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SCATTERING BY A MOVING PEC PLANE AND A DIELECTRIC HALF-SPACE IN HERTZIAN ELECTRODYNAMICS

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Abstract. For a demonstration of the predictions of Hertzian Electrodynamics in scattering problems, a general formulation is provided which is followed by various applications of the presented methodology to 2-D canonical problems involving a Perfect Electrical Conductor (PEC) plane and a dielectric half-space in uniform and harmonic motions under plane wave incidence.

Keywords: Maxwell Equations, Moving Media, Hertz Equations, Continuum Mechanics, Frame Indifference, Progressive Derivatives.

AMS Subject Classification: 78A25, 35Q60, 74A05.

1. Introduction

With the wide acceptance of Special and General Relativity Theories (SRT, GRT) as experimental facts in early 20th century, problems of electromagnetic wave propagation and scattering for moving bodies have been handled and solved mostly in a relativistic frame in literature till date. While it is impossible to provide an entire account of the immense literature in this area, a considerable number of works with relevance to electromagnetics engineering applications handled in this work can be reached at ([1],[2],[3],[4],[5],[6],[7],[8]) and the references cited therein.

In the present work we provide a demonstration of the predictions of Hertzian Electrodynamics (HE) in scattering problems. It is structured as a sequel of [9] where we reviewed and extended certain aspects of the mathematical foundations, axiomatic structure and principles of HE. In Section 2 a general formulation of the scattering problems for material bodies with an arbitrary velocity is provided. Specifically, we consider the scenario in Figure 1 where, according to an observer in Cartesian reference configuration $Ox_1x_2x_3t$, the incident electromagnetic wave with fields $(\vec{E}_{inc}(\vec{r};t), \vec{H}_{inc}(\vec{r};t))$ and sources $(\rho_{Tx}(\vec{r};t), \vec{J}_{Tx}(\vec{r};t))$ generated by a stationary transmitter in an ambient medium I is impinging on an object occupying a region D and in arbitrary relative motion with instantaneous velocity $\vec{v}(\vec{r};t)$. This is followed in the subsequent sections by various applications of the presented methodology to 2-D canonical problems involving a Perfect Electrical Conductor (PEC) plane and a dielectric half-space in uniform and harmonic

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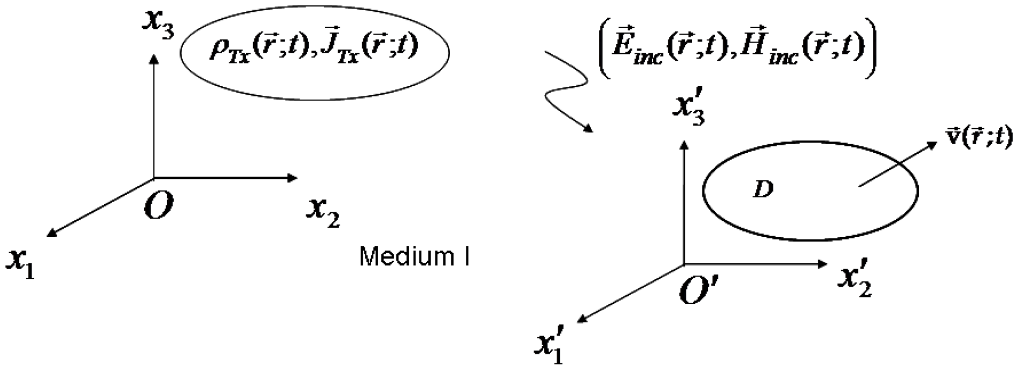

Figure 1. An illustration of a scattering problem

motions and under plane wave incidence. Finally, we provide a theoretical comparison of the results derived with HE and SRT for the special case of uniform motion. The reader is assumed to be already familiar with the terminology, definitions, postulates, theorems, etc. in [9] so that many of them shall not be repeated herein for practical reasons. In particular we shall frequently use the terms "E-frame" and "L-frame" as abbreviations of Eulerian and Lagrangian frames from fluid mechanics for denoting reference (spatial) and current (material) configurations for brevity.

2. The General Formulation of a Scattering Problem

The frame indifferent structure of Hertzian field equations require that the field expressions in E- frame in any scenario of moving bodies can be obtained via the maps of the end results obtained in the corresponding Maxwells theory of stationary media (in other words, in L- frame). Therefore we can solve a scattering problem from an isolated moving body formally by "frame hopping"[1] following the steps below:

I) Map the incoming field from E- to L-frame

II) Solve the scattered field from the associated boundary value problem in L-frame

III) Map the scattered field back from L- to E-frame

In constructing the boundary value problem in L-frame, the corresponding spatial/temporal jump and edge conditions are obtained from the distributional investigation of the field equations along with complementary conditions such as radiation condition, periodicity, boundedness, *etc.*

2.1. The Incoming Wave.

In E-frame the incident fields satisfy the Maxwell equations of stationary media

$$\text{curl}\vec{E}_{inc}(\vec{r};t) + \frac{\partial}{\partial t}\vec{B}_{inc}(\vec{r};t) = \vec{0},\ \text{curl}\vec{H}_{inc}(\vec{r};t) - \frac{\partial}{\partial t}\vec{D}_{inc}(\vec{r};t) = \vec{J}_{Tx}(\vec{r};t) \qquad (2.1\text{a,b})$$

$$\text{div}\vec{D}_{inc}(\vec{r};t) = \rho_{Tx}(\vec{r};t), \text{div}\vec{B}_{inc}(\vec{r};t) = 0 \qquad (2.1\text{c,d})$$

Let us assume medium I simple and lossless with constitutive parameters (ε, μ). Then the incident time domain (& phasor) fields in E-frame satisfy the stationary wave (&

[1] A methodology first employed by Van Bladel for relativistic scattering problems (cf.[1]).

Helmholtz) equations

$$\left(\text{lap} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\begin{pmatrix} \vec{E}_{inc}(\vec{r};t) \\ \vec{H}_{inc}(\vec{r};t) \end{pmatrix} = \begin{pmatrix} (1/\varepsilon)\,\text{grad}\rho_{Tx}(\vec{r};t) \\ \vec{0} \end{pmatrix} \tag{2.2a}$$

$$\left(\text{lap} + k^2\right)\begin{pmatrix} \vec{E}_{inc}(\vec{r}) \\ \vec{H}_{inc}(\vec{r}) \end{pmatrix} = \begin{pmatrix} (1/\varepsilon)\,\text{grad}\rho_{Tx}(\vec{r}) \\ \vec{0} \end{pmatrix} \tag{2.2b}$$

where $c = 1/\sqrt{\mu\varepsilon}$ is the phase velocity and $k = \omega_{inc}/c = 2\pi/\lambda$ is the wave number with angular frequency $\omega_{inc} = 2\pi f_{inc}$ and time dependence taken as $\exp(-i\omega_{inc}t)$. For an observer in L-frame $Ox_1'x_2'x_3't$, which is not necessarily Cartesian, the object is stationary and the surrounding medium I is in relative motion with an instantaneous velocity $\vec{v}'(\vec{r}';t)$. Accordingly, in L-frame the incident fields satisfy the Hertzian field and wave equations

$$\text{curl}'\vec{E}'_{inc}(\vec{r}';t) + \frac{\Diamond'}{\Diamond' t}\vec{B}'_{inc}(\vec{r}';t) = \vec{0},\ \text{curl}'\vec{H}'_{inc}(\vec{r}';t) - \frac{\Diamond'}{\Diamond' t}\vec{D}'_{inc}(\vec{r}';t) = \vec{J}'_{Tx}(\vec{r}';t) \tag{2.3a,b}$$

$$\text{div}'\vec{D}'_{inc}(\vec{r}';t) = \rho'_{Tx}(\vec{r}';t),\ \text{div}'\vec{B}'_{inc}(\vec{r}';t) = 0 \tag{2.3c,d}$$

$$\left(\text{lap}' - \frac{1}{c^2}\frac{\Diamond'^2}{\Diamond' t^2}\right)\begin{pmatrix} \vec{E}'_{inc}(\vec{r}';t) \\ \vec{H}'_{inc}(\vec{r}';t) \end{pmatrix} = \begin{pmatrix} (1/\varepsilon)\,\text{grad}'\rho'_{Tx}(\vec{r}';t) \\ \vec{0} \end{pmatrix}. \tag{2.4}$$

In (2.3) the comoving time derivative of a vector $\vec{A}'_{inc}(\vec{r}';t)$ is given by

$$\frac{\Diamond'}{\Diamond' t}\vec{A}'_{inc} = \frac{\partial}{\partial t}\vec{A}'_{inc} + \vec{v}'\cdot\text{grad}'\vec{A}'_{inc} - \vec{A}'_{inc}\cdot\text{grad}'\vec{v}' + \vec{A}'_{inc}\,\text{div}'\vec{v}'. \tag{2.5}$$

When the incident fields and sources in L-frame are monochromatic with an arbitrary time dependence, say $\exp(-i\omega'_{inc}t)$, then their phasors satisfy the reduced field equations (see [9], Section 7)

$$\text{curl}'\vec{E}'_{inc}(\vec{r}') - i\omega_{inc}\vec{B}'_{inc}(\vec{r}') = \vec{0},\ \text{curl}'\vec{H}'_{inc}(\vec{r}') + i\omega_{inc}\vec{D}'_{inc}(\vec{r}') = \vec{J}'_{Tx}(\vec{r}') \tag{2.6a,b}$$

$$\text{div}'\vec{D}'_{inc}(\vec{r}') = \rho'_{Tx}(\vec{r}'),\ \text{div}'\vec{B}'_{inc}(\vec{r}') = 0 \tag{2.6c,d}$$

and the Helmholtz equations

$$\left(\text{lap}' + k^2\right)\begin{pmatrix} \vec{E}'_{inc}(\vec{r}') \\ \vec{H}'_{inc}(\vec{r}') \end{pmatrix} = \begin{pmatrix} (1/\varepsilon)\,\text{grad}'\rho'_{Tx}(\vec{r}') \\ \vec{0} \end{pmatrix}. \tag{2.7}$$

2.2. **The Scattered Wave.** Let us express the total field in space in E- and L-frames respectively as

$$(\vec{E}_{tot}, \vec{H}_{tot}) = \begin{cases} (\vec{E}_{inc}, \vec{H}_{inc}) + (\vec{E}_{sc}, \vec{H}_{sc}), \text{in medium I} \\ (\vec{E}_d, \vec{H}_d), \text{in region D} \end{cases}$$

$$\text{and}\ \ (\vec{E}'_{tot}, \vec{H}'_{tot}) = \begin{cases} (\vec{E}'_{inc}, \vec{H}'_{inc}) + (\vec{E}'_{sc}, \vec{H}'_{sc}), \text{in medium I} \\ (\vec{E}'_d, \vec{H}'_d), \text{in region D} \end{cases}$$

In L-frame of the scattered wave, i.e., with reference to the motion of region D, an L-observer senses the entire space (constituting the ambient *source free* medium I and region D) in motion with instantaneous velocity $-\tilde{v}'(\vec{r}';t)$. Accordingly, the scattered fields in medium I satisfy the Hertzian equations

$$\text{curl}'\vec{E}'_{sc}(\vec{r}';t) + \frac{\bar{\Diamond}'}{\bar{\Diamond}' t}\vec{B}'_{sc}(\vec{r}';t) = \vec{0},\ \text{curl}'\vec{H}'_{sc}(\vec{r}';t) - \frac{\bar{\Diamond}'}{\bar{\Diamond}' t}\vec{D}'_{sc}(\vec{r}';t) = \vec{0} \tag{2.8a,b}$$

$$\text{div}'\vec{D}'_{sc}(\vec{r}';t) = 0,\ \text{div}'\vec{B}'_{sc}(\vec{r}';t) = 0 \tag{2.8c,d}$$

$$\left(\text{lap}' - \frac{1}{c^2}\frac{\bar{\Diamond}'^2}{\bar{\Diamond}'t^2}\right)\begin{pmatrix}\vec{E}'_{sc}(\vec{r}';t)\\ \vec{H}'_{sc}(\vec{r}';t)\end{pmatrix} = \vec{0} \tag{2.9}$$

where the accompanying comoving time derivative of a vector $\vec{A}'_{sc}$ is defined as

$$\frac{\bar{\Diamond}'}{\bar{\Diamond}'t}\vec{A}'_{sc} = \frac{\partial}{\partial t}\vec{A}'_{sc} - \vec{v}' \cdot \text{grad}'\vec{A}'_{sc} + \vec{A}'_{sc} \cdot \text{grad}'\vec{v}' - \vec{A}'_{sc}\,\text{div}'\,\vec{v}' \tag{2.10}$$

When the scattered fields in L-frame are monochromatic with an arbitrary time dependence, say $\exp(-i\omega'_{sc}t)$, then their phasors satisfy the Hertzian equations

$$\text{curl}\vec{E}_d(\vec{r}') - i\omega_{inc}\vec{B}_d(\vec{r}) = \vec{0},\ \text{curl}\vec{H}_d(\vec{r}) + i\omega_{inc}\vec{D}_d(\vec{r}) = \vec{J}_d(\vec{r}) \tag{2.11a,b}$$

$$\text{div}\vec{D}_d(\vec{r}) = \rho_d(\vec{r}),\ \text{div}\vec{B}_d(\vec{r}) = 0 \tag{2.11c,d}$$

and the Helmholtz equations

$$\left(\text{lap} + k_d^2\right)\begin{pmatrix}\vec{E}_d(\vec{r})\\ \vec{H}_d(\vec{r})\end{pmatrix} = \begin{pmatrix}(1/\varepsilon_d)\,\text{grad}\rho_d(\vec{r})\\ \vec{0}\end{pmatrix} \tag{2.12}$$

When the incident fields in L-frame are not monochromatic but *possess* arbitrary waveforms which can be expressed as a superposition of monochromatic components in terms of a Fourier series or integral representation, then their each (discrete or continuous) component satisfies (2.7) individually, while similar arguments also hold in (2.12), (2.16) and (2.21).

It should be noticed that the wave numbers k, k_d remain invariant in E- and L-frames. Since the canonical examples in the present investigation are restricted to rigid bodies with $\vec{v} = \vec{v}(t)$, one may set $\text{div}'\vec{v}' = 0$, $\text{div}\vec{v} = 0$, $\text{grad}\vec{v} = \bar{\bar{0}}$, $\text{grad}'\vec{v}' = \bar{\bar{0}}$ in (2.10) and (2.19).

2.3. **Total Field inside the Moving Object.** In L-frame of the fields $(\vec{E}'_d, \vec{H}'_d)$ and sources $(\rho'_d, \vec{J}'_d)$ inside the moving object, the region D is sensed as stationary since the ambient medium I is observed as source-free. Therefore in region D the field equations of stationary media

$$\text{curl}'\vec{E}'_d(\vec{r}';t) + \frac{\partial}{\partial t}\vec{B}'_d(\vec{r}';t) = \vec{0}, \text{curl}'\vec{H}'_d(\vec{r}';t) - \frac{\partial}{\partial t}\vec{D}'_d(\vec{r}';t) = \vec{J}'_d(\vec{r}';t) \tag{2.13a,b}$$

$$\text{div}'\vec{D}'_d(\vec{r}';t) = \rho'_d(\vec{r}';t), \text{div}'\vec{B}'_d(\vec{r}';t) = 0 \tag{2.13c,d}$$

are satisfied. When the region D simple with constitutive parameters $(\varepsilon_d, \mu_d, \sigma_d)$, (2.13) yield the stationary wave equations

$$\left(\text{lap}' - \frac{1}{c_d^2}\frac{\partial^2}{\partial t^2} - \sigma_d\mu_d\frac{\partial}{\partial t}\right)\begin{pmatrix}\vec{E}'_d(\vec{r}';t)\\ \vec{H}'_d(\vec{r}';t)\end{pmatrix} = \begin{pmatrix}(1/\varepsilon_d)\,\text{grad}'\rho'_d(\vec{r}';t)\\ \vec{0}\end{pmatrix}, \tag{2.14}$$

with $c_d = 1\big/\sqrt{\mu_d\varepsilon_d}$. When the transmitted fields in L-frame are monochromatic with an arbitrary time dependence, say $\exp(-i\omega'_d t)$, then their phasors satisfy the reduced field equations

$$\text{curl}'\vec{E}'_d(\vec{r}') - i\omega_{inc}\vec{B}'_d(\vec{r}') = \vec{0}, \text{curl}'\vec{H}'_d(\vec{r}') + i\omega_{inc}\vec{D}'_d(\vec{r}') = \vec{J}'_d(\vec{r}') \tag{2.15a,b}$$

$$\text{div}'\vec{D}'_d(\vec{r}') = \rho'_d(\vec{r}'), \text{div}'\vec{B}'_d(\vec{r}') = 0 \tag{2.15c,d}$$

and the Helmholtz equations

$$\left(\text{lap}' + k_d^2\right)\begin{pmatrix}\vec{E}'_d(\vec{r}')\\ \vec{H}'_d(\vec{r}')\end{pmatrix} = \begin{pmatrix}(1/\varepsilon_d)\,\text{grad}'\rho'_d(\vec{r}')\\ \vec{0}\end{pmatrix} \tag{2.16}$$

with $k_d^2 = \omega_{inc}^2 \varepsilon_d \mu_d + i\omega_{inc}\sigma_d\mu_d$. For E-observer the field and wave equations (2.13), (2.14) read

$$\mathrm{curl}\vec{E}_d(\vec{r};t) + \frac{\Diamond}{\Diamond t}\vec{B}_d(\vec{r};t) = \vec{0}, \mathrm{curl}\vec{H}_d(\vec{r};t) - \frac{\Diamond}{\Diamond t}\vec{D}_d(\vec{r};t) = \vec{J}_d(\vec{r};t) \quad (2.17\text{a,b})$$

$$\mathrm{div}\vec{D}_d(\vec{r};t) = \rho_d(\vec{r};t), \mathrm{div}\vec{B}_d(\vec{r};t) = 0 \quad (2.17\text{c,d})$$

$$\left(\mathrm{lap} - \frac{1}{c_d^2}\frac{\Diamond^2}{\Diamond t^2} - \sigma_d\mu_d\frac{\Diamond}{\Diamond t}\right)\begin{pmatrix} \vec{E}_d(\vec{r};t) \\ \vec{H}_d(\vec{r};t) \end{pmatrix} = \begin{pmatrix} (1/\varepsilon_d)\,\mathrm{grad}\rho_d(\vec{r};t) \\ \vec{0} \end{pmatrix}, \quad (2.18)$$

where the accompanying comoving time derivative of a vector $\vec{A}_d$ is defined as

$$\frac{\Diamond}{\Diamond t}\vec{A}_d = \frac{\partial}{\partial t}\vec{A}_d + \vec{\mathrm{v}}\cdot\mathrm{grad}\vec{A}_d - \vec{A}_d\cdot\mathrm{grad}\,\vec{\mathrm{v}} + \vec{A}_d\,\mathrm{div}\vec{\mathrm{v}}. \quad (2.19)$$

When the transmitted fields in E-frame are monochromatic with an arbitrary time dependence, say $\exp(-i\omega_{tr}t)$, then their phasors satisfy the reduced field equations

$$\mathrm{curl}\vec{E}_d(\vec{r}) - i\omega_{inc}\vec{B}_d(\vec{r}) = \vec{0}, \mathrm{curl}\vec{H}_d(\vec{r}) + i\omega_{inc}\vec{D}_d(\vec{r}) = \vec{J}_d(\vec{r}) \quad (2.20\text{a,b})$$

$$\mathrm{div}\vec{D}_d(\vec{r}) = \rho_d(\vec{r}), \mathrm{div}\vec{B}_d(\vec{r}) = 0 \quad (2.20\text{c,d})$$

and the Helmholtz equations

$$\left(\mathrm{lap} + k_d^2\right)\begin{pmatrix} \vec{E}_d(\vec{r}) \\ \vec{H}_d(\vec{r}) \end{pmatrix} = \begin{pmatrix} (1/\varepsilon_d)\,\mathrm{grad}\rho_d(\vec{r}) \\ \vec{0} \end{pmatrix}. \quad (2.21)$$

When the incident fields in L-frame are not monochromatic but possess *arbitrary* waveforms which can be expressed as a superposition of monochromatic components in terms of a Fourier series or integral representation, then their each (discrete or continuous) component satisfies (2.7) individually, while similar arguments also hold in (2.12), (2.16) and (2.21).

It should be noticed that the wave numbers k, k_d remain invariant in E- and L-frames. Since the canonical examples in the present investigation are restricted to rigid bodies with $\vec{\mathrm{v}} = \vec{\mathrm{v}}(t)$, one may set $\mathrm{div}'\,\vec{\mathrm{v}}'= 0$, $\mathrm{div}\vec{\mathrm{v}}= 0$, $\mathrm{grad}\,\vec{\mathrm{v}}=\bar{\bar{0}}$, $\mathrm{grad}'\,\vec{\mathrm{v}}'=\bar{\bar{0}}$ in (2.10) and (2.19).

2.4. **Boundary Relations on the Moving Object.** In the context of the scattering problems investigated in the subsequent sections we shall assume the enclosure $S = \partial D$ of the moving medium a simple interface, which might be a PEC or a dielectric interface supporting surface charges and currents $\rho'_S(\vec{r}'_S;t)$, $\vec{J}'_S(\vec{r}'_S;t)$. In these cases the distributional form of stationary field (Maxwell) equations in L- frame respectively read

$$\hat{n}'\times\left[\vec{E}'_{inc}(\vec{r}'_S;t) + \vec{E}'_{sc}(\vec{r}'_S;t)\right] = \vec{0} \quad (2.22\text{a})$$

$$\hat{n}'\times\left[\vec{H}'_{inc}(\vec{r}'_S;t) + \vec{H}'_{sc}(\vec{r}'_S;t)\right] = \vec{J}'_S(\vec{r}'_S;t) \quad (2.22\text{b})$$

$$\hat{n}'\cdot\left[\vec{D}'_{inc}(\vec{r}'_S;t) + \vec{D}'_{sc}(\vec{r}'_S;t)\right] = \rho'_S(\vec{r}'_S;t) \quad (2.22\text{c})$$

$$\hat{n}'\cdot\left[\vec{B}'_{inc}(\vec{r}'_S;t) + \vec{B}'_{sc}(\vec{r}'_S;t)\right] = 0 \quad (2.22\text{d})$$

and

$$\hat{n}' \times \left[\vec{E}'_{inc}(\vec{r}'_S;t) + \vec{E}'_{sc}(\vec{r}'_S;t)\right] = \hat{n}' \times \vec{E}'_d(\vec{r}'_S;t) \tag{2.23a}$$

$$\hat{n}' \times \left[\vec{H}'_{inc}(\vec{r}'_S;t) + \vec{H}'_{sc}(\vec{r}'_S;t)\right] = \hat{n}' \times \vec{H}'_d(\vec{r}'_S;t) \tag{2.23b}$$

$$\hat{n}' \cdot \left[\vec{D}'_{inc}(\vec{r}'_S;t) + \vec{D}'_{sc}(\vec{r}'_S;t)\right] = \hat{n}' \cdot \vec{D}'_d(\vec{r}'_S;t) \tag{2.23c}$$

$$\hat{n}' \cdot \left[\vec{B}'_{inc}(\vec{r}'_S;t) + \vec{B}'_{sc}(\vec{r}'_S;t)\right] = \hat{n}' \cdot \vec{B}'_d(\vec{r}'_S;t) \tag{2.23d}$$

Along with constitutive relations and radiation, edge, tip, periodicity, boundedness etc. type complementary conditions, the associated boundary value problem can be solved uniquely to yield the L-fields $(\vec{E}'_{sc}, \vec{H}'_{sc})$ and $(\vec{E}'_d, \vec{H}'_d)$, whose maps also yield the E-fields $(\vec{E}_{sc}, \vec{H}_{sc})$ and $(\vec{E}_d, \vec{H}_d)$.

3. TE Plane Wave Scattering by a Moving PEC Plane

In this section we shall investigate the scattering of uniform homogeneous TE plane waves by a PEC plane as depicted in Figure 2 for four different modes of motion of practical interest. Common to all cases is the expression of the incident wave in half-space

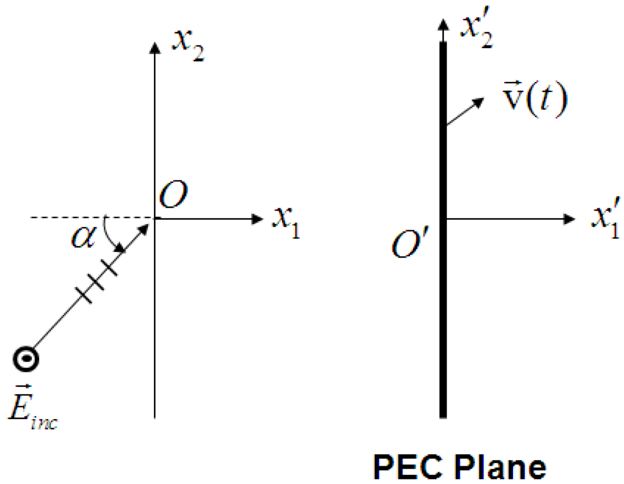

Figure 2. An illustration of TE plane wave scattering by a moving PEC plane

$x_1 < 0$ (medium I), which propagates along $\hat{n}_{inc} = (\cos\alpha, \sin\alpha)$ direction in (x_1, x_2) plane with fields represented by

$$\vec{E}_{inc}(\vec{r};t) = \hat{x}_3 f(\hat{n}_{inc} \cdot \vec{r} - ct), \; \vec{H}_{inc}(\vec{r};t) = (1/Z)\, \hat{n}_{inc} \times \vec{E}_{inc}(\vec{r};t) \tag{3.1a,b}$$

with $\hat{n}_{inc} \cdot \vec{r} = x_1 \cos\alpha + x_2 \sin\alpha$, where $\alpha \in [0, \pi/2)$ is the incidence angle and $Z = \sqrt{\mu/\varepsilon}$ stands for the characteristic impedance of lossless medium I. For the special case of a monochromatic source the incident electrical field is assumed to have the general form

$$\vec{E}_{inc}(\vec{r};t) = \hat{x}_3 g(k\hat{n}_{inc} \cdot \vec{r} - \omega_{inc} t) \tag{3.1c}$$

while (3.1b) still holds.

In the first two special cases below we carry out the investigation for general time harmonic and monochromatic waves simultaneously.

3.1. **Case I: Uniform Motion Parallel to the Plane.** For E-observer we assume the PEC plane at $x_1 = 0$ in uniform rectilinear motion with velocity $\vec{v} = G\hat{x}_2$, $G = const.$ G is assumed to take negative values for motion along $-\hat{x}_2$ direction.

For L-observer the PEC plane is stationary and it is half-space $-\hat{x}_2$ (medium I) moving with linear velocity $\vec{v}' = -G\hat{x}_2'$. Incorporating the Galilean transformations $x_2 = x_2' + Gt$, $x_{1,3} = x_{1,3}'$; $\hat{x}_i = \hat{x}_i'$, $i = 1, 2, 3$, the incoming fields in L-frame read

$$\begin{aligned} \vec{E}'_{inc}(\vec{r}'; t) &= \hat{x}_3' f(\hat{n}'_{inc} \cdot \vec{r}' - c'_{inc} t) = \hat{x}_3' g(k\hat{n}'_{inc} \cdot \vec{r}' - \omega'_{inc} t), \\ \vec{H}'_{inc}(\vec{r}'; t) &= (1/Z)\, \hat{n}'_{inc} \times \vec{E}'_{inc}(\vec{r}'; t) \end{aligned} \tag{3.2a,b}$$

with $\hat{n}'_{inc} \cdot \vec{r}' = x_1' \cos\alpha + x_2' \sin\alpha$ and

$$\begin{aligned} c'_{inc} &= c - G\sin\alpha = c(1 - \beta\sin\alpha), \omega'_{inc} = \omega_{inc}(1 - \beta\sin\alpha), \\ f'_{inc} &= f_{inc}(1 - \beta\sin\alpha) \end{aligned} \tag{3.2c-e}$$

where $\beta = G/c$. While the wave number k remains invariant in E- and L-frames as mentioned in Section 2, the phase velocity and the angular frequency of the incident wave as observed in L- frame are scaled proportionally, obeying $k = \omega_{inc}/c = \omega'_{inc}/c'_{inc}$.

One observes

$$c'_{inc}, \omega'_{inc}, f'_{inc} > 0, \text{ i.e., } 1 - \beta\sin\alpha > 0 \text{ or } \beta < 1/\sin\alpha \tag{3.3}$$

as a kinematical upper limit on for the the incident wave to catch the moving plane. (3.3) is satisfied for $\forall\alpha$, $\beta \le 1$ & $\alpha \in \left[0, \sin^{-1}(1/\beta)\right)$, $\beta \ge 1$. Unless this condition is satisfied the incident wave is observed to propagate away in L-frame in the direction $(-\cos\alpha, -\sin\alpha)$.

Let the scattered[2] field be given in the form

$$\begin{aligned} \vec{E}'_{sc}(\vec{r}'; t) &= \hat{x}_3' R_{TE} f(\hat{n}'_{sc} \cdot \vec{r}' - c'_{sc} t) = \hat{x}_3' R_{TE} g(k\hat{n}'_{sc} \cdot \vec{r}' - \omega'_{sc} t), \\ \vec{H}'_{sc}(\vec{r}'; t) &= (1/Z)\, \hat{n}'_{sc} \times \vec{E}'_{sc}(\vec{r}'; t) \end{aligned} \tag{3.4}$$

with $\hat{n}'_{sc} \cdot \vec{r}' = -x_1' \cos\alpha_{sc} + x_2' \sin\alpha_{sc}$. The unknown quantities c'_{sc}, α_{sc}, R_{TE} are solved from the boundary value problem

$$\left\{ \begin{array}{l} \left(\mathrm{lap}' - \frac{1}{c^2} \frac{\bar{\Diamond}'^2}{\Diamond' t^2}\right) \vec{E}'_{sc}(x_1', x_2'; t) = \vec{0}, \text{in medium I} \\ \text{Boundary Condition} : \vec{E}'_{inc}(0, x_2'; t) + \vec{E}'_{sc}(0, x_2'; t) = \vec{0}, \forall x_2', t \\ \text{Radiation Condition as } x_1' \to -\infty \end{array} \right. \tag{3.5a-c}$$

The wave equation (3.5a) in medium I, namely,

$$\begin{aligned} \left(\mathrm{lap}' - \frac{1}{c^2} \frac{\bar{\Diamond}'^2}{\bar{\Diamond}' t^2}\right) \vec{E}'_{sc} &= \hat{x}_3' \left(\mathrm{lap}' - \frac{1}{c^2}\left(\frac{\partial}{\partial t} + G\frac{\partial}{\partial x_2'}\right)^2\right) f(\hat{n}'_{sc} \cdot \vec{r}' - c'_{sc} t) \\ &= \hat{x}_3' \left(1 - \frac{1}{c^2}\left(-c'_{sc} + G\sin\alpha_{sc}\right)^2\right) f = \vec{0} \end{aligned}$$

requires

$$c'_{sc} = c(1 + \beta\sin\alpha_{sc}),\ \omega'_{sc} = \omega_{inc}(1 + \beta\sin\alpha_{sc}), \tag{3.6a,b}$$

while $\omega'_{sc}/c'_{sc} = k$. Boundary condition (3.5b) requires

$$f(x_2' \sin\alpha - c'_{inc} t) + R_{TE} f(x_2' \sin\alpha_{sc} - c'_{sc} t) = 0, \forall x_2', t \tag{3.7}$$

from which one uniquely obtains

[2]We use the terminology "scattering" in the general sense, by which a merely reflection mechanism should be understood in the cases scrutinized in Sections 3.1, 3.2, 4.1 and 4.2.

I) phase invariance:

$$dx_2'/dt = c_{inc}'/\sin\alpha = c_{sc}'/\sin\alpha_{sc} \tag{3.8a}$$

or

$$(1-\beta\sin\alpha)/\sin\alpha = (1+\beta\sin\alpha_{sc})/\sin\alpha_{sc} \tag{3.8b}$$

which reads

$$\sin\alpha_{sc} = \sin\alpha/(1-2\beta\sin\alpha) \tag{3.8c}$$

II)full reflection:

$$R_{TE} = -1 \tag{3.8d}$$

Substituting (3.8c) into (3.6) reads

$$c_{sc}' = c_{inc}'(1-2\beta\sin\alpha), \omega_{sc}' = \omega_{inc}'(1-2\beta\sin\alpha) \tag{3.8e,f}$$

The maps of $(\vec{E}_{sc}', \vec{H}_{sc}')$ into E-frame read

$$\begin{aligned}\vec{E}_{sc}(\vec{r};t) &= -\hat{x}_3 f(\hat{n}_{sc}\cdot\vec{r} - c_{sc}t) = -\hat{x}_3 g(k\hat{n}_{sc}\cdot\vec{r} - \omega_{sc}t),\\ \vec{H}_{sc}(\vec{r};t) &= (1/Z)\,\hat{n}_{sc}\times\vec{E}_{sc}(\vec{r};t)\end{aligned} \tag{3.9}$$

with $\hat{n}_{sc}\cdot\vec{r} = -x_1\cos\alpha_{sc} + x_2\sin\alpha_{sc}$ and

$$\begin{aligned}c_{sc} &= c\,(1+2\beta\sin\alpha_{sc}) = c/(1-2\beta\sin\alpha)\\ \omega_{sc} &= \omega_{inc}\,(1+2\beta\sin\alpha_{sc}) = \omega_{inc}/(1-2\beta\sin\alpha)\end{aligned} \tag{3.10}$$

revealing a Doppler effect due to the component of the incident wave parallel to, i.e., in the same direction with the motion of the boundary, while $\omega_{sc}/c_{sc} = k$.

It is seen that the realization of a scattering phenomenon requires

$$c_{sc}, \omega_{sc}, \alpha_{sc} > 0, \text{ i.e. } 1-2\beta\sin\alpha > 0, \text{ or } \beta < 1/(2\sin\alpha) \tag{3.11}$$

which is satisfied for $\forall\alpha$, $\beta \le 1/2$ & $\alpha \in \left[0, \sin^{-1}(1/2\beta)\right)$, $\beta \ge 1/2$. Depending on α and β, there are two different scattering wave mechanisms observed:

$$\begin{cases}\text{Space wave mode :}\\ 0 \le \sin\alpha_{sc} < 1, 1-2\beta\sin\alpha > \sin\alpha,\\ \text{i.e.}\forall\alpha, \beta \le 0 \;\&\; \alpha \in \left[0, \sin^{-1}(1/(2\beta+1))\right), \beta \ge 0\\ \text{Evanescent wave modes :}\\ \sin\alpha_{sc} \ge 1, 0 < 1-2\beta\sin\alpha \le \sin\alpha,\\ \text{i.e.}\alpha \in \left[\sin^{-1}(1/(2\beta+1)), \pi/2\right), -1/2 < \beta \le 1/2\\ \&\alpha \in \left[\sin^{-1}(1/(2\beta+1)), \sin^{-1}(1/(2\beta))\right), \beta \ge 1/2\end{cases}$$

while, in space wave mode, the parameters of the scattered wave are related to those of the incident wave through the relations

$$\begin{cases}\sin\alpha_{sc} \le \sin\alpha, \alpha_{sc} \le \alpha < \pi/2, \omega_{sc} \le \omega_{inc}, c_{sc} \le c \text{ when } 1-2\beta\sin\alpha \ge 1, \text{i.e.}, \beta \le 0;\\ \sin\alpha_{sc} \ge \sin\alpha, \alpha \le \alpha_{sc} < \pi/2, \omega_{sc} \ge \omega_{inc}, c_{sc} \ge c \text{ when } \sin\alpha < 1-2\beta\sin\alpha \le 1,\\ \text{i.e.}, 0 \le \beta < (1/2)(1/\sin\alpha - 1)\end{cases}$$

The special case $\alpha_{sc} = \alpha$ only occurs for the stationary case $\beta = 0$. When we express the frequency of the scattered wave as

$$f_{sc} = f_{inc} + \Delta f \tag{3.12a}$$

we get

$$\Delta f/f_{inc} = 2\beta\sin\alpha/(1-2\beta\sin\alpha) \tag{3.12b}$$

which indicates that Δf is directly proportional with α and β parameters.

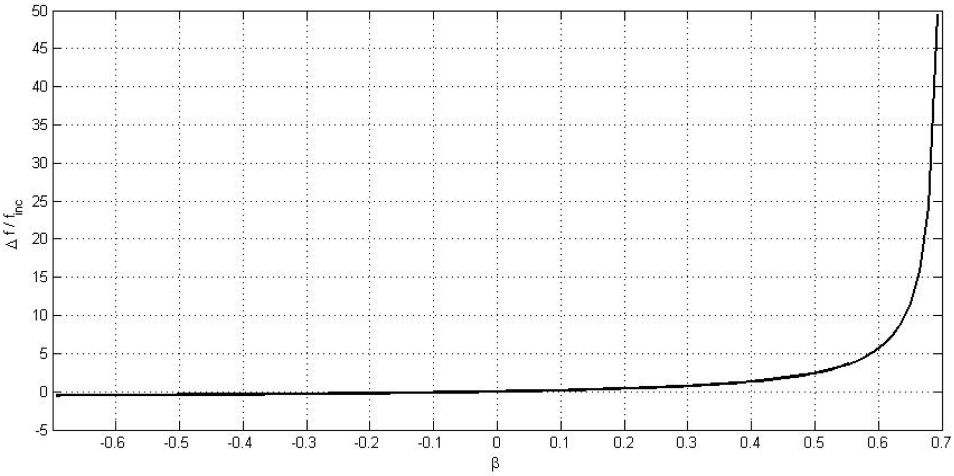

Figure 3. An illustration of (3.12b) for $\alpha = \pi/4$. Evanescent wave mode starts at $\beta = 0.207$.

When a monochromatic incidence is considered, in the evanescent wave mode we observe scattered evanescent (surface) waves with a pattern

$$e^{ik(-x_1 \cos\alpha_{sc} + x_2 \sin\alpha_{sc})} = e^{kx_1\sqrt{\sin^2\alpha_{sc}-1}} e^{ikx_2 \sin\alpha_{sc}}, \quad x_1 < 0$$

In the limiting case $\alpha = \sin^{-1}(1/2\beta)$ the scattered evanescent wave totally vanishes. In TM mode the angle and the frequency of the scattered wave are the same as in TE mode, while the reflection coefficient is calculated as $R_{TM} = \cos\alpha / \cos\alpha_{sc}$, which provides

$$\begin{cases} R_{TM} \leq 1 \text{ when } \alpha_{sc} \leq \alpha < \pi/2, \text{i.e.}, \beta \leq 0 \\ R_{TM} \geq 1 \text{ when } \alpha \leq \alpha_{sc} < \pi/2, \text{i.e.}, 0 \leq \beta \leq (1/2)(1/\sin\alpha - 1) \end{cases}$$

in space wave mode. For both TE and TM modes, the parallel motion of the plane has no influence on the scattered wave under normal incidence $\alpha = 0$.

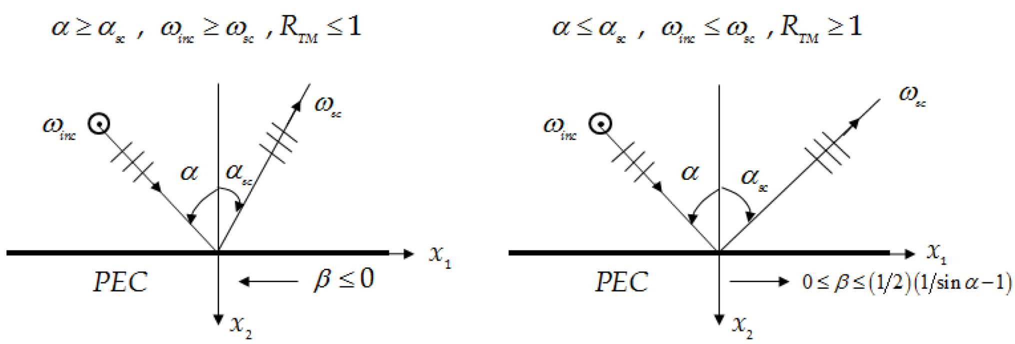

Figure 4. An illustration of scattering mechanism for opposite directions of uniform motion parallel to the plane

3.2. **Case II: Uniform Motion Perpendicular to the Plane.** For E-observer we assume the PEC plane at $x_1 = 0$ in uniform rectilinear motion with velocity $\vec{v} = G\hat{x}_1$, $G = const.$, while for L-observer the PEC plane is stationary and it is half-space $x_1' < 0$ (medium I) moving with linear velocity $\vec{v}' = -G\hat{x}_1'$. Incorporating the Galilean transformations $x_1 = x_1' + Gt$, $x_{2,3} = x_{2,3}'$; $\hat{x}_i = \hat{x}_i'$, $i = 1, 2, 3$, the incoming fields in L-frame have the same form as (3.2a,b), while

$$c'_{inc} = c - G\cos\alpha = c(1-\beta\cos\alpha),\ \omega'_{inc} = \omega_{inc}(1-\beta\cos\alpha),\ f'_{inc} = f_{inc}(1-\beta\cos\alpha) \tag{3.13a-c}$$

with the requirement

$$c'_{inc}, \omega'_{inc}, f'_{inc} > 0,\ \text{i.e.,}\ 1-\beta\cos\alpha > 0,\ \text{or}\ \beta < 1/\cos\alpha \tag{3.13d}$$

as a kinematical upper limit on β for the the incident wave to catch the moving plane. (3.13d) is satisfied for $\forall\alpha$, $\beta \le 1$ & $\alpha \in \left(\cos^{-1}(1/\beta), \pi/2\right)$, $\beta \ge 1$. Unless this condition is satisfied the incident wave is observed to propagate *away* in L-frame in the direction $(-\cos\alpha, -\sin\alpha)$. Similarly, the scattered fields in L-frame have the same form as (3.4), while the unknown quantities c'_{sc}, α_{sc}, R_{TE} are solved from the same boundary value problem as (3.5).

The wave equation (3.5a), namely,

$$\left(\text{lap}' - \frac{1}{c^2}\frac{\bar{\Diamond}'^2}{\bar{\Diamond}' t^2}\right)\vec{E}'_{sc} = \hat{x}'_3\left(\text{lap}' - \frac{1}{c^2}\left(\frac{\partial}{\partial t} + G\frac{\partial}{\partial x'_1}\right)^2\right) f(\hat{n}'_{sc}\cdot\vec{r}' - c'_{sc}t)$$

$$= \hat{x}'_3\left(1 - \frac{1}{c^2}\left(-c'_{sc} - G\cos\alpha_{sc}\right)^2\right) f = \vec{0}$$

requires

$$c'_{sc} = c(1-\beta\cos\alpha_{sc}),\ \omega'_{sc} = \omega_{inc}(1-\beta\cos\alpha_{sc}) \tag{3.14a,b}$$

while $\omega'_{sc}/c'_{sc} = k$. From the boundary condition (3.5b) one uniquely obtains

I) phase invariance as in (3.8a), which reads

$$(1-\beta\cos\alpha)/\sin\alpha = (1-\beta\cos\alpha_{sc})/\sin\alpha_{sc} \tag{3.15}$$

II) full reflection as in (3.8d). (3.15) can be written equivalently as $\sin\alpha_{sc} - \sin\alpha = \beta\sin(\alpha_{sc}-\alpha)$. Setting $\xi = \cos\alpha_{sc} \in (0,1]$ and using basic trigonometric relations shapes (3.15) into the quadratic equation

$$\left(1+\beta^2 A\right)\xi^2 - 2\beta A\xi + A - 1 = 0 \tag{3.16a}$$

with $A = \sin^2\!\alpha \big/ (1-\beta\cos\alpha)^2$. In virtue of (3.3), the square-root of the discriminant of (3.16a) reads $\sqrt{\Delta} = 2(\cos\alpha - \beta)/(1-\beta\cos\alpha) > 0$ and one obtains the roots as

$$\xi_{1,2} = \left[\beta\sin^2\!\alpha \pm (1-\beta\cos\alpha)(\cos\alpha-\beta)\right]\big/\left(1-2\beta\cos\alpha+\beta^2\right). \tag{3.16b}$$

The only root that falls in the described range $\xi \in (0,1]$ is the one with upper plus sign, which exactly reads

$$\xi = \cos\alpha,\ \text{i.e.}\ \alpha_{sc} = \alpha. \tag{3.16c}$$

Substituting (3.16c) into (3.14) reads

$$c'_{sc} = c'_{inc},\ \omega'_{sc} = \omega'_{inc}. \tag{3.16d,e}$$

It is observed that in L-frame the wave is reflected with the same phase velocity and frequency when the motion is perpendicular to the plane.

Finally, the maps of $(\vec{E}'_{sc}, \vec{H}'_{sc})$ into E-frame read the same equations as (3.9), where

$$c_{sc} = c\,(1-2\beta\cos\alpha),\ \omega_{sc} = \omega_{inc}\,(1-2\beta\cos\alpha),\ \Delta f/f_{inc} = -2\beta\cos\alpha \tag{3.17a-c}$$

revealing a Doppler effect due to the component of the incident wave in the same direction with the motion of the boundary, while $\omega_{sc}/c_{sc} = k$. (3.17c) indicates that Δf is inversely proportional with α, while it is linearly proportional with $-\beta$. It is seen that the realization of a scattering phenomenon requires

$$c_{sc}, \omega_{sc} > 0, \text{ i.e. } 1 - 2\beta\cos\alpha > 0, \text{ or } \beta < 1/(2\cos\alpha) \tag{3.17d}$$

which is satisfied for $\forall\alpha, \beta \le 1/2$ & $\alpha \in \left(\cos^{-1}(1/2\beta), \pi/2\right)$, $\beta \ge 1/2$.

In TM mode, as in Section 3.1, the angle and frequency of the scattered wave is the same as in TE mode, while one also observes full reflection since $R_{TM} = \cos\alpha/\cos\alpha_{sc} = 1$.

3.3. **Case III: Harmonic Motion Parallel to the Plane.**

We consider the special case of harmonic motion

$$\vec{\text{v}}(t) = G(t)\hat{x}_2, \; G(t) = G\cos(\omega t), \; G = const \tag{3.18}$$

with coordinate transformations

$$x_2 = x_2' + F(t), \; F(t) = (G/\omega)\sin(\omega t), \; x_1 = x_1', \; x_3 = x_3', \; \hat{x}_i = \hat{x}_i', \; i = 1, 2, 3 \tag{3.19}$$

$$\vec{E}_{inc}(\vec{r}; t) = \hat{x}_3 \, e^{ik\hat{n}_{inc}\cdot\vec{r}} e^{-i\omega_{inc}t}, \; \vec{H}_{inc}(\vec{r}; t) = (1/Z)\, \hat{n}_{inc} \times \vec{E}_{inc}(\vec{r}; t) \tag{3.20}$$

In virtue of the well known Bessel property

$$e^{i\Omega\sin(\omega t)} = \sum_{-\infty}^{\infty} J_m(\Omega) e^{im\omega t}, \tag{3.21}$$

the map of (3.20) into L-frame can be written as

$$\begin{aligned} \vec{E}'_{inc}(\vec{r}'; t) &= \sum_{-\infty}^{\infty} \vec{E}'^{(m)}_{inc}(\vec{r}'; t) = \sum_{-\infty}^{\infty} \vec{E}'^{(m)}_{inc}(\vec{r}') e^{-i\omega'^{(m)}_{inc}\, t}, \\ \vec{E}'^{(m)}_{inc}(\vec{r}') &= \hat{x}_3' J_m(\Omega) e^{ik\hat{n}'_{inc}\cdot\vec{r}'} \end{aligned} \tag{3.22a}$$

$$\begin{aligned} \vec{H}'_{inc}(\vec{r}'; t) &= \sum_{-\infty}^{\infty} \vec{H}'^{(m)}_{inc}(\vec{r}'; t) = \sum_{-\infty}^{\infty} \vec{H}'^{(m)}_{inc}(\vec{r}') e^{-i\omega'^{(m)}_{inc}\, t}, \\ \vec{H}'^{(m)}_{inc}(\vec{r}')t &= (1/Z)\, \hat{n}'_{inc} \times \vec{E}'^{(m)}_{inc}(\vec{r}') \end{aligned} \tag{3.22b}$$

with $\hat{n}'_{inc} \cdot \vec{r}' = x_1' \cos\alpha + x_2' \sin\alpha$ and $\Omega = (G/\omega)\, k \sin\alpha$. Based on the principle of superposition for sources and fields, the incident wave (3.22) can be considered as an infinite sum of hypothetical plane wave modes with amplitude $J_m(\Omega)$, angular frequency $\omega'^{(m)}_{inc} = \omega_{inc} - m\omega$, while for each mode the plane moves with a uniform velocity $\vec{\text{v}}^{(m)} = G^{(m)}\hat{x}_2 = c\beta^{(m)}\hat{x}_2$. which is determined via (3.2d) by writing $\omega'^{(m)}_{inc} = \omega_{inc} - m\omega = \omega_{inc}(1 - \beta^{(m)} \sin\alpha)$ to get $\beta^{(m)} = (m\omega/\omega_{inc})/\sin\alpha$. Then the scattered fields in L-frame can be written directly by substituting $\omega'^{(m)}_{inc}$, $\beta^{(m)}$ for ω'_{inc}, β in the available results in Section 3.1 to get

$$\begin{aligned} \vec{E}'_{sc}(\vec{r}'; t) &= \sum_{-\infty}^{\infty} \vec{E}'^{(m)}_{sc}(\vec{r}'; t) = \sum_{-\infty}^{\infty} \vec{E}'^{(m)}_{sc}(\vec{r}') e^{-i\omega'^{(m)}_{sc}\, t}, \\ \vec{E}'^{(m)}_{sc}(\vec{r}') &= -\hat{x}_3' J_m(\Omega) R^{(m)}_{TE} e^{ik\hat{n}'^{(m)}_{sc}\cdot\vec{r}'} \end{aligned} \tag{3.23a}$$

$$\begin{aligned} \vec{H}'_{sc}(\vec{r}'; t) &= \sum_{-\infty}^{\infty} \vec{H}'^{(m)}_{sc}(\vec{r}'; t) = \sum_{-\infty}^{\infty} \vec{H}'^{(m)}_{sc}(\vec{r}') e^{-i\omega'^{(m)}_{sc}\, t}, \\ \vec{H}'^{(m)}_{sc}(\vec{r}') &= (1/Z)\, \hat{n}'^{(m)}_{sc} \times \vec{E}'^{(m)}_{sc}(\vec{r}') \end{aligned} \tag{3.23b}$$

with $\hat{n}'^{(m)}_{sc} \cdot \vec{r}' = -x'_1 \cos\alpha^{(m)}_{sc} + x'_2 \sin\alpha^{(m)}_{sc}$, $R^{(m)}_{TE} = -1$ and

$$\sin\alpha^{(m)}_{sc} = \sin\alpha \Big/ \left(1 - 2\beta^{(m)} \sin\alpha\right) = \sin\alpha/(1 - 2m\omega/\omega_{inc}) \tag{3.24a}$$

$$\omega'^{(m)}_{sc} = \omega'^{(m)}_{inc} \left(1 - 2\beta^{(m)} \sin\alpha\right) = \omega'^{(m)}_{inc} (1 - 2m\omega/\omega_{inc}) \tag{3.24b}$$

while in E-frame one has

$$\vec{E}_{sc}(\vec{r};t) = \sum_{-\infty}^{\infty} \vec{E}^{(m)}_{sc}(\vec{r};t) = \sum_{-\infty}^{\infty} \vec{E}^{(m)}_{sc}(\vec{r}) e^{-i\omega^{(m)}_{sc} t},$$

$$\vec{E}^{(m)}_{sc}(\vec{r}) = -\hat{x}_3 J_m(\Omega) e^{ik\hat{n}^{(m)}_{sc} \cdot \vec{r}} \tag{3.25a}$$

$$\vec{H}_{sc}(\vec{r};t) = \sum_{-\infty}^{\infty} \vec{H}^{(m)}_{sc}(\vec{r};t) = \sum_{-\infty}^{\infty} \vec{H}^{(m)}_{sc}(\vec{r}) e^{-i\omega^{(m)}_{sc} t},$$

$$\vec{H}^{(m)}_{sc}(\vec{r}) = (1/Z)\, \hat{n}^{(m)}_{sc} \times \vec{E}^{(m)}_{sc}(\vec{r}) \tag{3.25b}$$

with $\hat{n}^{(m)}_{sc} \cdot \vec{r} = -x_1 \cos\alpha^{(m)}_{sc} + x_2 \sin\alpha^{(m)}_{sc}$ and

$$\omega^{(m)}_{sc} = \omega_{inc} \Big/ \left(1 - 2\beta^{(m)} \sin\alpha\right) = \omega_{inc}/(1 - 2m\omega/\omega_{inc}) \tag{3.26}$$

In TM mode the angle and the frequency of the scattered wave modes are the same as in TE mode, while the modal reflection coefficient is calculated as $R^{(m)}_{TM} = \cos\alpha \Big/ \cos\alpha^{(m)}_{sc}$.

For both TE and TM modes, the parallel harmonic motion of the plane has no influence on the scattered wave under normal incidence $\alpha = 0$. The special case $\omega = 0$ coincides with Case I.

Evanescent modes are observed for $\sin\alpha > 1 - 2m\omega/\omega_{inc}$ or $m > (\omega_{inc}/2\omega)(1 - \sin\alpha)$, while only space waves occur under normal incidence.

3.4. **Case IV: Harmonic Motion Perpendicular to the Plane.** In this example we consider the special case of harmonic motion

$$\vec{\mathrm{v}}(t) = G(t)\hat{x}_1,\ G(t) = G\cos(\omega t),\ G = const \tag{3.27}$$

with coordinate transformations

$$x_1 = x'_1 + F(t),\ F(t) = (G/\omega)\sin(\omega t),\ x_2 = x'_2,\ x_3 = x'_3,\ \hat{x}_i = \hat{x}'_i,\ i = 1,2,3 \tag{3.28}$$

and under monochromatic TE plane wave incidence as in (3.20). In L-frame the incident electrical field is expressed by $\vec{E}'_{inc}(\vec{r}';t) = \hat{x}'_3\, e^{ik\hat{n}'_{inc} \cdot \vec{r}'} e^{i\Omega \sin(\omega t)} e^{-i\omega_{inc} t}$ with $\hat{n}'_{inc} \cdot \vec{r}' = x'_1 \cos\alpha + x'_2 \sin\alpha$, while $\Omega = (G/\omega)\, k \cos\alpha$. The scattered electrical field satisfies the boundary value problem

$$\begin{cases} \left(\mathrm{lap}' - \frac{1}{c^2}\left(\frac{\partial}{\partial t} + G(t)\frac{\partial}{\partial x'_1}\right)^2\right) \vec{E}'_{sc} \ \ (\vec{r}';t) = \vec{0}, \text{or } \left(\mathrm{lap}' + k^2\right) \vec{E}'_{sc}(\vec{r}';t) = \vec{0}, \text{ in medium I} \\ \text{Boundary Condition} : \vec{E}'_{inc}(0, x'_2;t) + \vec{E}'_{sc}(0, x'_2;t) = \vec{0}, \forall x'_2, t \\ \text{Radiation Condition as } x'_1 \to -\infty \end{cases} \tag{3.29}$$

A solution in the form $\vec{E}'_{sc}(\vec{r}';t) = \hat{x}'_3\, R_{TE} e^{ik\hat{n}'_{sc} \cdot \vec{r}'} e^{i\Omega \sin(\omega t)} e^{-i\omega'_{sc} t}$ with $\hat{n}'_{sc} \cdot \vec{r}' = -x'_1 \cos\alpha_{sc} + x'_2 \sin\alpha_{sc}$ directly requires $R_{TE} = -1$, $\omega'_{sc} = \omega_{inc}$, $\alpha_{sc} = \alpha$, which reads

$$\vec{E}'_{sc}(\vec{r}';t) = -\hat{x}'_3\, e^{ik\hat{n}'_{sc} \cdot \vec{r}'} e^{i\Omega \sin(\omega t)} e^{-i\omega_{inc} t} \tag{3.30}$$

The same result can also be obtained by expanding the incident field in L-frame into an infinite sum of hypothetical plane wave modes and substituting $\omega'^{(m)}_{inc} = \omega_{inc} - m\omega$,

$\beta^{(m)} = (m\omega/\omega_{inc})/\cos\alpha$ for ω'_{inc}, β in the available results in Section 3.2. Then in E-frame one has

$$\vec{E}_{sc}(\vec{r};t) = -\hat{x}_3\, e^{ik\hat{n}_{sc}\cdot\vec{r}} e^{i\Omega\sin(2\omega t)} e^{-i\omega_{inc}t} \tag{3.31}$$

with $\hat{n}_{sc}\cdot\vec{r} = -x_1\cos\alpha + x_2\sin\alpha$. In virtue of (3.21), (3.31) can also be expressed in terms of plane wave harmonics as

$$\vec{E}_{sc}(\vec{r};t) = -\hat{x}_3 \sum_{-\infty}^{\infty} J_m(\Omega) e^{ik\hat{n}_{sc}\cdot\vec{r}} e^{-i(\omega_{inc}-2m\omega)t}. \tag{3.32}$$

In TM mode the angle and the frequency of the scattered wave modes are the same as in TE mode, while the modal reflection coefficient is calculated as $R_{TM} = \cos\alpha/\cos\alpha_{sc} = 1$. The special case coincides with Case II.

4. TM Plane Wave Scattering by a Moving Dielectric Half Space

In this section we shall investigate the scattering of uniform homogeneous TM plane waves by a lossless dielectric half space for the same four modes of motion as in Section 3 in a similar fashion. Therefore the same quantities already described for the corresponding problem in Section 3 will not be repeated.

We assume the incident wave propagates along $\hat{n}_{inc} = (\cos\alpha, \sin\alpha)$ direction in (x_1, x_2) plane in half-space $x_1 < 0$ (medium I) with fields represented by

$$\vec{H}_{inc}(\vec{r};t) = \hat{x}_3 f(\hat{n}_{inc}\cdot\vec{r} - ct),\ \vec{E}_{inc}(\vec{r};t) = Z\vec{H}_{inc}(\vec{r};t)\times\hat{n}_{inc}. \tag{4.1a,b}$$

For the special case of monochromatic source the incident magnetic field is assumed to have the general form

$$\vec{H}_{inc}(\vec{r};t) = \hat{x}_3 g(k\hat{n}_{inc}\cdot\vec{r} - \omega_{inc}t). \tag{4.1c}$$

Without losing generality, let us assume the half-space $x_1 > 0$ (region D) lossless with constitutive parameters (ε_d,μ_d), characteristic impedance $Z_d = \sqrt{\mu_d/\varepsilon_d}$, wave number $k_d = \omega_{inc}\sqrt{\varepsilon_d\mu_d}$ and refractivity defined by $n = \sqrt{\varepsilon_d\mu_d}/\sqrt{\varepsilon\mu} = c/c_d = k_d/k$.

4.1. **Case I: Uniform Motion Parallel to the Plane.**

For E-observer we assume the dielectic half-space in uniform rectilinear motion with velocity $\vec{v} = G\hat{x}_2$, $G = const.$ Based on the same coordinate transformations as in Section 3.1 the incoming fields in L-frame read

$$\begin{aligned} &\vec{H}'_{inc}(\vec{r}';t) = \hat{x}'_3 f(\hat{n}'_{inc}\cdot\vec{r}' - c'_{inc}t) = \hat{x}'_3 g(k\hat{n}'_{inc}\cdot\vec{r}' - \omega'_{inc}t),\\ &\vec{E}'_{inc}(\vec{r}';t) = Z\vec{H}'_{inc}(\vec{r}';t)\times\hat{n}'_{inc}\end{aligned} \tag{4.2}$$

while (3.2c-e) still holds. Let the scattered field in medium I and the total field in region D be given in the form

$$\begin{aligned} &\vec{H}'_{sc}(\vec{r}';t) = \hat{x}'_3 R_{TM} f(\hat{n}'_{sc}\cdot\vec{r}' - c'_{sc}t) = \hat{x}'_3 R_{TM} g(k\hat{n}'_{sc}\cdot\vec{r}' - \omega'_{sc}t),\\ &\vec{E}'_{sc}(\vec{r}';t) = Z\vec{H}'_{sc}(\vec{r}';t)\times\hat{n}'_{sc}\end{aligned} \tag{4.3}$$

$$\begin{aligned} &\vec{H}'_{d}(\vec{r}';t) = \hat{x}'_3 T_{TM} f(\hat{n}'_{d}\cdot\vec{r}' - c'_{d}t) = \hat{x}'_3 T_{TM} g(k_d\hat{n}'_{d}\cdot\vec{r}' - \omega'_{d}t),\\ &\vec{E}'_{d}(\vec{r}';t) = Z_d\vec{H}'_{d}(\vec{r}';t)\times\hat{n}'_{d}\end{aligned} \tag{4.4}$$

with $\hat{n}'_{sc}\cdot\vec{r}' = -x'_1\cos\alpha_{sc}+x'_2\sin\alpha_{sc}$, $\hat{n}'_d\cdot\vec{r}' = x'_1\cos\alpha_d+x'_2\sin\alpha_d$. The unknown quantities c'_{sc}, α_{sc}, c'_d, α_d, R_{TM}, T_{TM} are solved from the boundary value problem

$$\begin{cases} \left(\text{lap}' - \frac{1}{c^2}\frac{\bar{\Diamond}'^2}{\Diamond' t^2}\right)\vec{H}'_{sc}(x'_1, x'_2; t) = \vec{0}, \text{in medium I} \\ \left(\text{lap}' - \frac{1}{c_d^2}\frac{\partial^2}{\partial t^2}\right)\vec{H}'_d(x'_1, x'_2; t) = \vec{0}, \text{in region D} \\ \vec{H}'_{inc}(0, x'_2; t) + \vec{H}'_{sc}(0, x'_2; t) = \vec{H}'_d(0, x'_{23}; t), \forall x'_2, t \\ Z\left[\vec{H}'_{inc}(0, x'_2; t)\times\hat{n}'_{inc} + \vec{H}'_{sc}(0, x'_2; t)\times\hat{n}'_{sc}\right] = Z_d\vec{H}'_d(0, x'_2; t)\times\hat{n}'_d, \forall x'_2, t \\ \text{Radiation Conditions as } x'_1\to\pm\infty \end{cases} \quad (4.5\text{a,e})$$

The wave equations (4.5a,b) require

$$c'_{sc} = c(1+\beta\sin\alpha_{sc}),\ \omega'_{sc} = \omega_{inc}(1+\beta\sin\alpha_{sc}),\ c'_d = c_d,\ \omega'_d = \omega_{inc} \quad (4.6)$$

while the boundary relations (4.5c,d) read

$$f(x'_2\sin\alpha - c'_{inc}t) + R_{TM}f(x'_2\sin\alpha_{sc} - c'_{sc}t) = T_{TM}f(x'_2\sin\alpha_d - c'_dt) \quad (4.7\text{a})$$

$$Z\left[\cos\alpha\, f(x'_2\sin\alpha - c'_{inc}t) - \cos\alpha_{sc}\, R_{TM}f(x'_2\sin\alpha_{sc} - c'_{sc}t)\right] = \cos\alpha_d\, Z_dT_{TM}f(x'_2\sin\alpha_d - c'_dt) \quad (4.7\text{b})$$

for $\forall x'_2, t$, from which one uniquely obtains

I) phase invariance:

$$dx'_2/dt = c'_{inc}/\sin\alpha = c'_{sc}/\sin\alpha_{sc} = c'_d/\sin\alpha_d \quad (4.8\text{a})$$

or

$$(1-\beta\sin\alpha)/\sin\alpha = (1+\beta\sin\alpha_{sc})/\sin\alpha_{sc} = 1/(n\sin\alpha_d) \quad (4.8\text{b})$$

which reads

$$\sin\alpha_{sc} = \sin\alpha/(1-2\beta\sin\alpha),\ \sin\alpha_d = \sin\alpha/[n(1-\beta\sin\alpha)] \quad (4.8\text{c,d})$$

as well as

$$c'_{sc} = c'_{inc}(1-2\beta\sin\alpha),\ \omega'_{sc} = \omega'_{inc}(1-2\beta\sin\alpha) \quad (4.8\text{e,f})$$

II) the reflection and transmission coefficients:

$$R_{TM} = \frac{Z\cos\alpha - Z_d\cos\alpha_d}{Z\cos\alpha_{sc} + Z_d\cos\alpha_d},\ T_{TM} = \frac{Z(\cos\alpha + \cos\alpha_{sc})}{Z\cos\alpha_{sc} + Z_d\cos\alpha_d} \quad (4.9\text{a,b})$$

The maps of $(\vec{E}'_{sc}, \vec{H}'_{sc})$ and $(\vec{E}'_d, \vec{H}'_d)$ into E-frame read

$$\vec{H}_{sc}(\vec{r};t) = \hat{x}_3R_{TM}f(\hat{n}_{sc}\cdot\vec{r} - c_{sc}t) = \hat{x}_3R_{TM}g(k\hat{n}_{sc}\cdot\vec{r} - \omega_{sc}t),\ \vec{E}_{sc}(\vec{r};t) = Z\vec{H}_{sc}(\vec{r};t)\times\hat{n}_{sc} \quad (4.10)$$

$$\vec{H}_d(\vec{r};t) = \hat{x}_3T_{TM}f(\hat{n}_d\cdot\vec{r} - c_{tr}t) = \hat{x}_3T_{TM}g(k_d\hat{n}_d\cdot\vec{r} - \omega_{tr}t),\ \vec{E}_d(\vec{r};t) = Z_d\vec{H}_d(\vec{r};t)\times\hat{n}_d \quad (4.11)$$

with $\hat{n}_{sc}\cdot\vec{r} = -x_1\cos\alpha + x_2\sin\alpha$, $\hat{n}_d\cdot\vec{r} = x_1\cos\alpha_d + x_2\sin\alpha_d$ and

$$c_{sc} = c\,(1+2\beta\sin\alpha_{sc}) = c/(1-2\beta\sin\alpha),$$

$$\vec{E}_d(\vec{r};t) = Z_d\vec{H}_d(\vec{r};t)\times\hat{n}_d \quad (4.12\text{a,b})$$

$$\Delta f/f_{inc} = 2\beta\sin\alpha/(1-2\beta\sin\alpha) \quad (4.12\text{c})$$

$$c_{tr} = c_d + G\sin\alpha_d = c_d(1 + \beta n\sin\alpha_d) = c_d/(1 - \beta\sin\alpha) \tag{4.13a}$$

$$\omega_{tr} = \omega_{inc} + kG\sin\alpha_d = \omega_{inc}(1 + \beta n\sin\alpha_d) = \omega_{inc}/(1 - \beta\sin\alpha) \tag{4.13b}$$

while $\omega_{sc}/c_{sc} = k$, $\omega_{tr}/c_{tr} = k_d$. In (4.12) one observes the same Doppler effect as in Section 3.1 for a PEC plane regardless of the constitutive parameters of the dielectric half-space.

The Brewster angle α_B for which one has zero reflection coefficient ($R_{TM} = 0$) is calculated from the equation

$$Z\cos\alpha_B = Z_d\cos\alpha_d \tag{4.14a}$$

which, upon substituting (4.8d), shapes into the transcendental equation

$$n^2\left(Z_d^2 - Z^2\cos^2\alpha_B\right)(1 - \beta\sin\alpha_B)^2 - Z_d^2\sin^2\alpha_B = 0. \tag{4.14b}$$

A change of variables $\xi = \sin\alpha_B \in [0, 1)$ provides a compact closed form representation

$$(Z_d/Z)^2\left[n^2(1 - \beta\xi)^2 - \xi^2\right] - n^2(1 - \beta\xi)^2\left(1 - \xi^2\right) = 0 \tag{4.14c}$$

For $\forall n, \beta$ there is always one and only one root of the quartic (fourth order) polynomial (4.14c) that falls into the described range of ξ. For the special case $\mu_d = \mu$ one has $Z/Z_d = n$ and (4.14c) simplifies as

$$n^2(1 - \beta\xi)^2\left[1 - n^2(1 - \xi^2)\right] - \xi^2 = 0. \tag{4.14d}$$

While the roots of (4.14c,d) can always be obtained analytically through Cardanos formulas, under the asymptotic condition

$$n^4\beta^2\xi^4 << 2|\beta|\,n^4\xi^3, \text{namely } |\beta|\,\xi << 2, \text{(roughly equivalent to } |\beta| < 0.2), \tag{4.14e}$$

(4.14d) can be approximated by the cubic polynomial

$$-2\beta n^2\xi^3 + (\beta^2(1 - n^2) + n^2 - 1/n^2)\xi^2 - 2\beta\left(1 + n^2\right)\xi + 1 - n^2 = 0 \tag{4.14f}$$

The limiting case $\beta = 0$ yields the classical result $\xi = n\big/\sqrt{n^2 + 1}$. As depicted in Figure 5, one observes an opposite variation in Brewster angle with increasing values of β for $n < 1$ and $n > 1$.

In TE mode the angle and the frequency of the scattered and transmitted waves are the same as in TM mode, while the reflection and transmission coefficients are calculated as

$$R_{TE} = \frac{(1/Z)\cos\alpha - (1/Z_d)\cos\alpha_d}{(1/Z)\cos\alpha_{sc} + (1/Z_d)\cos\alpha_d},\ T_{TE} = \frac{(1/Z)(\cos\alpha + \cos\alpha_{sc})}{(1/Z)\cos\alpha_{sc} + (1/Z_d)\cos\alpha_d} \tag{4.15a,b}$$

In this case (4.14d) is replaced by

$$\beta^2\xi^4 - 2\beta\xi^3 + (n^2 - 1)(\beta^2\xi^2 - 2\beta\xi + 1) = 0$$

the roots of which do not permit the Brewster angle mechanism for any β value.

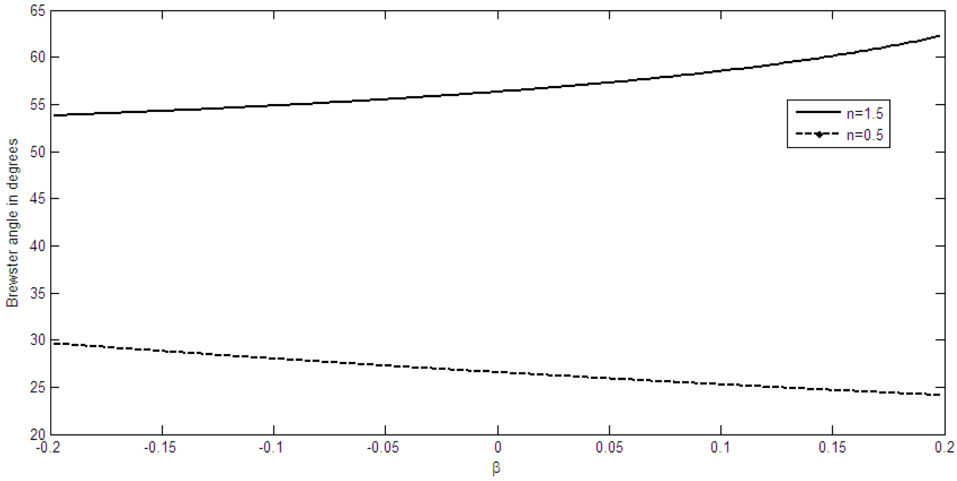

Figure 5. A MATLAB^{TM} calculation of Brewster angle from (4.14d)

4.2. **Case II: Uniform Motion Perpendicular to the Plane.** In this case the incident wave has the same expression as (4.2) along with (3.13). We may assume the scattered field in medium I and the total field in region D in the form (4.4), where the unknown quantities c'_{sc}, α_{sc}, c'_d, α_d, R_{TM}, T_{TM} are solved from the same boundary value problem as (4.5) which reads

$$c'_{sc} = c(1-\beta\cos\alpha_{sc}),\ \omega'_{sc} = \omega_{inc}(1-\beta\cos\alpha_{sc}),\ c'_d = c_d,\ \omega'_d = \omega_{inc} \tag{4.16a-d}$$

$$\alpha_{sc} = \alpha,\ \sin\alpha_d = \sin\alpha/[n(1-\beta\cos\alpha)] \tag{4.17a,b}$$

and the same R_{TM}, T_{TM} values as in (4.9) which read

$$R_{TM} = \frac{Z\cos\alpha - Z_d\cos\alpha_d}{Z\cos\alpha + Z_d\cos\alpha_d},\ T_{TM} = \frac{2Z\cos\alpha}{Z\cos\alpha + Z_d\cos\alpha_d}$$

. In E-frame the fields have the same expressions as (4.10) where

$$c_{sc} = c\left(1-2\beta\cos\alpha\right),\ \omega_{sc} = \omega_{inc}\left(1-2\beta\cos\alpha\right),\ \Delta f/f_{inc} = -2\beta\cos\alpha \tag{4.18a-c}$$

$$c_{tr} = c_d + G\cos\alpha_d = c_d(1+\beta n\cos\alpha_d),\ \omega_{tr} = \omega_{inc} + kG\cos\alpha_d = \omega_{inc}(1+\beta n\cos\alpha_d) \tag{4.18d,e}$$

In (4.18a-c) one observes the same Doppler effect as in Section 3.2 for a PEC plane regardless of the constitutive parameters of the dielectric half-space.

The Brewster angle α_B is calculated from the same equation as (4.14a), which, upon substituting (4.17b), shapes into the transcendental equation

$$n^2\left(Z_d^2 - Z^2\cos^2\alpha_B\right)(1-\beta\cos\alpha_B)^2 - Z_d^2\sin^2\alpha_B = 0. \tag{4.19a}$$

A change of variables $\xi = \cos\alpha_B \in (0,1]$ provides a compact closed form representation

$$(Z_d/Z)^2\left[n^2(1-\beta\xi)^2 - (1-\xi^2)\right] - n^2\xi^2(1-\beta\xi)^2 = 0. \tag{4.19b}$$

For $\forall n,\beta$ there is always one and only one root of the fourth order polynomial (4.19b) that falls into the described range of ξ. For the special case $\mu_d = \mu$ one has $Z/Z_d = n$ and

(4.19b) simplifies as

$$n^2(1-\beta\xi)^2\left(1-n^2\xi^2\right)-(1-\xi^2)=0. \tag{4.19c}$$

While the roots of (4.19b,c) can always be obtained analytically through Cardanos formulas, under the asymptotic condition (4.14e), (4.19c) can be approximated by the cubic polynomial

$$2\beta n^4\xi^3+(-n^4+n^2\beta^2+1)\xi^2-2\beta n\xi+n^2-1=0. \tag{4.19d}$$

The limiting case $\beta=0$ yields the classical result $\xi=1\Big/\sqrt{n^2+1}$. Again, as depicted in Figure 6, an opposite variation in Brewster angle is observed with increasing values of β for $n<1$ and $n>1$.

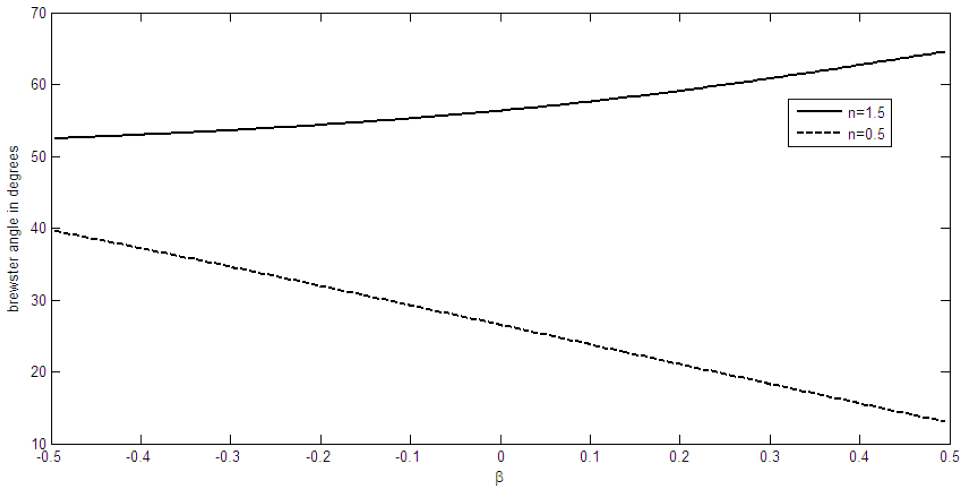

Figure 6. A MATLAB^{TM} calculation of Brewster angle from (4.14c)

The angle of total reflection α_{TR} is observed for $R_{TM}=1$ and $\alpha_d=\pi/2$ and calculated from the relation $\sin\alpha_{TR}=n(1-\beta\cos\alpha_{TR})$. Taking the square of each side and setting $\xi=\cos\alpha_{TR}$ reads the quadratic equation

$$\left(1+n^2\beta^2\right)\xi^2-2\beta n^2\xi+n^2-1=0. \tag{4.20a}$$

The nonnegative discriminant requirement brings the physical restriction

$$n\le 1\Big/\sqrt{1-\beta^2}\equiv\gamma\le 1 \tag{4.20b}$$

on refractrive index and a lower limit on β as $-1<\beta<\min\left(1,1/(2\cos\alpha)\right)$. The positive roots of (4.20a) yields the angle of total reflection uniquely as

$$\cos\alpha_{TR}=\left[\beta n^2+\sqrt{1-n^2/\gamma^2}\right]\Big/\left(1+n^2\beta^2\right). \tag{4.20c}$$

From Figure 7 it is observed that is inversely proportional with varying in the interval $\alpha_{TR}\in\left(0,\cos^{-1}\left(\frac{1-n^2}{1+n^2}\right)\right)$.

In TE mode the angle and the frequency of the scattered and transmitted waves are the same as in TM mode, while the reflection and transmission coefficients are calculated as

$$R_{TE}=\frac{(1/Z)\cos\alpha-(1/Z_d)\cos\alpha_d}{(1/Z)\cos\alpha+(1/Z_d)\cos\alpha_d},\ T_{TE}=\frac{(2/Z)\cos\alpha}{(1/Z)\cos\alpha+(1/Z_d)\cos\alpha_d}$$

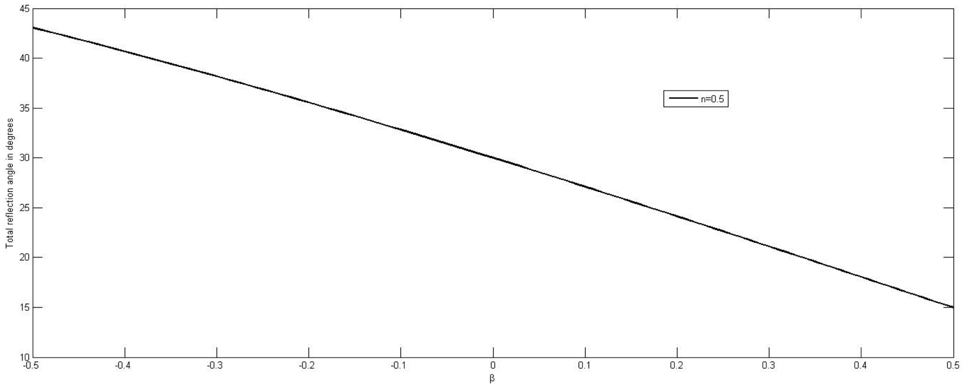

Figure 7. A MATLAB^{TM} calculation of total reflection angle from (4.20a) for $n = 0.5$.

In this case (4.19c) is replaced by

$$\beta^2\xi^4 - 2\beta\xi^3 + (1 - n^2\beta^2)\xi^2 + 2\beta n^2\xi + 1 - n^2 = 0$$

the roots of which do not permit the Brewster angle mechanism for any β value.

4.3. **Case III: Harmonic Motion Parallel to the Plane.** In this case we consider the velocity field and coordinate transformations given in (3.18) and (3.19). Then the fields of the incident TM plane wave are written as

$$\vec{H}'_{inc}(\vec{r}';t) = \sum_{-\infty}^{\infty} \vec{H}'^{(m)}_{inc}(\vec{r}';t) = \sum_{-\infty}^{\infty} \vec{H}'^{(m)}_{inc}(\vec{r}')e^{-i\omega'^{(m)}_{inc}t},\ \vec{H}'^{(m)}_{inc}(\vec{r}') = \hat{x}'_3 J_m(\Omega)e^{ik\hat{n}'_{inc}\cdot\vec{r}'} \tag{4.21}$$

$$\vec{E}'_{inc}(\vec{r}';t) = \sum_{-\infty}^{\infty} \vec{E}'^{(m)}_{inc}(\vec{r}';t) = \sum_{-\infty}^{\infty} \vec{E}'^{(m)}_{inc}(\vec{r}')e^{-i\omega'^{(m)}_{inc}t},\ \vec{E}'^{(m)}_{inc}(\vec{r}') = Z\vec{H}'^{(m)}_{inc}(\vec{r}')\times\hat{n}'_{inc} \tag{4.22}$$

with $\hat{n}'_{inc}\cdot\vec{r}' = x'_1\cos\alpha + x'_2\sin\alpha$, $\Omega = (G/\omega)\,k\sin\alpha$ and $\omega'^{(m)}_{inc} = \omega_{inc} - m\omega$. Then it is satisfactory that one substitutes $\omega'^{(m)}_{inc} = \omega_{inc} - m\omega$, $\beta^{(m)} = (m\omega/\omega_{inc})/\sin\alpha$ for ω'_{inc}, β in the available results in Section 4.1 to get

$$\vec{H}'_{sc}(\vec{r}';t) = \sum_{-\infty}^{\infty} \vec{H}'^{(m)}_{sc}(\vec{r}';t) = \sum_{-\infty}^{\infty} \vec{H}'^{(m)}_{sc}(\vec{r}')e^{-i\omega'^{(m)}_{sc}t},\ \vec{H}'^{(m)}_{sc}(\vec{r}') = \hat{x}'_3 J_m(\Omega)R^{(m)}_{TM}e^{ik\hat{n}'^{(m)}_{sc}\cdot\vec{r}'} \tag{4.23}$$

$$\vec{E}'_{sc}(\vec{r}';t) = \sum_{-\infty}^{\infty} \vec{E}'^{(m)}_{sc}(\vec{r}';t) = \sum_{-\infty}^{\infty} \vec{E}'^{(m)}_{sc}(\vec{r}')e^{-i\omega'^{(m)}_{sc}t},\ \vec{E}'^{(m)}_{sc}(\vec{r}') = Z\vec{H}'^{(m)}_{sc}(\vec{r}')\times\hat{n}'^{(m)}_{sc} \tag{4.24}$$

with

$$\hat{n}'^{(m)}_{sc}\cdot\vec{r}' = -x'_1\cos\alpha^{(m)}_{sc} + x'_2\sin\alpha^{(m)}_{sc},\ \hat{n}'^{(m)}_{d}\cdot\vec{r}' = x'_1\cos\alpha^{(m)}_{d} + x'_2\sin\alpha^{(m)}_{d} \tag{4.25}$$

$$\sin\alpha^{(m)}_{sc} = \sin\alpha/(1 - 2m\omega/\omega_{inc}),\ \sin\alpha^{(m)}_{d} = \sin\alpha/[n(1 - m\omega/\omega_{inc})] \tag{4.26}$$

$$\omega'^{(m)}_{sc} = \omega'^{(m)}_{inc}\ (1 - 2m\omega/\omega_{inc}),\ \omega'^{(m)}_{d} = \omega_{inc} \tag{4.27}$$

$$R^{(m)}_{TM} = \frac{Z\cos\alpha - Z_d\cos\alpha^{(m)}_{d}}{Z\cos\alpha^{(m)}_{sc} + Z_d\cos\alpha^{(m)}_{d}},\ T^{(m)}_{TM} = \frac{Z(\cos\alpha + \cos\alpha^{(m)}_{sc})}{Z\cos\alpha^{(m)}_{sc} + Z_d\cos\alpha^{(m)}_{d}}, \tag{4.28}$$

while in E-frame one has

$$\vec{H}_{sc}(\vec{r};t)=\sum_{-\infty}^{\infty}\vec{H}_{sc}^{(m)}(\vec{r};t)=\sum_{-\infty}^{\infty}\vec{H}_{sc}^{(m)}(\vec{r})e^{-i\omega_{sc}^{(m)}t},\ \vec{H}_{sc}^{(m)}(\vec{r})=\hat{x}_3J_m(\Omega)R_{TM}^{(m)}e^{ik\hat{n}_{sc}^{(m)}\cdot\vec{r}} \tag{4.29}$$

$$\vec{E}_{sc}(\vec{r};t)=\sum_{-\infty}^{\infty}\vec{E}_{sc}^{(m)}(\vec{r};t)=\sum_{-\infty}^{\infty}\vec{E}_{sc}^{(m)}(\vec{r})e^{-i\omega_{sc}^{(m)}t},\ \vec{E}_{sc}^{(m)}(\vec{r})=Z\vec{H}_{sc}^{(m)}(\vec{r})\times\hat{n}_{sc}^{(m)} \tag{4.30}$$

$$\vec{H}_{d}(\vec{r};t)=\sum_{-\infty}^{\infty}\vec{H}_{d}^{(m)}(\vec{r};t)=\sum_{-\infty}^{\infty}\vec{H}_{d}^{(m)}(\vec{r})e^{-i\omega_{tr}^{(m)}t},\ \vec{H}_{d}^{(m)}(\vec{r})=\hat{x}_3J_m(\Omega)T_{TM}^{(m)}e^{ik_d\hat{n}_{d}^{(m)}\cdot\vec{r}} \tag{4.31}$$

$$\vec{E}_{d}(\vec{r};t)=\sum_{-\infty}^{\infty}\vec{E}_{d}^{(m)}(\vec{r};t)=\sum_{-\infty}^{\infty}\vec{E}_{d}^{(m)}(\vec{r})e^{-i\omega_{tr}^{(m)}t},\ \vec{E}_{d}^{(m)}(\vec{r})=Z_d\vec{H}_{d}^{(m)}(\vec{r})\times\hat{n}_{d}^{(m)} \tag{4.32}$$

with

$$\hat{n}_{sc}^{(m)}\cdot\vec{r}=-x_1\cos\alpha_{sc}^{(m)}+x_2\sin\alpha_{sc}^{(m)},\ \hat{n}_{d}^{(m)}\cdot\vec{r}=x_1\cos\alpha_{d}^{(m)}+x_2\sin\alpha_{d}^{(m)} \tag{4.33}$$

$$\omega_{sc}^{(m)}=\omega_{inc}/(1-2m\omega/\omega_{inc}),\ \omega_{tr}^{(m)}=\omega_{inc}/(1-m\omega/\omega_{inc}). \tag{4.34}$$

In TE mode the angle and the frequency of the scattered and transmitted wave modes are the same as in TM mode, while the modal reflection and transmission coefficients are calculated as

$$R_{TE}^{(m)}=\frac{(1/Z)\cos\alpha-(1/Z_d)\cos\alpha_d^{(m)}}{(1/Z)\cos\alpha_{sc}^{(m)}+(1/Z_d)\cos\alpha_d^{(m)}},\ T_{TE}^{(m)}=\frac{(1/Z)(\cos\alpha+\cos\alpha_{sc}^{(m)})}{(1/Z)\cos\alpha_{sc}^{(m)}+(1/Z_d)\cos\alpha_d^{(m)}}. \tag{4.35}$$

For both modes, the parallel harmonic motion of the plane has no influence on the scattered wave under normal incidence $\alpha=0$. The special case $\omega=0$ coincides with Case I.

4.4. **Case IV: Harmonic Motion Perpendicular to the Plane.** In this case we consider the velocity field and coordinate transformations given in (3.27) and (3.28). The fields of the incident TM plane wave are given as (4.21) and (4.22), while $\Omega=(G/\omega)\,k\cos\alpha$. Then it is satisfactory that one substitutes $\beta^{(m)}=(m\omega/\omega_{inc})/\cos\alpha$ for β in the available results in Section 4.2 to get the same scattered and transmitted fields as in (4.23), (4.24), (4.27)-(4.30) where

$$\alpha_{sc}^{(m)}=\alpha,\sin\alpha_d^{(m)}=\sin\alpha/\left[n(1-m\omega/\omega_{inc})\right] \tag{4.36}$$

$$\begin{aligned}\hat{n}_{sc}^{\prime(m)}\cdot\vec{r}'&=-x_1'\cos\alpha_{sc}^{(m)}+x_2'\sin\alpha_{sc}^{(m)}=-x_1'\cos\alpha+x_2'\sin\alpha,\\ \hat{n}_{d}^{\prime(m)}\cdot\vec{r}'&=x_1'\cos\alpha_{d}^{(m)}+x_2'\sin\alpha_{d}^{(m)}\end{aligned} \tag{4.37}$$

$$\omega_{sc}^{\prime(m)}=\omega_{inc}-m\omega,\omega_{d}^{\prime(m)}=\omega_{inc} \tag{4.38}$$

$$R_{TM}^{(m)}=\frac{Z\cos\alpha-Z_d\cos\alpha_d^{(m)}}{Z\cos\alpha+Z_d\cos\alpha_d^{(m)}},T_{TM}^{(m)}=\frac{2Z\cos\alpha}{Z\cos\alpha+Z_d\cos\alpha_d^{(m)}} \tag{4.39}$$

$$\hat{n}_{sc}^{(m)}\cdot\vec{r}=-x_1\cos\alpha+x_2\sin\alpha,\hat{n}_{d}^{(m)}\cdot\vec{r}=x_1\cos\alpha_d^{(m)}+x_2\sin\alpha_d^{(m)} \tag{4.40}$$

$$\omega_{sc}^{(m)}=\omega_{inc}-2m\omega,\omega_{tr}^{(m)}=\omega_{inc}(1+\beta^{(m)}n\cos\alpha_d^{(m)}). \tag{4.41}$$

Evanescent transmitted modes with pattern $e^{ik_d\hat{n}_d^{(m)}\cdot\vec{r}}=e^{-k_dx_1\sqrt{\sin^2\alpha_d^{(m)}-1}}e^{ik_dx_2\sin\alpha_d^{(m)}}$, $x_1>0$ are observed for $\sin\alpha_d^{(m)}>1$, i.e., $m>(\omega_{inc}/\omega)\left[1-(\sin\alpha)/n\right]$.

In TE mode the angle and the frequency of the scattered and transmitted wave modes are the same as in TM mode, while the modal reflection and transmission coefficients are calculated as

$$R_{TE}^{(m)} = \frac{(1/Z)\cos\alpha - (1/Z_d)\cos\alpha_d^{(m)}}{(1/Z)\cos\alpha + (1/Z_d)\cos\alpha_d^{(m)}}, T_{TE}^{(m)} = \frac{(1/Z)(\cos\alpha + \cos\alpha_{sc}^{(m)})}{(1/Z)\cos\alpha + (1/Z_d)\cos\alpha_d^{(m)}} . \quad (4.42)$$

The special case $\omega = 0$ coincides with Case II.

5. A Comparison with Same Results Derived with Special Relativity Theory

In Table 1 we provide a theoretical comparison of the results derived with HE and SRT[3] for the special cases of plane wave scattering by a PEC plane and a dielectric half space in uniform motion as in Sections 3.1, 3.2, 4.1 and 4.2. In these cases the Euclidean transformations of HE for rigid bodies reduce to the Galilean transformations

$$x' = x - Gt,\ t' = t \quad (5.1a)$$

while SRT assumes the standard Lorentz transformations

$$x' = \gamma\,(x - Gt)\,,\ t' = \gamma\,(t - \beta x/c) \quad (5.1b)$$

with $\gamma = 1\Big/\sqrt{1-\beta^2}$. Here the axes x',x signify x'_2,x_2 and x'_1,x_1 when the motion is parallel and perpendicular to the plane, respectively. A first order approximation in β reads first order Lorentz transformations

$$x' = x - Gt,\ t' = t - \beta x/c, \quad (5.1c)$$

which also indicates that a first or higher order departure in should be expected between the physical quantities to be calculated with HE and SRT.

The different nature of frame indifferent and form invariant formulations reveals itself dramatically in both cases of the motion parallel and perpendicular to the plane:

When the PEC plane (or dielectric half-space) moves parallel to its own plane, SRT predicts that the reflected wave is not affected by motion with $\alpha_{sc} = \alpha$, $\omega_{sc} = \omega_{inc}$, while HE indicates an increase/decrease in α_{sc}, ω_{sc} values w.r.t. α, ω_{inc} with the same scaling factor $1/(1-2\beta\sin\alpha)$, which yields a first order departure in β between the results calculated by these two methods. On the other hand, in both theories the transmission angle $\alpha_d 4$ is predicted to differ from its value at rest.

When the PEC plane (or dielectric half-space) moves perpendicular to its own plane, HE predicts that $\alpha_{sc} = \alpha$, i.e., the angle of reflection is unaffected from motion, while SRT indicates an aberration $\alpha_{sc} \neq \alpha$, which yields a first order departure in β between the results calculated by these two methods. On the other hand, both methods predict a Doppler shift $\omega_{sc} \neq \omega_{inc}$, while the departure inbetween the corresponding results is of order β^2.

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[3]Plane wave scattering by a PEC plane and a dielectric half space in uniform motion constitute the most basic canonical problems in the context of electrodynamics of moving bodies and it has been investigated intensely in literature by SRT as initiated in the original paper of Einstein [10]. The results given in Table 1 are adopted from ([11], Section 7.5) and [2].

6. Concluding Remarks

With the scope of reviving interest in HE, the present work is planned to be extended to demonstrate the predictions of HE for a broad set of canonical problems with important applications. (cf.[12]). Of primary interest are

i.the study of Doppler spectrum of moving sources;

ii.3-D radiation and scattering problems of practical interest for various combinations of different types of sources, propagation media, scattering objects with/without edges, and modes of Euclidean motion;

iii.the predictions of HE for various rotating electrical devices as well as for interferometry and GPS experiments, etc.;

iv.the investigation of progressive derivatives, the field equations and solutions to boundary value problems of practical interest for bodies in non-Euclidean motion, especially involving radial expansion/contraction mechanisms with nonsolenoidal and irrotational velocity fields.

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TABLE 1. A comparison of the results derived with HE and SRT.

Canonical Problem	Solution by HE	Solution by SRT
Section 3.1	$R_{TE/TM}=\mp 1$ $\frac{\sin\alpha_{sc}}{\sin\alpha}=\frac{\omega_{sc}}{\omega_{inc}}=\frac{1}{1-2\beta\sin\alpha}$	$R_{TE/TM}=\mp 1, \alpha_{sc}=\alpha,\ \omega_{sc}=\omega_{inc}$ No Doppler shift
	$(\sin\alpha_{sc}/\sin\alpha)_{HE}-(\sin\alpha_{sc}/\sin\alpha)_{SRT}=(\omega_{sc}/\omega_{inc})_{HE}-(\omega_{sc}/\omega_{inc})_{SRT}$ $=2\beta\sin\alpha+\textit{higher order terms in }\beta$	
Section 3.2	$R_{TE/TM}=\mp 1\ (\alpha=0)$ $\alpha_{sc}=\alpha,\ \frac{\omega_{sc}}{\omega_{inc}}=1-2\beta\cos\alpha$	$R_{TE/TM}=\mp(1-\beta)^2/(1-\beta^2)\ (\alpha=0)$ $\frac{\omega_{sc}}{\omega_{inc}}=\frac{1-2\beta\cos\alpha+\beta^2}{1-\beta^2}$ $\cos\alpha_{sc}=\frac{(1+\beta^2)\cos\alpha-2\beta}{1-2\beta\cos\alpha+\beta^2}$
	$(\cos\alpha_{sc}-\cos\alpha)_{HE}-(\cos\alpha_{sc}-\cos\alpha)_{SRT}=-2\beta\sin^2\alpha+h.o.t.$ $(\omega_{sc}/\omega_{inc})_{HE}-(\omega_{sc}/\omega_{inc})_{SRT}=-2\beta^2+h.o.t$ $\left(R_{TE/TM}\right)_{HE}-\left(R_{TE/TM}\right)_{SRT}=\mp 2\beta+h.o.t.\ (\alpha=0)$	
Section 4.1	$\frac{\sin\alpha_{sc}}{\sin\alpha}=\frac{\omega_{sc}}{\omega_{inc}}=\frac{1}{1-2\beta\sin\alpha}$ $\frac{\omega_{tr}}{\omega_{inc}}=\frac{1}{1-\beta\sin\alpha},$ $\frac{\sin\alpha_d}{\sin\alpha}=\frac{1}{n(1-\beta\sin\alpha)}$	$\alpha_{sc}=\alpha,\ \omega_{sc}=\omega_{tr}=\omega_{inc}$ $\frac{\sin\alpha_d}{\sin\alpha}=\frac{1}{\sqrt{\gamma^2(n^2-1)(1-\beta\sin\alpha)^2+1}}$ No Doppler shift
	$(\sin\alpha_{sc}/\sin\alpha)_{HE}-(\sin\alpha_{sc}/\sin\alpha)_{SRT}=(\omega_{sc}/\omega_{inc})_{HE}-(\omega_{sc}/\omega_{inc})_{SRT}$ $=2\beta\sin\alpha+h.o.t.$ $(\sin\alpha_d/\sin\alpha)_{HE}-(\sin\alpha_d/\sin\alpha)_{SRT}=n^{-3}\beta\sin\alpha+h.o.t.$ $(\omega_{tr}/\omega_{inc})_{HE}-(\omega_{tr}/\omega_{inc})_{SRT}=\beta\sin\alpha+h.o.t.$	
Section 4.2	$\alpha_{sc}=\alpha$ $\frac{\sin\alpha_d}{\sin\alpha}=\frac{1}{n(1-\beta\cos\alpha)}$ $\omega_{sc}/\omega_{inc}=1-2\beta\cos\alpha$ $\omega_{tr}/\omega_{inc}=1+\beta n\sqrt{1-\frac{\sin^2\alpha}{\left[n^2(1-\beta\cos\alpha)^2\right]}}$	$\cos\alpha_{sc}=\frac{(1+\beta^2)\cos\alpha-2\beta}{1-2\beta\cos\alpha+\beta^2}$ $\frac{\sin\alpha_d}{\sin\alpha}=\left\{\begin{array}{l}\sin^2\alpha\\+\gamma^2\left[\begin{array}{l}\beta(1-\beta\cos\alpha)\\\times[\beta(1-\beta\cos\alpha)\\+2q]+q^2\end{array}\right]\end{array}\right\}^{-1/2}$ $\frac{\omega_{sc}}{\omega_{inc}}=\frac{1-2\beta\cos\alpha+\beta^2}{1-\beta^2},\frac{\omega_{tr}}{\omega_{inc}}=\frac{1-\beta\cos\alpha+\beta q}{1-\beta^2}$ $q=\left\{\begin{array}{l}\frac{\left[\gamma^2(n^2-1)(1-\beta\cos\alpha)^2-\sin^2\alpha\right]}{\gamma^2}\\+(1-\beta\cos\alpha)^2\end{array}\right\}^{1/2}$
	$(\cos\alpha_{sc}-\cos\alpha)_{HE}-(\cos\alpha_{sc}-\cos\alpha)_{SRT}=-2\beta\sin^2\alpha+h.o.t.$ $(\omega_{sc}/\omega_{inc})_{HE}-(\omega_{sc}/\omega_{inc})_{SRT}=-2\beta^2+h.o.t$	

Burak Polat, for a photograph and biography, see TWMS Journal of Applied and Engineering Mathematics, Volume 1, No.2, 2011.
