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COINCIDENCE AND COMMON FIXED POINT THEOREMS FOR FAINTLY COMPATIBLE MAPS

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Abstract. The paper is aimed to generalize and improve the results of Bisht and Shahzad [Faintly compatible mappings and common fixed points, fixed point theory and applications, 2013, 2013:156]. The significance of this paper lies in the fact that coincidence and common fixed point theorems under Ćirić type contractive condition via faint compatibility and conditional reciprocal continuity is established without using continuity of even single map and containment requirement of the range space of involved maps. Illustrative examples are furnished to highlight the realized improvement of our results.

Keywords: Coincidence point; conditional reciprocal continuity; common fixed point; faintly compatible.

AMS Subject Classification: 47H10; 54H25; 55M20

1. Introduction

Fixed point theory plays a significant role in non-linear analysis as many real-world problems in applied science, economics, physics and engineering can be reformulated as a problem of finding fixed points of non-linear maps. Common fixed point theorem commonly require commutativity, continuity, completeness together with a suitable condition on containment of ranges of involved maps beside an appropriate contraction condition. Thus, research in this field is aimed at weakening one or more of these conditions (see for instance, [5], [8], [9], [10], [13], [15], [17]). Ćirić [4] introduced the following contractive condition to establish fixed point theorem:
$d(Tx, Ty) \leq \lambda\ M(x, y)$, where $0 \leq \lambda < 1$, for all $x, y \in X$,
where $M(x, y) = max\{d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\}$, which is known as Ćirić type contractive condition.
The purpose of this paper is to establish coincidence and common fixed point theorems for maps satisfying Ćirić type contractive condition via faint compatibility and conditional reciprocal continuity without using continuity of even single map and containment requirement of the range space of involved maps. In the process, we emphasize on the role of faint compatibility for the existence of common fixed point for a pair of maps satisfying non-contractive condition which admits the possibility of more than one common fixed point. Obtained results for non-compatible discontinuous self maps in non-complete metric space generalize and improve the results of Bisht and Shahzad [3].

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2. Preliminaries

A point x in metric space (X, d) is coincidence point of a pair of self map (f, g) iff $fx = gx = w$, w is a point of coincidence of f and g. Further x is common fixed point if $fx = gx = x$.

Definition 2.1. [6] *A pair of self maps (f, g) of a metric space (X, d) is compatible if $lim_n d(fgx_n, gfx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $lim_n fx_n = lim_n gx_n = t$ for some t in X.*
A pair of self maps (f, g) is non compatible if there exists at least one sequence $\{x_n\}$ in X such that $lim_n fx_n = lim_n gx_n = t$ for some t in X, but $lim_n d(fgx_n, gfx_n)$ is either non zero or non-existent.

Definition 2.2. [7] *A pair (f, g) of self maps of a metric space (X, d) is weakly compatible if the pair commutes on the set of coincidence points, i.e. $fx = gx$ $(x \in X)$ implies $fgx = gfx$.*

Definition 2.3. [1] *A pair of self maps (f, g) of a metric space (X, d) satisfies the property (E.A.) if there exists a sequence $\{x_n\}$ in X such that $lim_n fx_n = lim_n gx_n = t$ for some t in X.*

Non-compatible pair of maps satisfies the property (E.A.) however converse is not essentially true. In fact the property (E.A) can be viewed as slight unification of compatible and non-compatible maps. Here it is worth mentioning that a similar notion specifically 'tangential maps' was introduced by Sastry and Murthy [14] but appears to be escaped from the notice of the researchers of this area.

Definition 2.4. [16] *A pair of self maps (f, g) on a metric space (X, d) satisfies the common limit in the range of g property $(CLRg)$ if there exists a sequence $\{x_n\}$ in X such that $lim_n fx_n = lim_n gx_n = gt$ for some $t \in X$.*

Example 2.1. *Let $X = [1, 15)$ and d be the usual metric on X. Let the pair of self map (f, g) of X be defined as*

$$fx = \begin{cases} 1, & if\ x \in 1 \bigcup (3, 15) \\ 12, & if\ x \in (1, 3]. \end{cases}$$

$$and\ gx = \begin{cases} 1, & if\ x = 1 \\ 5, & if\ x \in (1, 3] \\ \frac{x+1}{4}, & if\ x \in (3, 15). \end{cases}$$

Consider a sequence $\{x_n\}$ in X satisfying $x_n = 3 + \frac{1}{n}$ and $\lim_n fx_n = \lim_n gx_n = 1 = g1$ where $1 \in X$. Hence the pair of maps (f, g) satisfies both property (E.A.) and property $(CLRg)$.

Example 2.2. *Let $X = [1, 15)$ and d be the usual metric on X. Let the pair of self map (f, g) of X be defined as*

$$fx = \begin{cases} 1, & if\ x \in 1 \bigcup (3, 15) \\ 14, & if\ x \in (1, 3]. \end{cases}$$

$$and\ gx = \begin{cases} 3, & if\ x = 1 \\ 6, & if\ x \in (1, 3] \\ \frac{x+1}{4}, & if\ x \in (3, 15). \end{cases}$$

Consider a sequence $\{x_n\}$ *in* X *satisfying* $x_n = 3+\frac{1}{n}$ *and* $\lim_n fx_n = \lim_n gx_n = 1 \neq g1$ *where* $1 \in X$. *Hence a pair of maps* (f, g) *satisfies property (E.A) but does not satisfy property* $(CLRg)$.

Clearly a pair of self maps satisfying the property (E.A.) along with closedness of the subspace always enjoys the common limit in the range property. Also the pair of maps satisfying property $(CLRg)$ need not be continuous, i.e. continuity is not the necessary condition for maps to satisfy property $(CLRg)$. Evidently, in above examples maps f and g satisfying property $(CLRg)$ are discontinuous at 1 and 3.

Definition 2.5. [11] *A pair of self maps* (f, g) *of a metric space* (X, d) *is reciprocally continuous iff* $lim_n fgx_n = ft$ *and* $lim_n gfx_n = gt$ *whenever* $\{x_n\}$ *is a sequence in* X *such that* $lim_n fx_n = lim_n gx_n = t$ *for some* t *in* X.

If f and g are both continuous, then they are obviously reciprocally continuous but the converse need not be true.

Definition 2.6. [2] *A pair of self maps* (f, g) *of a metric space* (X, d) *is conditionally reciprocally continuous iff whenever the set of sequences* $\{x_n\}$ *satisfying* $lim_n fx_n = lim_n gx_n$ *is non empty, there exists a sequence* $\{y_n\}$ *satisfying* $lim_n fy_n = lim_n gy_n = t$ *(say) for some* t *in* X *such that* $lim_n fgy_n = ft$ *and* $lim_n gfy_n = gt$.

It is interesting to point out here that a pair of reciprocally continuous self maps (f, g) is conditionally reciprocally continuous but the converse need not be true [2].

Definition 2.7. [12] *A pair of self maps* (f, g) *of a metric space* (X, d) *is conditionally compatible if whenever the sequence* $\{x_n\}$ *satisfying* $lim_n fx_n = lim_n gx_n$ *is non-empty, there exists a sequence* $\{y_n\}$ *in* X *such that* $lim_n fy_n = lim_n gy_n = t$ *and* $lim_n d(fgy_n, gfy_n) = 0$.

Definition 2.8. [3] *A pair of self maps* (f, g) *of a metric space* (X, d) *is faintly compatible iff* f *and* g *are conditionally compatible and* f *and* g *commute on a non empty subset of coincidence points whenever the set of coincidences is non empty.*

It is interesting to mention here that faint compatibility, compatibility and non-compatibility are independent concepts. In fact faint compatibility, like most of the weaker forms of compatibility existing in literature [15, 3] does not reduce to the class of compatibility in the presence of common fixed point (or coincidence point) and is applicable for maps satisfying both contractive and non contractive condition.

3. Main results

Theorem 3.1. *Let conditionally reciprocally continuous, faintly compatible pair of self maps* (f, g) *of a metric space* (X, d) *satisfy the property (E.A.). Then* f *and* g *have a coincidence point. Moreover* f *and* g *have a unique common fixed point provided that the pair satisfies:*

$$d(fx, fy) \leq \lambda \; max\{d(gx, gy), d(fx, gx), d(fy, gy), d(fx, gy), d(fy, gx)\}, 0 \leq \lambda < 1. \quad (1)$$

Proof. Since the pair of self maps (f, g) satisfies the property (E.A.), there exists a sequence $\{x_n\}$ in X such that $lim_n fx_n = lim_n gx_n = t$ for some $t \in X$.
Faint compatibility of the pair (f, g) implies that there exists a sequence $\{y_n\}$ in X satisfying $lim_n fy_n = lim_n gy_n = v$ such that $lim_n d(fgy_n, gfy_n) = 0$.

As pair (f, g) is also conditionally reciprocally continuous,
$lim_n fgy_n = fv$ and $lim_n gfy_n = gv$.
Hence $fv = gv$, i.e. f and g have a coincidence point.
Further, since the pair (f, g) is faintly compatible, $fgv = gfv$.
Hence $ffv = fgv = gfv = ggv$.
Now we claim $fv = ffv$. If not, using (1) we get
$d(fv, ffv) \leq \lambda\ max\ \{d(gv, gfv), d(fv, gv), d(ffv, gfv), d(fv, gfv), d(ffv, gv)\}$,
i.e. $d(fv, ffv) \leq \lambda\ d(fv, ffv)$, a contradiction.
So $fv = ffv = gfv$, i.e. fv is a common fixed point of f and g.
The uniqueness of the common fixed point is an easy consequence of the condition (1).
□

Example 3.1. *Let $X = [0, 10]$ and d be the usual metric on X . Let the pair of self map (f, g) of X be defined as*

$$fx = \begin{cases} 1, \ if\ x \leq 1 \\ \frac{x+7}{2} \quad, \ if\ 1 < x \leq 10 \end{cases}$$

$$and\ gx = \begin{cases} 2 - x, \ if\ x \leq 1 \\ \frac{x+10}{2} \quad, \ if\ 1 < x \leq 10 \end{cases}$$

Then one may verify that f and g satisfy condition (1) of theorem (3.1) for $0 \leq \lambda < 1$. Let $\{x_n\}$ be a sequence in X where $x_n = 1 - \frac{1}{n}$, then the pair of self map (f, g) satisfies the property (E.A.), since $lim_n fx_n = lim_n gx_n = 1$.
With $y_n = 1 \in X$, we get $lim_n fy_n = lim_n gy_n = 1$ and $lim_n fgy_n = lim_n gfy_n = 1$, i.e. $lim_n d(fgy_n, gfy_n) = 0$, so the pair (f, g) is conditionally compatible. Also f and g commute at the only coincidence point $x = 1 \in X$. Therefore, the pair of self maps (f, g) is faintly compatible.
Further $lim_n gf1 = 1 = g1$ and $lim_n fg1 = 1 = f1$, i.e. f and g are conditionally reciprocally continuous.
Hence f and g satisfies all the conditions of theorem $3 \cdot 1$ and have a unique common fixed point at $x = 1$. Moreover both the self maps are discontinuous and a pair (f, g) is neither compatible nor reciprocally continuous as $lim_n fgx_n = 4 \neq f4$ and $lim_n gfx_n = 1 = g1$ and hence $lim_n fgx_n \neq lim_n gfx_n$. Further $fX \not\subset gX$.

Theorem 3.2. *Let conditionally reciprocally continuous, faintly compatible pair of self maps (f, g) of a metric space (X, d) satisfy the property (E.A.). Then f and g have a coincidence point. Moreover f and g have a common fixed point provided that the pair of self maps satisfies:*

$$d(fx, fy) < max\{d(gx, gy), d(fx, gx), d(fy, gy), d(fx, gy), d(fy, gx)\}, \ x, y \in X. \tag{2}$$

Proof. Proof of Theorem $3 \cdot 2$ follows on the similar lines as of Theorem $3 \cdot 1$. □

Now we prove a result, which is different than the common fixed point theorems for contractive type as well as non-expansive and Lipschitz-type pair of maps. While contractive type pairs of maps cannot possess more than one common fixed point, this theorem admits the possibility of more than one common fixed point.

Theorem 3.3. *Let conditionally reciprocally continuous, faintly compatible pair of self maps of (f,g) a metric space (X,d) satisfy the property (E.A.). Then f and g have a* *coincidence point. Moreover f and g have a common fixed point provided that the pair of self maps (f, g) satisfies:*

$$d(gx, ggx) \neq max\{d(gx, fgx), d(fgx, ggx)\}, \text{ whenever the right hand side is non zero.} \quad (3)$$

Proof. On the similar lines as in Theorem $3 \cdot 1$ we may prove that $ffv = fgv = gfv = ggv$.
Now we claim $gv = ggv$. if $gv \neq ggv$,
From (4), $d(gv, ggv) \neq max\{d(gv, fgv), d(fgv, ggv)\}$,
i.e. $d(gv, ggv) \neq d(gv, ggv)$, a contradiction. So $gv = ggv = fgv$, i.e. gv is a common fixed point of f and g. This completes the proof of the theorem. □

Now we furnish an example to demonstrate the fact that the notion of faint compatibility allows the existence of multiple common fixed points or coincidence points .

Example 3.2. *Let $X = [0, 1]$ and d be the usual metric on X and the pair of self map (f, g) of X be defined as*

$$fx = \begin{cases} x, \ if \ x \ \leq \ \frac{1}{2} \\ \frac{x+1}{2}, \quad if \ \frac{1}{2} < x \ \leq 1. \end{cases}$$

$$and \ gx = \begin{cases} 1 - x, \ if \ x \leq \frac{1}{2} \\ 1, \quad if \ \frac{1}{2} < x \ \leq 1. \end{cases}$$

Then one may verify that f and g satisfy the condition (3) of theorem $(3 \cdot 3)$. Let $\{x_n\}$ be a sequence in X where $x_n = \frac{1}{2}$-$\frac{1}{n}$, then the pair of self map (f, g) satisfies the property (E.A.), since $lim_n fx_n = lim_n gx_n = \frac{1}{2}$.
With $y_n = 1 - \frac{1}{n} \in X$, we get $lim_n fy_n = lim_n gy_n = 1$ and $lim_n fgy_n = lim_n gfy_n = 1$ i.e. $lim_n d(fgy_n, gfy_n) = 0$, so the pair (f, g) is conditionally compatible. Also f and g commute at both the coincidence points $x = 1$ and $x = \frac{1}{2} \in X$. Therefore, the pair of self maps (f, g) is faintly compatible.
Further $lim_n gfy_n = 1 = g1$ and $lim_n fgy_n = 1 = f1$, i.e. f and g are conditionally reciprocally continuous.
Hence f and g satisfy all the conditions of theorem $(3 \cdot 3)$ and have two common fixed points at $x = 1$ and $x = \frac{1}{2}$. Moreover both the self maps are discontinuous and a pair (f, g) is neither compatible nor reciprocally continuous as $lim_n fgx_n = \frac{3}{4} \neq f\frac{1}{2}$ and $lim_n gfx_n = \frac{1}{2} = g\frac{1}{2}$ and hence $lim_n fgx_n \neq gfx_n$. Further $fX \not\subset gX$.

The notions of property (E.A.) and property $(CLRg)$ are suitable for studying common fixed points of a pair of maps satisfying strict contractive, nonexpansive or Lipschitz type conditions in a metric spaces, which are not even complete. Now we prove our next result using property $(CLRg)$ and weak compatibility.

Theorem 3.4. *Let a pair of self maps (f, g) of a metric space (X, d) satisfying the common limit in the range of g property $(CLRg)$satisfy:*

$$d(fx, fy) \leq \lambda \ max\{d(gx, gy), d(fx, gx), d(fy, gy), d(fx, gy), d(fy, gx)\}, \ 0 \leq \lambda < 1 \ . \quad (1)$$

Then f and g have a coincidence point. Moreover f and g have a unique common fixed point provided that the pair of selfmaps is weakly compatible.

Proof. Since the pair of self maps (f, g) satisfies the property $(CLRg)$, there exists a sequence $\{x_n\}$ in X such that $lim_n fx_n = lim_n gx_n = gx$ for some $x \in X$.
Taking $x = x_n$ and $y = x$ in (1)
$d(fx_n, fx) \leq \lambda \ max\{d(gx_n, gx), d(fx_n, gx_n), d(fx, gx), d(fx_n, gx), d(fx, gx_n)\}$.

As $n \to \infty$, $d(gx, fx) \leq \lambda\ d(fx, gx)$, a contradiction.
Hence $fx = gx$, i.e. f and g have a coincidence point.
Let $z = fx = gx$. So weak compatibility of a pair of map implies $fz = fgx = gfx = gz$.
Now we claim $fz = z$. If not, using (1) we have
$d(fz, z) = d(fz, fx)$
$\leq \lambda\ max\{d(gz, gx), d(fz, gz), d(fx, gx), d(fz, gx), d(fx, gz)\}$,
$= \lambda\ d(fz, z)$, a contradiction, i.e. z is a common fixed point of f and g.
The uniqueness of the common fixed point is an easy consequence of the condition (1).
□

Example 3.3. *Let $X = (0, 5)$ and d be the usual metric on X. Let the pair of self map (f, g) of X be defined as*

$$fx = \begin{cases} 1, & if\ 0 < x\ \leq 1 \\ 2, & if\ x \geq 1. \end{cases}$$

$$and\ gx = \begin{cases} 2 - x, & if\ 0 < x\ \leq 1 \\ 4, & if\ x \geq 1. \end{cases}$$

Then one may verify that f and g satisfy the condition (1) of theorem (3.4). Let $\{x_n\}$ be a sequence in X where $x_n = 1$ then the pair of self map (f, g) satisfies the property $(CLRg)$, as $lim_n fx_n = lim_n gx_n = 1 = g1$ for $1 \in X$.
f and g commute at the coincidence point $x = 1 \in X$ such that $fg1 = gf1$.Therefore, the pair of self maps (f, g) is weakly compatible.
Hence, f and g satisfy all the conditions of theorem (3.4) and have a unique common fixed point at $x = 1$. Moreover both the self maps are discontinuous and a pair (f, g) is neither compatible nor reciprocally continuous as there exist a sequence $y_n = 1 - \frac{1}{n}$ such that $lim_n fy_n = lim_n gy_n = 1$ and $lim_n fgy_n = 2 \neq f1$ and $lim_n gfy_n = 1 = g1$, so $lim_n d(fgy_n, gfy_n) \neq 0$. Further $fX \not\subset gX$.

Remark 3.1. *(i) Theorem $3 \cdot 1$, $3 \cdot 2$ and $3 \cdot 3$ generalize and improve the results of Bisht and Shahzad [3], which is demonstrated well by a suitable example $3 \cdot 1$ and $3 \cdot 2$. Theorems $3 \cdot 1$, $3 \cdot 2$ and $3 \cdot 3$ reveal the prominence of conditional reciprocal continuity over continuity of single map when the given pair of maps is not even compatible and marks supremacy over all those results wherein the continuity of even single map, containment of range space of involved maps and completeness (or closedness) of the whole space/subspaces are assumed for the existence of coincidence point (or common fixed point).*
(ii) Ćirić type strict contractive condition used to establish coincidence and unique common fixed point, is more general than, used by Bisht and Shahzad [3]. However it is believed that the strict contractive condition does not guarantee the existence of common fixed points without taking the space to be compact/complete or assuming some sequence of iterates to be a Cauchy sequence.

Remark 3.2. *It may be observed that we have established Theorem $3 \cdot 3$ without continuity of even single map for more general condition (non-contractive) (3) which include contractive type as well as non-expansive and Lipschitz-type condition. Moreover none of the map is continuous in the illustrating example $3 \cdot 3$. Hence our results generalize, extend and improve the several well-known results existing in literature.*

Remark 3.3. *Theorem $3 \cdot 3$ remain true if one replaces inequality $(3 \cdot 3)$ by any one of the following conditions*
(a) $d(fx, fgx) \neq d(gx, gfx)$;
(b) $d(x, fx) \neq max\{d(x, gx), d(fx, gx)\}$;

(c) $d(x, gx) \neq max\{d(x, fx), d(fx, gx)\}$;
(d) $d(gx, ggx) \neq max\{d(gx, fgx), d(fgx, ggx)\}$;
(e) $d(fx, ffx) \neq max\{d(gx, gfx), d(fx, gx), d(ffx, gfx), d(fx, gfx), d(gx, ffx)\}$;
(f) $d(gx, ggx) \neq max\{d(fx, fgx), d(fx, gx), d(ggx, fgx), d(gx, fgx), d(fx, ggx)\}$; *whenever the right-hand side is non-zero.*

Conclusion. Weak compatibility is most widely used concept among all weaker forms of commuting maps and remains the minimal condition of commutativity for the existence of common fixed point for a long time. For a development of weaker forms of commuting maps and relationship between them one may refer to Singh and Tomar [15]. It is worth mentioning here that the notion of weak compatibility is not applicable when a pair of self maps (f, g) has more than one coincidence points. Whereas the notion of faint compatibility allows the existence of a common fixed point or multiple common fixed points or coincidence points under contractive, strict contractive as well as non-contractive conditions. Nevertheless contractivity of maps is not sufficient for the existence of fixed point. For instance: If $X = [0, \infty)$ and $fx = x + e^{-2x}$, then obviously f is contractive but do not have a fixed point. In such cases either the space is taken to be complete or compact, some sequence of iterates is presumed to be Cauchy sequence or some strong condition is presumed on the maps for the existence of fixed point.

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