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Covering-based Generalized Intuitionistic Fuzzy-Rough Set Models for A Selecting HVAC System

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Abstract: In today's urban life, modern buildings such as apartments, schools, hospitals, and work offices are quite important for people's life. Since people spend most of their time in these buildings, a significant decision-making process is required for the design and selection of heating (H), ventilation (V), and air conditioning (AC) systems. The most important criteria in these decision-making processes are the low effect on the environment, high comfort, low cost, and high energy productivity. These basic parameters considered to select the most suitable HVAC systems in the buildings may not have crisp values every time. Since some of the criteria for HVAC systems are described as linguistic, it is not possible to evaluate the systems with traditional methods using crisp values. Therefore, we propose new and flexible method called covering-based generalized intuitionistic fuzzy (IF)-rough set models based on IF-technique for order preference by similarity to ideal solution (TOPSIS) principles. The upper and lower approximations in the rough set theory, and IF-implicator and IF-t norms operators in the IF-neighborhoods are utilized for the proposed methodology. In this study, nine different HVAC systems are investigated according to nine different criteria for four main factors in the selection of HVAC systems. According to the obtained results, it can be seen that the proposed method is a suitable multi criteria decision-making (MCDM) approach that considers the linguistic uncertainties in order to determine the most suitable HVAC system.

Bir HVAC Sistemi Seçimi için Örtme Tabanlı Genelleştirilmiş Sezgisel Bulanık-Kaba Küme Modelleri

Anahtar Kelimeler
HVAC,
Sezgisel Bulanık-Örtme,
Kaba Küme Teorisi,
Bulanık Küme Yaklaşımları

Öz: Günümüz kent yaşamında apartman, okul, hastane, işyeri gibi modern yapılar insan yaşamı için oldukça önemlidir. İnsanlar zamanlarının çoğunu bu binalarda geçirdikleri için ısıtma (H), havalandırma (V) ve iklimlendirme (AC) sistemlerinin tasarımı ve seçimi için önemli bir karar verme süreci gerekmektedir. Bu karar verme süreçlerinde en önemli kriterler; çevreye az etki, yüksek konfor, düşük maliyet ve yüksek enerji verimliliğidir. Binalarda en uygun HVAC sistemlerini seçmek için düşünülen bu temel parametreler her zaman net değerlere sahip olmayabilir. HVAC sistemleri için bazı kriterler dilsel olarak tanımlandığından, sistemlerin kesin değerler kullanılarak geleneksel yöntemlerle değerlendirilmesi mümkün değildir. Bu nedenle, IF-TOPSIS prensiplerine dayalı, kapsama tabanlı genelleştirilmiş sezgisel bulanık-kaba küme modelleri adı verilen yeni ve esnek bir yöntem öneriyoruz. Önerilen metodoloji için kaba küme teorisindeki üst ve alt yaklaşımlar ve sezgisel bulanık-komşularındaki sezgisel bulanık-anlamlandırıcı ve sezgisel bulanık-t norm operatörleri kullanılmıştır. Bu çalışmada, HVAC sistemlerinin seçiminde dört ana faktör için dokuz farklı kritere göre dokuz farklı HVAC sistemi incelenmiştir. Elde edilen sonuçlara göre, önerilen yöntemin en uygun HVAC sistemini belirlemek için dilsel belirsizlikleri dikkate alan uygun çok kriterli karar verme yaklaşımı olduğu görülmektedir.

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1. Introduction

Buildings are very important for people in modern city life. Due to developing cities with the industrial revolution, the population of cities increased and socio-economic structure changed. This situation affects the shape of the structure and the purpose of the materials used. The population growth in the cities forced people to build a multi-storey building. This caused the problems of heating, lighting and ventilation in multi-storey buildings. Because heating, cooling, ventilation and lighting services must be equally provided to many households at the same time. With the development of technology, the increase in living standards positively affected the comfort of people's living and working areas. Heating, lighting, ventilation and air conditioning in buildings are the main factors that determine the comfort level. Through the today's technology, HVAC systems can control these factors. We can say that HVAC systems have four main evaluation criteria. These are energy consumption, comfort, environmental impacts and costs. Most of the energy consumed in buildings is used for heating, ventilation, air conditioning and lighting. The International Energy Agency states that the construction sector has an important position in global energy consumption. although HVAC systems have an important function to meet the basic need of the buildings, the environmental and climate problems caused by energy used cannot be ignored. Especially, greenhouse gases and air pollution (foggy, smoke, etc.) occurring because of energy consumption may cause undesirable environmental and climatic problems while providing comfort in structures. Moreover, considering that one third of global CO_2 emissions are due to buildings, the importance of HVAC systems to be used in structures becomes even more important. It is important to evaluate HVAC systems in terms of human comfort such as thermal comfort, noise, and pollution. In addition, setup costs, operational and maintenance costs are important criteria to be taken into consideration for setup and use of HVAC systems. While evaluating a HVAC system, there are many factors as mentioned above and, there is a holistic and dynamic interaction between these factors [1]. HVAC systems to be used in buildings must be integrated with cost, energy and environment.

MCDM methods, which give very effective results in decision-making problems with crisp values, are widely applied in the literature. However, real-life decision-making processes may not always depend on exact decisions and values. In the absence of crisp values when using only multi-criteria decision-making (MCDM) methods, it is very difficult to make a clear evaluation using traditional methods [2]. The methods defined with a binary function may remain incapable in problems involving human judgment. Therefore, the fuzzy set theory used for solving the problems including imprecise values was first presented by Zadeh [3]. Membership degrees can be between 0 and 1 in the fuzzy set theory, which gives good results for problems in many branches such as engineering, economy, and management, instead of the sets consisting of 0 or 1 value such as classical methods. In the fuzzy set theory, the sum of membership and non-membership degrees equals to 1. In this situation, an alternative is either member or not member to a set at a certain degree. That is, while membership degrees in the fuzzy set theory are considered, the uncertainty of the membership situations is not defined. In the fuzzy set theory, the sum of membership and non-membership degrees equals to 1. In this situation, an alternative is either member or not member to a set at a certain degree. That is, while membership degrees in the fuzzy set theory are considered, the uncertainty of the membership situations is not defined. An alternative has no information of any degree about the related set. Therefore, the intuitionistic fuzzy (IF) set approach is presented by Atanassov [4]. He takes into account the degree of 'indeterminacy or hesitancy' along with the membership and non-membership degrees. It has been determined by studies that IF set theory is more effective than traditional fuzzy set theory in overcoming uncertainty [5]. Therefore, intuitionistic fuzzy (IF) set models can be used to evaluate uncertain linguistic expressions. Traditional MCDM problems with IF information are mainly focused on an IF binary relation [6]. However, some complicated problems cannot be effectively solved by an IF relation [6]. Although some alternatives have IF logic, some alternatives do not. Therefore, the IF approach may not obtain effective and consistent results. There is a need to apply new methods that make important contributions and offer different perspectives to MCDM problems by increasing the effect of IF set models and reducing their negative effects. Accordingly, in this study, covering-based generalized IF-rough models based on IF-TOPSIS principles are used. IF-neighborhoods are created by using IF-implicator and IF-triangular norm (t norm) operators. Upper and lower approximation values are calculated according to these neighborhoods. Obtained upper and lower approximations are integrated into the IF-TOPSIS principle and the most suitable system is determined. It is aimed to determine the most suitable HVAC system by using the upper and lower approximation in the rough set theory in cases where the traditional MCDM methods and IF models are ineffective.

In the literature, considering the cost, electricity consumption, and comfort, HVAC systems become MCDM problems [7]. Decision-making methods in building energy management are one of the commonly used methods [8]. However, there are very limited papers in the literature about the related subject. Balcomb [9] used the decision-making methods to compare design alternatives of the building strategies. The study is related to insulation, glazing, duct leakage, thermal mass. De Wit and Augenbroe [10] made a choice in order to compare two design strategies using Bayesian decision theory. They discussed whether adding a cooling system in thermal zone considering uncertainties. Wang et al. [11] proposed a fuzzy-MCDM method for the selection of the cool storage system. The subjective judgments (dependent on the decision-maker) and objective (dependent on numerical values) values are considered together to determine the most suitable alternative. They evaluate the storage systems according to high-temperature water cooling storage, phase change material cooling storage, ice storage, chilled water storage, and air conditioning criteria. Hopfe [12] adopted the analytic hierarchy process, one of the MCDM methods, in order to make a decision about set-up cost, architectural form, and symbolism performances criteria of two buildings. Kim and Augenbroe [13] studied a multi-criterion assessment considering organizational behavior for ventilation operation in a hospital isolation room. They adapted a variable air volume (VAV) in response to the complaint that the current operation was not sufficient since the related fan material caused excessive energy consumption. The study evaluated the current practice by supporting a rational selection of ventilation operation through a set of objective performance criteria in order to demonstrate the efficiencies of the adaptive VAV operation. Zhang et al. [14] apply fuzzy-MCDM for the scheme selection processes of heating and cooling recourses in HVAC systems. They consider initial investment cost, annual operating expense, cycle life, and reliability as the evaluation criteria. Kim et al. [15] studied the decision making of the HVAC system using Bayesian Markov Chain Monte Carlo method. They present MCDM of HVAC systems under uncertainty. Using EnergyPlus 6.0, they studied about construction cost and total energy consumption criteria for two HVAC candidates. Former HVAC candidate has VAV for the interior zone, fan coil unit for perimeter zone gas boiler and electric chiller. Latter candidate has VAV for the interior zone, fan coil unit for perimeter zone, gas boiler and electric chiller as well as ice thermal storage system [15]. Huang et al. [16] decided a HVAC system design under peak load prediction uncertainty using MCDM technique. Case studies are used to illustrate the design procedure, and the result is compared with that of a conventional design method. Baki et al. [17] consider environmental, economic, social, and competency criteria by using an approach based on fuzzy-MCDM and best-worst methods. Poongavanam et al. [18] evaluate 14 different automotive air conditioning systems by using TOPSIS, EDAS, and MOORA methods. They use the latent heat of vaporization, thermal conductivity, vapor pressure, saturated fluid density, specific heat capacity, dynamic viscosity, GWP, ozone depletion potential, and cost per pound as performance criteria. Wan et al. [19] evaluate the effects of the supply vane angles and supply air temperature on ventilation performance by considering both 13 different heat comfort scales and human factors. They propose the TOPSIS method based on set pair analysis.

Unlike the above studies, it is aimed to select the most suitable HVAC systems by using the MCDM method based on IF and rough set theories. The IF and rough set methods are integrated with each other to obtain more consistent solutions in real-life problems. Finally, the MCDM method based on the IF-TOPSIS principle has been applied. The investment cost, operational cost, maintenance cost, green gas effect, energy consumption effect, thermal comfort, air quality, noise, and smog are considered as performance criteria for nine different HVAC systems.

Rest of the paper is structured as follows. Section 2 includes a brief review of the relevant literature. In Section 3, a brief definition of IF-TOPSIS and the covering-based generalized IF-rough set models are given. Section 4 presents IF-TOPSIS and covering-based generalized IF-rough set model for selecting HVAC system. Finally, a conclusion is presented in Section 5.

2. Material and Method

2.1. IF sets

Attonasov [20] defines the degree of the membership $\mu_A(x)$, degree of non-membership $v_A(x)$ as well as degree of indeterminacy $\pi_A(x)$ in IF sets. In IF set theory, sum of $\mu_A(x)$ and $v_A(x)$ is smaller than 1. If sum of $\mu_A(x)$ and $v_A(x)$ equals 1, IF set returns fuzzy set. The lower value of $\mu_A(x)$, the more accurate the as relative crisp information about the x element. Assume that X is a non-empty set. An IF set in X is shown in equations (1), (2) and (3).

$$A = \{\langle x, \mu_A(x), v_A(x) \rangle x \in X\} \tag{1}$$

$$0 \leq \mu_A(x) + v_A(x) \leq 1 \tag{2}$$

$$\pi_A(x) = 1 - \mu_A(x) + v_A(x) \tag{3}$$

2.2. IF-neighborhood, IF-implicator, and IF-t norm

IF-neighborhood is a model which determines distances of alternatives for each criterion. Assume that $C = \{C_1, C_2, \dots, C_n\}$ is an IF-covering set representing criteria; $U = \{x_1, x_2, \dots, x_m\}$ is a set of alternatives, I is IF-implicator operator and $N^C(x_i)$ is IF-neighborhood of alternative i. $N^C(x_i)$ is shown with formula equation (4) in [21].

$$N^C(x_i)(x_j) = \bigwedge_{A \in C} I\left(A(x_i), A(x_j)\right) \qquad i, j \in \{1, \dots, m\} \tag{4}$$

where N^C is reflective, T-transitive, serial, and IF-covering. $I\big(A(x_i), A(x_j)\big) = \big(\min\big(1 + \mu_{xj} - \mu_{xi}, 1 + v_{xi} - v_{xj}\big), \max\big(0, v_{xj} - v_{xi}\big)\big)$, where I is IF-implicator operator. $T\big(A(x_i), A(x_j)\big) = \big(\max\big(0, \mu_{xi} + \mu_{xj} - 1\big), \min\big(1, v_{xi} + v_{xj}\big)\big)$, where T is IF-t norm operator.

2.3. Lower and upper approximations for covering based IF-rough set models

Assume that C is IF-covering set, U is a non-empty universe, I is an implicator, and T is a t-norm. In [6, 7], two pair of covering-based generalized IF-rough approximations are defined as follows:

$$C_U(A)(x_i) = \bigvee_{x_j \in U} T\left(N^C(x_j)(x_i), \bigwedge_{x_k \in U} I\left(N^C(x_j)(x_k), A(x_k)\right)\right) \tag{5}$$

$$C_L(A)(x_i) = \bigwedge_{x_j \in U} I\left(N^C(x_j)(x_i), \bigvee_{x_k \in U} T\left(N^C(x_j)(x_k), A(x_k)\right)\right) \tag{6}$$

$$C_L'(A)(x_i) = \bigwedge_{x_j \in U} I\left(N^C(x_j)(x_i), \bigwedge_{x_k \in U} I\left(N^C(x_j)(x_k), A(x_k)\right)\right) \tag{7}$$

$$C_U'(A)(x_i) = \bigvee_{x_j \in U} T\left(N^C(x_j)(x_i), \bigvee_{x_k \in U} T\left(N^C(x_j)(x_k), A(x_k)\right)\right) \tag{8}$$

2.4. IF-TOPSIS decision making approach

After C IF-covering set is built, $N^C(x_i)$ is calculated for each alternative $x_i \in U$. According to IF-covering C set, positive ideal solution A^+ and negative ideal solution A^- required for TOPSIS approach are calculated. The formulation for A^+ and A^- are shown in equation (9) and (10), respectively.

$$\mu_{A^+}(x_i), v_{A^+}(x_i) = \max_{j \in C}\left(\mu_{C_j}(x_i)\right), \min_{j \in C}\left(v_{C_j}(x_i)\right) \tag{9}$$

$$\mu_{A^-}(x_i), v_{A^-}(x_i) = \min_{j \in C}\left(\mu_{C_j}(x_i)\right), \max_{j \in C}\left(v_{C_j}(x_i)\right) \tag{10}$$

C_L, C_U, C_L', and C_U' are calculated by using A^+ and A^-. According to obtained lower and upper approximations, two ranking functions for each x_i using formula in equation (11), and (12) in [6, 7]. $\big(\mu(x_i), v(x_i)\big)(+)\big(\mu(x_j), v(x_j)\big)$= $\mu(x_i) + \mu(x_j) - \mu(x_i) * \mu(x_j), v(x_i) * v(x_j)$ is IF set summation formula, where i and $j \in \{1, \dots, m\}$.

$$S^-(x_i) = \alpha\left(\boldsymbol{L}\big(C_L(A^-)(x_i)(+)C_U(A^-)(x_i)\big)\right) + (1 - \alpha)\left(\boldsymbol{L}\big(C_L'(A^-)(x_i)(+)C_U'(A^-)(x_i)\big)\right) \tag{11}$$

$$S^+(x_i) = \alpha\left(\boldsymbol{L}\big(C_L(A^+)(x_i)(+)C_U(A^+)(x_i)\big)\right) + (1 - \alpha)\left(\boldsymbol{L}\big(C_L'(A^+)(x_i)(+)C_U'(A^+)(x_i)\big)\right) \tag{12}$$

where, α is a level adjustment value. $\boldsymbol{L}(A)(x_i) = \mu(x_i) + v(x_i) * \pi(x_i)$ is score function. Finally, relative closeness coefficient C^* is calculated by using the TOPSIS method principle for each alternative with equation (13).

$$C^* = \frac{S^-(x_i)}{S^-(x_i) + S^+(x_i)} \tag{13}$$

3. Results

In this study, it is aimed to determine the most suitable system among nine different HVAC systems for a public building HVAC systems have four basic assessment factors as mentioned Section 1. According to these factors, nine criteria were determined. These are investment cost, operational cost, maintenance cost, green gas effect, energy consume effect, thermal comfort, air quality, noise, and smog labeled $C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9$, respectively. Since the calculations are quite complicated, the solution structure was coded in MATLAB®. Also, a framework was formed over MATLAB in order to apply the software to different problems. The results were easily obtained. Linguistic terms and their IF degrees in terms of criteria for each HVAC system are shown in Table 1.

Table 1. Linguistic terms to evaluate the alternatives

Linguistic Terms	IF degrees		
	μ	ν	π
Excellent	1	0	0
Quite good	0.75	0.1	0.15
Good	0.6	0.25	0.15
Medium	0.5	0.5	0
Bad	0.25	0.6	0.15
Quite bad	0.1	0.75	0.15

In order to properly operate the decision-making process, the decision matrix must be carefully created by the decision-makers. IF-decision matrix determined according to linguistic terms is shown in Table 2.

Table 2. IF-decision matrix

HVAC	Assessment Criteria								
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
X_1	0.6;0.25; 0.15	0.75;0.1; 0.15	0.1;0.75; 0.15	0.5;0.5;0	1;0;0	0.5;0.5;0	0.5;0.5;0	0.25;0.6; 0.15	0.75;0.1; 0.15
X_2	0.5;0.5;0	1;0;0	0.6;0.25; 0.15	0.6;0.25; 0.15	0.6;0.25; 0.15	0.6;0.25; 0.15	0.5;0.5;0	0.5;0.5;0	0.6;0.25; 0.15
X_3	0.75;0.1; 0.15	0.1;0.75; 0.15	0.5;0.5;0	0.5;0.5;0	0.75;0.1; 0.15	0.25;0.6; 0.15	0.25;0.6; 0.15	0.5;0.5;0	1;0;0
X_4	0.1;0.75; 0.15	0.75;0.1; 0.15	0.6;0.25; 0.15	0.75;0.1; 0.15	0.75;0.1; 0.15	0.6;0.25; 0.15	1;0;0	0.75;0.1; 0.15	0.25;0.6; 0.15
X_5	0.5;0.5;0	0.25;0.6; 0.15	0.5;0.5;0	0.5;0.5;0	0.75;0.1; 0.15	0.75;0.1; 0.15	0.6;0.25; 0.15	1;0;0	0.75;0.1; 0.15
X_6	0.25;0.6; 0.15	0.6;0.25; 0.15	1;0;0	0.25;0.6; 0.15	0.1;0.75; 0.15	0.6;0.25; 0.15	0.6;0.25; 0.15	0.75;0.1; 0.15	0.75;0.1; 0.15
X_7	1;0;0	0.25;0.6; 0.15	0.5;0.5;0	0.1;0.75; 0.15	0.5;0.5;0	0.5;0.5;0	0.5;0.5;0	0.25;0.6; 0.15	0.6;0.25; 0.15
X_8	0.1;0.75; 0.15	0.5;0.5;0	0.6;0.25; 0.15	0.75;0.1; 0.15	0.75;0.1; 0.15	1;0;0	0.75;0.1; 0.15	0.6;0.25; 0.15	0.75;0.1; 0.15
X_9	0.25;0.6;0 .15	0.75;0.1;0 .15	0.6;0.25;0 .15	1;0;0	0.5;0.5;0	0.6;0.25;0 .15	0.75;0.1;0 .15	0.6;0.25;0 .15	0.6;0.25; 0.15

As shown in Table 2, at least one criterion equals to the value $(1; 0; 0)$ for each alternative. Therefore, decision matrix s an IF-covering set. That is, the methodology mentioned in Section 3 is proper for this application. Firstly, IF-neighborhood $N^C(x_i)$ is calculated for each alternative x_i by using equation (4). IF-neighborhood results are obtained in Table 3.

Table3. IF-neighborhood relations

Relation	HVAC Alternatives								
	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
$N^C(x_1)$	1;0;0	0.6;0.25; 0.15	0.35;0.65; 0	0.5;0.5;0	0.5;0.5;0	0.1;0.75;0 .15	0.5;0.5;0	0.5;0.5;0	0.5;0.5;0
$N^C(X_2)$	0.5;0.5;0	1;0;0	0.1;0.75;0 .15	0.6;0.25;0 .15	0.25;0.65; 0.15	0.5;0.5;0	0.25;0.6;0 .15	0.5;0.5;0	0.75;0.25 ;0
$N^C(X_3)$	0.6;0.25; 0.15	0.6;0.4;0	1;0;0	0.35;0.65; 0	0.6;0.4;0	0.35;0.65; 0	0.6;0.4;0	0.35;0.65; 0	0.5;0.5;0
$N^C(X_4)$	0.5;0.5;0	0.85;0.15; 0	0.35;0.65; 0	1;0;0	0.5;0.5;0	0.35;0.65; 0	0.35;0.65; 0	0.6;0.4;0	0.6;0.4;0
$N^C(X_5)$	0.6;0.25;	0.85;0.15;	0.85;0.15;	0.6;0.25;0	1;0;0	0.35;0.65;	0.6;0.4;0	0.6;0.25;0	0.6;0.4;0

	0.15	0	0	.15		0		.15	
$N^C(X_6)$	0.1;0.75; 0.15	0.6;0.25; 0.15	0.5;0.5;0	0.6;0.25;0.15	0.5;0.5;0	1;0;0	0.5;0.5;0	0.6;0.25;0.15	0.6;0.25; 0.15
$N^C(X_7)$	0.6;0.25; 0.15	0.5;0.5;0	0.75;0.15; 0.1	0.1;0.75;0.15	0.5;0.5;0	0.25;0.6;0.15	1;0;0	0.1;0.75;0.15	0.25;0.6; 0.15
$N^C(X_8)$	0.5;0.5;0	0.85;0.15; 0	0.6;0.4;0	1;0;0	0.6;0.4;0	0.35;0.65; 0	0.35;0.65; 0	1;0;0	0.6;0.4;0
$N^C(X_9)$	0.5;0.5;0	0.6;0.25; 0.15	0.35;0.65; 0	0.75,0.15; 0.1	0.5;0.5;0	0.25;0.6;0.15	0.1;0.7;0.15	0.6;0.4;0	1;0;0

After the IF-neighborhood relations are calculated with equation (4), positive ideal solution A^+ and negative ideal solution A^- are computed. For both the ideal solutions, each lower and upper approximation is calculated by using equations (5), (6), (7), and (8). It is obvious that A^+ is $(1;0;0)$ for each alternative x_i. Therefore, all approximations equal to U for A^+. The results of the ideal solutions and each A^- approximation values are shown in Table 4.

Table 4. Ideal solutions and approximations

Relation	HVAC Alternatives								
	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
A^+	1;0;0	1;0;0	1;0;0	1;0;0	1;0;0	1;0;0	1;0;0	1;0;0	1;0;0
A^-	0.1;0.75 ; 0.15	0.5;0.5;0	0.1;0.75; 0.15	0.1;0.75; 0.15	0.25;0.6; 0.15	0.1;0.75; 0.15	0.1;0.75; 0.1	0.1;0.75; 0.15	0.25;0.6; 0.15
$C_L(A^-)(x_i)$	0.1;0.75 ; 0.15	0.5;0.5;0	0.1;0.75; 0.15	0.1;0.75; 0.15	0.25;0.6; 0.15	0.1;0.75; 0.15	0.1;0.75; 0.15	0.1;0.75; 0.15	0.25;0.6;0.15
$C_U(A^-)(x_i)$	0.1;0.75 ; 0.15	0.5;0.5,0	0.1;0.75; 0.15	0.1;0.75; 0.15	0.25;0.6; 0.15	0.1;0.75; 0.15	0.1;0.75; 0.15	0.1;0.75; 0.15	0.25;0.6;0.15
$C_L'(A^-)(x_i)$	0.1;0.75 ; 0.15	0.5;0.5;0	0.1;0.75; 0.15	0.1;0.75; 0.15	0.25;0.6; 0.15	0.1;0.75; 0.15	0.1;0.75; 0.15	0.1;0.75; 0.15	0.25;0.6;0.15
$C_L'(A^-)(x_i)$	0.1;0.75 ;0.15	0.5;0.5;0	0.1;0.75; 0.15	0.35;0.65; 0	0.35;0.65; 0	0.1;0.75; 0.15	0.1;0.75; 0.15	0.35;0.65;0	0.25;0.6;0.15

According to equations (11) and (12), two ranking functions S^+ and S^- are calculated by using lower and upper approximation values. Here, α is a level adjustment value, and is assumed as 0.5. Then, the values of the S^+, S^-, and closeness function C^* are calculated in Table 5.

Table 5. Ranking values and closeness coefficient

Relation	HVAC Alternatives								
	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9
S^-	0.3292	0.6987	0.3489	0.4625	0.5584	0.3292	0.3292	0.4625	0.5104
S^+	1	1	1	1	1	1	1	1	1
C^*	0.2477	0.4113	0.2587	0.3163	0.3583	0.2477	0.2477	0.3163	0.3379

Finally, alternative having the highest closeness coefficient is the best choice, and alternative having the lowest closeness is the worst choice. According to the value of C^*, nine HVAC systems are ranked as follows: $X_2 > X_5 > X_9 > X_8 \approx X_4 > X_3 > X_7 \approx X_6 \approx X_1$.

If traditional IF methods are used for the problems with an IF-covering structure, the effective results may not be obtained. Many conventional IF operators are available in the literature. One of the most widely used operators is fuzzy weighted averaging (IFWA) operator proposed by Xu [21] in equation (14).

$$X_i = \left[1 - \prod_{j=1}^{n}\left(1-\mu_{ij}\right)^{\alpha_j}; \prod_{j=1}^{n}\left(v_{ij}\right)^{\alpha_j}; \prod_{j=1}^{n}\left(1-\mu_{ij}\right)^{\alpha_j} - \prod_{j=1}^{n}\left(v_{ij}\right)^{\alpha_j}\right] \tag{14}$$

where, x_i is IF value of alternative i, n is number of the performance criteria $(j = 1, \dots, n)$ and α_i is the weight of each criterion for IFWA operator.

Assume that the weights equal to each other for each criterion in Table 2. Once the IFWA operator is applied for IF matrix in table 2, we obtain following results for X_1,$(j = 1, \dots, 9)$:

$X_1(\mu_{1j}) = 1 - (1-0.6)^{0.111}(1-0.75)^{0.111}(1-0.1)^{0.111}(1-0.5)^{0.111}(1-1)^{0.111}(1-0.5)^{0.111}(1-0.5)^{0.111}(1-0.25)^{0.111}(1-0.75)^{0.111} = 1.$

$X_1(v_{1j}) = (0.25)^{0.111}(0.1)^{0.111}(0.75)^{0.111}(0.5)^{0.111}(0)^{0.111}(0.5)^{0.111}(0.5)^{0.111}(0.6)^{0.111} = 0.$

$X_1(\pi_{1j}) = (1-0.6)^{0.111}(1-0.75)^{0.111}(1-0.1)^{0.111}(1-0.5)^{0.111}(1-1)^{0.111}(1-0.5)^{0.111}(1-0.5)^{0.111}(1-0.25)^{0.111}(1-0.75)^{0.111} - (0.25)^{0.111}(0.1)^{0.111}(0.75)^{0.111}(0.5)^{0.111}(0)^{0.111}(0.5)^{0.111}(0.5)^{0.111}(0.6)^{0.111} = 0.$

If the same computation is applied for other alternatives, we obtain the same results. That is, scores of all alternatives equal to each other such as $X_2 \approx X_5 \approx X_9 \approx X_8 \approx X_4 \approx X_3 \approx X_7 \approx X_6 \approx X_1$. The IFWA operator cannot make a ranking. Therefore, we cannot choose the most suitable alternative for problems with the IF-covering structure.

4. Discussion and Conclusion

It is very important that the building provides comfort to people in terms of HVAC systems. In addition, HVAC systems should not cause problems in the climate and environment. Since some of the criteria for HVAC systems are described as linguistic, it is not possible to evaluate the systems with traditional methods using crisp values. Moreover, even traditional IF operators may not provide an effective solution for the problems described as linguistic such as IF-covering. In this study, covering-based generalized IF-rough set model was utilized for selecting HVAC system to be used in a public building. Also, According to IF-decision matrix generated, IF-neighborhood was calculated. Approximation operators in rough set theory adapted covering-based IF structure. Through IF-TOPSIS principle and using approximation values, closeness coefficients were calculated for each HVAC alternative system. HVAC systems were ranked according to the results and the best solution has been determined. If there is the IF-covering structure under the linguistic expressions, the proposed structure offers appropriate solutions. Although the IFWA operator, which is one of the IF methods for certain alternatives, cannot find a solution, the proposed covering-based generalized IF-rough set model provides a solution. According to the solution obtained, alternative 2 is obtained as the best option. Alternatives 1, 6, and 7 are determined as the worst options. In future studies, the scope of the study can be expanded by making comparisons with different methods. In addition, the proposed approach can be applied to different problems in construction, industry, economics, social, and management branches.

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