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Abstract

In this addendum we give an example to show that there is an error in Theorem 3.7 in “Ideal Rothberger spaces” [Hacet. J. Math. Stat. 47(1), 69-75, 2018]. We also prove the theorem with different hypothesis.

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We use notation and terminology from [2]. In [2], the author gave the following theorem for inverse invariant.

A function \( f \) from a topological space \( X \) to a space \( Y \) is said to be perfect map [1] if

1. \( f \) is onto
2. \( f \) is continuous
3. \( f \) is closed map
4. \( f^{-1}(y) \) is compact in \( X \) for each \( y \in Y \).

**Theorem 1** ([2]). Let \( f : X \rightarrow Y \) be a perfect map and \( I \) be an ideal in \( Y \). If \( Y \) is Rothberger modulo \( I \), then \( X \) is Rothberger modulo \( f^{-1}(I) \).

Here we give an example which contradicts the Theorem 1 given in [2].

**Example 2.** Let \( \mathbb{R} \) be set of real numbers with usual topology and \( I = \{ \phi \} \) be an ideal in \( \{a\} \). Take a constant function \( f \) from \([0, 1]\) to one point Rothberger space or \( \{a\} \), where \([0, 1]\) is compact closed subspace of \( \mathbb{R} \). Then \( f \) is closed, open, onto and continuous map. Also \( f^{-1}(a) = [0, 1] \) is compact but \([0, 1]\) is not Rothberger [3] since \( \{a\} \) is Rothberger.

Now we give positive result regarding this and provide maps under which Rothberger modulo an ideal spaces are inverse invariant.

**Theorem 3.** Let \( f \) be an open bijective map from a space \( X \) to \( Y \) and \( I \) be an ideal in \( Y \). If \( Y \) is Rothberger modulo \( I \), then \( X \) is Rothberger modulo \( f^{-1}(I) \).

**Proof.** Let \( \{U_n : n \in \omega \} \) be a sequence of open covers of \( X \). Then for each \( n \),

\[
\mathcal{V}_n = \{f(U) : U \in \mathcal{U}_n\}
\]

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is an open cover of $Y$. Since $Y$ is Rothberger modulo $\mathcal{I}$, there are $J \in \mathcal{I}$ and a sequence $\langle W_n : n \in \omega \rangle$ such that for each $n$, $W_n$ is a singleton subset of $\mathcal{U}_n$ and for each $y \in Y \setminus J$, belongs to $\bigcup W_n$ for some $n$. Now assume that for each $n$,

$$W_n = \{ f(U_{n,1}) \} \text{ and } \mathcal{G}_n = \{ U_{n,1} \}.$$ 

Then $f^{-1}(J) \in f^{-1}(\mathcal{J})$ and sequence $\langle \mathcal{G}_n : n \in \omega \rangle$ witnesses Rothberger modulo $f^{-1}(\mathcal{I})$ property of $X$ for the sequence $\langle \mathcal{U}_n : n \in \omega \rangle$. Let $x \in X \setminus f^{-1}(J)$. Then

$$y = f(x) \in Y \setminus J \text{ and } y \in \bigcup W_n \text{ for some } n.$$ 

This implies that $y \in f(U_{n,1})$. Since $f$ is one-to-one, $x \in U_{n,1}$. So $x \in \bigcup \mathcal{G}_n$ for some $n$. This completes the proof. \qed

References