
Manoj Bhardwaj
Department of Mathematics, University of Delhi, New Delhi-110007, India

Abstract
In this addendum we give an example to show that there is an error in Theorem 3.7 in “Ideal Rothberger spaces” [Hacet. J. Math. Stat. 47(1), 69-75, 2018]. We also prove the theorem with different hypothesis.

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We use notation and terminology from [2]. In [2], the author gave the following theorem for inverse invariant.

A function \( f \) from a topological space \( X \) to a space \( Y \) is said to be perfect map [1] if

1. \( f \) is onto
2. \( f \) is continuous
3. \( f \) is closed map
4. \( f^{-1}(y) \) is compact in \( X \) for each \( y \in Y \).

Theorem 1 ([2]). Let \( f : X \to Y \) be a perfect map and \( I \) be an ideal in \( Y \). If \( Y \) is Rothberger modulo \( I \), then \( X \) is Rothberger modulo \( f^{-1}(I) \).

Here we give an example which contradicts the Theorem 1 given in [2].

Example 2. Let \( \mathbb{R} \) be set of real numbers with usual topology and \( J = \{ \phi \} \) be an ideal in \( \{ a \} \). Take a constant function \( f \) from \( [0, 1] \) to one point Rothberger space or \( \{ a \} \), where \( [0, 1] \) is compact closed subspace of \( \mathbb{R} \). Then \( f \) is closed, open, onto and continuous map. Also \( f^{-1}(a) = [0, 1] \) is compact but \( [0, 1] \) is not Rothberger [3] since \( \{ a \} \) is Rothberger.

Now we give positive result regarding this and provide maps under which Rothberger modulo an ideal spaces are inverse invariant.

Theorem 3. Let \( f \) be an open bijective map from a space \( X \) to \( Y \) and \( I \) be an ideal in \( Y \). If \( Y \) is Rothberger modulo \( J \), then \( X \) is Rothberger modulo \( f^{-1}(J) \).

Proof. Let \( \langle U_n : n \in \omega > \) be a sequence of open covers of \( X \). Then for each \( n \),

\[ V_n = \{ f(U) : U \in U_n \} \]

is an open cover of \( Y \). Since \( Y \) is Rothberger modulo \( J \), there are \( \tilde{J} \in J \) and a sequence \( \langle W_n : n \in \omega > \) such that for each \( n \), \( W_n \) is a singleton subset of \( U_n \) and for each \( y \in Y \setminus \tilde{J} \), belongs to \( \bigcup W_n \) for some \( n \). Now assume that for each \( n \),

Email address: manojmnj27@gmail.com
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\[ W_n = \{ f(U_{n,1}) \} \] and \[ S_n = \{ U_{n,1} \}. \]

Then \( f^{-1}(J) \in f^{-1}(J) \) and sequence \( < S_n : n \in \omega > \) witnesses Rothberger modulo \( f^{-1}(J) \) property of \( X \) for the sequence \( < U_n : n \in \omega > \). Let \( x \in X \setminus f^{-1}(J) \). Then

\[ y = f(x) \in Y \setminus J \] and \( y \in \bigcup W_n \) for some \( n \).

This implies that \( y \in f(U_{n,1}) \). Since \( f \) is one-to-one, \( x \in U_{n,1} \). So \( x \in \bigcup S_n \) for some \( n \).

This completes the proof. \( \square \)

**References**

