# Increase of Spur Gears Power Transmission Capacity by Slip Torque Criterion in off Road Machines

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**Abstract**: Main goal of this research is appropriate criterion choosing for modification and increase power transmission capacity of gears that applied in off road machines and choosing of appropriate model for bending and contact analysis of modified gears. Finite elements method used for surveys of stress distribution and stableness in this gears, so in contact analysis of meshing gears used the contact model of one gear pair due to contact ratio. Comparison of finite elements method results by theory method results show that stresses of this contact model were accordance by theory method (AGMA) results. Calculation criterion and suitable analysis model for survey of gears mechanically behavior, presented by using of results. Results show that Slip Torque can be a criterion for increase of power transmission capacity of gears which applied in off road machines Gearbox. Finally, Survey of elements behavior that placed between contacts surfaces show that contact stress in tooth contact locate being after than specific duration.

Key words: Finite element, gearbox, gear modification, slip torque, spur gears

## INTRODUCTION

Capacity of power transmission gearbox depends on the size and dimensions of the gears. Therefore, to increase power transmission capacity must be changes in their dimensions. Gear drives transmit motion and power by tooth mesh. Mostly in the form of involutes profiles, gear tooth mesh is a complex process involving, e.g. multi-tooth engagement, multi-point contact and varying load conditions. To achieve improved static and dynamic characteristics of gear drives and enhanced load carrying capacity and reliability, accurate determination of the tooth load distribution, mesh stiffness as well as the deformation and stress of tooth face is an important part in gear drive design. A small weight reduction for each gear can reduce a significant amount of the total system weight. On the other hand, a small increase in stress can cause a significant change in the expected fatigue life. For the case of mild steel, a 10% reduction in stress range can cause about a 50% change in fatigue life (Shigley et al., 2004). New gear designs are needed because of the increasing performance requirements, such as high load capacity, high endurance, low cost, long life, and high speed. Therefore, it is critically important to predict stress accurately in gears.

The design of gear strength is based on two models: bending and contact stress models. The contact stress accounts for the wear and pitting failure while bending stress accounts for bending failure (Chaudhuri et al., 2010). Although there are, standards such as AGMA for gears design, But due to the multitude of relations and their parameters, Numerical methods used widely for design and analysis of gears. In the numerical approach, computers are used to solve the gear problem. Simulation models are created and these are then divided into a number of segments called elements. Accurate prediction of loads, deformations and stresses in complex bodies such as gears requires a numerical method such as the Finite element method (FEM). Static analysis of gear with the finite element method performed without contact modelina phenomenon and Numerical results compared with results photo elasticity method (Andrews, 1991).

Other researchers had also studied the dynamic bending stress loaded to the teeth with simulated finite element method (Wallace and Seireg, 1973; Vijayarangan and Ganesan, 1993). Contact models for calculating bending teeth spur gears developed using finite element method (Wilocox and Coleman, 1993; Chabert et al., 1994). Finite element model proposed to study stress in tooth root and compare numerical results with the results of empirical relationships (Huseyin and Eyercioglou, 1995). In addition, studies had done also by researchers on the contact stress distribution in the gears (Arikan, 1991; Refaat and Meguid, 1995). Calculated contact force and deformation at the beginning of engagement teeth spur gears conducted using two-dimensional finite element model with contact element sheet (Park and Yoo, 2004). For the dynamic behavior of spur gears were used in the teeth of the three pairs of two-dimensional model of tooth contact with the sheet elements Lin, (Lin et al., 2002; Mao, 2007; Krapat et al., 2008). Two-dimensional models did not show contact stresses generated in the tooth root as real, therefore, in addition to three dimensional contact models; force applied only to a pair of engagement teeth (Sfakiotakis et al., 2001; Marunić, 2008). However, this method of force applied, is suitable for gears with a contact ratio between 1 and 2. Body contact model used to analyze the teeth in spur gears under load (Li, 2002).

The researcher used the contact area model in other studies for bending and contact stresses analyzing of spur gears with one to two contact ratios, and compare the results, has reported high accuracy of the contact body model (Li, 2007; Li, 2007). Survey contact model types of spur gears bending and contact analysis, show that tooth pair contact model have reliable results than the whole contact model (Arikan, 1987; Çelik, 1999; faydor et al., 2000). Conforms to findings based on NASA experimental results (Rebbechi et al., 1991) confirm the advantage of pair of tooth contact model. Used the tooth pair contact model for spur gears contact Tangential force and displacement analysis. constraints were imposed on the driven gears and the driver gears tooth, respectively (Wonchai, 2009). Recent research that was conducted on the modified spur gears, which has demonstrated high accuracy

and confidence are three-dimensional models of contact pairs of teeth (Huang and Su, 2010). Select a network or suitable mesh is one of the important points in the numerical analysis of mechanical problems. Main goal of this research is choosing an analytical model, element and appropriate and accurate mesh, so this method can be used for further research will be done about the bending and contact analysis of spur gears.

#### MATERIALS and METHOD

In this paper we studied the off road machine (Harvest Combine) gearbox's gears for power transmission capacity increase by 130hp in 180hp. Slip torgue is transferring maximum torgue to wheels. Due to dependence on cars to move between machines and soil relationship the criteria used for the design of power transmission system of road machines. Slip torque is a final torque limit for off road machines such as tractors, combine harvesting, etc and calculated with the graphical and analytical methods (Thomas, 1990). Slip torque obtained by graphical methods, is greater than the analytical method. For this reason, for sure, the graphic method was used in calculations. Differential and gearbox output torque in graphical method are equal to respectively:

$$T_{WSG} = r_{R} \times F = F_{S} \left( \frac{W_{0} \left( X_{1} - X_{2} \right)}{X_{1} \times m_{A}} \right) r_{R}$$
(1)

$$T_{WSGB} = r_{R} \times F = F_{S} \left( \frac{W_{0} \left( X_{1} - X_{2} \right)}{X_{1} \times m_{A} \times m_{G}} \right) r_{R}$$
(2)

Where  $T_{WSG}$  is the output slip torque of differential (N.m),  $r_R$  is the rolling radius (m),  $F_S$  is the friction coefficient between wheel and surface motion (%),  $W_0$  is the Total weight (kg),  $X_1$ ,  $X_2$  is the distance between the axle and rear axle to the center of gravity respectively (m),  $m_A$  is the final drive speed ratio Force,  $T_{WSGB}$  is the output slip torque of gearbox (N.m) and  $m_G$  is the differential speed ratio.

torque acting on the gears will be obtained by calculating the slip torque and gears transmission ratios. Rolling radius of tire dynamic state will be obtained from the following equation (Artamonov et al., 1976):

$$r = 0.5D_r + B_t (1 - \lambda_t)$$
(3)

Where;  $D_r$  is the wheel diameter (m),  $B_t$  is the tire profile height (m) and  $\lambda_t$  is the Coefficient of radial deformation (%). Loads into the components of power transmission system depend on the capacity of power supply and its consumption method. Based on calculations performed, will enter the transmission combine harvesting in about 42-40 percent of engine power produced (Kutzbach, and Quick, 1999). Considering that, this research has the maximum hardness of gears (620-720 HB). Therefore, changing the dimensions of the gears were used to increase power transmission capacity. AGMA standard has proposed the following equations for the design and analysis of gear (Shigly et al., 2004):

$$\sigma_{t} = \frac{W_{t}}{FJm} \times \frac{K_{a} \times K_{B} \times K_{s} \times K_{m}}{K_{v}} \prime \sigma_{t_{all}} \leq \frac{S_{t}K_{l}}{K_{T}K_{R}}$$

$$\sigma_{c} = C_{P} \sqrt{\frac{W_{t} \times C_{a} \times C_{s} \times C_{m} \times C_{f}}{C_{v} \times F \times d \times I}} \prime \sigma_{C_{al}} \leq \frac{S_{C} \times C_{L} \times C_{H}}{C_{T} \times C_{R}}$$

$$W_{r} = \frac{2T}{D_{n}}$$
(5)

Where  $W_t$  is the tangential (transmitted) load,  $D_P$  is the pitch diameter of the pinion and T equal to the power transmitted divided by the rotational speed that resultant by equation 2 in above.

#### Gears modeling

Before the simulated conjugate of two gears with CAD software, gears involute curve was constructed using SPLINE method of GEARTRAX software. In this method, the contact between the gears is as an integrated surface, therefore application of this method, there will be no restrictions in choosing the type of meshing and contact surfaces for the gears in the finite element method analysis.

#### Finite element model

In literature, generally, single tooth, one full and two semi teeth, and three teeth models are used to analyze the gear problem. Some authors using FEM software such as ANSYS, ABAQUS etc, also analyze gear body models containing all teeth. However, the solution technique which is used for this type of gear model shows accuracy and CPU time problems. In this paper, we use an FEA program, ANSYS, to evaluate the stresses acting on the gear. By using FEA software the effect of detailed geometry as well as complex loading conditions can be considered accurately. It allows us to calculate the variation of the bending and contact stresses during the rotation of gears. To model spur gears using FEM, a threedimensional model is adopted. The stress of spur external gears is determined by means of threedimensional finite element method and by use of the developed pinion-wheel numerical model. The development of 3D models that can simulate tooth pairs in contact, gear bodies and transmission shafts, has offered more possibilities for the analysis and improvement of gear strength calculation. In order to simplify the computation of tooth deflection, only one tooth was considered. Such model is widely used and accepted in literatures (Chaari et al., 2009; Pimsarn and Kazerounian, 2002; Wang, 2003; Sirichai, 1999). Effect of assembly errors on the high contact stress analysis of gears has been reported (Li, 2007), therefore, in this study were simulated gear pair conjugate without assembly error. Numerical simulation of conjugate action requires establishment of geometric relation and kinematic constrains describing both operation and load carrying characteristics of this equipment. These relationships concern analytic mechanics of gearing, obey standards and rules depend on manufacturing and design methodologies. Without loss of generality and for simplicity reasons, the present study assumes standard gear profiles, 20° pressure angle, and one pair of gear teeth in contact any time. Pair of gears that were conjugate in the transmission mode heaviest was selected for analysis of bending and contact stresses. Due to limited memory on computer systems, will not investigate the full model of the gears, therefore, a fourth driven gear and a pair of conjugate teeth, was selected for the analysis of bending and contact respectively. Analysis was conducted as three-dimensional and nonlinear, and of course, considering the teeth conjugates time. In the numerical approach, computers are used to solve the gear problem. Simulation models are created and these are then divided into a number of segments called elements. For analysis of gears were used 95Nod20Solid Elements and the two types of area and body contact model was used to analyze the contact stresses. In addition, contact elements were used for analysis where the teeth contact. In order to obtain high quality 3D element, the mapped mesh is used for tooth root and contact surface. Figure 1 and 2 illustrate the gear geometry that was created in ANSYS. Contact between the gear and pinion is established by defining contact elements (CONTA171) Increase of Spur Gears Power Transmission Capacity by Slip Torque Criterion in off Road Machines

on the edge of the gear teeth and target elements (TARGE169) on the edge of the pinion teeth. Since contact pressures can only be calculated on the contact elements, the same procedure is repeated with contact elements on the pinion teeth and target elements on the gear teeth. Element contact will be resolved gap elements flaw that some researchers (Ken, 2007) have used them. GAP elements are causing slip between the teeth surfaces, while the slip velocity is zero, where two teeth in contact spur gears (Chawathe, 2001). Most of the existing researchers utilized gap elements to simulate the surface contact behavior and estimated the contact pressures and deformations. To use the gap element method, the location of the contact must be calculated first by classic Hertzian theory. For loaded gear tooth contact, the contact path can only be known after the model has been run. The other limitations of using gap elements are that they usually allow for only relatively small sliding between the contact surfaces.

#### Stages of tooth contact

During operation, the teeth of two gears pass through three stages of contact: First phase: includes mating of the first contact point of the dedendum of the driving gear shown in Fig. 3a with the addition of the driven gear. Here, both rolling and sliding movements are present (Wright and Kukureka, 2001; Walton and Goodwin, 1998). At the same time, the two teeth share the tooth load. The rolling direction is from root to tip (A-E) for the driving gear. For the driven tooth, the rolling direction is from tip to root (E–A). Sliding and rolling orientation for the driving gear at the tooth tip is in the same direction, although it is the opposite in the tooth root. The sliding orientation is always towards the outside direction from the mating line. In contrast with the previous situation, the sliding velocity is from root to tip and towards inside from the mating line. Second phase: At full mesh, when the two teeth meet at their common or "operating" pitch-line, there is only a rolling motion, and no sliding. However, this stage produces the greatest tooth loading (Fig. 3b), meaning that just one tooth holds the full tooth load. Third phase: Coming out of mesh, the two mating teeth also move with a sliding action, opposite to the initial contact stage (Fig. 3c). In this region, the two teeth share the tooth load (Fernandes and Mcduling, 1997).

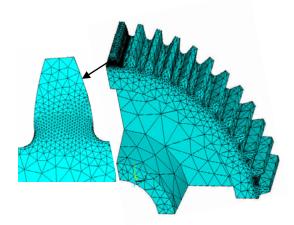


Figure 1. Finite element model of the wheel.

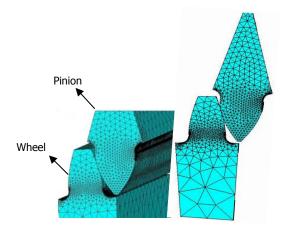


Figure 2. Finite element model for spur gear assembly.

In spur gears with low-contact ratio, during one mesh period, there is one tooth pair in contact and two tooth pairs in contact, occurring separately. If gears contact ratio is less than two (such as this paper), Calculation will perform based on a pair of conjugate. For analysis of bending, force was applied to the middle tooth, Surface that was located on the pitch circle. For analysis of contact stress, torque According to gear contact ratio as force was applied on the pinion gear tooth. Loading was performed along the action line or pressure angle in both analyses. Assembly errors are one of the main causes of differences between numerical results and theoretical results. In the meshing simulation, assembly errors prevent the creation of a real contact between the teeth, thus, stress distribution is asymmetric in two teeth (Li, 2007). In this study was removed assembly error, consequently, the symmetrical distribution of stress was observed on both sides gear.

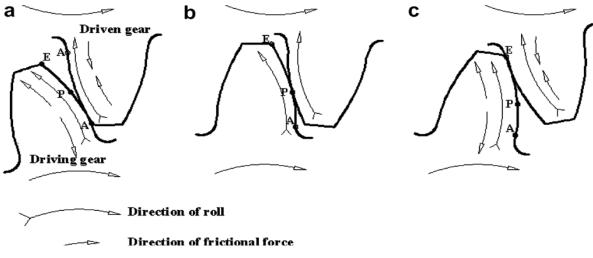


Figure 3. Mechanics of gear tooth contact, (a) at point of first contact, (b) at pitch point and (c) at last point of contact (Walton and Goodwin, 1998).

Table 1. Gears geometric before and after	r changes and AGMA stress values.
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Contact Stress (Mpa)	Bending Stress (Mpa)	Pitch Diameter (mm)	Moud ol (mm)	Face Width <sup>**</sup> (mm)	Face Width <sup>*</sup> (mm)	Gear
540**	325**	192	4	32	24	Wheel
570**	305**	44	4	32	26	Pinion

\*Befor change,\*\*After change

#### **RESULTS and DISCUSSION**

Slip torque is calculated using graphical method and then tangential force and torgue transfer values were determined for each of the different gears in the transmission status. In harvesting machines about 40-42 percent of engine power transferred to the gearbox (Kutzbach and Quick, 1999), therefore, to use this gearbox in new power source its transmission capacity must be increased approximately 16 percent. In fact, with increasing power supply from 130 to 180 hp is needed gears transmission capacity increased 16 percent. To increase the capacity of power transmission gears, geometry of new gears obtained accurately using Geartrax software. Table 1 gives the gears geometric before and after changes also results for maximum bending and contact stresses due to AGMA for pair of gears that have been analyzed.

#### **Bending stress**

Spur gear is an important part of machine, which is usually damaged due to the bending stress during its operating period. The bending stress occurs in the gear tooth when the pinion meshes with the gear, as a result, the gear tooth failure could damage its teeth's root seriously. The pinion tooth and the gear tooth always contact along the pitch circle line theoretically. In this paper Software analysis indicated that the gears ability to work with the new power source. Figure 4 illustrates in three-dimensional view of the gear that has been analyzed. Maximum stress or stress concentration was observed in the area of tooth root and tooth fillet although cross-sectional maximum stress is created at the site loading (384 Mpa). Stress distribution profile in all analysis is quite similar to the stress distribution profile of Photoelasticity method. This indicates that the location and method of loading this research can be used in other studies. Of course, the available literature shows that the tooth-root stress as a criterion for the tooth strength evaluation has been mainly the aim of research. Stress distribution Symmetrical profile on both sides gear (Fig. 4 and 5) shows high accuracy and analyzed by applying proper boundary conditions (force applied widely and quite uniform). As most failures occur through tension in the gear (Dolan and Broghamer, 1942).

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### **Contact analysis**

Contact analysis also has similarities with the theoretical results. The contact between the pinion and wheel was accomplished on the engagement teeth surfaces with geometrical contact line posotioned approximately at the middle. Teeth contact time is approximately 0.004 sec. of course, the contact time is is calculated according to the linear speed, transmission ratios and circular pitch gears. Contact analysis showed that the highest stress have been created in the teeth contact area (581 Mpa in Fig. 6). Because the pinion pitch diameter is smaller than gear pitch diameter and the number of its loading is higher, so that the created contact stress in the pinion is larger (Chawathe, 2001). This situation also was observed in this study. Although the contact for analysis, stress concentration was observed around the gear tooth root, but this phenomenon is due to the base of pinion was constrained radially and axially, and in tangential direction was left free.

Survey of elements behavior that placed between contacts surfaces show that contact stress in tooth contact locate being after than specific duration. Contact stress begins after 0.0008 sec. from the time of engagement teeth and is increasing to the end of contact time (Fig. 6).

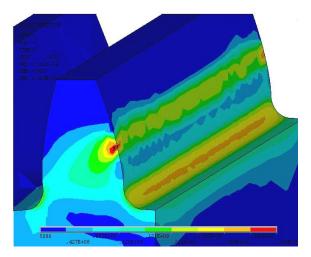


Figure 4. 3D Contour of bending stress for wheel (Mpa)

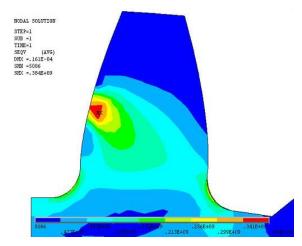


Figure 5. Contour of bending stress for wheel in right view (Mpa)

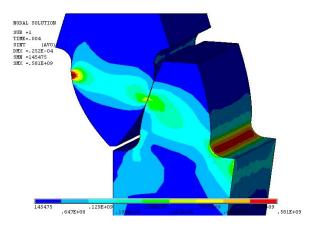


Figure 6. Contour of contact stress for gear pair (Mpa)

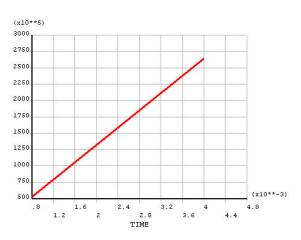


Figure 7. Contact stress creating process in the teeth contact area

## CONCLUSION

The followings were concluded from the study:

- The results showed that the slip torque is suitable for gear design in gearbox of non-road machinery.
- Slip velocity is zero at the contact of spur gear teeth therefore, by Application of contact elements in contact analysis the friction coefficient will not have important role on stress values.
- The results showed that the maximum bending stress is created at the root of tooth (teeth Fillet) and maximum contact stress at the teeth contact.

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- Volumetric (body) contact model results compared with results of Area contact model is closer to the results of theoretical methods
- Contact fatigue caused by stress, is higher in the pinion and is due to the higher number of loading cycles.
- contact stress in tooth contact locate being after than specific duration.
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