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SOME PROPERTIES OF HYPERBOLIC CONTACT MANIFOLD IN A QUASI SASAKIAN MANIFOLD

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Abstract. The present paper deals with different geometrical properties of hyperbolic contact manifold of a quasi sasakian manifold. In the end we studies the properties of the Integrability of the distributions on a hyperbolic contact manifold of a quasi sasakian manifold.

Keywords: Hyperbolic contact manifold, Quasi- Sasakian Manifold.

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1. Introduction

The hyperbolic contact manifold was given by Sasaki (1960), A. Al-Aqeel, A. Hamoli and M. D. Upadhyay (1987) have studied some properties of such manifold.CR-submanifold of a Kahlerian manifold has been defined by A. Bejancu[1]. Later, A. Bejancu and N. Papaghiue [3], introduced and studied the notion of semi-invariant submanifold of a Sasakian manifold.
The purpose of the paper is to define and study different geometrical properties of hyperbolic contact manifold of quasi sasakian manifold.
In Section 2, we recall some results and formula for later use. In Section 3, we prove that the existence of a globally metric frame f-stracture. In Section 4, we show that the integrability of distributions on and geometry of their leaves.
We find the characteristic properties of existence of a globally metric frame f-stracture, integrability of distribution on M and geometry of their leaves on hyperbolic contact manifold of quasi sasakian manifold.

2. Definition and Identities

Let $\bar{M}$ be a real $2n + 1$ dimensional differentiable manifold, endowed with an almost contact metric structure $(\mathrm{f}, \xi, \eta, g)$. Then we have from [5]

$$(a) \quad f^2 = I + \eta \otimes \xi$$

$$(b) \quad \eta(\xi) = -1$$

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$$(c) \quad \eta \circ f = 0 \tag{2.1}$$
$$(d) \quad f(\xi) = 0$$

for every vector field X, then $\bar{M}$ is called a hyperbolic contact manifold.

Let the manifold $\bar{M}$ admits a contact metric g such that

$$(e) \quad \eta(X) = g(X, \xi)$$
$$(f) \quad g(\ fX,\ fY) = g(X, Y) - \eta(X)\eta(Y)$$

then $\bar{M}$ is said to be a hyperbolic contact metric manifold.
A hyperbolic contact metric manifold is said to be normal (Blair, 1976) if

$$N^{(1)} = [f, f] + d\eta \otimes \xi = 0 \tag{2.2}$$

we call $\bar{M}$ to be a certain class of hyperbolic contact metric manifold if

$$(\bar{\nabla}_X f)Y = g(fX, Y) + \eta(Y)fX \tag{2.3}$$

and

$$(\bar{\nabla}_X f)Y + (\bar{\nabla}_Y f)X = \eta(Y)fX + \eta(X)fY \tag{2.4}$$

where $\bar{\nabla}$ denotes the covariant differentiation with respect to g. Let $\bar{M}$ be a hyperbolic contact metric manifold $\bar{M}$. According to [6] we say that $\bar{M}$ is a quasi-Sasakian manifold if and only if ξ is a Killing vector field and

$$(\bar{\nabla}_X f)Y = g(\bar{\nabla}_{fX}\xi, Y)\xi - \eta(Y)\bar{\nabla}_{fX}\xi \quad \forall X, Y \epsilon \Gamma(T\bar{M}) \tag{2.5}$$

Next we define a tensor field F of type $(1, 1)$ by

$$FX = -\bar{\nabla}_X \xi \quad \forall X \epsilon \Gamma(T\bar{M}) \tag{2.6}$$

From [6] we recall

Lemma 2.1. Let $\bar{M}$ be a quasi-Sasakian manifold. Then we have

$$(a) \quad (\bar{\nabla}_\xi f)X = 0 \quad \forall X \epsilon \Gamma(T\bar{M})$$
$$(b) \quad f \circ F = F \circ f$$
$$(c) \quad F\xi = 0 \tag{2.7}$$
$$(d) \quad g(FX, Y) + g(X, FY) = 0 \quad \forall X, Y \epsilon \Gamma(T\bar{M})$$
$$(e) \quad \eta \circ F = 0$$
$$(f) \quad (\bar{\nabla}_X F)Y = \bar{R}(\xi, X)Y \quad \forall X, Y \epsilon \Gamma(T\bar{M})$$

let M be a hyperbolic contact manifold of a quasi-sasakian manifold $\bar{M}$ and denote by N the unit vector field normal to M. Denote by the same symbol g the induced tensor metric on M, by ∇ the induced Levi-Civita connection on M and by $TM^{\perp}$ the normal vector bundle to M.The Gauss and Weingarten formulae are

$$(a) \quad \bar{\nabla}_X Y = \nabla_X Y + B(X, Y)N, \tag{2.8}$$

$$(b) \quad \bar{\nabla}_X N = -AX \quad \forall X, Y \epsilon \Gamma(TM)$$

where A is the shape operator with respect to the section N . It follows that

$$B(X,Y) = g(AX,Y) \quad \forall X, Y \epsilon \Gamma(TM) \tag{2.9}$$

Because the position of the structure vector field with respect to M is very important we prove the following results.

Theorem 2.1 Let M be a hyperbolic contact manifold of a quasi-sasakian manifold $\bar{M}$. If the structure vector field ϵ is normal to M then $\bar{M}$ is cosympletic manifold and M is totally geodesic immersed in $\bar{M}$.

Proof. Because $\bar{M}$ is hyperbolic contact manifold in a quasi-Sasakian manifold , then it is normal and $d\phi = 0$ ([4]). By direct calculation using (2.8) (b), we infer

$$2d\eta(X,Y) = g(\bar{\nabla}_X \xi, Y) - g(\bar{\nabla}_Y \xi, X) = g(AY, X) - g(AX, Y) = 0 \quad \forall X, Y \epsilon \Gamma(TM) \tag{2.10}$$

From (2.8) (b) and (2.10) we deduce

$$0 = d\eta(X,Y) = g(Y, \bar{\nabla}_X \xi) = -g(AX,Y) = 0 \quad \forall X, Y \epsilon \Gamma(TM) \tag{2.11}$$

which proves that M is totally geodesic. From (2.11) we obtain $\bar{\nabla}_X \xi = 0 \quad \forall X \epsilon \Gamma(TM)$ By using (2.6),(2.7)(b)and (2.1) (d) from the above relation we state

$$\bar{\nabla}_X \xi = -f \bar{\nabla}_{fX} \xi = 0 \quad \forall X \epsilon \Gamma(T\bar{M}) \tag{2.12}$$

because $fX \epsilon \Gamma(T\bar{M}) \quad \forall X \epsilon \Gamma(T\bar{M})$. Using (2.12) and the fact that ξ is a Killing vector field, we deduce $d\eta = 0$ that is $\bar{M}$ is a cosympletic manifold.

The proof is complete.

Next we consider only the hyperbolic contact manifold which are tangent to ξ. Denote by $U = fN$ and from (2.1) (f), we deduce $g(U,U) = -1$. Moreover, it is easy to see that $U \epsilon \Gamma(TM)$ .Denote by $D^{\perp} = Span(U)$ the 1-dimensional distribution generated by U, and by D the orthogonal complement of $D^{\perp} \oplus (\xi)$ in TM. It is easy to see that

$$fD = D; \quad fD^{\perp} \subseteq TM^{\perp}; \quad TM = D \oplus D^{\perp} \oplus (\xi) \tag{2.13}$$

where $\oplus$ denote the orthogonal direct sum. According with [1] from (2.13) we deduce that M is a CR-submanifold of $\bar{M}$.

A CR-submanifold M of a quasi-Sasakian manifold $\bar{M}$ is called CR-product if both distributions $D \oplus (\xi)$ and $D^{\perp}$ are integrable and their leaves are totally geodesic submanifold of M.

Denote by P the projection morphism of TM to D and using the decomposion in (2.13) we deduce

$$X = PX + a(X)U + \eta(X)\xi \quad \forall X \epsilon \Gamma(TM) \tag{2.14}$$

Where a is a 1-form on M defined by $a(X) = g(X,U), \quad X \epsilon \Gamma(TM)$. From (2.14) using (2.1) (a) we infer

$$fX = tX + a(X)N \quad \forall X \epsilon \Gamma(TM) \tag{2.15}$$

Where t is a tensor field defined by $tX = fPX, \quad X \epsilon \Gamma(TM)$

It is easy to see that

$$(a)\quad t\xi = 0 \qquad (b)\quad tU \neq 0 \tag{2.16}$$

3. Induced structures on a hyperbolic contact manifold in a quasi-sasakian manifold

Let M be hyperbolic contact manifold in a quasi-sasakian manifold. From (2.1) (a), (2.15) and (2.16) we obtain

$$t^2X = X - a(X)U - \eta(X)\xi \quad \forall X\epsilon\Gamma(TM) \tag{3.1}$$

LEMMA 3.1. On a hyperbolic contact manifold M of a quasi-Sasakian manifold $\bar{M}$ the tensor field t satisfies

$$(a)\quad g(tX, tY) = -g(X, Y) - \eta(X)\eta(Y) - a(X)a(Y) \tag{3.2}$$

$$(b)\quad g(tX, Y) + g(X, tY) = 0 \quad \forall X, Y\epsilon\Gamma(TM)$$

Proof. From (2.1) (f), and (2.15) we deduce

$$-g(X, Y) - \eta(X)\eta(Y) = g(fX, fY) = g(tX + a(X)N, tY + a(Y)N)$$

$$= g(tX, tY) + a(Y)g(tX, N) + a(X)g(N, tY) + a(X)a(Y)g(N, N)$$

$$= g(tX, tY) + a(X)a(Y) \quad \forall X, Y\epsilon\Gamma(TM)$$

$$g(tX, Y) + g(X, tY) = g(fX - a(X)N, Y) + g(X, fY - a(Y)N)$$

$$= g(fX, Y) - a(X)g(N, Y) + g(X, fY) - a(Y)g(X, N)$$

$$= g(fX, Y) + g(X, fY) = 0$$

LEMMA 3.2. Let M be a hyperbolic contact manifold in a quasi-sasakian manifold $\bar{M}$. Then we have

$$(a)\quad FU = -fA\xi \quad (b)\quad FN = -A\xi \quad (c)\quad [U, \xi] \neq 0 \tag{3.3}$$

Proof. We take $X = U$, and $Y = \xi$ in (2.1) and obtain

$$f\bar{\nabla}_U\xi = -\bar{\nabla}_N\xi$$

Then using (2.1) (a), (2.6), (2.8)(b), we deduce the assertion (a). The assertion (b) follows from (2.1) (a), (2.7) (b) and (2.8) (b) we derive

$$\bar{\nabla}_\xi U = (\bar{\nabla}_\xi f)N + f\bar{\nabla}_\xi N = -fA\xi = FU = -\bar{\nabla}_U\xi,$$

$$[U, \xi] = \bar{\nabla}_U\xi - \bar{\nabla}_\xi U = \bar{\nabla}_U\xi + \bar{\nabla}_U\xi \neq 0$$

Which prove assertion (c). By using the decomposition $T\bar{M} = TM \oplus TM^\perp$, we deduce

$$FX = \alpha X - \eta(AX)N, \quad \forall X\epsilon\Gamma(T\bar{M}) \tag{3.4}$$

where α is a tensor field of type (1, 1) on M, since $g(FX, N) = -g(X, FN) = g(X, A\xi) = \eta(AX) \quad \forall X\epsilon\Gamma(T\bar{M})$. By using (2.5), (2.6), (2.8), (2.15) and (3.1), we obtain

THEOREM 3.2. Let M be a hyperbolic contact manifold in a quasi-sasakian manifold $\bar{M}$. Then the covariant derivative of a tensors t, a, η and α are given by

$$\begin{aligned} &(a)\quad (\nabla_X t)Y = g(FX, fY)\xi + \eta(Y)[\alpha tX - \eta(AX)U] \\ &\qquad -a(Y)AX + B(X,Y)U \\ &(b)\quad (\nabla_X a)Y = B(X, tY) + \eta(Y)\eta(AtX) \\ &(c)\ (\nabla_X \eta)(Y) = g(Y, \nabla_X \xi) \qquad \text{and} \\ &(d)\quad (\nabla_X \alpha)Y = R(\xi, X) - \eta(AY)AX - g(AX, Y)A\xi \quad \forall X, Y \epsilon \Gamma(TM) \end{aligned} \tag{3.5}$$

respectively, where R is the curvature tensor field of M.
From (2.5), (2.6), (2.16) (a) (b) and (3.5) (a) we get
PROPOSITION 3.1 On a hyperbolic contact manifold M of a quasi-sasakian manifold $\bar{M}$, we have

$$\begin{aligned} &(a)\quad \nabla_X U = -tAX + \eta(AtX)\xi \\ &(b)\quad B(X, U) = a(AX) \quad \forall X \epsilon \Gamma(TM) \end{aligned} \tag{3.6}$$

Next we state
THEOREM 3.3 Let M be a hyperbolic contact manifold in a quasi-sasakian manifold $\bar{M}$. The tensor field t is a parallel with respect to the Levi Civita connection ∇ on M iff

$$\begin{aligned} &(a)\quad AX = -\eta(AX)\xi - a(AX)U \quad \text{and} \\ &(b)\quad FX = \eta(AtX)U - \eta(AX)N \quad \forall X \epsilon \Gamma(TM) \end{aligned} \tag{3.7}$$

Proof. Suppose that the tensor field t is parallel with respect to ∇, that is $\nabla t = 0$. By using (3.5) (a), we deduce

$$\eta(Y)[\alpha tX - \eta(AX)U] + g(FX, fY)\xi - a(Y)AX + B(X,Y)U = 0 \quad \forall X, Y \epsilon \Gamma(TM) \tag{3.8}$$

Take $Y = U$ in (3.8) and using (2.8) (b), (2.9), (3.6) (b) we infer

$$\eta(U)[\alpha tX - \eta(AX)U] - a(U)AX + g(FX, fU)\xi + B(X,U)U = 0$$

$$\eta(U) = g(U, \xi) = 0, \quad a(U) = -1 \text{ and } g(X, N) = 0$$

$$\begin{aligned} AX &= -g(FX, fU)\xi - a(AX)U = -g(FX, N)\xi - a(AX)U, \qquad fU = N \\ &= -\eta(AX)\xi - a(AX)U \end{aligned}$$

Next let $Y = fZ$, $\quad Z\epsilon\Gamma(D)$ in (2.12) and using (2.1) (f), (2.7) (b), (3.3) (a) (b), (3.7) (a), we deduce

$$\eta(fZ)[\alpha tX - \eta(AX)U] + g(fX, fFZ)\xi - a(fZ)AX + B(X, fZ)U = 0$$

$$g(fX, fFZ)\xi + B(X, fZ)U = 0 \quad \eta(fZ) = 0$$

$$-g(X, FZ)\xi - \eta(X)\eta(FZ)\xi + B(X, fZ)U = 0$$

$$-g(X, FZ)\xi + a(AX)a(fZ)U = 0$$

$$g(X, FZ) = 0 \Rightarrow FX = \eta(AtX)U - \eta(AX)N \quad \forall X \epsilon \Gamma(TM)$$

The proof is complete.

PROPOSITION 3.2. Let M be a hyperbolic contact manifold in a quasi-sasakian manifold $\bar{M}$. Then we have the assertions

$$(a) \quad (\nabla_X a)Y = 0 \Leftrightarrow \nabla_X U = 0$$

$$(b) \quad (\nabla_X \eta)Y = 0 \Leftrightarrow \nabla_X \xi = 0 \quad \forall X, Y \epsilon \Gamma(TM)$$

Proof. Let $X, Y \epsilon \Gamma(TM)$ and using (2.9), (3.2) (b), (3.5) (b) and (3.6) (a) we obtain

$$g(\nabla_X U, Y) = g(-tAX + \eta(AtX)\xi, Y)$$

$$= g(-tAX, Y) + \eta(AtX)g(\xi, Y)$$

$$= g(AX, tY) + \eta(AtX)\eta(Y) = B(X, tY) + \eta(AtX)\eta(Y)$$

$$= (\nabla_X a)Y$$

$$g(\nabla_X U, Y) = (\nabla_X a)Y \quad if \quad \nabla_X U = 0$$

$$(b) \quad (\nabla_X \eta) = g(Y, \nabla_X \xi)$$

Killing vector $\nabla_X \xi = 0$

$$(\nabla_X \eta)Y = 0$$

4. Integrability of distributions on a hyperbolic contact manifold in a quasi-sasakian manifold $\bar{M}$

From Lemma 3.2 we obtain

COROLLARY 4.1 On a hyperbolic contact manifold M of a quasi-sasakian manifold $\bar{M}$ there exists a 2-dimensional foliation determined by the integral distribution $D^{\perp} \oplus (\xi)$

THEOREM 4.1 Let M be a hyperbolic contact manifold in a quasi-sasakian manifold $\bar{M}$. Then (a) the distribution $D \oplus (\xi)$ is integrable iff

$$g(AfX + fAX, Y) = 0, \quad \forall X, Y \epsilon \Gamma(D), \tag{4.1}$$

(b) the distribution D is integrable iff (3.7) holds and

$$FX = \eta(AtX)U - \eta(AX)N, \quad (equivalent \quad with \quad FD \perp D) \quad \forall X \epsilon \Gamma(D)$$

(c) The distribution $D \oplus D^{\perp}$ is integrable iff $FX = 0, \quad \forall X \epsilon \Gamma(D)$.

Proof. Let $X, Y \epsilon \Gamma(D)$. Since ∇ is a torsion free and ξ is a Killing vector field, we infer

$$g([X, \xi], U) = g(\bar{\nabla}_X \xi, U) - g(\bar{\nabla}_\xi X, U) \tag{4.2}$$

$$= g(\nabla_X \xi + B(X, \xi)N, U) - g(\nabla_\xi X + B(\xi, X)N, U) = 0 \quad \forall X \epsilon \Gamma(D)$$

$$\begin{aligned} B(X, \xi) &= g(AX, \xi) = \eta(AX) \\ &= g(\nabla_X \xi, U) + B(X, \xi)g(N, U) - g(\nabla_\xi X, U) - B(\xi, X)g(N, U) \\ &= g(\nabla_X \xi, U) - g(\nabla_\xi X, U) = 0 \end{aligned}$$

Using (2.1) (a), (2.8) (a) we deduce

$$g([X,Y],U) = g(\bar{\nabla}_X Y - \bar{\nabla}_Y X, U) = g(\bar{\nabla}_X Y - \bar{\nabla}_Y X, fN) \tag{4.3}$$
$$= g(\bar{\nabla}_Y fX, N) - g(\bar{\nabla}_X fY, N) = g(-\bar{\nabla}_{fX} N, Y) + g(f(-\bar{\nabla}_X N, Y)$$
$$= g(AfX + \eta(fX)N, Y) + g(AfX + f\eta(X)N, Y)$$
$$= g(AfX, Y) + g(fAX, Y) + \eta(X) g(fN, Y) = g(AfX + fAX, Y) \quad \forall X, Y \epsilon \Gamma(D)$$

Next by using (2.6) (2.7) (d) and the fact that ∇ is a metric connection we get

$$g([X,Y],\xi) = g(\bar{\nabla}_X Y, \xi) - g(\bar{\nabla}_Y X, \xi) \tag{4.4}$$
$$= 2g(-\bar{\nabla}_X \xi, Y) = 2g(FX, Y) \quad \forall X, Y \epsilon \Gamma(D)$$

The assertion (a) follows from (4.2), (4.3) and assertion (b) follows from (4.2)-(4.4). Using (2.6) and (2.7) we obtain

$$g([X,U],\xi) = g(\bar{\nabla}_X U, \xi) - g(\bar{\nabla}_U X, \xi) \tag{4.5}$$
$$= g(-\bar{\nabla}_X \xi, U) - g(\bar{\nabla}_X \xi, U) = 2g(FX, U) \quad \forall X \epsilon \Gamma(D)$$

Taking into account that

$$g(FX, N) = g(FfX, fN) = g(FfX, U) \quad \forall X \epsilon \Gamma(D) \tag{4.6}$$

The assertion (c) follows from (4.4) and (4.5).

THEOREM 4.2 Let M be a hyperbolic contact manifold in a quasi-sasakian manifold $\bar{M}$. Then we have

(a) the distribution D is integrable and its leaves are totally geodesic immersed in M if and only if

$$FD \perp D \quad and \quad AX = a(AX)U + \eta(AX)\xi, \quad \forall X \epsilon \Gamma(D) \tag{4.7}$$

(b) the distribution $D \oplus (\xi)$ is integrable and its leaves are totally geodesic immersed in if and only if

$$AX = a(AX)U, \quad X \epsilon \Gamma(D) \quad and \quad FU = 0 \tag{4.8}$$

(c) the distribution $D \oplus D^{\perp}$ is integrable and its leaves are totally geodesic immersed in M if and only if

$$FX = 0, \quad X \epsilon \Gamma(D).$$

Proof. Let M_1^* be a leaf of integrable distribution D and h_1^* the second fundamental form of immersion $M_1^* \to M$. Then by direct calculation we infer

$$g(h_1^*(X,Y),U) = g(\bar{\nabla}_X Y, U) = -g(Y, \nabla_X U) = -g(AX, tY), \tag{4.9}$$

and

$$g(h_1^*(X,Y),\xi) = g(\bar{\nabla}_X Y, \xi) = g(FX, Y) \quad \forall X, Y \epsilon \Gamma(D) \tag{4.10}$$

Now suppose M_1^* is a totally submanifold of M. Then (4.7) follows from (4.9) and (4.10). Conversely suppose that (4.7) is true. Then using the assertion (b) in Theorem 4.1 it is easy to see that the distribution D is integrable. Next the proof follows by using (4.9) and (4.10). Next, suppose that the distribution $D \oplus (\xi)$ is integrable and its leaves are totally geodesic submanifolds of M. Let M_1 be a leaf of $D \oplus (\xi)$ and h_1 the second fundamental form of immersion $M_1 \rightarrow M$. By direct calculations, using (2.6), (2.8) (b), (3.2) (b) and (3.6) (c), we deduce

$$g(h_1(X,Y),U) = g(\bar{\nabla}_X Y, U) = -g(AX, tY), \quad \forall X, Y \epsilon \Gamma(D) \tag{4.11}$$

and

$$g(h_1(X,\xi),U) = g(\bar{\nabla}_X \xi, U) = -g(FU, X), \quad \forall X \epsilon \Gamma(D) \tag{4.12}$$

Then the assertion (b) follows from (4.6), (4.11), (4.12) and the assertion (a) of Theorem 4.1 . Next let $\bar{M_1}$ a leaf of the integrable distribution $D \oplus D^{\perp}$ and $\bar{h_1}$ the second fundamental form of the immersion $\bar{M_1} \rightarrow M$. By direct calculation we get

$$g(\bar{h_1}(X,Y),\xi) = g(FX,Y), \quad \forall X \epsilon \Gamma(D), Y \epsilon \Gamma(D \oplus D^{\perp}). \tag{4.13}$$

The assertion (c) follows from (2.7) (c), (4.6) and (4.13)

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