TWMS J. App. Eng. Math. V.7, N.2, 2017, pp. 343-353

DIRICHLET SERIES AND APPROXIMATE ANALYTICAL METHOD FOR THE SOLUTION OF MHD BOUNDARY LAYER FLOW OF CASSON FLUID OVER A STRETCHING/SHRINKING SHEET

VISHWANATH B. AWATI [1], §

Abstract. The paper presents analytical and semi-numerical solution for magnetohydrodynamic (MHD) boundary layer flow of Casson fluid over a exponentially permeable shrinking sheet. The governing partial differential equations of momentum equations are reduced to ordinary differential equations by using a classical similarity transformation along with appropriate boundary conditions. Both nonlinearity and infinite interval demand novel mathematical tools for their analysis. We use fast converging Dirichlet series and approximate analytical solution by the Method of stretching of variables for the solution of the nonlinear differential equation. These methods have the advantages over pure numerical methods for obtaining the derived quantities accurately for various values of the parameters involved at a stretch and also they are valid in much larger parameter domain as compared with HAM, HPM, ADM and the classical numerical schemes.

Keywords: magnetohydrodynamics (MHD), boundary layer flow, Casson fluid, shrinking /stretching sheet, wall mass transfer, Dirichlet series, Powells method, method of stretching of variables.

AMS Subject Classification: 76XX, 76Dxx, 76Wxx

1. Nomenclature:

: U_w shrinking velocity
: U_0 shrinking constant
: V_w mass transfer velocity
: B strength of the magnetic field $[wm^{-2}]$
: f similarity function
: P_y yield stress
: M^2 Hartmann number
: u velocity component along the x-axis $[ms^{-1}]$
: v velocity component normal to the y-axis $[ms^{-1}]$
: f_w suction / blowing parameter
: x coordinate along the sheet $[m]$
: y coordinate normal to the sheet $[m]$
: l characteristic length $[m]$

Greek symbols

: α amplification factor

[1] Department of Mathematics, Rani Channamma University, Belagavi-591 156, India.
e-mail: : awati_vb@yahoo.com, ORCID: http://orcid.org/0000-0002-2547-1058;

§ Manuscript received: March 28, 2016; accepted: January 25, 2017.
TWMS Journal of Applied and Engineering Mathematics Vol.7, No.2;

: τ stress tensor
: μ dynamic viscosity
: μ_B plastic dynamic viscosity
: ν kinenatic viscosity $[m^2s^{-1}]$
: σ electrical conductivity $[mhom^{-1}]$
: ρ density$[kgm^{-3}]$
: β Casson fluid parameter
: η similarity variable

2. Introduction

The MHD boundary layer flow of stretching/shrinking sheet problem has important application in several engineering fields. The influence of magnetic field on the viscous flow of an electrically conducting fluid is applicable in many industrial processes, such as in magnetic material processing, purification of crude oil, MHD electrical power generation, glass manufacturing, geophysics and paper production etc. The phenomena of velocities on the boundary towards a fixed point are known as shrinking phenomena, as in the situation of a rising shrinking balloon. Limited attention has been paid on the study of shrinking phenomena [1-8]. In certain situations, the shrinking sheet solutions does not exist, since the velocity cannot be confined in a boundary layer. Solutions exist if either the magnetic field or the stagnation flow is taken into account. Crane [9] found an exact similarity solution for steady two-dimensional stretching where the velocity on the boundary is away and proportional to the distance from the fixed point. Carragher and Carane [10] studied heat transfer on a continuous stretching sheet. Vajravelu and Rollins [11] discussed the heat transfer in an electrically conducting fluid over a stretching surface. Other important investigations of the stretching sheet flow have been made by Salem and Fathy [12], who analysed the effects of variable properties on MHD heat and mass transfer flow near a stagnation point towards a stretching sheet in a porous medium with a thermal radiation. Bhattacharya and Layek [13] discussed the slip effects on diffusion of chemically flow over a stretching sheet with suction or blowing. Hayat et al. [14] discussed MHD flow and heat transfer over a permeable shrinking sheet with slip conditions. Wang [15] introduced a new type of flow, one which is due to a shrinking sheet which is quite different from stretching sheet flow, as the generated verticity due to shrinking does not remain inside the boundary layer. Miklavcic and Wang [16] discussed steady flow due to a shrinking sheet, a certain amount of wall mass suction is required to restrain the generated vorticity. Fang et al. [3, 17] analysed the MHD viscous flow of Newtonian flow over a shrinking sheet with wall mass transfer, taking no-slip as well as slip boundary conditions. Ali et al. [18] analysed the boundary layer flow and heat transfer due to permeable shrinking sheet with prescribed surface heat flux by Keller-box method. Noor et al. [19] discussed simple non-perturbative solution for MHD viscous flow due to a shrinking sheet by series solution using adomain decomposition method (ADM). Raftari and Yildirim [20], investigated MHD viscous flow due to a shrinking sheet by employing the homotopy perturbation method (HPM) and Pade approximants. Bhattacharyya [21] examined the effects of heat source/sink on the steady two dimensional MHD boundary layer flow and heat transfer over a shrinking sheet with wall mass suction using finite difference method.

In all the above investigations, the classical Newtonian fluid flows are discussed. The non-Newtonian fluid theory is much more relevant in modern engineering and industrial applications. Casson fluid is one such Non-Newtonian fluids, where flow occurs if the shear stress magnitude is greater than yield shear stress. Fredrickson [22], examined the study flow behaviour of a Casson fluid in a tube. Mustafa et al. [23], analysed the stagnation point flow of a Casson fluid towards a stretching sheet. Thiagarajan and Senthilkumar [24], discussed the DTM-Pade approximants of MHD boundary layer flow of a Casson fluid over a shrinking sheet. Nadeem et al [25], analysed the MHD flow of a Casson fluid over an exponentially shrinking sheet. Abel et al. [26] have discussed the study of visco-elastic fluid flow and heat transfer over a stretching sheet with variable viscosity.Prasad et al. [27] analyzed the flow and heat transfer at a nonlinearly shrinking porous sheet: the case of asymptotically large power law shrinking rates. Recently, Prasad et al. [28] examined the MHD Casson nanofluid flow and heat transfer at a stretching sheet with variable thickness.

The present paper discusses the steady MHD boundary layer of electrically conducting Casson fluid fast a shrinking sheet. The semi-numerical and closed form analytical solution of the resulting third order nonlinear boundary value problem with an infinite interval is obtained using Dirichlet series method and method of stretching of variables.

We seek solution of the general equation of the type

$$f''' + Aff'' + Bf'^2 + Cf' = 0 \tag{1}$$

The relevant boundary conditions for the flow problem are

$$f(0) = \alpha_1 = f_w, f'(0) = \beta_1, f'(\infty) = 0 \tag{2}$$

where A, B, and C are constants and prime denotes derivative with respect to the independent variable η. This equation admits a Dirichlet series solution; necessary conditions for the existence and uniqueness of these solutions may be found in [29, 30]. For a specific type of boundary conditions i.e. $f'(\infty) = 0$, the Dirichlet series solution is particularly useful for obtaining the derived quantities exactly. A general discussion of the convergence of the Dirichlet series may also be found in Riesz [31]. The accuracy as well as uniqueness of the solution can be confirmed using other powerful semi-numerical schemes. Sachdev et al. [32] have analysed various problems from fluid dynamics of stretching sheet using this approach and have found more accurate solution compared with earlier numerical findings. Recently, Vishwanath et al [33, 34] and Ramesh et al [35] have analysed the problems from MHD boundary layer flow with nonlinear stretching sheet using the above methods and have found more accurate results compared with the classical numerical methods. In this article, we also present Dirichlet series solution and an approximate analytical method called method of stretching of variables. This method is quite easy to use especially for nonlinear ordinary differential equations and requires less computing time compared with pure numerical methods and easy to solve compared with other approximate methods(for example, Homotopy analysis method (HAM)).

3. Mathematical Formulation of the problem

Consider a two-dimensional incompressible flow of a Casson fluid over an exponentially shrinking sheet. The fluid is electrically conducting in the presence of a uniform magnetic field applied normal to the sheet. Under the approximation of small Reynolds number, the induced magnetic field is neglected. The rheological equation of extra stress tensor (τ) for an isotropic and incompressible flow of a Casson fluid, reported by Mustafa [23] as

$$\tau^{1/n} = \tau_0^{1/n} + \mu\gamma^{1/n} \tag{3}$$

or

$$\tau_{ij} = [\mu_B + (\frac{P_y}{\sqrt{2\pi}})^{1/n}]^n 2e_{ij} \tag{4}$$

where μ is the dynamic viscosity, μ_B is plastic dynamic viscosity of the non-Newtonian fluid, P_y is the yield stress of fluid, π is the product component of deformation rate with itself (i.e. $\pi = e_{ij}e_{ij}$), e_{ij} is the (i,j)th component of deformation rate and π_c is the critical value of π based on the non-Newtonian model. Under these conditions the MHD boundary layer equations for the steady flow of Casson fluid over a exponentially shrinking sheet problem is governed by the following equations

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{5}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu(1 + \frac{1}{\beta})\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho}u \tag{6}$$

The boundary conditions are defined as

$$u = U_w(x) = -U_0 exp(\frac{x}{l}) \quad and \quad v = V_w(x) = V_0 exp(\frac{x}{2l}) \quad at \quad y = 0; u \to 0 \quad as \quad y \to \infty \tag{7}$$

where u and v are the velocity components in the x and y directions respectively, x is the distance along the sheet, y is the distance perpendicular to the sheet, ν is the kinematic viscosity of the fluid, ρ is the density of the fluid, $\beta = \mu_B\sqrt{2\pi_c}/P_y$ is the non-Newtonian Casson fluid parameter, σ is the electrical conductivity of the fluid, U_w is the shrinking velocity with U_0 (shrinking constant) and V_w is the mass transfer velocity with ($V_0 > 0$ for mass injection and $V_0 < 0$ for mass suction). Let us assumed that the magnetic field $B(x)$ is of the form $B = B_0 exp(\frac{x}{l})$ where B_0 is the constant magnetic field. Introducing the dimensionless variables f and η as

$$\psi = a\sqrt{2\nu l U_0}xf(\eta)exp(\frac{x}{2l}) \quad and \quad \eta = y\sqrt{\frac{U_0}{2\nu l}}exp(\frac{x}{2l}) \tag{8}$$

Substituting Eqn.(8) into (5)-(8), continuity Eqn.(5) is automatically satisfied and the momentum Eqn.(6) reduces to

$$(1 + \frac{1}{\beta})f''' + ff'' - 2f'^2 - M^2 f' = 0 \tag{9}$$

The relevant boundary conditions (7) becomes

$$f(0) = f_w, \quad f'(0) = -1, \quad f'(\infty) = 0 \tag{10}$$

where $M^2 = \frac{2\sigma B^2 l}{\rho U_0}$ is the Hartmann number and β is the Casson fluid parameter.

4. Dirichlet Series Solution

We seek Dirichlet series solution of Eqn.(1) satisfying last boundary condition $f'(\infty) = 0$ automatically in the form ([29, 30])

$$f = \gamma_1 + \frac{6\gamma}{A}\sum_{i=1}^{\infty} b_i a^i e^{-i\gamma\eta} \tag{11}$$

where γ and a are parameters which are to be determined. Substituting Eqn.(11) into Eqn.(1), we get

$$\sum_{i=1}^{\infty}\{-\gamma^2 i^3 + A\gamma\gamma_1 i^2 - Ci\}b_i a^i e^{-i\gamma\eta} + \frac{6\gamma^2}{A}\sum_{i=2}^{\infty}\sum_{k=1}^{i-1}\{Ak^2 + Bk(i-k)\}b_k b_{i-k} a^i e^{-i\gamma\eta} = 0 \tag{12}$$

for $i = 1$, we have

$$\gamma_1 = \frac{\gamma^2 + C}{A}. \tag{13}$$

Substituting Eqn.(13) into Eqn.(12) the recurrence relation for obtaining coefficients is given by

$$b_i = \frac{6\gamma^2}{Ai(i-1)\{\gamma^2 i - C\}} \sum_{k=1}^{i-1} \{Ak^2 + Bk(i-k)\} b_k b_{i-k} \tag{14}$$

for i=2, 3, 4,..... . If the Eqn.(11) converges absolutely when $\gamma > 0$ for some η_0, this series converges absolutely and uniformly in the half plane $Re\eta \geq Re\eta_0$ and represents an analytic $\frac{2\pi}{\gamma}$ periodic function $f = f(\eta_0)$ such that $f'(\infty) = 0$ ([29]).

The Eqn.(11) contains two free parameters namely a and γ. These unknown parameters are determined from the remaining boundary conditions of Eqn.(2) at $\eta = 0$.

$$f(0) = \frac{\gamma^2 + C}{A\gamma} + \frac{6\gamma}{A} \sum_{i=1}^{\infty} b_i a^i = \alpha_1 \tag{15}$$

and

$$f'(0) = \frac{6\gamma^2}{A} \sum_{i=1}^{\infty} (-i) b_i a^i = \beta_1 \tag{16}$$

The solution of the above transcendental Eqn.(15) and Eqn.(16) yield constants a and γ. The solution of the above transcendental equations is equivalent to the unconstrained minimization of the functional

$$[\frac{\gamma^2 + C}{A\gamma} + \frac{6\gamma}{A} \sum_{i=1}^{\infty} b_i a^i - \alpha_1]^2 + [\frac{6\gamma^2}{A} \sum_{i=1}^{\infty} (-i) b_i a^i - \beta_1]^2 \tag{17}$$

We use Powell's method of conjugate directions (Press et al [36]) which is one of the most efficient techniques for solving unconstrained optimization problems. This helps in finding the unknown parameters a and γ uniquely for different values of the parameters A, B, C, α_1 and β_1. Alternatively, Newton's method is also used to determine the unknown parameters a and γ accurately. The shear stress at the surface of the problem is given by

$$f''(0) = \frac{6\gamma}{A} \sum_{i=1}^{\infty} b_i a^i (i\gamma)^2 \tag{18}$$

The velocity profiles of the problem is given by

$$f'(\eta) = \frac{6\gamma^2}{A} \sum_{i=1}^{\infty} (-i) b_i a^i e^{-i\gamma\eta} \tag{19}$$

5. Method of Stretching of Variables

Many nonlinear ODEs arising in MHD problems are not amenable for obtaining analytical solutions. In such situations, attempts have been made to develop an approximate method for the solution of these problems. The numerical approach is always based on the idea of stretching of variables of the flow problems. Method of stretching of variables is used here for the solution of such problems. In this method, we choose suitable derivative function H' such that the derivative boundary conditions are automatically satisfied and integration of H' will satisfy the remaining boundary condition. Substituting the resulting function into the given equation we get the residual of the form $R(\xi, \alpha)$ which is called defect function. Using Least squares approximation method, the residual of the defect function can be minimized. For details see (Ariel, [37]).

Using the transformation $f = f_w + F$ into Eqn.(1), we get

$$F''' + A(f_w + F)F'' + BF'^2 + CF' = 0, \qquad ' = \frac{d}{d\eta} \tag{20}$$

The boundary conditions (2) become

$$F(0) = 0, \quad F'(0) = -1, \quad F'(\infty) = 0 \tag{21}$$

We introduce two variables ξ and G in the form

$$G(\xi) = \alpha F(\eta) \quad and \quad \xi = \alpha\eta \tag{22}$$

where $\alpha > 0$, is an amplification factor. In view of Eqn.(22), the system (20-21) are transformed to the form

$$\alpha^2 G''' + A(f_w\alpha + G)G'' + BG'^2 + CG' = 0, \quad ' = \frac{d}{d\eta} \tag{23}$$

and the boundary conditions in Eqn.(21) become

$$G(0) = 0, \quad G'(0) = -1, \quad G'(\infty) = 0 \tag{24}$$

We choose a trail velocity profile

$$G' = -exp(-\xi) \tag{25}$$

Which satisfies the derivative conditions in Eqn.(24). Integrating Eqn.(25) with respect to ξ from 0 to ξ using conditions (24), we get

$$G = exp(-\xi) - 1 \tag{26}$$

Substituting Eqn.(26) into Eqn.(23), we get the residual of defect function

$$R(\xi, \alpha) = (-\alpha^2 + Af_w\alpha - A - C)exp(-\xi) + (A + B)exp(-2\xi) \tag{27}$$

By using the least squares approximation method as discussed in Ariel [37], the Eqn.(27) can be minimized for which

$$\frac{\partial}{\partial\alpha}\int_0^\infty R^2(\xi, \alpha)d\xi = 0 \tag{28}$$

Substituting Eqn.(27) into Eqn.(28) and solving cubic equation in α for a positive root, we get

$$\alpha = \frac{1}{6}(3Af_w \pm \sqrt{3}\sqrt{-4A + 8B - 12C + 3A^2 f_w^2}) \tag{29}$$

Once the amplification factor is calculated, then using Eqn.(20), original function f can be written as

$$f = f_w + \frac{1}{\alpha}(exp(-\alpha\eta) - 1) \tag{30}$$

with α defined in Eqn.(29). Thus Eqn.(30) gives the solution of Eqn.(1) for all A,B,C, f_w and β_1.

6. Results and Discussion

In the present paper we discuss the semi-numerical solution of MHD boundary layer flow of a Casson fluid over an exponentially permeable shrinking sheet. The governing equations are simplified by suitable similarity transformations and the reduced third order nonlinear boundary value problems with infinite domain are solved semi-numerically using an elegant powerful technique which are Dirichlet series method and an approximate analytical method by the method of stretching of variables. We have given exact analytical solution of the nonlinear boundary value problem in more general form. In this method it is important that the edge boundary layer $\eta \to \infty$ automatically satisfied. Numerical computations are performed for various values of the physical parameters involved in the equation viz. the Hartmann number M^2, Casson fluid parameter β and the wall mass transfer parameter $\alpha_1 = f_w$. The present solution is also validated by comparing it with the previously published work of Thiagarajan and Senthilkumar [24] and Nadeem et al. [25].

Table 1 shows that $f''(0)$ for various values of the Casson fluid parameter β is obtained by Dirichlet series, Method of stretching of variables for $f_w = 1$ and $M = 2$ and the results are compared with the numerical solutions which are comparable. Table 2 and Table 3 shows the $f''(0)$ for different values of Casson fluid parameter β, shrinking parameter f_w, magnetic parameter and the results are compared with DTM-Pade and Numerical methods, the results are comparable. Table 4 presents the $f''(0)$ for the case of $\beta = 1$ for different values of the shrinking parameter f_w and different values of the Hartmann number M which are compared with those of DTM-Pade and Nadeem [25].

Fig. 1 shows the dimensionless velocity profiles $f'(\eta)$ for different values of the Casson fluid parameter β when $f_w = 1$ and $M = 2$. It is observed that the Casson fluid parameter increases the velocity profiles decreases. Fig. 2 displays the dimensionless velocity profiles $f'(\eta)$ for various values of the shrinking parameter f_w when $M = 2$ and $\beta = 1$. The effect of the shrinking parameter decreases and the velocity profiles are also decreases. The Fig. 3 demonstrate the dimensionless velocity profiles $f'(\eta)$ for various values of the Hartmann number M when $f_w = 1$ and $\beta = 3$. It shows that the Hartmann number increase, velocity profiles decreases.

7. Conclusions

In this article, we describe the analysis of boundary value problem for third order nonlinear ordinary differential equation over an infinite domain arising in MHD boundary layer flow of Casson fluid over a exponentially permeable shrinking sheet. The semi-numerical schemes described here offer advantages over solutions obtained by DTM and numerical methods etc. The convergence of the Dirichlet series method and approximate analytical solution is given. The results are presented in Tables and graphically, the effects of the emerging parameters are discussed semi-numerically.

TABLE 1. Various values of Casson fluid parameter β , $f''(0)$ is obtained by Dirichlet series, Method of stretching of variables and its numerical solutions for $f_w = 1$ and M=2 .

β	Dirichlet Series method			MSV	DTM-Pade [24]	Numerical
	a	γ	$f''(0)$	$f''(0)$	$f''(0)$	$f''(0)$
1	0.0410778	1.4905952	1.3553910	1.35867789	1.3666795	1.3666779
2	0.0384806	1.7758458	1.6193250	1.62432778	1.6350828	1.6350794
3	0.0371173	1.9110404	1.7565830	1.75000000	1.7620987	1.7620938
5	0.0360615	2.0414439	1.8801230	1.87202076	1.8854407	1.8854341

TABLE 2. Various values of shrinking parameter f_w, $f''(0)$ is obtained by Dirichlet series, Method of stretching of variables and its numerical solutions for $\beta = 1$ and M=2

f_w	Dirichlet Series method			MSV	DTM-Pade [24]	Numerical
	a	γ	$f''(0)$	$f''(0)$	$f''(0)$	$f''(0)$
0	0.0659299	1.2090263	1.0319460	1.0801234	1.0777210	1.0777204
1	0.0410778	1.4905952	1.3553910	1.3586779	1.3666795	1.3666779
2	0.0270015	1.8164093	1.6987860	1.6902381	1.7051154	1.7051124
3	0.0183583	2.1819218	2.0780220	2.0649778	2.0833004	2.0832948

TABLE 3. Various values of magnetic parameter M ,$f''(0)$ is obtained by Dirichlet series, Method of stretching of variables and its numerical solutions for $f_w = 1$ and $\beta = 3$.

M	Dirichlet Series method			MSV	DTM-Pade [24]	Numerical
	a	γ	$f''(0)$	$f''(0)$	$f''(0)$	$f''(0)$
2	0.0371173	1.9110404	1.7565830	1.7500000	1.7620987	1.7620938
3	0.0159199	2.8504119	2.7531100	2.7500000	2.75465746	2.7546363
3.5	-	-	-	3.2172043	3.22055648	3.2205252

TABLE 4. Comparison of values of $f''(0)$ for $\beta = 1$, $f''(0)$ is obtained by Dirichlet series, Method of stretching of variables and DTM-Pade.

f_w	M	Dirichlet Series method			MSV	DTM-Pade	Numerical
		a	γ	$f''(0)$	$f''(0)$	$f''(0)$	$f''(0)$
0.5	2.0	0.05156954	1.344266	1.193946	1.2123324	1.2155035	1.2155025
1.0	2.0	0.04107781	1.490595	1.355393	1.3586779	1.3666795	1.3666779
1.0	3.0	0.01688239	2.263239	2.179553	2.1811050	2.1841832	2.1841832

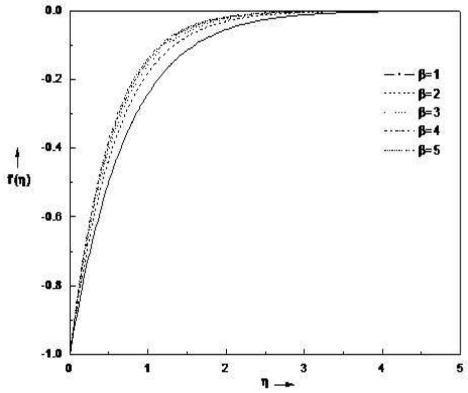

FIGURE 1. Velocity profiles for various values of Casson fluid parameter β for $f_w = 1$ and $M = 2$.

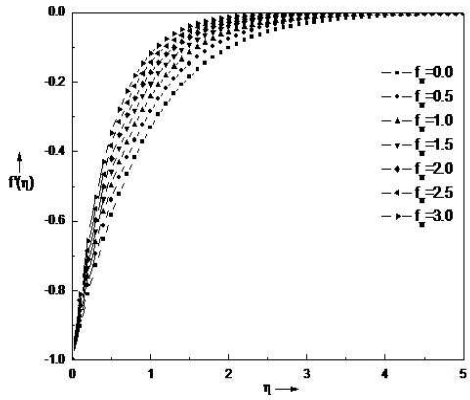

FIGURE 2. Velocity profiles for different values of shrinking parameter f_w for $\beta = 1$ and M=2.

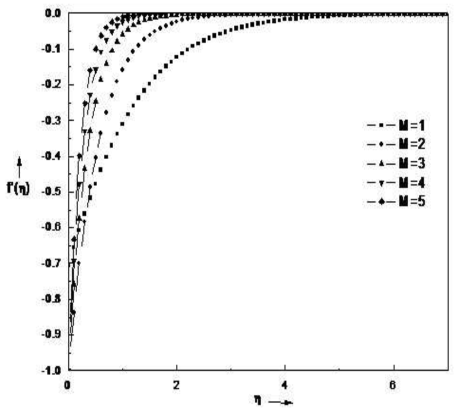

FIGURE 3. Effect of increasing magnetic field intensity on velocity profile for $f_w = 0.1$.

REFERENCES

[1] Sajid,M., and Hayat,T., (2009),The application of homotopy analysis method for MHD viscous flow due to shrinking sheet, Chaos Solitons and Fractals, 39, pp.1317-1323.

[2] Hayat,T., Abbas,Z., Javed,T., and Sajid,M., (2009),Three-dimensional rotating flow induced by a shrinking sheet for suction, Chaos Solitons and Fractals, 39(4), pp.1615-1626.

[3] Fang,T.,and Zhang,J., (2009), Closed-form exact solutions of MHD viscous flow over a shrinking sheet, Commun Nonlinear Sci. Neumer Simulat, 14(7), pp.2853-2857.

[4] Fang,T., (2008),Boundary layer flow over a shrinking sheet with power-law velocity, Int. J. Heat Mass Tran, 51(25-26), pp.5838-5843.

[5] Nadeem,S., and Awais,M., (2008), Thin film flow of an unsteady shrinking sheet through porous medium with variable viscosity, Phys. Lett. A, 372(30), pp.4965-4972.

[6] Hayat,T., Javed,T., and Sajid,M., (2008), Analytic solution for MHD rotating flow of a second grade fluid over a shrinking surface, Phys. Lett. A., 372(18), pp.3264-3273.

[7] Wang C.Y., (2008), Stagnation flow towards a shrinking sheet, Int. J. Non-Linear Mech, 43(5), pp.377-382.

[8] Hayat,T., Abbas,Z., and Ali,N., (2008), MHD flow and mass transfer of an upper-convected Maxwell fluid past a porous shrinking sheet with chemical reaction species, Phys. Lett.A., 732(26), pp.4698-4704.

[9] Crane,L.J., (1970), Flow past a stretching plate, Z Angew Math. Mech., 21, pp.645-647.

[10] Carragher,P., and Carane,L.J., (1982), Heat transfer on a continuous stretching sheet, Z Angew Math. Mech., 62, pp.564-565.

[11] Vajravelu,K., and Rollins,D.,(1992), Heat transfer in an electrically conducting fluid over a stretching surface, Int. J. Nonlinear Mech., 27, pp.265-277.

[12] Salem,A.M., and Fathy,R., (2012), Effects of variable properties on MHD heat and mass transfer near a stagnation point towards a stretching sheet in a porous medium with thermal radiation, Chin Phys. B, 21, 054701.

[13] Battacharyya,K., and Layek,G.C.,(2011), Slip effects on diffusion of chemically reactive species in boundary layer flow over a vertical stretching sheet with suction and blowing, Chem. Eng. Commun., 198, pp.1354-1365.

[14] Hayat,T., Qasim,M., and Mesloub,S., (2011),MHD flow and heat transfer over permeable stretching sheet with slip conditions, Int. J. Numer. Methods Fluids, 66, pp.963-975.

[15] Wang,C.Y., (1990), Liquid film on an unsteady stretching sheet, Quart Appl Math , 48, pp.601-610.

[16] Miklavcic,M., and Wang,C. Y.,(2006), Viscous flow due to a shrinking sheet, Quart. Appl. Math. 64, pp.283-290.

[17] Fang,T., Zhang,J.,and Yao,S., (2010), Slip magnetohydrodynami viscous flow over a permeable shrinking sheet, Chin. Phys. Lett. 27, 124702.

[18] Ali,F., Nazar,R., Arifin,N., (2010), MHD viscous flow and heat transfer induced by permeable shrinking sheet with prescribed surface heat flux, WSAS Trans Math, 5(9), pp.365-375.

[19] Noor,N.F.M., Kechil,S.A.,and Hashim,I., (2010), Simple non-perturbative solution for MHD viscous flow due to a shrinking sheet, Commun Nonlinar Sci. Numer Simulat , 15, pp.144-148.

[20] Raftari,B., Yildirim,A.,(2011), A series solution of nonlinear ODE arising in MHD by HPM-Pade technique, Comp. Math. Appl., 61, pp.1676-1681.

[21] Bhattacharyya, K., (2011),Effects of heat source/ sink on MHD flow and heat transfer over a shrinking sheet with mass suction, Chem. Eng. Res. Bull 15, pp.12-17.

[22] Fredrickson,A.G., (1964), Principles and applications of rheology, (1964) (Englewood Cliffs, N.J; Prentice-Hall).

[23] Mustafa,M., Hayat,T., Pop,I. and Hendi,A., (2012), Stagnation point flow and heat transfer of a Casson fluid towards a stretching, Z. Naturforsch, 67a, pp.70-76.

[24] Thiagarajan,M., and Senthilkumar,K., (2013), DTM-Pade approximants of MHD boundary layer flow of a Casson fluid over a shrinking sheet, United States of America Research Journal (USARJ), Vol.1, No.1, pp.1-7.

[25] Nadeem, S., Haq, R. U., and Lee, C.,(2013), MHD flow of a Casson fluid over an exponentially shrinking sheet, Scientia Iranica B, 19(6), pp.1550-1553.

[26] Abel,M.S., Sujit Kumar Khan, and Prasad,K.V.,(2002), Study of visco-elastic fluid flow and heat transfer over a stretching sheet with variable viscosity, International journal of non-linear mechanics, 37(1), pp.81-88.

[27] Prasad,K.V., Vajravelu,K., and Pop,I., (2013) ,Flow and heat transfer at a nonlinearly shrinking porous sheet: the case of asymptotically large power law shrinking rates, IJAME, 18(3), pp.779-791.

[28] Prasad,K.V., Vajravelu,K., and Vaidya,H., (2016), MHD Casson Nanofluid Flow and Heat Transfer at a Stretching Sheet with Variable Thickness, Journal of Nanofluids, 5(3), pp.423-435.

[29] Kravchenko,T.K., and Yablonskii,A.I., (1965), Solution of an infinite boundary value problem for third order equation, Differentialnye Uraneniya. 327, 1.
[30] Kravchenko,T.K., and Yablonskii,A.I., (1972), A boundary value problem on a semi-infinite interval, DifferentialnyeUraneniya, 8(12), pp.2180-2186.
[31] Riesz,S., (1957), Introduction to Dirichlet series, Camb. Univ. Press.
[32] Sachdev,P.L., Bujurke,N.M., and Awati,V.B.,(2005), Boundary value problems for third order non-linear ordinary differential equations, Stud. Appl. Math, 115, pp.303-318.
[33] Awati,V.B., Bujurke,N.M., and Kudenatti,R.B., (2011), An exponential series method for the solution of the free convection boundary layer flow in a saturated porous medium. AJCM , 1, pp.104-110.
[34] Awati,V.B., Bujurke,N.M., and Kudenatti,R.B., (2011), Dirichlet series method for the solution of MHD flow over a nonlinear stretching sheet. IJAMES, 5(1), pp.07-12.
[35] Kudenatti,R.B., Awati,V.B., and Bujurke,N.M.,(2011), Exact analytical solutions of class of boundary layer equations for a stretching surface, Appl Math Comp, 218, pp.2952-2959.
[36] Press,W.H.H., Flannery,B.P., Teulosky,S.A., and Vetterling,W.T., (1987), Numerical Recipes in C, Camb. Univ. Press, UK.
[37] Ariel,P.D., (1994), Stagnation point flow with suction; an approximate solution, Journal of Applied Mechanics, 61(4), pp.976-978.

Dr. Vishwanath B. Awati was born in Bijapur District, Karnataka state. He graduated from Karnatak University, Dharwad and obtained his M.Sc. degrees in Mathematics from Karnatak University, Dharwad. He obtained his Ph.D. degree in Mathematics from the same University. He is working as an associate professor in the Mathematics Department at Rani Channamma University, Belagavi. His research interest includes the MHD boundary layer flows of a stretching/ shrinking sheet problems.