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SOME FIXED POINT THEOREMS IN PARTIAL METRIC SPACES

MUKTI GANGOPADHYAY[1], MANTU SAHA[2], A.P. BAISNAB[3] §

Abstract. Here we prove two fixed point theorems on partial metric space, which was defined by S. Matthews [8] in 1994. In the literature one can find fixed point theorems proved on such spaces by using Picard iteration schemes. Here our main ingredient is Cantor intersection type results.

Keywords: Partial metric space, contraction mapping, fixed points.

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1. Introduction

Partial metric spaces were originally developed by S. Matthews [8] in 1994 to provide a mechanism to generalize a metric space; If (X, p) is a partial metric space, then $p(x, x)$ is not necessary zero as $x \in X$. Partial metric spaces as defined has now found vast applications in topological structures in the study of computer science, information science and in biological sciences. Banach contraction principle [2] is a fundamental result in fixed point theory in a complete metric space and the same has been extended in many directions like inviting broader class of mappings or by taking more generalized domain or by making a combination of both.

We would like to present two fixed point theorems in a partial metric spaces. We cite some relevant references like S.J. O'Neill work [9] and [10] on Mathew's notion of partial metric spaces. O. Valero and S. Oltra in [11] and [12] has also proved several generalizations of Banach fixed point theorem in the setting of partial metric spaces and in consequence one finds Matthew's findings are rendered special cases of those of O. Valero's. One is also refereed to the work of S.K Chaterjee in [5]. In the above mentioned works the authors had used Picard iteration scheme in the proof of fixed point theorems. In our investigations we have avoided that track and instead employed Cantor's intersection like theorems in the setting of a partial metric space. Thus our findings may be looked upon as an alternative method of proving some useful fixed point theorems in partial metric spaces and the involved mappings as they appear underneath.

2. some useful definitions and results

We start with defining partial metric space.

[1] Calcutta Girls B.T. College, 6/1, Swinhoe Street , Kolkata 700019, India
e-mail: muktigangopadhyay@yahoo.com
[2] Department of Mathematics, The University of Burdwan, Burdwan –713104, India.
e-mail: mantusaha@yahoo.com
[3] Lady Brabourne College, Kolkata, India.
e-mail: apb@yahoo.com
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Definition 2.1. *A partial metric (PM) denoted by* p *on a nonempty set is a function* $p : X \times X \to \mathbb{R}^+$ *(set of non-negative reals) such that for any* $x, y, z \in X$,

$(PM1)$ $\quad p(x,x) \leq p(x,y)$.

$(PM2)$ $\quad$ *If* $p(x,x) = p(x,y) = p(y,y)$, *then* $x = y$.

$(PM3)$ $\quad p(x,y) = p(y,x)$.

$(PM4)$ $\quad p(x,y) \leq p(x,z) + p(z,y) - p(z,z)$.

The pair (X,p) is called a partial metric space and will be denoted by PMS in short.

Clearly any metric space is always a partial metric space. If (X,p) is a partial metric space, then $d_p : X \times X \to \mathbb{R}$ given by $d_p(x,y) = 2p(x,y) - p(x,x) - p(z,z)$ for $x, y \in X$ is a metric on X. In a PMS (X,p) if $x \in X$ and $\varepsilon > 0$, the set $B_p(x,\varepsilon) = \{y \in X : p(x,y) < p(x,x) + \varepsilon\}$ is called a $p-$ open ball centered at x with radious ε. It can be easily verified that the family $\{B_p(x,\varepsilon)\}$ together with empty set forms a base for a topology τ_p on X which is T_0. Similarly, a $p-$ closed ball centered at x with radious ε is defined.

Example 2.1. *For a given positive integer* n, *let* $\wp$ *denotes the collection of all real polynomials like* $f(t) = a_0 + a_1 t + \ldots\ldots + a_n t^n, a_i \in \mathbb{R}$, $a_i \geq 0$ *with degree* $\leq n$. *If* $f_1, f_2 \in \wp$, *let*

$$p(f_1, f_2) = \max_i (a_i, b_i),$$

where a_i, b_i *are coefficients of the polynomials* f_1, f_2 *respectively. Then by routine verification we see that* $(\wp, p)$ *is a partial metric.*

Lemma 1. *A partial metric space* (X,p) *is first countable.*

Proof. For each rational $r(>0)$ let $B_r(x_0) = \{x \in X : p(x,x_0) < p(x_0,x_0) + r\}$. Then the family $\{B_r(x_0)\}, r > 0$ forms a neighbourhood base at x_0. So, (X,p) is first countable. □

We recall following well known definitions (viz. [3] and [6]). From now on (X,p) will always denote a PMS.

Definition 2.2. *(i) A sequence* $\{x_n\}$ *in* (X,p) *is said to converge to a point* $x \in X$ *if and only if*

$$\lim_{n\to\infty} p(x_n, x) = p(x,x).$$

(ii) A sequence $\{x_n\}$ *in* (X,p) *converges properly to a point* $x \in X$ *if and only if*

$$p(x,x) = \lim_{n\to\infty} p(x_n,x_n) = \lim_{n\to\infty} p(x,x_n).$$

That is to say if and only if

$$\lim_{n\to\infty} p^s(x,x_n) = 0 \text{ where } p^s(x,y) = 2p(x,y) - p(x,x) - p(y,y).$$

(iii) A sequence $\{x_n\}$ *in* (X,p) *is called a Cauchy sequence if*

$$\lim_{n\to\infty} p(x_n, x_m)$$

exists finitely.

(iv) (X,p) *is said to be complete if every Cauchy sequence* $\{x_n\}$ *in* X *converges to a point* $x \in X$, *i.e.,*

$$p(x,x) = \lim_{n\to\infty} p(x_n, x_m).$$

(v) A subset B *of* (X,p) *is called bounded if there is a positive number* K *such that* $p(u,v) \leq K$ *for all* $u, v \in B$.

(vi) Diameter of a bounded set B *is defined as*

$$\textit{diameter } (B) = \sup_{u,v\in B} p(u,v).$$

The following lemma is obvious.

Lemma 2. *(i) A sequence $\{x_n\}$ is Cauchy in (X,p) if and only if $\{x_n\}$ is Cauchy in the metric space (X,p^s).*

(ii) (X,p) is complete if and only if the metric space (X,p^s) is complete.

3. Results

We start this section with stating the following result.

Theorem 1. *A necessary and sufficient condition for a PMS (X,p) to complete is that every nested sequence of nonempty closed subsets $\{G_n\}$ in (X,p) with diameter $(G_n) \to 0$ as $n \to \infty$ has*

$$\bigcap_{n=1}^{\infty} G_n$$

as a singleton.

We prove the following lemma and then the proof of the above theorem (using the lemma) run parallel to classical Cantor's theorem in a usual metric space. Hence we omit the proof of the theorem.

Lemma 3. *If G is a non empty subset of (X,p) then diameter(G) = diameter($\bar{G}$), where $\bar{G}$ denotes τ_p- closure of G in (X,p).*

Proof. Clearly,

$$\text{diameter}(G) \leq \text{diameter}(\bar{G}). \tag{1}$$

Let $\varepsilon > 0$ be arbitarary. If $a, b \in \bar{G}$, we find $u, v \in G$ such that $p(u,a) < \varepsilon/2$ and $P(v,b) < \varepsilon/2$. Now

$$\begin{aligned} p(a,b) &\leq p(a,u) + p(b,u) - p(u,u) \\ &\leq p(a,u) + p(u,v) + p(v,b) - p(v,v) - p(u,u) \\ &\leq p(a,u) + p(u,v) + p(v,b) \\ &< \varepsilon/2 + \varepsilon/2 + p(u,v) \\ &= \varepsilon + p(u,v). \end{aligned}$$

This gives

$$p(a,b) \leq \varepsilon + \text{ diameter } (G)$$

and hence

$$\sup_{(a,b)\in\bar{G}} p(a,b) \leq \varepsilon + \text{ diameter } (G).$$

Thus diameter($\bar{G}$) $\leq \varepsilon +$ diameter(G) and hence one has

$$\text{diameter } (\bar{G}) \leq \text{ diameter } (G). \tag{2}$$

Now we have the result from (1) and (2). □

Now applying theorem 1 we prove the following two fixed point results.

Theorem 2. *Let (X,p) be a partial metric space and $f : X \to X$ satisfy the following*

$$p(f(x), f(y)) \leq \alpha p(x, f(x)) + \beta p(y, (f(y))) + \gamma p(x,y)$$

with $\alpha + \beta + \gamma < 1$ and $0 \leq \alpha, \beta, \gamma \forall x, y \in X$. Then f has a unique fixed point in X.

Proof. If x_0 is an arbitrary point in X and

$$x_n = f^n(x_0), n = 1, 2, \ldots\ldots\ldots, (x_0 = f^0(x_0)).$$

Then,

$$\begin{aligned} p(x_2, x_1) &= p(f(x_1), f(x_2)) \\ &\leq \alpha p(x_1, x_2) + \beta p(x_0, x_1) + \gamma p(x_0, x_1). \\ &\leq \frac{\beta + \gamma}{1 - \alpha} p(x_0, x_1). \end{aligned}$$

Similarly,

$$p(x_3, x_2) \leq \left(\frac{\beta+\gamma}{1-\alpha}\right)^2 p(x_0, x_1).$$

By induction,

$$p(x_n, x_{n+1}) \leq \left(\frac{\beta+\gamma}{1-\alpha}\right)^n p(x_0, x_1).$$

Hence

$$p(x_n, x_{n+1}) = \delta^n p(x_0, f(x_0)), \tag{3}$$

where $\delta = \frac{\beta+\gamma}{1-\alpha} < 1$. Therefore,

$$\lim_{n\to\infty} \delta^n = 0.$$

Let h_k be a sequence of positive real numbers so that

$$\lim_{k\to\infty} h_k = 0.$$

Without loss of generality we assume $h_k \geq h_{k+1} \geq \ldots$. Put $G_k = \{x \in X : p(x, f(x)) \leq h_k\}$. Now from 3 it follows that for large $k, G_k \neq \varphi$ for all k's. Clearly $\{G_k\}$ forms a decreasing chain of nonempty sets in X. We show that f maps G_k into itself. If $x \in G_k$ then

$$p(f(x), f(f(x))) \leq \alpha p(x, f(x)) + \beta p(f(x), f(f(x))) + \gamma p(x, f(x)).$$

This implies that,

$$p(f(x), f(f(x)) \leq \frac{\alpha+\gamma}{1-\alpha} p(x, f(x)) \leq \frac{\alpha+\gamma}{1-\alpha} h_k < h_k.$$

Hence $p : G_k \to G_k$.

Next we show that each G_k is closed. Let u be a limit point of G_k. Since a PMS is a first countable space, there is a sequence $\{x_j\}$ in G_k such that

$$\lim_{j} x_j = u.$$

Now we have

$$\begin{aligned}
p(u, f(u)) &\leq p(u, f(u)) + p(x_j, f(u)) - p(x_j, x_j) \\
&\leq p(u, x_j) + p(x_j, f(x_j)) + p(f(x_j), f(f(u)) - p((f(x_j), f(x_j)) \\
&\leq p(u, x_j) + p(x_j, f(x_j)) + p(f(x_j), f(f(u)). \\
(1-\beta)p(u, f(u)) &\leq h_k + \alpha h_k + (1+\alpha)p(x_j, u) \\
&\leq (\alpha+1)h_k + (1+\gamma)p(x_j, u). \\
p(u, f(u)) &\leq \frac{\alpha+1}{1-\beta} h_k + \frac{1+\alpha}{1-\beta} p(x_j, u).
\end{aligned}$$

Passing on to the limit $j \to \infty$, we find $p(u, f(u)) \leq \frac{\alpha+1}{1-\beta} h_k$. Since

$$\lim_{j\to\infty} p(x_j, u) = 0 \text{ (using lemma 1.5 in [1]).}$$

Now $0 \leq \alpha, \beta, \gamma < 1$ and $\alpha + \beta + \gamma < 1$ gives $\frac{\alpha+\gamma}{1-\beta} < 1$ and so

$$\sup_{\gamma} \left\{\frac{\alpha+\gamma}{1-\beta}\right\} \leq 1$$

and hence $\frac{\alpha+1}{1-\beta} \leq 1$. Thus $p(u, f(u)) \leq h_k$, and hence $u \in G_k$ and so G_k is closed.

Finally we need to show G_k is bounded and diameter $(G_k) \to 0$ as $k \to \infty$. Let $u, v \in G_k$ then,

$$\begin{aligned}
p(u, v) &\leq p(u, f(u)) + p(f(u), v) - p(f(u), f(u)) \\
&\leq h_k + p(f(u), f(v)) + p(v, f(v)) - p(f(v), f(v)) \\
&\leq 2h_k + \alpha p(u, f(u)) + \beta p(v, f(v)) + \gamma p(u, v).
\end{aligned}$$

Hence,

$$\begin{aligned} p(u,v) &\leq (2+\alpha+\beta)h_k \\ &\leq \frac{\alpha+\beta+2}{1-\gamma}h_k. \end{aligned}$$

Hence G_k is bounded and diameter $G_k \to 0$ as $k \to \infty$.

Thus $\{G_k\}$ is a decreasing chain of nonempty closed sets in (X,p) with diameter $(G_k) \to 0$ as $k \to \infty$. By Cantor's type intersection theorem 1,

$$\bigcap_{k=1}^{\infty} G_k$$

is a singleton, say u for some $u \in X$. Hence $f(u) = u$. Uniqueness of u is clear from the argument and thus the proof is complete. □

We have a couple of interesting corollaries.

Corollary 1. *If f is a contraction mapping from a complete PMS X into itself, then f has a unique fixed point in X*

(Taking $\alpha = \beta = 0$ and $0 < \gamma < 1$ in theorem 2 the above corollary will follow.)

Corollary 2. *If f is a Kannan type* [7] *mapping from a complete PMS X into itself, then f has a unique fixed point in X.*

(By taking $\alpha = \beta$ and $\gamma = 0$ in theorem 2 the above corollary will follow.)

Theorem 3. *If (X,p) is a complete PMS and $f : X \to X$ satisfy the condition that $p(f(x), f(y)) \leq \varphi[\max\{p(x,y), p(x,f(x)), p(y,f(y))\}]$ for all $x, y \in X$ where φ is an upper semi continuous function from right from $\mathbb{R}^+$ to $\mathbb{R}^+$ such that*

$$\underset{t>0}{Sup}\frac{t}{t-\varphi(t)} < \infty \text{ and } \varphi(t) \neq t.$$

Then f has a fixed point in X.

We first prove a lemma and using the lemma we prove theorem 3.

Lemma 4. *If $\alpha_n = p(x_n, x_{n+1})$ where $x_n = f^n(x), x \in X$, then*

$$\lim_{n\to\infty} \alpha_n = 0.$$

Proof. Suppose $\alpha_n > 0$ for all n. Then

$$\begin{aligned} \alpha_n &= p(x_n, x_{n+1}) = p(f(x_{n_1}), f(x_n)) \\ &\leq \varphi[\max\{p(x_{n-1}, x_n), p(x_{n-1}, f(x_{n-1}), p(x_n, f(x_n))\}] \\ &= \varphi[\max\{p(x_{n-1}, x_n), p(x_{n-1}, x_n), p(x_n, x_{n+1})\}] \\ &= \varphi[\max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\}]. \end{aligned}$$

If the maximum value of φ equals $p(x_n, x_{n+1})$, then one has $\alpha_n \leq \varphi(\alpha_{n-1}) < \alpha_{n-1}$. That means $\{\alpha_n\}$ is strictly decreasing and let

$$\lim_{n\to\infty} \alpha_n = \alpha.$$

If $\alpha > 0$ then $\varphi(\alpha) < \alpha$. Since φ is upper semi continuous from right we get,

$$\overline{\lim}\varphi(\alpha_n) \leq \varphi\left(\overline{\lim_n}\alpha_n\right) = \varphi(\alpha) < \alpha.$$

Which contradicts the fact that $\alpha_n = \varphi(\alpha_{n-1})$ for all n. Hence one concludes

$$\lim_n \alpha_n = \alpha = 0.$$

□

Proof. (Proof of theorem 3) Let $G_n = \left\{x \in X : p(x, f(x)) \leq \frac{1}{n}\right\}$ and as usual Φ denotes the null set. We show that for large values of $n, G_n \neq \Phi$. We take $x \in X$ and consider $x_n = f^n(x)$. Now lemma 4 says that $p(x_n, x_{n+1}) \to 0$ as $n \to \infty$. So, we get

$$\lim_{n\to\infty} p(x_n, x_{n+1}) = 0.$$

Thus given G_k it follows that $p(x_n, f(x_n)) < \frac{1}{k}$ for large values of n. Hence $G_k \neq \Phi$. We verify that f maps G_n into G_n, and for that take $x \in G_n$. Then

$$\begin{aligned} p(f(x), f(f(x))) &\leq \varphi[\max\{p(x, f(x))p(f(x), f(f(x))), p(x, f(x))\}] \\ &= \varphi[\max\{p(x, f(x)), p(f(x), f(f(x)))\}]. \end{aligned}$$

Now if the maximum value equals to $p(f(x), f(f(x))$, then

$$p(f(x), f(f(x)) \leq \varphi\{p(f(x), f(f(x)))\} < p(f(x), f(f(x))),$$

a contradiction. Therefore

$$p(f(x), f(f(x))) \leq \varphi(p(x, f(x))) < p(x, f(x)) \leq \frac{1}{n}.$$

Hence $f(x) \in G_n$. Thus f maps G_n into itself. Now we check that each G_n is closed in (X, p). Let $\{x_{n_k}\} \in G_n$ satisfy

$$p - \lim_{k\to\infty} x_{n_k} = x_0 \in X$$

or,

$$\lim_{k\to\infty} p(x_{n_k}, x_0) = 0.$$

Clearly, $p(x_{n_k}, f(x_{n_k})) \leq \frac{1}{n}$ for all k. Now,

$$\begin{aligned} &p(x_0, f(x_0)) \leq p(x_0, x_{n_k}) + p(f(x_0), x_{n_k}) - p(x_{n_k}, x_{n_k}) \\ &\leq p(x_0, x_{n_k}) + p(f(x_0), x_{n_k}) \\ &\leq p(x_0, x_{n_k}) + p(f(x_0), f(x_{n_k-1})) \\ &\leq p(x_0, x_{n_k}) + \varphi[\max\{p(x_{n_k-1}, x_0), p(x_{n_k-1}, x_{n_k}), p(x_0, f(x_0))\}]. \end{aligned}$$

Since

$$\lim_{k\to\infty} p(x_0, x_{n_k}) = p(x_0, x_0) = \lim_{k\to\infty} p(x_{n_k-1}, x_0) = 0,$$

and

$$p(x_{n_k-1}, x_{n_k}) \leq \frac{1}{n},$$

we have

$$p(x_0, f(x_0)) \leq \varphi[\max\left\{\frac{1}{n}, p(x_0, f(x_0))\right\}].$$

If $\max\left\{\frac{1}{n}, p(x_0, f(x_0))\right\} = p(x_0, f(x_0))$ then

$$p(x_0, f(x_0)) \leq \varphi\{p(x_0, f(x_0)\} < p(x_0, f(x_0)),$$

which is untenable, so we conclude that

$$p(x_0, f(x_0)) \leq \varphi\left\{\frac{1}{n}\right\} < \frac{1}{n}.$$

Thus $x_0 \in G_n$ and G_n is shown to be closed. Next we show that G_n is bounded. For $x, y \in G_n$, Clearly $p(x, f(x)) \leq \frac{1}{n}$;$p(y, f(y)) \leq \frac{1}{n}$. Thus we have

$$\begin{aligned} p(x, y) &\leq p(x, f(x)) + p(y, f(x)) - p(f(x), f(x)) \\ &\leq p(x, f(x)) + p(y, f(y)) + p(f(x), f(y)) - p(f(y), f(y)) \\ &\leq p(x, f(x)) + p(y, f(y)) + p(f(x), f(y)) \\ &\leq \frac{2}{n} + \varphi \max\left\{p(x, y), \frac{1}{n}, \frac{1}{n}\right\}. \end{aligned}$$

If $\max\left\{p(x,y)\frac{1}{n}\right\} = \frac{1}{n}$, then $p(x,y) \leq \frac{2}{n} + \varphi\left(\frac{2}{n}\right) < \frac{2}{n} + \frac{1}{n} = \frac{3}{n}$. Thus in this case G_n is bounded with diameter $(G_n) \leq \left\{\frac{3}{n}\right\} \to 0$ as $n \to \infty$. Or,

$$p(x,y) \leq \frac{2}{n} + \varphi(p(x,y)).$$

Else, $\max\left\{p(x,y), \frac{1}{n}\right\} = p(x,y)$, then

$$p(x,y)\left(1 - \frac{\varphi(p(x,y))}{p(x,y)}\right) \leq \frac{2}{n}.$$

Thus,

$$p(x,y) \leq \frac{2}{n}\frac{p(x,y)}{p(x,y) - \varphi(p(x,y))}.$$

Now,

$$\frac{2}{n}\sup_{t>0}\frac{t}{t-\varphi(t)} = \frac{2}{n}R,$$

where

$$\sup_{t>0}\frac{t}{t-\varphi(t)} = R < \infty.$$

So, diameter $(G_n) \leq \frac{2R}{n}$ and hence G_n is bounded and diameter $(G_n) \to 0$ as $n \to \infty$. Thus $\{G_n\}$ is a decreasing chain of nonempty closed sets in (X,p) with diameter $(G_n) \to 0$ as $n \to \infty$. By theorem 3 we have,

$$\bigcap_{n=1}^{\infty} G_n$$

is a singleton, say $\{u\}$ for some $u \in X$. Thus $f(u) = u$ and the proof is complete. □

4. Conclusion

Our findings as presented in this paper shall inspire researchers in fixed point theory to explore applications of Cantor type theorems as new contour instead of following Picard iterative schemes in generalized metric spaces like non symmetric metric spaces, partially ordered cone metric spaces etc. Operators may be invited there in form of a class strictly larger than those containing contractive type like Banach, Kannan and Ciric operator.

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M. Gangopadhyay received her Ph.D from the Department of Mathematics, The University of Burdwan, Burdwan, West Bengal, India. M.ed, Department of Education, The University of Burdwan, Burdwan, West Bengal, India. Employment History: Associate Professor in Mathematics Calcutta GirlsB.T College, 6/1 Swinhoe Street, Kolkata-19, West Bengal, India ,from 15.01.1993 to 31.08.13. Principal-Panihati Mahavidyalaya Degree college Sodepur, West Bengal, India. More than 10 research publication to the credit Doing research work in the area of Functional Analysis and General Topology.

Mantu Saha for photograph and biography, see Volume 3, No.1, 2013.

A.P. Baisnab is teaching in Post Graduate department of Mathematics in University of Burdwan, West Bengal (India) for more than four decades, and after superannuation in 2003 now engaged in P.G. teaching in Lady Brabourne College, Kolkata since 2005. More than a dozen students had already been awarded Ph.D degree under supervision of A.P. Baisnab in areas like functional analysis and general topology, and currently working with four more students in above areas, also got more than fifty research publications to the credit, and ii) approved more than four text books in mathematics in Higher secondary and graduate levels. Other activities: Already served and also at present serving in bound of studies in mathematics in several university in India