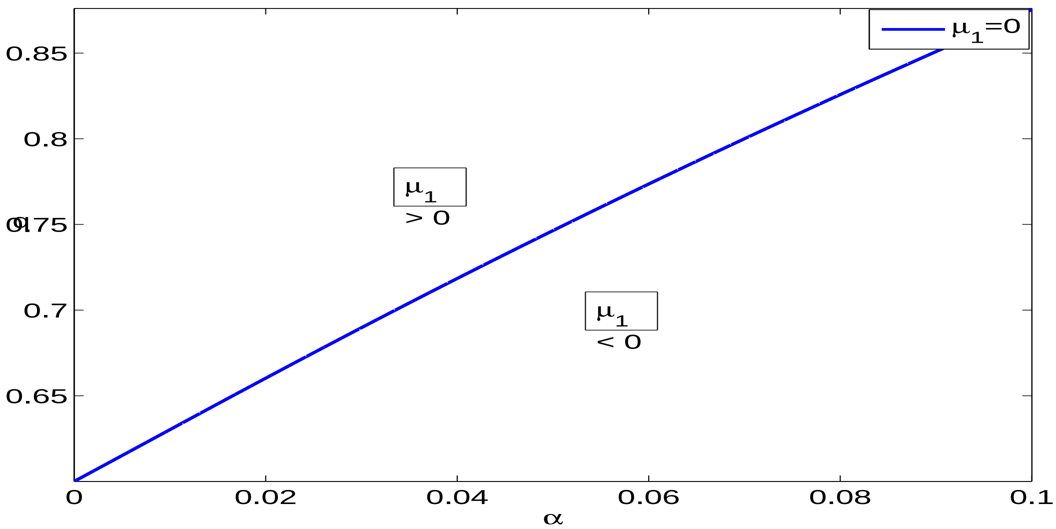

Figure 1. The variation of nonextensvity (q) with nonthermal parameter (α) for $\mu_1 = 0$.

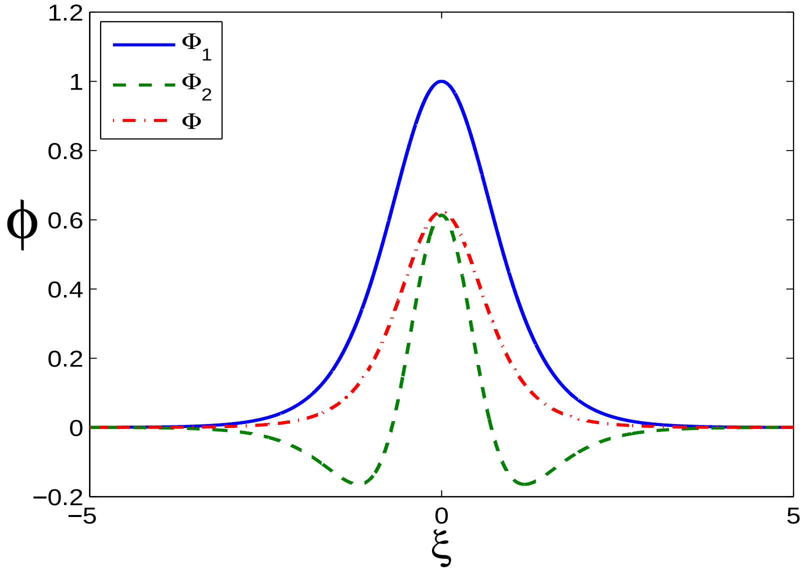

FIGURE 2. The variations of the compressive soliton profiles with ζ for $\epsilon = 0.5,\ a = 1,\ \alpha = 0$ and $q = 0.8$.

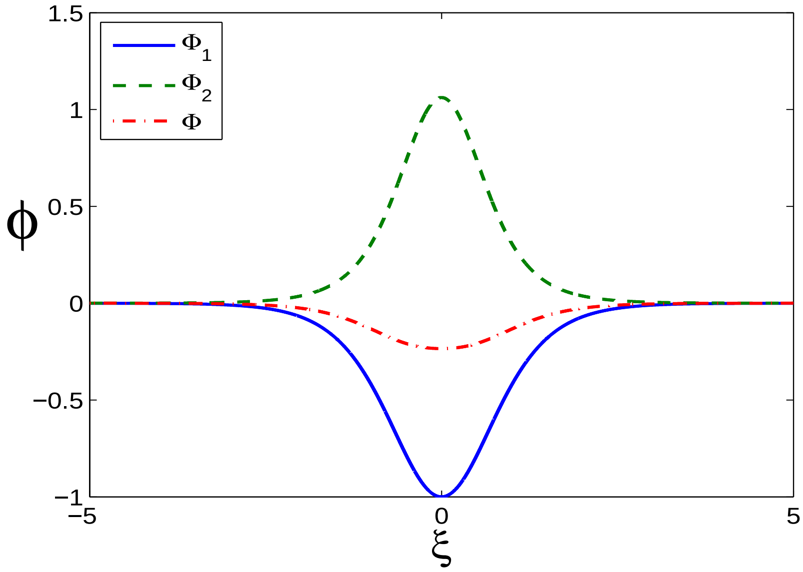

FIGURE 3. The variations of the rarefactive soliton profiles with ζ for $\epsilon = 0.5,\ a = -1,\ \alpha = 0.1$ and $q = 0.7$.

REFERENCES

[1] Washimi, H. and Taniuti, T.(1966)Propagation of ion-acoustic solitary waves of small amplitude, Phys. Rev. Lett.,17, 996-998.

[2] Tribeche, M. and Sabry,R.(2012) Electron-acoustic solitary waves in a magnetized plasma with hot electrons featuring Tsallis distribution, Astrophys. Space Sci., 341, 579-85.

[3] Bostrom, R.(1992) Observation of weak double layers on auroral field lines, IEEE Trans. Plasma Sci., 20, 756-763.

[4] Dovner, P.,Erikson, E.,Bostrom, R.and Holback, B.(1994) Freja multiprobe observations of electrostatic solitary structures, Geophys. Res. Lett., 21, 1827-1830.

[5] Cairn, R., Mamun, A., Bingham, R. ,Bostrom, R., Dendy, R., Nairn, C.and Shukla, P.(1995) Electrostatics solitary structures in nonthermal plasmas, J. Geophys. Res., 22, 2709-2712.

[6] C. Tsallis, C. (1988)Possible generalization of Boltzman-Gibbs statistics, J. Statistical Physics, 52, 479-487.

[7] Tribeche, M.,Djebarni, L. and Armour, R.(2010)Ion-acoustic solitary waves in a plasma with a q-nonextensive electron velocity distribution, Phys. Plasmas, 17, 042114.

[8] N. Akhtar, N.,El-Taibany, W. and Mahmoud, S.(2013) Electrostatic double layers in a warm negative ion plasma with non-extensive electrons, Phys. Lett. A, 377, 1282-1289.

[9] Tribeche, M., Amour, R. and Shukla, P. K. (2012) Ion acoustic solitary waves in a plasma with nonthermal electrons featuring Tsallis distribution, Phys. Rev. E, 85, 037401.

[10] Amour, R., Tribeche, M. and Shukla,P. K.(2012) Electron-acoustic solitary waves in a plasma with nonthermal electrons featuring Tsallis distribution, Asrophys. Space Sci. 338, 287-294.

[11] Wang, Y. Y., Li, J. T.,Dai, C. Q.,Chen, X. F. and Zhang,Z. F.(2013) Solitary waves and rogue waves in a plasma with nonthermal electrons featuring Tsallis distribution, Phys. Lett. A ,377, 2097-2014.

[12] Bouzit, O., Gougam, L. A. and Tribeche, M.(2014) Solitons and freak waves in a mixed nonextensive high energy-tail electron distribution, Phys. Plasmas, 21, 062101

[13] Bouzit, O.,Tribeche, M. and Bains, A. S.(2015) Modulational instability of ion-acoustic waves in a plasma with a q-nonextensive nonthermal electron velocity distribution, Phys. Plasmas, 22,084506.

[14] Ikezi, H., Taylor,R. and Baker, D.(1970)Formation and interaction of ion-acoustic solitons, Phys. Rev. Lett. 25, 11-14 .

[15] Ikezi, H.(1973)Experiments on ion-acoustic solitary waves, Phys. Fluids, 16, 1668-167.

[16] Kato,Y., Tajiri, M. and Taniuti, T.(1972) Precursor of ion-acoustic quasishock wave in collisionless plasma, Phys. Fluids, 15, 865-870.

[17] Taniuti, T.(1974) Reductive perturbation method and far fields of wave equations,Suppl. of Progress in Theoretical Phys., 55, 1-35.

[18] N. Sugimoto and T. Kakutani, Note on the higher order terms in reductive perturbation theory, J. Phys. Soc. Japan, 43, 1469-1470 (1977).

[19] Kodama, Y. and Taniuti, T.(1978) Higher order perturbation in the reductive perturbation method I:Weakly dispersive systems, J. Phys. Soc. Japan, 45, 298-310.

[20] Malfliet, M. and Wieers, E.(1999) Theory of ion-acoustic waves re-visited, J. Plasma Phys., 56, 441-450.

[21] Demiray, H.(1999) A modified reductive perturbation method as applied to nonlinear ion-acoustic waves, J. Phys. Soc. Japan, 68, 1833-1837.

[22] Demiray, H.(2012) Contribution of higher order terms in the nonlinear shallow water waves, TWMS J. Appl. Engr. Math., 2, 210-218.

[23] Williams, G., Kourakis, I., Verheest, F. and Hellberg, M. A.(2013) Re-examining the Cairns-Tsallis model for ion acoustic solitons, Physical Review E, 88, 023103.

Hilmi Demiray, for a photograph and biography, see TWMS Journal of Applied and Engineering Mathematics, Volume 1, No.1, 2011.