TWMS J. App. Eng. Math. V.10, N.2, 2020, pp. 338-345

EVEN VERTEX ODD MEAN LABELING OF TRANSFORMED TREES

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Abstract. Let $G = (V, E)$ be a graph with p vertices and q edges. A graph G is said have an even vertex odd mean labeling if there exists a function $f : V(G) \to \{0, 2, 4, ..., 2q\}$ satisfying f is 1-1 and the induced map $f^* : E(G) \to \{1, 3, 5, ..., 2q-1\}$ defined by $f^*(uv) = \frac{f(u)+f(v)}{2}$ is a bijection. A graph that admits even vertex odd mean labeling is called an even vertex odd mean graph. In this paper, we prove that T_p-tree (transformed tree), $T@P_n$, $T@2P_n$ and $\langle T\tilde{o}K_{1,n}\rangle$ (where T is a T_p-tree), are even vertex odd mean graphs.

Keywords: mean labeling, odd mean labeling, T_p-tree, even vertex odd mean labeling, even vertex odd mean graph.

AMS Subject Classification: 05C78

1. Introduction

Throughout this paper by a graph we mean a finite, simple and undirected one. The vertex set and the edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. Terms and notations not defined here are used in the sense of Harary [3]. There are several types of labeling. An excellent survey of graph labeling is available in [2]. The concept of mean labeling was introduced by Somasundaram and Ponraj [5].

A graph $G(V, E)$ with p vertices and q edges is called a mean graph if there is an injective function f that maps $V(G)$ to $\{0, 1, 2, ..., q\}$ such that for each edge uv, labeled with $\frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $\frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$ is odd. Then the resulting edge labels are distinct. The notion odd mean labeling was introduced by Manickam and Marudai in [4].

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§ Manuscript received: August 18, 2018; accepted: August 24, 2019.
TWMS Journal of Applied and Engineering Mathematics, Vol.10, No.2

Let $G(V, E)$ be a graph with p vertices and q edges. A graph G is said to be odd mean if there exists a function $f : V(G) \to \{0, 1, 2, 3, ..., 2q-1\}$ satisfying f is 1-1 and the induced map $f^* : E(G) \to \{1, 3, 5, ..., 2q-1\}$ defined by

$$f^*(uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

The concept of even vertex odd mean labeling was introduced in [6]. Let $G(V, E)$ be a graph with p vertices and q edges. A graph G is said have an even vertex odd mean labeling if there exists a function $f : V(G) \to \{0, 2, 4, ..., 2q\}$ satisfying f is 1-1 and the induced map $f^* : E(G) \to \{1, 3, 5, ..., 2q-1\}$ defined by $f^*(uv) = \frac{f(u)+f(v)}{2}$ is a bijection. A graph that admits even vertex odd mean labeling is called an even vertex odd mean graph. Motivated by the concepts of even vertex odd mean labeling [6] and T_p-tree [1], in this paper we prove that T_p-tree, $T@P_n$, $T@2P_n$ $\langle T\tilde{o}K_{1,n}\rangle$ admit even vertex odd mean labeling.

We use the following definitions in the subsequent sequel.

2. Definition

Definition 2.1. *Let T be a graph and u_o and v_o be two adjacent vertices in $V(T)$. Let there be two pendant vertices u and v in T such that the length of u_o - u path is equal to the length v_o - v path. If the edge u_ov_o is deleted from T and u, v are joined by an edge uv, then such a transformation of T is called an elemantary parallel transformation (or an EPT) and the edge u_ov_o is called a transformable edge. If by a sequence of EPT's T can be reduced to a path, then T is called a T_p-tree (transformed tree) and any such sequence regarded as a composition of mappings (EPT's) denoted by P, is called a parallel transformation of T. The path, the image of T under P is denoted as $P(T)$.*

Definition 2.2. *Let T be a T_p-tree with m vertices. Let $T@P_n$ be the graph obtained from T and m copies of P_n by identifying one pendant vertex of i^{th} copy of P_n with i^{th} vertex of T, where P_n is a path of length $n-1$. Let $T@2P_n$ be the graph obtained from T by identifying the pendant vertices of two vertex disjoint paths of equal lengths $n-1$ at each vertex of the T_p-tree T.*

Definition 2.3. *Let T be a T_p-tree with m vertices. Let $\langle T\tilde{o}K_{1,n}\rangle$ be a graph obtained from T and m copies of $K_{1,n}$ by joining the central vertex of i^{th} copy of $K_{1,n}$ with i^{th} vertex of T by an edge.*

3. Even Vertex Odd Mean Labeling of Transformed Trees

Theorem 3.1. *Every T_p-tree T is an even vertex odd mean graph.*

Proof. Let T be a T_p-tree with n vertices.
By the definition of T_p-tree there exists a parallel transformation P of T such that for the path $P(T)$ we have (i) $V(P(T)) = V(T)$ and
(ii) $E(P(T)) = (E(T)\backslash E_d) \cup E_P$,
where E_d is the set of edges deleted from T and E_P is the set of edges newly added through the sequence $P = (P_1, P_2, ..., P_K)$ of the EPT's P used to arrive at the path $P(T)$. Clearly E_d and E_p have the same number of edges. Now, denote the vertices of $P(T)$ successively as $v_1, v_2, v_3, ..., v_n$ starting from one pendant vertex of $P(T)$ right up to the other.
Define $f : V(T) \to \{0, 2, 4, ..., 2q\}$ as follows:
$f(v_i) = 2(i-1)$ for $1 \le i \le n$.

Let v_iv_j be a transformed edge in T for some indices i and j, $1 \leq i \leq j \leq m$ and P_1 be the EPT that deletes the edge v_iv_j and adds the edge $v_{i+t}v_{j-t}$ where t is the distance of v_i from v_{i+t} and also the distance of v_j from v_{j-t}.
Let P be a parallel transformation of T that contains P_1 as one of the constituent EPT's. Since $v_{i+t}v_{j-t}$ is an edge in the path $P(T)$, $i+t+1 = j-t$ which implies $j = i+2t+1$.
The induced label of the edge v_iv_j is given by,
$f^*(v_iv_j) = f^*(v_iv_{i+2t+1}) = \frac{f(v_i)+f(v_{i+2t+1})}{2} = 2(i+t)-1$ ......(1)
and $f^*(v_{i+t}v_{j-t}) = f^*(v_{i+t}v_{i+t+1}) = \frac{f(v_{i+t})+f(v_{i+t+1})}{2} = 2(i+t)-1$ .....(2)
Therefore from (1) and (2), $f^*(v_iv_j) = f^*(v_{i+t}v_{j-t})$.
Let $e_j = v_jv_{j+1}$ for $1 \leq j \leq n-1$.
For the vertex labeling f, the induced edge label f^* is defined as follows:
$f^*(e_j) = 2j-1$ for $1 \leq j \leq n-1$.
Therefore, f is an even vertex odd mean labeling of T.
Hence, T is an even vertex odd mean graph.
For example, an even vertex odd mean labeling of a T_P-tree with 14 vertices is given in Figure 1. □

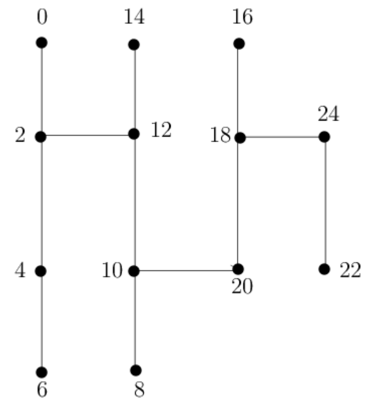

FIGURE 1. T_P-tree with 14 vertices.

Theorem 3.2. *Let T be a T_p-tree with m vertices. Then the graph $T@P_n$ is an even vertex odd mean graph.*

Proof. Let T be a T_p-tree with m vertices. By the definition of a T_p-tree there exists a parallel transformation P of T such that for the path $P(T)$ we have (i) $V(P(T)) = V(T)$ and (ii) $E(P(T)) = (E(T)\backslash E_d) \cup E_p$, where E_d is the set of edges deleted from T and E_p is the set of edges newly added through the sequence $P = (P_1, P_2, ..., P_K)$ of the EPT's P used to arrive at the path $P(T)$. Clearly E_d and E_p have the same number of edges. Now denote the vertices of $P(T)$ successively as $v_1, v_2, v_3, ..., v_m$ starting from one pendant vertex of $P(T)$ right up to other. Let $u_1^j, u_2^j, u_3^j, ..., u_n^j$ $(1 \leq j \leq m)$ be the vertices of j^{th} copy of P_n. Then $V(T@P_n) = \{u_i^j : 1 \leq i \leq n, 1 \leq j \leq m$ with $u_n^j = v_j\}$.
Define $f : V(T@P_n) \to \{0, 2, 4, ..., 2q\}$ as follows:
$f(u_i^j) = 2n(j-1) + 2(i-1)$ if j is odd, $1 \leq i \leq n$, $1 \leq j \leq m$,
$f(u_i^j) = 2n(j-1) + 2(n-i)$ if j is even, $1 \leq i \leq n$, $1 \leq j \leq m$.
Let v_iv_j be a transformed edge in T for some indices i and j, $1 \leq i \leq j \leq m$ and P_1 be

the EPT that deletes the edge v_iv_j and adds the edge $v_{i+t}v_{j-t}$ where t is the distance of v_i from v_{i+t} and also the distance of v_j from v_{j-t}.
Let P be a parallel transformation of T that contains P_1 as one of the constituent EPT's. Since $v_{i+t}v_{j-t}$ is an edge in the path $P(T)$, $i+t+1=j-t$ which implies $j=i+2t+1$. The induced label of the edge v_iv_j is given by,
$f^*(v_iv_j)=f^*(v_iv_{i+2t+1})=\frac{f(v_i)+f(v_{i+2t+1})}{2}=2n(i+t)-1$ ......(3)
$f^*(v_{i+t}v_{j-t})=f^*(v_{i+t}v_{i+t+1})=\frac{f(v_i)+f(v_{i+2t+1})}{2}=2n(i+t)-1$......(4)
Therefore from (3) and (4), $f^*(v_iv_j)=f^*(v_{i+t}v_{j-t})$.
Let $e_i^j=u_i^ju_{i+1}^j$ for $1\leq i\leq n-1$, $1\leq j\leq m$ and $e_j=v_jv_{j+1}$ for $1\leq j\leq m-1$.
For the vertex labeling f, the induced edge label f^* is defined as follows:
$f^*(e_i^j)=2n(j-1)+2i-1$ if j is odd, $1\leq i\leq n-1$, $1\leq j\leq m$,
$f^*(e_i^j)=2(nj-i)-1$ if j is even, $1\leq i\leq n-1$, $1\leq j\leq m$,
$f^*(e_j)=2nj-1$ for $1\leq j\leq m-1$.
Therefore,f is an even vertex odd mean labeling of $T@P_n$. Hence $T@P_n$ is an even vertex odd mean graph. For example, an even vertex odd mean labeling of $T@P_4$, where T is a T_p-tree with 8 vertices, is given in Figure 2. □

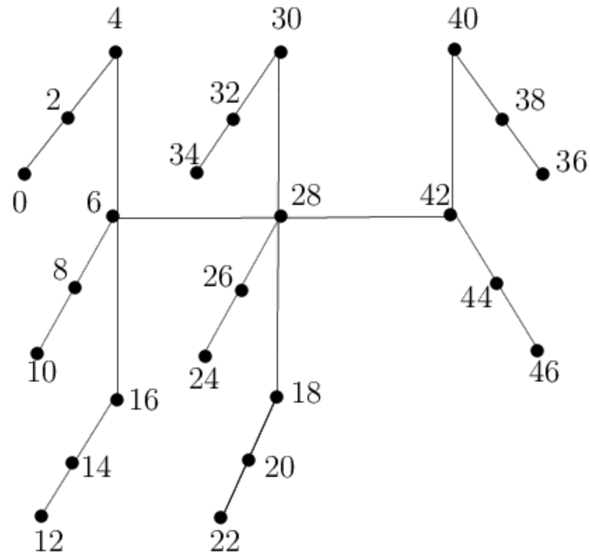

FIGURE 2. $T@P_4$

Theorem 3.3. *Let T be a T_p-tree with m vertices. Then the graph $T@2P_n$ is an even vertex odd mean graph.*

Proof. Let T be a T_p-tree with m vertices. By the definition of a T_p-tree there exists a parallel transformation P of T such that for the path $P(T)$ we have (i) $V(P(T))=V(T)$ and (ii) $E(P(T))=(E(T)\backslash E_d)\cup E_p$, where E_d is the set of edges deleted from T and E_p is the set of edges newly added through the sequence $P=(P_1,P_2,...,P_K)$ of the EPT's P used to arrive at the path $P(T)$. Clearly E_d and E_p have the same number of edges.
Now denote the vertices of $P(T)$ successively as $v_1,v_2,v_3,...,v_m$ starting from one pendant vertex of $P(T)$ right up to other. Let $u_{1,1}^j,u_{1,2}^j,u_{1,3}^j,...,u_{1,n}^j$ and $u_{2,1}^j,u_{2,2}^j,u_{2,3}^j,...,u_{2,n}^j$

$(1 \leq j \leq m)$ be the vertices of the two vertex disjoint paths identified with j^{th} vertex of T such that $v_j = u^j_{1,n} = u^j_{2,n}$. Then $V(T@2P_n) = \{v_j, u^j_{1,i}, u^j_{2,i} : 1 \leq i \leq n, 1 \leq j \leq m$ with $v_j = u^j_{1,n} = u^j_{2,n}\}$. Define $f : V(T@2P_n) \to \{0, 2, 4, ..., 2q\}$ as follows:
$f(u^j_{1,i}) = 2((2n-1)j - 2n + i)$ for $1 \leq i \leq n$, $1 \leq j \leq m$,
$f(u^j_{2,i}) = 2((2n-1)j - i)$ for $1 \leq i \leq n$, $1 \leq j \leq m$.
Let v_iv_j be a transformed edge in T for some indices i and j, $1 \leq i \leq j \leq m$ and P_1 be the EPT that deletes the edge v_iv_j and adds the edge $v_{i+t}v_{j-t}$ where t is the distance of v_i from v_{i+t} and also the distance of v_j from v_{j-t}.
Let P be a parallel transformation of T that contains P_1 as one of the constituent EPT's. Since $v_{i+t}v_{j-t}$ is an edge in the path $P(T)$, $i + t + 1 = j - t$ which implies $j = i + 2t + 1$. The induced label of the edge v_iv_j is given by,
$f^*(v_iv_j) = f^*(v_iv_{i+2t+1}) = \frac{f(v_i)+f(v_{i+2t+1})}{2} = 2(2n-1)(i+t) - 1$ ......(5)
$f^*(v_{i+t}v_{j-t}) = f^*(v_{i+t}v_{i+t+1}) = \frac{f(v_i)+f(v_{i+2t+1})}{2} = 2(2n-1)(i+t) - 1$......(6)
Therefore from (5) and (6), $f^*(v_iv_j) = f^*(v_{i+t}v_{j-t})$.
$e^j_{1,i} = u^j_{1,i}u^j_{1,i+1}$ for $1 \leq i \leq n-1$, $1 \leq j \leq m$
$e^j_{2,i} = u^j_{2,i}u^j_{2,i+1}$ for $1 \leq i \leq n-1$, $1 \leq j \leq m$ and
$e_j = v_jv_{j+1}$ for $1 \leq j \leq m-1$.
For the vertex labeling f, the induced edge label f^* is defined as follows:
$f^*(v_jv_{j+1}) = 2j(2n-1) - 1$ for $1 \leq j \leq m-1$,
$f^*(e^j_{1,i}) = 2(2n-1)j + 2i - 4n + 1$ for $1 \leq i \leq n-1$, $1 \leq j \leq m$,
$f^*(e^j_{2,i}) = 2(2n-1)j - 2i - 1$ for $1 \leq i \leq n-1$, $1 \leq j \leq m$,
Therefore,f is an even vertex odd mean labeling of $T@2P_n$.
Hence $T@2P_n$ is an even vertex odd mean graph.
For example, an even vertex odd mean labeling of $T@2P_3$, where T is a T_p-tree with 12 vertices, is given in Figure 3. □

Theorem 3.4. *Let T be a T_p-tree with $2m$ vertices. Then the graph $\langle T\tilde{o}K_{1,n}\rangle$ is an even vertex odd mean graph.*

Proof. Let T be a T_p-tree with $2m$ vertices.
By the definition of T_p-tree there exists a parallel transformation P of T such that for the path $P(T)$, we have (i) $V(P(T)) = V(T)$ and (ii) $E(P(T)) = (E(T)\backslash E_d) \cup E_p$,
where E_d is the set of edges deleted from T and E_p is the set of edges newly added through the sequence $P = (P_1, P_2, ..., P_K)$ of the EPT's P used to arrive at the path $P(T)$.
Clearly E_d and E_p have the same number of edges.
Now denote the vertices of $P(T)$ successively as $v_1, v_2, v_3, ..., v_{2m}$ starting from one pendant vertex of $P(T)$ right up to other.
Let $u^i_0, u^i_1, u^i_2, ..., u^i_n$ be the vertices of the i^{th} copy of $K_{1,n}$, attached with v_i of T by an edge.
Define $f : V(\langle T\tilde{o}K_{1,n}\rangle) \to \{0, 2, 4, ..., 2q\}$ as follows:
$f(v_j) = 2(n+2)(j-1)$ if j is odd and $1 \leq j \leq 2m$,
$f(v_j) = 2(n+2)(j-2) + 4n + 6$ if j is even and $1 \leq j \leq 2m$,
$f(u^j_0) = 2(n+2)(j-1) + 2$ if j is odd and $1 \leq j \leq 2m$,
$f(u^j_0) = 2(n+2)(j-2) + 4n + 4$ if j is even and $1 \leq j \leq 2m$,
$f(u^j_i) = 2(n+2)(j-1) + 4i$ if j is odd and $1 \leq j \leq 2m$, $1 \leq i \leq n$,
$f(u^j_i) = 2(n+2)(j-2) + 4i + 2$ if j is even and $1 \leq j \leq 2m$, $1 \leq i \leq n$.
Let v_iv_j be a transformed edge in T for some indices i and j, $1 \leq i \leq j \leq 2m$ and

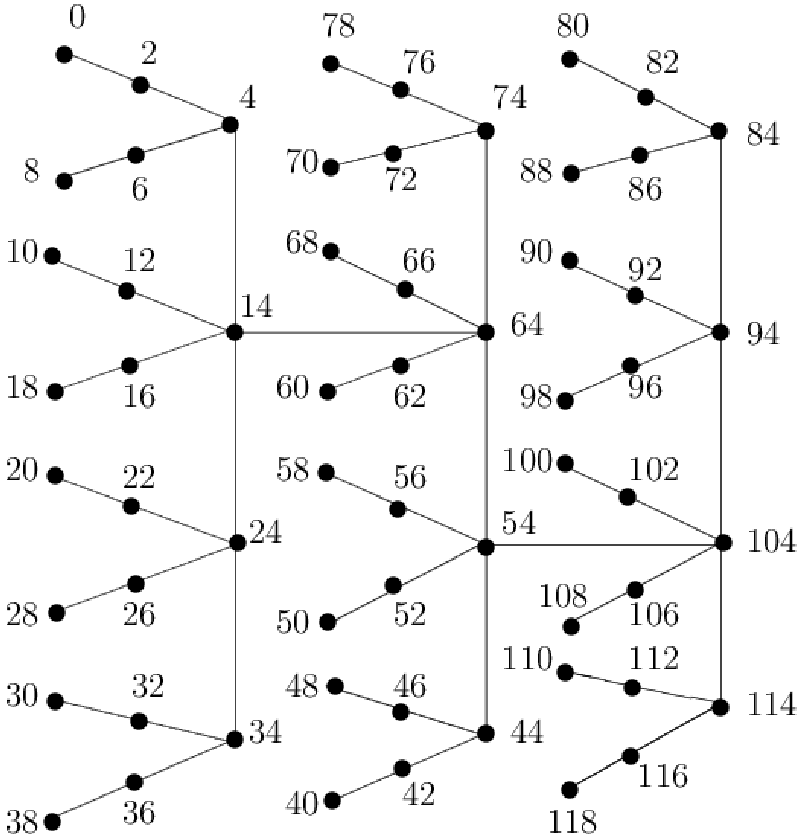

FIGURE 3. $T@2P_3$

P_1 be the EPT that deletes the edge v_iv_j and adds the edge $v_{i+t}v_{j-t}$
where t is the distance of v_i from v_{i+t} and also the distance of v_j from v_{j-t}.
Let P be a parallel transformation of T that contains P_1 as one of the constituent EPT's.
Since $v_{i+t}v_{j-t}$ is an edge in the path $P(T)$, $i+t+1 = j-t$ which implies $j = i+2t+1$.
The induced label of the edge v_iv_j is given by,
$f^*(v_iv_j) = f^*(v_iv_{i+2t+1}) = \frac{f(v_i)+f(v_{i+2t+1})}{2} = 2(n+2)(i+t-1)+2n+3$ ......(7)
$f^*(v_{i+t}v_{j-t}) = f^*(v_{i+t}v_{i+t+1}) = \frac{f(v_i)+f(v_{i+2t+1})}{2} = 2(n+2)(i+t-1)+2n+3$ ......(8).
Therefore from (7) and (8), $f^*(v_iv_j) = f^*(v_jv_{j-t})$.
Let $e_i^j = u_0^ju_i^j$ for $1 \le j \le 2m$, $1 \le i \le n$.
For the vertex labeling f, the induced edge label f^* is defined as follows:
$f^*(v_jv_{j+1}) = 2(n+2)(j-1)+2n+3$ for $1 \le j \le 2m-1$,
$f^*(v_ju_0^j) = 2(n+2)(j-1)+1$ if j is odd and $1 \le j \le 2m$,
$f^*(v_ju_0^j) = 2(n+2)j-3$ if j is even and $1 \le j \le 2m$,
$f^*(u_0^ju_i^j) = 2(n+2)(j-1)+2i+1$ if j is odd and $1 \le j \le 2m$, $1 \le i \le n$,
$f^*(u_0^ju_i^j) = 2(n+2)(j-2)+2(n+i)+3$ if j is even and $1 \le j \le 2m$,$1 \le i \le n$.
Therefore, f is an even vertex odd mean labeling of $\langle T\tilde{o}K_{1,n}\rangle$.
Hence $\langle T\tilde{o}K_{1,n}\rangle$ is an even vertex odd mean graph.
For example, an even vertex odd mean labeling of $\langle T\tilde{o}K_{1,5}\rangle$, where T is a T_p-tree with 8 vertices, is given in Figure 4. □

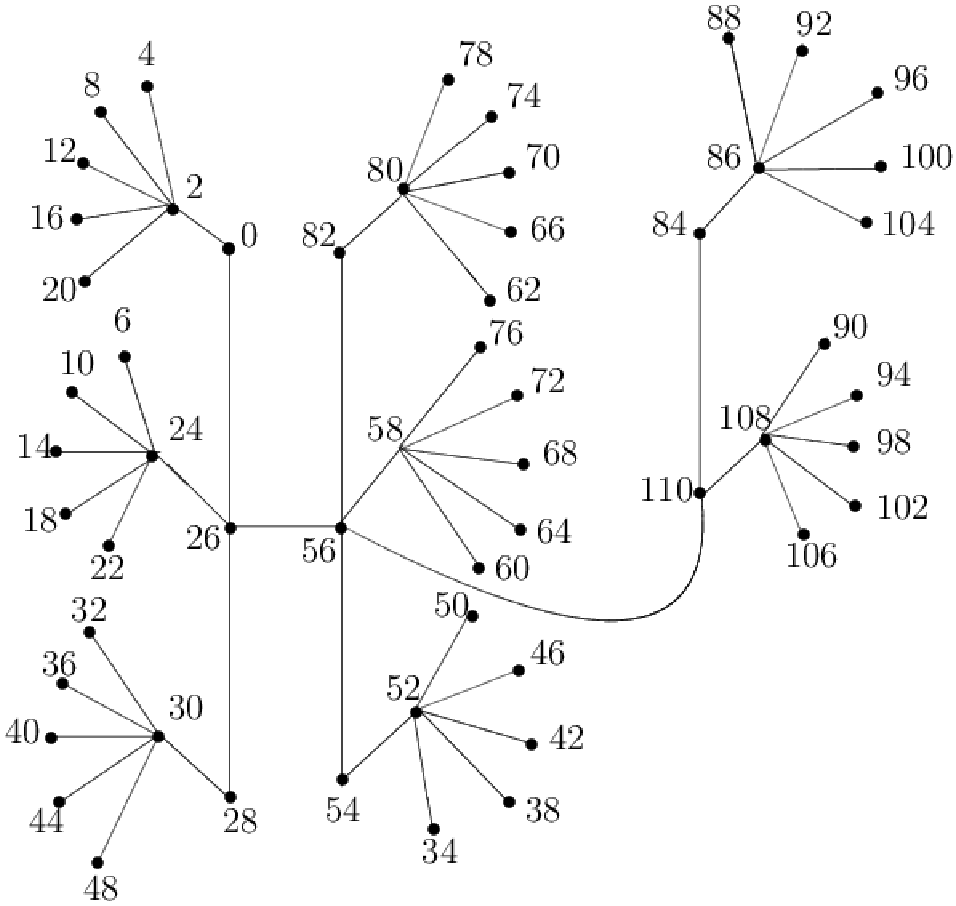

FIGURE 4. $\langle T\tilde{o}K_{1,5}\rangle$

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