



Optimal synthesis of function-generating slider-crank mechanism based on a closed-form solution using five design parameters

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ABSTRACT

Kinematic dimensional synthesis is one of the essential steps during the process of design. It is classified into three tasks: function generation, path generation, and motion generation. In this paper, a design method is presented to solve the synthesis of the slider-crank mechanism for function generation based on a closed-form solution with five design parameters. The success criterion of a method used for function generation depends on the structural error function within an operational domain of the slider-crank mechanism. The structural error reduction is related to the number of design parameters utilized in mechanism synthesis, and the effective maximum number of design parameters for the slider-crank mechanism is five. In this study, the closed-form synthesis is found using three methods: precision point method, sub-domain method, and galerkin method. The system of equations is reduced to a twelfth-degree univariate polynomial equation, and thus all the available solutions are obtained. The effectiveness of this design method is tested with some examples of commonly used test functions, namely e^x , $\sin(x)$, $\tan(x)$ and $\ln(x)$, using a developed computer program. This design method gives lower structural error than traditional methods in kinematics literature.

1. INTRODUCTION

In kinematic synthesis, the shape and size of mechanical elements are determined to develop a flow of power through the system between the input and the output links, according to the desired performance (McCarthy and Jostkowicz 2009; Söylemez 1985). The motion equation of the output link is to be a function of the input link's motion or a function of time, and it is formed from nonlinear algebraic and/or high order equations (Dhingra et al. 2000a; Wampler et al. 1990; Wampler et al. 1992; Almadi et al. 1996; Dhingra et al. 2001a; Dhingra et al. 2000b; Dhingra et al. 2001b; Akçalı 1984). These mathematical relationships between the input and output links are so complex; hence, graphical approaches are commonly utilized in solving such problems (Shigley 1961). However, graphical approaches are time-consuming and lack accuracy.

With the evolution of computers, many analytical methods have been developed, and they approximately

express the desired function. Therefore, the structural error function which is the difference between the desired function and the function produced by a mechanism is taken into account and minimized using mathematical approaches depending on iteration such as the precision point method (Akçalı and Dittrich 1989), sub-domain method (Dhingra et al. 2000a), galerkin method (Hartenberg and Denavit 1964) and the least-squares method (Nolle 1997; Singh et al. 2005). Here, the most important factor in minimizing the structural error is the excess number of unknown parameters of a mechanism.

However, since the aforementioned methods are based on iteration, the excess number of unknown parameters causes great difficulties, and thus the solution may be impossible. For instance, in the solution of the precision point method of synthesis applied to nine positions in reference (Almadi et al. 1995), the Newton-Raphson technique was used in a solution strategy called 'Bootstrap.' And, due to the complexity of the equation,

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the drawing for each initial value does not exist. Even if a solution exists, different results are obtained from different initial values.

For kinematic synthesis problems, the precision point method and the least-squares method are extensively used to minimize the structural error (Singh et al. 2005; Tinubu and Gupta 1984). In this way, the structural error is zero at the given precision points while, at the mean values, it is small as possible. When the number of precision points increases, the structural error decreases at the mean values. The number of points that can be selected depends on the number of kinematic dimensions of the active mechanism in the design. However, when the number of parameters used in the design increases, it becomes difficult to obtain a closed-form solution. And if the number of unknown parameters exceeds five, the direct application of closed-form solutions to mechanism designs will probably lead to undesirable results (Akçalı 1984; Özkul and Mutlu 2018).

In this study, given five design parameters as a maximum, the design of the slider-crank mechanism for function generation with minimum error has been presented. The system of equations has been reduced to a twelfth-degree univariate polynomial equation, and in this way, all the possible solutions have been obtained. Because three methods (precision point method, sub-domain method, and galerkin method) are used in the solution, the designer has been provided with alternative solutions. Additionally, the effectiveness of the presented mathematical model is tested on some commonly used test functions by using a developed computer program. Accordingly, we found that our design method gives lower structural error than traditional methods in kinematics literature.

2. FUNCTION-GENERATING SLIDER-CRANK MECHANISM

For creating a required function using the sliding-crank mechanism, first, Fig. 1, which shows the mechanism's design parameters, is given. Here, x_1 is the crank length, x_2 is the connecting rod length, and x_3 is the vertical distance (AO_1). ψ_0 is the angle when the crank starts rotating and s_n is the distance (AC), vertical on the AO_1 line, at the starting position when the angle ψ_0 occurs. While designing a function-generating slider-crank mechanism, two different functional relationships can be considered. The first functional relationship is between the crank rotation angle (ψ) and the linear horizontal displacement of the piston (s). The second functional relationship is between the crank rotation angle (ψ) and the angle of the connecting rod with the horizontal line (δ).

If the ψ - s functional relation is taken into consideration, the vector expression written from the closed-loop AO_1BC in Fig. 1 is squared and added side by side, and the angle of the connecting rod with the horizontal line (δ) is eliminated, the following motion equation, Eq. (1), is generated.

$$x_1^2 - 2sx_1\cos\psi + 2x_1x_3\sin\psi + x_3^2 + s^2 - x_2^2 = 0 \quad (1)$$

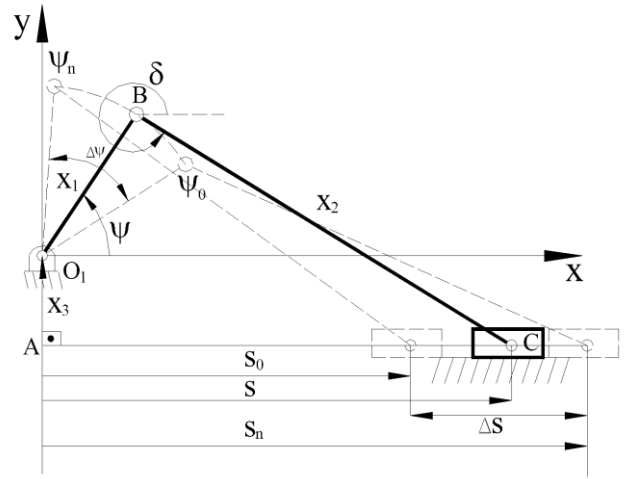


Figure 1. The design parameters of a function-generating slider-crank mechanism

Also, the following linear relations of the slider-crank mechanism, the angle (ψ) with the independent variable (x) of the desired function and the piston displacement (s) with the function's dependent variable (y), can be established as in expressions (2) and (3), respectively.

$$\begin{aligned} \psi &= \psi_0 + \psi' \\ \psi' &= R_x(x - x_0) \\ R_x &= \frac{\Delta\psi}{\Delta x} \end{aligned} \quad (2)$$

$$\begin{aligned} s &= s_0 + s' \\ s' &= R_y(y - y_0) \\ R_y &= \frac{\Delta s}{\Delta y} \end{aligned} \quad (3)$$

Here, the operational domains of the crank link and piston, determined by the designer, are ($\Delta\psi = \psi_n - \psi_0$) and ($\Delta s = s_n - s_0$), respectively. Also, the change amounts of the independent variable and dependent variable of the generated function are ($\Delta x = x_n - x_0$) and ($\Delta y = y_n - y_0$), respectively. If Eq. (2) and Eq. (3) are written in Eq. (1), the motion equation can be obtained in the shape of $G(s_0, \psi_0, x_1, x_2, x_3, x, y) = 0$ as shown in Eq. (4).

$$\begin{aligned} 2s's_0 + s'^2 + Z_3(\sin\psi_0\cos\psi' + \cos\psi_0\sin\psi') \\ + Z_1(-s_0\cos\psi_0\cos\psi' \\ + s_0\sin\psi_0\sin\psi' \\ - s'\cos\psi_0\cos\psi' \\ + s'\sin\psi_0\sin\psi') + Z_2 = 0 \end{aligned} \quad (4)$$

where,

$$\begin{aligned} Z_1 &= 2x_1 \\ Z_2 &= x_1^2 + x_3^2 + s_0^2 - x_2^2 \\ Z_3 &= 2x_1x_3 \end{aligned}$$

For solving the unknown parameters ($s_0, \psi_0, x_1, x_2, x_3$) in Eq. (4), a nonlinear equation with five unknowns is obtained as shown in Eq. (5).

$$(a_i s_0 + g_i) + Z_3(c_i \sin \psi_0 + b_i \cos \psi_0) + Z_1(-c_i s_0 \cos \psi_0 + b_i s_0 \sin \psi_0 - d_i \cos \psi_0 + e_i \sin \psi_0 + f_i Z_2) = 0 \quad i = 1, 2, 3, 4, 5 \quad (5)$$

The coefficients ($a_i, b_i, c_i, d_i, e_i, f_i, g_i$) in Eq.(5) can be solved using the three methods: precision point method, sub-domain method, and galerkin method.

Precision Point Method:

$$\begin{aligned} a_i &= 2s'_i \\ b_i &= \sin \psi'_i \\ c_i &= \cos \psi'_i \\ d_i &= s' \cos \psi'_i \\ e_i &= s'_i \sin \psi'_0 \\ f_i &= 1 \\ g_i &= s'^2_i \\ i &= 1, 2, 3, 4, 5 \end{aligned} \quad (6)$$

Sub-domain Method:

$$\left. \begin{aligned} a_i &= 2 \int_{x_{i-1}}^{x_i} s' dx \\ b_i &= \int_{x_{i-1}}^{x_i} \sin \psi' dx \\ c_i &= \int_{x_{i-1}}^{x_i} \cos \psi' dx \\ d_i &= \int_{x_{i-1}}^{x_i} s' \cos \psi' dx \\ e_i &= \int_{x_{i-1}}^{x_i} s' \sin \psi' dx \\ f_i &= \int_{x_{i-1}}^{x_i} dx \\ g_i &= \int_{x_{i-1}}^{x_i} s'^2 dx \end{aligned} \right\} i = 1, 2, 3, 4, 5 \quad (7)$$

Galerkin Method:

$$\left. \begin{aligned} a_i &= 2 \int_{x_0}^{x_n} s' w_i(x) dx \\ b_i &= \int_{x_0}^{x_n} \sin \psi' w_i(x) dx \\ c_i &= \int_{x_0}^{x_n} \cos \psi' w_i(x) dx \\ d_i &= \int_{x_0}^{x_n} s' \cos \psi' w_i(x) dx \\ e_i &= \int_{x_0}^{x_n} s' \sin \psi' w_i(x) dx \\ f_i &= \int_{x_0}^{x_n} w_i(x) dx \\ g_i &= \int_{x_0}^{x_n} s'^2 w_i(x) dx \end{aligned} \right\} i = 1, 2, 3, 4, 5 \quad (8)$$

To achieve the closed-form solution, if Z_2, Z_3 and Z_1 are eliminated from Eq. (5) and the equation is rearranged, two nonlinear equations depending on the unknown parameters $\tan(\psi_0)$ and s_0 are obtained, as in Eq. (9).

$$\begin{aligned} a_{pk} s_0^2 + (b_{pk} + c_{pk} \cos 2 \psi_0 + d_{pk} \sin 2 \psi_0) s_0 + e_{pk} + f_{pk} \cos 2 \psi_0 + g_{pk} \sin 2 \psi_0 &= 0 \\ k &= 1, 2 \end{aligned} \quad (9)$$

The computable coefficients in Eq. (9) are given below in Appendix A in Eqs. (10-16). If the term (s_0) is removed using the intersection function mentioned in

(Akçalı et al. 2006), the 12th-degree univariate polynomial equation dependent on t can be obtained as in Eq. (17).

$$\begin{aligned} w_0 + w_1 t + w_2 t^2 + w_3 t^3 + w_4 t^4 + w_5 t^5 + w_6 t^6 - w_5 t^7 + w_4 t^8 - w_3 t^9 + w_2 t^{10} - w_1 t^{11} + w_0 t^{12} &= 0 \end{aligned} \quad (17)$$

$$\text{where, } t = \tan \frac{\psi_0}{2} \quad (18)$$

The coefficients of Eq. (17) are given also in Appendix A in Eqs. (19-25). As a result of finding all the roots of Eq. (17), a set of 12 solutions (real or complex) for the ψ_0 angle is found as given in Eq. (26).

$$\psi_{0i} = 2 \arctan t_i; \quad i = 1, 2, \dots, 12 \quad (26)$$

For calculating the other kinematic quantities (s_0, x_1, x_2, x_3) of the function-generating slider-crank mechanism, firstly, if the term s_0^2 is removed from Eq. (9), s_0 will be found as in Eq. (27).

$$s_{0i} = - \frac{a_{p1} e_{p2} - a_{p2} e_{p1} + (a_{p1} f_{p2} - a_{p2} f_{p1}) \cos 2 \psi_{0i} + (a_{p1} g_{p2} - a_{p2} g_{p1}) \sin 2 \psi_{0i}}{a_{p1} b_{p2} - a_{p2} b_{p1} + (a_{p1} c_{p2} - a_{p2} c_{p1}) \cos 2 \psi_{0i} + (a_{p1} d_{p2} - a_{p2} d_{p1}) \sin 2 \psi_{0i}} \quad i = 1, \dots, 12 \quad (27)$$

And, for calculating the other three dimensional parameters (x_1, x_2 and x_3), the following path in Eqs. (28-33) is followed.

$$Z_{3i} = - \frac{\left\{ \begin{aligned} &2 \cos \psi_{0i} (-d_k f_2 g_1 + d_2 f_k g_1 + d_k f_1 g_2 - d_1 f_k g_2 - d_2 f_1 g_k + d_1 f_2 g_k) + \\ &2 \sin \psi_{0i} (e_k f_2 g_1 - e_2 f_k g_1 - e_k f_1 g_2 + e_1 f_k g_2 + e_2 f_1 g_k - e_1 f_2 g_k) + \\ &s_0 (2 \sin \psi_{0i} (a_k e_2 f_1 - a_2 e_k f_1 - a_k e_1 f_2 + a_1 e_k f_2 + a_2 e_1 f_k - a_1 e_2 f_k) + \\ &b_k f_2 g_1 - b_2 f_k g_1 - b_k f_1 g_2 + b_1 f_k g_2 + b_2 f_1 g_k - b_1 f_2 g_k) + \\ &2 \cos \psi_{0i} (-a_k d_2 f_1 + a_2 d_k f_1 + a_k d_1 f_2 - a_1 d_k f_2 - a_2 d_1 f_k + a_1 d_2 f_k - c_k f_2 g_1 + c_2 f_k g_1 + c_k f_1 g_2 - c_1 f_k g_2 - c_2 f_1 g_k + c_1 f_2 g_k) + \\ &s_0^2 (2 \sin \psi_{0i} (a_k b_2 f_1 - a_2 b_k f_1 - a_k b_1 f_2 + a_1 b_k f_2 + a_2 b_1 f_k - a_1 b_2 f_k) + \\ &2 \cos \psi_{0i} (-a_k c_2 f_1 + a_2 c_k f_1 + a_k c_1 f_2 - a_1 c_k f_2 - a_2 c_1 f_k + a_1 c_2 f_k) + \end{aligned} \right\}}{\left\{ \begin{aligned} &-b_k d_2 f_1 + b_2 d_k f_1 + c_k e_2 f_1 - c_2 e_k f_1 + b_k d_1 f_2 - b_1 d_k f_2 - c_k e_1 f_2 + c_2 e_k f_2 + \\ &c_1 e_k f_2 - b_2 d_1 f_k + b_1 d_2 f_k + c_2 e_1 f_k - c_1 e_2 f_k + \sin 2 \psi_{0i} (-c_k d_2 f_1 + c_2 d_k f_1 + b_k e_2 f_1 - b_2 e_k f_1 + c_k d_1 f_2 - c_1 d_k f_2 - b_k e_1 f_2 + b_1 e_k f_2 - c_2 d_1 f_k + c_1 d_2 f_k + b_2 e_1 f_k - b_1 e_2 f_k) + \\ &\cos 2 \psi_{0i} (-b_k d_2 f_1 + b_2 d_k f_1 - c_k e_2 f_1 + c_2 e_k f_1 + b_k d_1 f_2 - b_1 d_k f_2 - c_k e_1 f_2 + c_2 e_k f_2 - b_2 d_1 f_k + b_1 d_2 f_k - c_2 e_1 f_k + c_1 e_2 f_k) + \\ &2 s_{0i} (-b_k c_2 f_1 + b_2 c_k f_1 + b_k c_1 f_2 - b_1 c_k f_2 - b_2 c_1 f_k + b_1 c_2 f_k) \end{aligned} \right\}} \quad i = 1, \dots, 12 \quad k = 3, 4, 5 \quad (28)$$

$$Z_{1i} = - \frac{\begin{cases} f_1 g_k - f_k g_1 + s_{0i}(a_k f_1 - a_1 f_k) + \\ Z_{3i} \cos \psi_{0i}(b_k f_1 - b_1 f_k) + \\ Z_{3i} \sin \psi_{0i}(c_k f_1 - c_1 f_k) \end{cases}}{\begin{cases} \cos \psi_{0i}(s_{0i}(c_1 f_k - c_k f_1) + \\ d_1 f_k - d_k f_1) + \\ \sin \psi_{0i}(s_{0i}(b_k f_1 - b_1 f_k) + \\ e_k f_1 - e_1 f_k) \end{cases}} \quad (29)$$

$$\begin{aligned} i &= 1, \dots, 12 \\ k &= 1, 2, 3, 4, 5 \end{aligned} \quad (30)$$

$$x_{1i} = \frac{Z_{1i}}{2} \quad i = 1, 2, \dots, 12 \quad (31)$$

$$x_{3i} = \frac{Z_{3i}}{2x_{1i}} \quad i = 1, 2, \dots, 12 \quad (32)$$

$$x_{2i} = \sqrt{x_{1i}^2 + x_{3i}^2 + s_{0i}^2 - Z_{2i}} \quad i = 1, 2, \dots, 12 \quad (33)$$

The structural error (e) is given in Eq. (34) to determine the accuracy of the actual solutions (x_{1j}, x_{2j}, x_{3j}, s_{0j}, ψ_{0j}), j=1, 2, ..., 12, in the solution set.

$$e = s_{th} - s_{ac} \quad (34)$$

where s_{th} is the theoretical piston displacement calculated in Eq. (3), and s_{ac} represents the actual piston displacement calculated from Eq. (35) below. Finally only one solution from Eq. (35) will correspond to the desired function.

$$s_{ac\pm} = \frac{2x_1 \cos(\psi_0 + \psi') \pm \sqrt{(2x_1 \cos(\psi_0 + \psi'))^2 - 4 \left(x_1^2 + x_3^2 - x_2^2 + 2x_1 x_3 \sin(\psi_0 + \psi') \right)}}{2} \quad (35)$$

The compatibility of the designed slider-crank mechanism to the desired function is determined by the distribution of the structural error function in Eq. (34).

3. NUMERICAL EXAMPLES

The above-mentioned design approaches were employed using a developed computer program for generating commonly used functions such as e^x, sin(x), tan(x), and ln(x) by the slider-crank mechanism. Taking into consideration the structural error values of the real solutions from among the various solutions obtained as well as the compatibility of the aspect ratios with manufacturing, the structural error functions of the three approaches are presented. Here, four examples of test functions generated by the slider-crank mechanism are given.

3.1. Example A

In this example, the desired function is y=e^x over 0≤x≤1. The design results of the slider-crank mechanism that produces this function while the input link rotates Δψ = 90° and the output link moves Δs = -1.0 (dimensionless, as a scale) have been obtained according to the three different methods as shown in Table 1.

Table 1. The design results for y=e^x over 0≤x≤1 using three methods

Precision Point Method	
Precision points	x _{pi} , (i=1, ..., 5) = 0.0, 0.2, 0.5, 0.8, 1.0
Mechanism dimensions	x ₁ = -1.241043 x ₂ = 2.123150 x ₃ = -0.814295 ψ ₀ = 150.324° s ₀ = -0.492244
Maximum absolute error	0.00027071
Sub-domain Method	
Sub-domains	x _{si} , (i=1, ..., 6) = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0
Mechanism dimensions	x ₁ = -1.223522 x ₂ = 2.119717 x ₃ = -0.820609 ψ ₀ = 150.6590° s ₀ = -0.506569
Maximum absolute error	-0.00069492
Galerkin Method	
Weighting functions	w _i , (i=1, ..., 5) = 1, x, x ² , x ³ , x ⁴
Mechanism dimensions	x ₁ = -1.240452 x ₂ = 2.137685 x ₃ = -0.824552 ψ ₀ = 149.7941° s ₀ = -0.499602
Maximum absolute error	-0.00037269

The structural error distribution for this example is shown in Fig. (2).

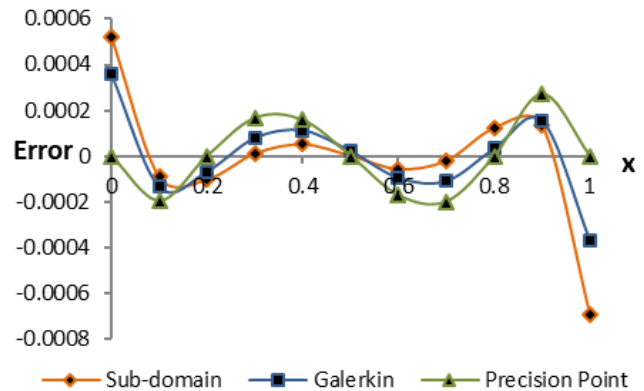


Figure 2. Structural error curve for e^x

3.2. Example B

The desired function of this example is y=sin(x) over 0° ≤ x ≤ 90°. The design results of the slider-crank mechanism that produces this function while the input link rotates Δψ = 80° and the output link moves Δs = -1.0 have been obtained according to the three different methods as shown in Table 2.

The structural error distribution for this example is as shown in Fig. (3).

Table 2. The design results for $y=\sin(x)$ over $0^\circ \leq x \leq 90^\circ$ using three methods

Precision Point Method	
Precision points	$x_{pi}, (i=1, \dots, 5) = 0.000, 0.436, 0.873, 1.309, 1.571$
Mechanism dimensions	$x_1 = -1.086305$ $x_2 = 7.310604$ $x_3 = 0.917419$ $\psi_0 = -73.7607^\circ$ $s_0 = -7.346641$
Maximum absolute error	-0.00014533
Sub-domain Method	
Sub-domains	$x_{si}, (i=1, \dots, 6) = 0.000, 0.349, 0.698, 1.047, 1.396, 1.571$
Mechanism dimensions	$x_1 = -1.088700$ $x_2 = 7.504273$ $x_3 = 0.941629$ $\psi_0 = -73.7607^\circ$ $s_0 = -7.541173$
Maximum absolute error	0.00025793
Galerkin Method	
Weighting functions	$w_i, (i=1, \dots, 5) = 1, x, x^2, x^3, x^4$
Mechanism dimensions	$x_1 = -1.077320$ $x_2 = 7.185135$ $x_3 = 1.089057$ $\psi_0 = -72.5057^\circ$ $s_0 = -7.190281$
Maximum absolute error	-0.00010383

Table 3. The design results for $y = \tan(x)$ over $0^\circ \leq x \leq 45^\circ$ using three methods

Precision Point Method	
Precision points	$x_{pi}, (i=1, \dots, 5) = 0.000, 0.175, 0.349, 0.524, 0.785$
Mechanism dimensions	$x_1 = 3.002218$ $x_2 = 2.470431$ $x_3 = -0.531810$ $\psi_0 = 92.8363^\circ$ $s_0 = -0.013371$
Maximum absolute error	-0.00011431
Sub-domain Method	
Sub-domains	$x_{si}, (i=1, \dots, 6) = 0.000, 0.175, 0.349, 0.524, 0.698, 0.785$
Mechanism dimensions	$x_1 = 3.458880$ $x_2 = 2.928213$ $x_3 = -0.530865$ $\psi_0 = 94.2682^\circ$ $s_0 = -0.019198$
Maximum absolute error	-0.00102642
Galerkin Method	
Weighting functions	$w_i, (i=1, \dots, 5) = 1, x, x^2, x^3, x^4$
Mechanism dimensions	$x_1 = 3.278928$ $x_2 = 2.747495$ $x_3 = -0.531536$ $\psi_0 = 93.7131^\circ$ $s_0 = -0.017070$
Maximum absolute error	-0.00053734

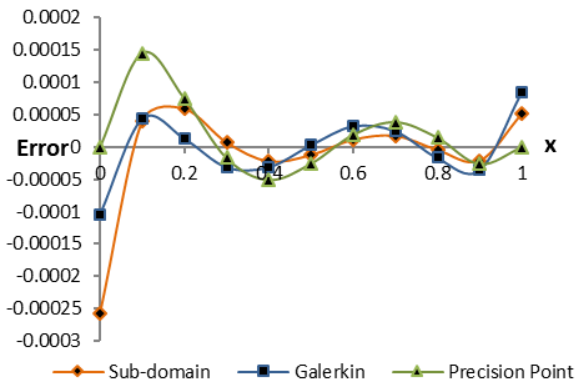


Figure 3. Structural error curve for $\sin(x)$

3.3. Example C

In this example, the desired function is $y=\tan(x)$ over $0^\circ \leq x \leq 45^\circ$. The design results of the slider-crank mechanism that produces this function while the input link rotates $\Delta\psi = 80^\circ$ and the output link moves $\Delta s = -0.5$ have been obtained according to the three different methods as shown below in Table 3.

The structural error distribution for this example is as shown in Fig. (4).

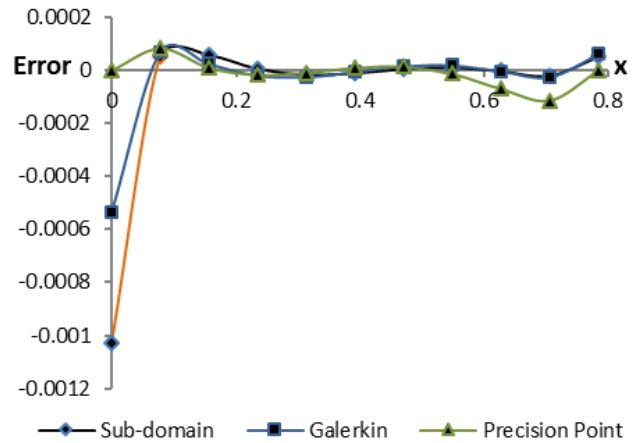


Figure 4. Structural error curve for $\tan(x)$

3.4. Example D

Finally, in this example, the desired function is $y=\ln(x)$ over $1 \leq x \leq 2$. The design results of the slider-crank mechanism that produces this function while the input link rotates $\Delta\psi = 90^\circ$ and the output link moves $\Delta s = -0.1$ have been obtained according to the following three different methods shown in Table 4.

The structural error distribution for this example is as shown in Fig. (5).

Table 4. The design results for $y=\ln(x)$ over $1 \leq x \leq 2$ using three methods

Precision Point Method	
Precision points	$x_{pi}, (i=1, \dots, 5) = 1.00, 1.20, 1.50, 1.80, 2.00$
Mechanism dimensions	$x_1= 1.505762$ $x_2= 2.608397$ $x_3= -1.094561$ $\psi_0= 151.2138^\circ$ $s_0= 1.262414$
Maximum absolute error	0.00025632
Sub-domain Method	
Sub-domains	$x_{si}, (i=1, \dots, 6) = 1.00, 1.200, 1.40, 1.60, 1.80, 2.00$
Mechanism dimensions	$x_1= 1.475092$ $x_2= 2.590246$ $x_3= -1.105152$ $\psi_0= 151.1454^\circ$ $s_0= 1.268790$
Maximum absolute error	-0.00080016
Galerkin Method	
Weighting functions	$w_i, (i=1, \dots, 5) = 1, x, x^2, x^3, x^4$
Mechanism dimensions	$x_1= 1.507783$ $x_2= 2.620459$ $x_3= -1.103462$ $\psi_0= 151.8690^\circ$ $s_0= 1.261505$
Maximum absolute error	-0.00055926

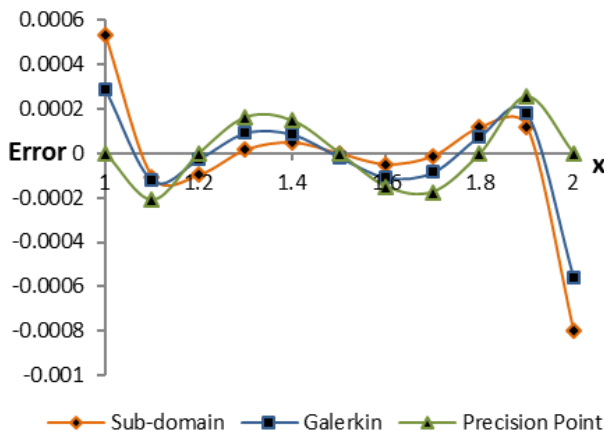


Figure 5. Structural error curve for $\ln(x)$

4. DISCUSSION AND CONCLUSION

During the examination of the abovementioned examples, the achieved maximum values in the structural error distributions of the three methods (precision point method, sub-domain method, and galerkin method) are compared with the results discussed in kinematics literature as given in Appendix B. Accordingly, our results demonstrated in this paper were found to be much better.

The structural error distributions of the three methods are given as shown in Figs. (2-5) and, accordingly, it can be said that there is no clear

superiority of one method over the others. However, in the sub-domain and the precision point methods, the structural error distribution is greatly influenced by the arbitrarily selected sub-domains and precision points, whereas in the galerkin method, the error distribution is not affected by the weighting functions selected arbitrarily. In this case, the galerkin method has superiority over the other two methods. Finally, the design algorithm discussed in this study can be applied to different types of mechanisms and also for path and motion synthesis, as well.

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APPENDIX A. Computed coefficients of the abovementioned equations

$$a_{pi} = 2a_k b_3 c_2 f_1 - 2a_3 b_k c_2 f_1 - 2a_k b_2 c_3 f_1 + 2a_2 b_k c_3 f_1 + 2a_3 b_2 c_k f_1 - 2a_2 b_3 c_k f_1 - 2a_k b_3 c_1 f_2 + 2a_3 b_k c_1 f_2 + 2a_k b_1 c_3 f_2 - 2a_1 b_k c_3 f_2 - 2a_3 b_1 c_k f_2 + 2a_1 b_3 c_k f_2 + 2a_k b_2 c_1 f_3 - 2a_2 b_k c_1 f_3 - 2a_k b_1 c_2 f_3 + 2a_1 b_k c_2 f_3 + 2a_2 b_1 c_k f_3 - 2a_1 b_2 c_k f_3 - 2a_3 b_2 c_1 f_k + 2a_2 b_3 c_1 f_k + 2a_3 b_1 c_2 f_k - 2a_1 b_3 c_2 f_k - 2a_2 b_1 c_3 f_k + 2a_1 b_2 c_3 f_k$$

$$i=1, 2 \quad k=i+1 \tag{10}$$

$$b_{pi} = a_k b_3 d_2 f_1 - a_3 b_k d_2 f_1 - a_k b_2 d_3 f_1 + a_2 b_k d_3 f_1 + a_3 b_2 d_k f_1 - a_2 b_3 d_k f_1 - a_k c_3 e_2 f_1 + a_3 c_k e_2 f_1 + a_k c_2 e_3 f_1 - a_2 c_k e_3 f_1 - a_3 c_2 e_k f_1 + a_2 c_3 e_k f_1 - a_k b_3 d_1 f_2 + a_3 b_k d_1 f_2 + a_k b_1 d_3 f_2 - a_1 b_k d_3 f_2 - a_3 b_1 d_k f_2 + a_1 b_3 d_k f_2 + a_k c_3 e_1 f_2 - a_3 c_k e_1 f_2 - a_k c_1 e_3 f_2 + a_1 c_k e_3 f_2 + a_3 c_1 e_k f_2 - a_1 c_3 e_k f_2 + a_k b_2 d_1 f_3 - a_2 b_k d_1 f_3 - a_k b_1 d_2 f_3 + a_1 b_k d_2 f_3 + a_2 b_1 d_k f_3 - a_1 b_2 d_k f_3 - a_k c_2 e_1 f_3 + a_2 c_k e_1 f_3 + a_k c_1 e_2 f_3 - a_1 c_k e_2 f_3 - a_2 c_1 e_k f_3 + a_1 c_2 e_k f_3 - a_3 b_2 d_1 f_k + a_2 b_3 d_1 f_k + a_3 b_1 d_2 f_k - a_1 b_3 d_2 f_k - a_2 b_1 d_3 f_k + a_1 b_2 d_3 f_k + a_3 c_2 e_1 f_k - a_2 c_3 e_1 f_k - a_3 c_1 e_2 f_k + a_1 c_3 e_2 f_k + a_2 c_1 e_3 f_k - a_1 c_2 e_3 f_k - 2b_k c_3 f_2 g_1 + 2b_k c_2 f_3 g_1 - 2b_k c_2 f_3 g_1 - 2b_3 c_2 f_k g_1 + 2b_2 c_3 f_k g_1 + 2b_k c_3 f_1 g_2 - 2b_3 c_k f_1 g_2 - 2b_k c_1 f_3 g_2 + 2b_1 c_k f_3 g_2 + 2b_3 c_1 f_k g_2 - 2b_1 c_3 f_k g_2 - 2b_k c_2 f_1 g_3 + 2b_2 c_k f_1 g_3 + 2b_k c_1 f_2 g_3 - 2b_1 c_k f_2 g_3 - 2b_2 c_1 f_k g_3 + 2b_1 c_2 f_k g_3 + 2b_3 c_2 f_1 g_k - 2b_2 c_3 f_1 g_k - 2b_3 c_1 f_2 g_k + 2b_2 c_1 f_3 g_k - 2b_1 c_2 f_3 g_k$$

$$i=1, 2 \quad k=i+1 \tag{11}$$

$$c_{pi} = a_k b_3 d_2 f_1 - a_3 b_k d_2 f_1 - a_k b_2 d_3 f_1 + a_2 b_k d_3 f_1 + a_3 b_2 d_k f_1 - a_2 b_3 d_k f_1 + a_k c_3 e_2 f_1 - a_3 c_k e_2 f_1 - a_k c_2 e_3 f_1 + a_2 c_k e_3 f_1 + a_3 c_2 e_k f_1 - a_2 c_3 e_k f_1 - a_k b_3 d_1 f_2 + a_3 b_k d_1 f_2 + a_k b_1 d_3 f_2 - a_1 b_k d_3 f_2 - a_3 b_1 d_k f_2 + a_1 b_3 d_k f_2 + a_k c_3 e_1 f_2 - a_3 c_k e_1 f_2 - a_k c_1 e_3 f_2 + a_1 c_k e_3 f_2 + a_3 c_1 e_k f_2 - a_1 c_3 e_k f_2 + a_k b_2 d_1 f_3 - a_2 b_k d_1 f_3 - a_k b_1 d_2 f_3 + a_1 b_k d_2 f_3 + a_2 b_1 d_k f_3 - a_1 b_2 d_k f_3 + a_k c_2 e_1 f_3 - a_2 c_k e_1 f_3 - a_k c_1 e_2 f_3 + a_1 c_k e_2 f_3 + a_2 c_1 e_k f_3 - a_1 c_2 e_k f_3 - a_3 b_2 d_1 f_k + a_2 b_3 d_1 f_k + a_3 b_1 d_2 f_k - a_1 b_3 d_2 f_k - a_2 b_1 d_3 f_k + a_1 b_2 d_3 f_k - a_3 c_2 e_1 f_k + a_2 c_3 e_1 f_k + a_3 c_1 e_2 f_k - a_1 c_3 e_2 f_k - a_2 c_1 e_3 f_k + a_1 c_2 e_3 f_k$$

$$i=1, 2 \quad k=i+1 \tag{12}$$

$$d_{pi} = a_k c_3 d_2 f_1 - a_3 c_k d_2 f_1 - a_k c_2 d_3 f_1 + a_2 c_k d_3 f_1 + a_3 c_2 d_k f_1 - a_2 c_3 d_k f_1 - a_k b_3 e_2 f_1 + a_3 b_k e_2 f_1 + a_k b_2 e_3 f_1 - a_2 b_k e_3 f_1 - a_3 b_2 e_k f_1 + a_2 b_3 e_k f_1 - a_k c_3 d_1 f_2 + a_3 c_k d_1 f_2 + a_k c_1 d_3 f_2 - a_1 c_k d_3 f_2 - a_3 c_1 d_k f_2 + a_1 c_3 d_k f_2 + a_k b_3 e_1 f_2 - a_3 b_k e_1 f_2 - a_k b_1 e_3 f_2 + a_1 b_k e_3 f_2 + a_3 b_1 e_k f_2 - a_1 b_3 e_k f_2 + a_k c_2 d_1 f_3 - a_2 c_k d_1 f_3 - a_k c_1 d_2 f_3 + a_1 c_k d_2 f_3 + a_2 c_1 d_k f_3 - a_1 c_2 d_k f_3 - a_k b_2 e_1 f_3 + a_2 b_k e_1 f_3 + a_k b_1 e_2 f_3 - a_1 b_k e_2 f_3 - a_2 b_1 e_k f_3 + a_1 b_2 e_k f_3 - a_3 c_2 d_1 f_k + a_2 c_3 d_1 f_k + a_3 c_1 d_2 f_k - a_1 c_3 d_2 f_k - a_2 c_1 d_3 f_k + a_1 c_2 d_3 f_k + a_3 b_2 e_1 f_k - a_2 b_3 e_1 f_k - a_3 b_1 e_2 f_k + a_1 b_3 e_2 f_k + a_2 b_1 e_3 f_k - a_1 b_2 e_3 f_k$$

$$i=1, 2 \quad k=i+1 \tag{13}$$

$$e_{pi} = -b_k d_3 f_2 g_1 + b_3 d_k f_2 g_1 + c_k e_3 f_2 g_1 - c_3 e_k f_2 g_1 + b_k d_2 f_3 g_1 - b_2 d_k f_3 g_1 - c_k e_2 f_3 g_1 + c_2 e_k f_3 g_1 - b_3 d_2 f_k g_1 + b_2 d_3 f_k g_1 + c_3 e_2 f_k g_1 - c_2 e_3 f_k g_1 + b_k d_3 f_1 g_2 - b_3 d_k f_1 g_2 - c_k e_3 f_1 g_2 + c_3 e_k f_1 g_2 - b_k d_1 f_3 g_2 + b_1 d_k f_3 g_2 + c_k e_1 f_3 g_2 - c_1 e_k f_3 g_2 + b_3 d_1 f_k g_2 - b_1 d_3 f_k g_2 - c_3 e_1 f_k g_2 + c_1 e_3 f_k g_2 - b_k d_2 f_1 g_3 + b_2 d_k f_1 g_3 + c_k e_2 f_1 g_3 - c_2 e_k f_1 g_3 + b_k d_1 f_2 g_3 - b_1 d_k f_2 g_3 - c_k e_1 f_2 g_3 + c_1 e_k f_2 g_3 - b_2 d_1 f_k g_3 + b_1 d_2 f_k g_3 + c_2 e_1 f_k g_3 - c_1 e_2 f_k g_3 + b_3 d_2 f_1 g_k - b_2 d_3 f_1 g_k - c_3 e_2 f_1 g_k + c_2 e_3 f_1 g_k - b_3 d_1 f_2 g_k + b_1 d_3 f_2 g_k + c_3 e_1 f_2 g_k - c_1 e_3 f_2 g_k + b_2 d_1 f_3 g_k - b_1 d_2 f_3 g_k - c_2 e_1 f_3 g_k + c_1 e_2 f_3 g_k$$

$$i=1, 2 \quad k=i+1 \tag{14}$$

$$f_{pi} = -b_k d_3 f_2 g_1 + b_3 d_k f_2 g_1 - c_k e_3 f_2 g_1 + c_3 e_k f_2 g_1 + b_k d_2 f_3 g_1 - b_2 d_k f_3 g_1 + c_k e_2 f_3 g_1 - c_2 e_k f_3 g_1 - b_3 d_2 f_k g_1 + b_2 d_3 f_k g_1 - c_3 e_2 f_k g_1 + c_2 e_3 f_k g_1 + b_k d_3 f_1 g_2 - b_3 d_k f_1 g_2 + c_k e_3 f_1 g_2 - c_3 e_k f_1 g_2 - b_k d_1 f_3 g_2 + b_1 d_k f_3 g_2 - c_k e_1 f_3 g_2 + c_1 e_k f_3 g_2 + b_3 d_1 f_k g_2 - b_1 d_3 f_k g_2 - c_3 e_1 f_k g_2 + c_1 e_3 f_k g_2 - b_k d_2 f_1 g_3 + b_2 d_k f_1 g_3 + c_k e_2 f_1 g_3 - c_2 e_k f_1 g_3 + b_k d_1 f_2 g_3 - b_1 d_k f_2 g_3 - c_k e_1 f_2 g_3 + c_1 e_k f_2 g_3 - b_2 d_1 f_k g_3 + b_1 d_2 f_k g_3 + c_2 e_1 f_k g_3 - c_1 e_2 f_k g_3 + b_3 d_2 f_1 g_k - b_2 d_3 f_1 g_k - c_3 e_2 f_1 g_k + c_2 e_3 f_1 g_k - b_3 d_1 f_2 g_k + b_1 d_3 f_2 g_k + c_3 e_1 f_2 g_k - c_1 e_3 f_2 g_k + b_2 d_1 f_3 g_k - b_1 d_2 f_3 g_k - c_2 e_1 f_3 g_k + c_1 e_2 f_3 g_k$$

$$c_1 e_3 f_k g_2 - b_k d_2 f_1 g_3 + b_2 d_k f_1 g_3 - c_k e_2 f_1 g_3 + c_2 e_k f_1 g_3 + b_k d_1 f_2 g_3 - b_1 d_k f_2 g_3 + c_k e_1 f_2 g_3 - c_1 e_k f_2 g_3 - b_2 d_1 f_k g_3 + b_1 d_2 f_k g_3 - c_2 e_1 f_k g_3 + c_1 e_2 f_k g_3 + b_3 d_2 f_1 g_k - b_2 d_3 f_1 g_k + c_3 e_2 f_1 g_k - c_2 e_3 f_1 g_k - b_3 d_1 f_2 g_k + b_1 d_3 f_2 g_k - c_3 e_1 f_2 g_k + c_1 e_3 f_2 g_k + b_2 d_1 f_3 g_k - b_1 d_2 f_3 g_k + c_2 e_1 f_3 g_k - c_1 e_2 f_3 g_k$$

$$i=1, 2 \quad k=i+1 \tag{15}$$

$$g_{pi} = -c_k d_3 f_2 g_1 + c_3 d_k f_2 g_1 + b_k e_3 f_2 g_1 - b_3 e_k f_2 g_1 + c_k d_2 f_3 g_1 - c_2 d_k f_3 g_1 - b_k e_2 f_3 g_1 + b_2 e_k f_3 g_1 - c_3 d_2 f_k g_1 + c_2 d_3 f_k g_1 + b_3 e_2 f_k g_1 - b_2 e_3 f_k g_1 + c_k d_3 f_1 g_2 - c_3 d_k f_1 g_2 - b_k e_3 f_1 g_2 + b_3 e_k f_1 g_2 - c_k d_1 f_3 g_2 + c_1 d_k f_3 g_2 + b_k e_1 f_3 g_2 - b_1 e_k f_3 g_2 + c_3 d_1 f_k g_2 - c_1 d_3 f_k g_2 - b_3 e_1 f_k g_2 + b_1 e_3 f_k g_2 - c_k d_2 f_1 g_3 + c_2 d_k f_1 g_3 + b_k e_2 f_1 g_3 - b_2 e_k f_1 g_3 + c_k d_1 f_2 g_3 - c_1 d_k f_2 g_3 - b_k e_1 f_2 g_3 + b_1 e_k f_2 g_3 - c_2 d_1 f_k g_3 + c_1 d_2 f_k g_3 + b_2 e_1 f_k g_3 - b_1 e_2 f_k g_3 + c_3 d_2 f_1 g_k - c_2 d_3 f_1 g_k - b_3 e_2 f_1 g_k + b_2 e_3 f_1 g_k - c_3 d_1 f_2 g_k + c_1 d_3 f_2 g_k + b_3 e_1 f_2 g_k - b_1 e_3 f_2 g_k - c_2 d_1 f_3 g_k - c_1 d_2 f_3 g_k - b_2 e_1 f_3 g_k + b_1 e_2 f_3 g_k$$

$$i=1, 2 \quad k=i+1 \tag{16}$$

$$w_0 = -4a_{p2} b_{p1} b_{p2} e_{p1} + 4a_{p1} b_{p2}^2 e_{p1} - 4a_{p2} b_{p2} c_{p1} e_{p1} - 4a_{p2} b_{p1} c_{p2} e_{p1} + 8a_{p1} b_{p2} c_{p2} e_{p1} - 4a_{p2} c_{p1} c_{p2} e_{p1} + 4a_{p1} c_{p2}^2 e_{p1} + 4a_{p2}^2 e_{p1}^2 + 4a_{p2} b_{p1}^2 e_{p2} - 4a_{p1} b_{p1} b_{p2} e_{p2} + 8a_{p2} b_{p1} c_{p1} e_{p2} - 4a_{p1} b_{p2} c_{p1} e_{p2} + 4a_{p2} c_{p1}^2 e_{p2} - 4a_{p1} b_{p1} c_{p2} e_{p2} - 4a_{p1} c_{p1} c_{p2} e_{p2} - 8a_{p1} a_{p2} e_{p1} e_{p2} + 4a_{p1}^2 e_{p2}^2 - 4a_{p2} b_{p1} b_{p2} f_{p1} + 4a_{p1} b_{p2}^2 f_{p1} - 4a_{p2} b_{p2} c_{p1} f_{p1} - 4a_{p2} b_{p1} c_{p2} f_{p1} + 8a_{p1} b_{p2} c_{p2} f_{p1} - 4a_{p2} c_{p1} c_{p2} f_{p1} + 4a_{p1} c_{p2}^2 f_{p1} + 8a_{p2}^2 e_{p1} f_{p1} - 8a_{p1} a_{p2} e_{p2} f_{p1} + 4a_{p2}^2 f_{p1}^2 + 4a_{p2} b_{p1}^2 f_{p2} - 4a_{p1} b_{p1} b_{p2} f_{p2} + 8a_{p2} b_{p1} c_{p1} f_{p2} - 4a_{p1} b_{p2} c_{p1} f_{p2} + 4a_{p2} c_{p1}^2 f_{p2} - 4a_{p1} b_{p1} c_{p2} f_{p2} - 4a_{p1} c_{p1} c_{p2} f_{p2} - 8a_{p1} a_{p2} e_{p1} f_{p2} + 8a_{p1}^2 e_{p2} f_{p2} - 8a_{p1} a_{p2} f_{p1} f_{p2} + 4a_{p1}^2 f_{p2}^2$$

$$(19)$$

$$w_1 = -16a_{p2} b_{p2} d_{p1} e_{p1} - 16a_{p2} c_{p2} d_{p1} e_{p1} - 16a_{p2} b_{p1} d_{p2} e_{p1} + 32a_{p1} b_{p2} d_{p2} e_{p1} - 16a_{p2} c_{p1} d_{p2} e_{p1} + 32a_{p1} c_{p2} d_{p2} e_{p1} + 32a_{p2} b_{p1} d_{p1} e_{p2} - 16a_{p1} b_{p2} d_{p1} e_{p2} + 32a_{p2} c_{p1} d_{p1} e_{p2} - 16a_{p1} c_{p2} d_{p1} e_{p2} - 16a_{p1} b_{p1} d_{p2} e_{p2} - 16a_{p2} b_{p2} d_{p1} f_{p1} - 16a_{p2} c_{p2} d_{p1} f_{p1} - 16a_{p2} b_{p1} d_{p2} f_{p1} + 32a_{p1} b_{p2} d_{p2} f_{p1} - 16a_{p2} c_{p1} d_{p2} f_{p1} + 32a_{p1} c_{p2} d_{p2} f_{p1} + 32a_{p2} b_{p1} d_{p1} f_{p2} - 16a_{p1} b_{p2} d_{p1} f_{p2} + 32a_{p2} c_{p1} d_{p1} f_{p2} - 16a_{p1} c_{p2} d_{p1} f_{p2} - 16a_{p1} b_{p1} d_{p2} f_{p2} - 16a_{p2} b_{p2} d_{p2} f_{p2} - 16a_{p2} b_{p1} b_{p2} g_{p1} + 16a_{p1} b_{p2}^2 g_{p1} - 16a_{p2} b_{p2} c_{p1} g_{p1} - 16a_{p2} b_{p1} c_{p2} g_{p1} + 32a_{p1} b_{p2} c_{p2} g_{p1} - 16a_{p2} c_{p1} c_{p2} g_{p1} + 16a_{p1} c_{p2}^2 g_{p1} + 32a_{p2}^2 e_{p1} g_{p1} - 32a_{p1} a_{p2} e_{p2} g_{p1} + 32a_{p2}^2 f_{p1} g_{p1} - 32a_{p1} a_{p2} f_{p2} g_{p1} + 16a_{p2} b_{p2}^2 g_{p2} - 16a_{p1} b_{p1} b_{p2} g_{p2} + 32a_{p2} b_{p1} c_{p1} g_{p2} - 16a_{p1} b_{p2} c_{p1} g_{p2} + 16a_{p2} c_{p1}^2 g_{p2} - 16a_{p1} b_{p1} c_{p2} g_{p2} - 16a_{p1} c_{p1} c_{p2} g_{p2} - 32a_{p1} a_{p2} e_{p1} g_{p2} + 32a_{p1}^2 e_{p2} g_{p2} - 32a_{p1} a_{p2} f_{p1} g_{p2} + 32a_{p1}^2 f_{p2} g_{p2}$$

$$(20)$$

$$w_2 = -24a_{p2} b_{p1} b_{p2} e_{p1} + 72a_{p1}^2 b_{p2}^2 e_{p1} + 8a_{p2} b_{p2} c_{p1} e_{p1} + 8a_{p2} b_{p1} c_{p2} e_{p1} - 16a_{p1} b_{p2} c_{p2} e_{p1} + 40a_{p2} c_{p1} c_{p2} e_{p1} - 40a_{p1} c_{p2}^2 e_{p1} - 64a_{p2} d_{p1} d_{p2} e_{p1} + 64a_{p1} d_{p2}^2 e_{p1} + 24a_{p2}^2 e_{p1}^2 + 24a_{p2} b_{p1}^2 e_{p2} - 24a_{p1} b_{p1} b_{p2} e_{p2} - 16a_{p2} b_{p1} c_{p1} e_{p2} + 8a_{p1} b_{p2} c_{p1} e_{p2} - 40a_{p2} c_{p1}^2 e_{p2} + 8a_{p1} b_{p1} c_{p2} e_{p2} + 40a_{p1} c_{p1} c_{p2} e_{p2} + 64a_{p2} d_{p1}^2 e_{p2} - 64a_{p1} d_{p1} d_{p2} e_{p2} - 48a_{p1} a_{p2} e_{p1} e_{p2} + 8a_{p2} b_{p1} b_{p2} f_{p1} - 24a_{p1}^2 b_{p2}^2 f_{p1} + 40a_{p2} b_{p2} c_{p1} f_{p1} + 40a_{p2} b_{p1} c_{p2} f_{p1} -$$

$$\begin{aligned}
 &80a_{p1}b_{p2}c_{p2}f_{p1} + 72a_{p2}c_{p1}c_{p2}f_{p1} - 72a_{p1}c_{p2}^2f_{p1} - \\
 &64a_{p2}d_{p1}d_{p2}f_{p1} + 64a_{p1}d_{p2}^2f_{p1} - 16a_{p2}^2e_{p1}f_{p1} + \\
 &16a_{p1}a_{p2}e_{p2}f_{p1} - 40a_{p2}^2f_{p1}^2 - 8a_{p2}b_{p1}^2f_{p2} + \\
 &8a_{p1}b_{p1}b_{p2}f_{p2} - 80a_{p2}b_{p1}c_{p1}f_{p2} + 40a_{p1}b_{p2}c_{p1}f_{p2} - \\
 &72a_{p2}c_{p1}^2f_{p2} + 40a_{p1}b_{p1}c_{p2}f_{p2} + 72a_{p1}c_{p1}c_{p2}f_{p2} + \\
 &64a_{p2}d_{p1}^2f_{p2} - 64a_{p1}d_{p1}d_{p2}f_{p2} + 16a_{p1}a_{p2}e_{p1}f_{p2} - \\
 &16a_{p1}^2e_{p2}f_{p2} + 80a_{p1}a_{p2}f_{p1}f_{p2} - 40a_{p1}^2f_{p2}^2 - \\
 &64a_{p2}b_{p2}d_{p1}g_{p1} - 64a_{p2}c_{p2}d_{p1}g_{p1} - 64a_{p2}b_{p1}d_{p2}g_{p1} + \\
 &128a_{p1}b_{p2}d_{p2}g_{p1} - 64a_{p2}c_{p1}d_{p2}g_{p1} + \\
 &128a_{p1}c_{p2}d_{p2}g_{p1} + 64a_{p2}^2g_{p1}^2 + 128a_{p2}b_{p1}d_{p1}g_{p2} - \\
 &64a_{p1}b_{p2}d_{p1}g_{p2} + 128a_{p2}c_{p1}d_{p1}g_{p2} - 64a_{p1}c_{p2}d_{p1}g_{p2} - \\
 &64a_{p1}b_{p1}d_{p2}g_{p2} - 64a_{p1}c_{p1}d_{p2}g_{p2} - \\
 &128a_{p1}a_{p2}g_{p1}g_{p2} + 64a_{p1}^2g_{p2}^2 \quad (21)
 \end{aligned}$$

$$\begin{aligned}
 w_3 = &-48a_{p2}b_{p2}d_{p1}e_{p1} + 80a_{p2}c_{p2}d_{p1}e_{p1} - \\
 &48a_{p2}b_{p1}d_{p2}e_{p1} + 96a_{p1}b_{p2}d_{p2}e_{p1} + 80a_{p2}c_{p1}d_{p2}e_{p1} - \\
 &160a_{p1}c_{p2}d_{p2}e_{p1} + 96a_{p2}b_{p1}d_{p1}e_{p2} - 48a_{p1}b_{p2}d_{p1}e_{p2} - \\
 &160a_{p2}c_{p1}d_{p1}e_{p2} + 80a_{p1}c_{p2}d_{p1}e_{p2} - 48a_{p1}b_{p1}d_{p2}e_{p2} + \\
 &80a_{p1}c_{p1}d_{p2}e_{p2} + 80a_{p2}b_{p2}d_{p1}f_{p1} + 208a_{p2}c_{p2}d_{p1}f_{p1} + \\
 &80a_{p2}b_{p1}d_{p2}f_{p1} - 160a_{p1}b_{p2}d_{p2}f_{p1} + 208a_{p2}c_{p1}d_{p2}f_{p1} - \\
 &416a_{p1}c_{p2}d_{p2}f_{p1} - 160a_{p2}b_{p1}d_{p1}f_{p2} + 80a_{p1}b_{p2}d_{p1}f_{p2} - \\
 &416a_{p2}c_{p1}d_{p1}f_{p2} + 208a_{p1}c_{p2}d_{p1}f_{p2} + 80a_{p1}b_{p1}d_{p2}f_{p2} + \\
 &208a_{p1}c_{p1}d_{p2}f_{p2} - 48a_{p2}b_{p1}b_{p2}g_{p1} + 48a_{p1}b_{p2}^2g_{p1} + \\
 &80a_{p2}b_{p2}c_{p1}g_{p1} + 80a_{p2}b_{p1}c_{p2}g_{p1} - 160a_{p1}b_{p2}c_{p2}g_{p1} + \\
 &208a_{p2}c_{p1}c_{p2}g_{p1} - 208a_{p1}c_{p2}^2g_{p1} - 256a_{p2}d_{p1}d_{p2}g_{p1} + \\
 &256a_{p1}d_{p2}^2g_{p1} + 96a_{p2}^2e_{p1}g_{p1} - 96a_{p1}a_{p2}e_{p2}g_{p1} - \\
 &160a_{p2}^2f_{p1}g_{p1} + 160a_{p1}a_{p2}f_{p2}g_{p1} + 48a_{p2}b_{p1}^2g_{p2} - \\
 &48a_{p1}b_{p1}b_{p2}g_{p2} - 160a_{p2}b_{p1}c_{p1}g_{p2} + 80a_{p1}b_{p2}c_{p1}g_{p2} - \\
 &208a_{p2}c_{p1}^2g_{p2} + 80a_{p1}b_{p1}c_{p2}g_{p2} + 208a_{p1}c_{p1}c_{p2}g_{p2} + \\
 &256a_{p2}d_{p1}^2g_{p2} - 256a_{p1}d_{p1}d_{p2}g_{p2} - 96a_{p1}a_{p2}e_{p1}g_{p2} + \\
 &96a_{p1}^2e_{p2}g_{p2} + 160a_{p1}a_{p2}f_{p1}g_{p2} - 160a_{p1}^2f_{p2}g_{p2} \quad (22)
 \end{aligned}$$

$$\begin{aligned}
 w_4 = &-60a_{p2}b_{p1}b_{p2}e_{p1} + 60a_{p1}b_{p2}^2e_{p1} + \\
 &68a_{p2}b_{p2}c_{p1}e_{p1} + 68a_{p2}b_{p1}c_{p2}e_{p1} - 136a_{p1}b_{p2}c_{p2}e_{p1} - \\
 &60a_{p2}c_{p1}c_{p2}e_{p1} + 60a_{p1}c_{p2}^2e_{p1} + 60a_{p2}^2e_{p1}^2 + \\
 &60a_{p2}b_{p1}^2e_{p2} - 60a_{p1}b_{p1}b_{p2}e_{p2} - 136a_{p2}b_{p1}c_{p1}e_{p2} + \\
 &68a_{p1}b_{p2}c_{p1}e_{p2} + 60a_{p2}c_{p1}^2e_{p2} + 68a_{p1}b_{p1}c_{p2}e_{p2} - \\
 &60a_{p1}c_{p1}c_{p2}e_{p2} - 120a_{p1}a_{p2}e_{p1}e_{p2} + 60a_{p1}^2e_{p2}^2 + \\
 &68a_{p2}b_{p1}b_{p2}f_{p1} - 68a_{p1}b_{p2}^2f_{p1} - 60a_{p2}b_{p2}c_{p1}f_{p1} - \\
 &60a_{p2}b_{p1}c_{p2}f_{p1} + 120a_{p1}b_{p2}c_{p2}f_{p1} - 444a_{p2}c_{p1}c_{p2}f_{p1} + \\
 &444a_{p1}c_{p2}^2f_{p1} + 512a_{p2}d_{p1}d_{p2}f_{p1} - 512a_{p1}d_{p2}^2f_{p1} - \\
 &136a_{p2}^2e_{p1}f_{p1} + 136a_{p1}a_{p2}e_{p2}f_{p1} + 60a_{p2}^2f_{p1}^2 - \\
 &68a_{p2}b_{p1}^2f_{p2} + 68a_{p1}b_{p1}b_{p2}f_{p2} + 120a_{p2}b_{p1}c_{p1}f_{p2} - \\
 &60a_{p1}b_{p2}c_{p1}f_{p2} + 444a_{p2}c_{p1}^2f_{p2} - 60a_{p1}b_{p1}c_{p2}f_{p2} - \\
 &444a_{p1}c_{p1}c_{p2}f_{p2} - 512a_{p2}d_{p1}^2f_{p2} + 512a_{p1}d_{p1}d_{p2}f_{p2} + \\
 &136a_{p1}a_{p2}e_{p1}f_{p2} - 136a_{p1}^2e_{p2}f_{p2} - 120a_{p1}a_{p2}f_{p1}f_{p2} + \\
 &60a_{p1}^2f_{p2}^2 + 512a_{p2}c_{p2}d_{p1}g_{p1} + 512a_{p2}c_{p1}d_{p2}g_{p1} - \\
 &1024a_{p1}c_{p2}d_{p2}g_{p1} - 1024a_{p2}c_{p1}d_{p1}g_{p2} + \\
 &512a_{p1}c_{p2}d_{p1}g_{p2} + 512a_{p1}c_{p1}d_{p2}g_{p2} \quad (23)
 \end{aligned}$$

$$\begin{aligned}
 w_5 = &-32a_{p2}b_{p2}d_{p1}e_{p1} + 96a_{p2}c_{p2}d_{p1}e_{p1} - \\
 &32a_{p2}b_{p1}d_{p2}e_{p1} + 64a_{p1}b_{p2}d_{p2}e_{p1} + 96a_{p2}c_{p1}d_{p2}e_{p1} - \\
 &192a_{p1}c_{p2}d_{p2}e_{p1} + 64a_{p2}b_{p1}d_{p1}e_{p2} - 32a_{p1}b_{p2}d_{p1}e_{p2} - \\
 &192a_{p2}c_{p1}d_{p1}e_{p2} + 96a_{p1}c_{p2}d_{p1}e_{p2} - 32a_{p1}b_{p1}d_{p2}e_{p2} + \\
 &96a_{p1}c_{p1}d_{p2}e_{p2} + 96a_{p2}b_{p2}d_{p1}f_{p1} - 800a_{p2}c_{p2}d_{p1}f_{p1} +
 \end{aligned}$$

$$\begin{aligned}
 &96a_{p2}b_{p1}d_{p2}f_{p1} - 192a_{p1}b_{p2}d_{p2}f_{p1} - 800a_{p2}c_{p1}d_{p2}f_{p1} + \\
 &1600a_{p1}c_{p2}d_{p2}f_{p1} - 192a_{p2}b_{p1}d_{p1}f_{p2} + \\
 &96a_{p1}b_{p2}d_{p1}f_{p2} + 1600a_{p2}c_{p1}d_{p1}f_{p2} - \\
 &800a_{p1}c_{p2}d_{p1}f_{p2} + 96a_{p1}b_{p1}d_{p2}f_{p2} - 800a_{p1}c_{p1}d_{p2}f_{p2} - \\
 &32a_{p2}b_{p1}b_{p2}g_{p1} + 32a_{p1}b_{p2}^2g_{p1} + 96a_{p2}b_{p2}c_{p1}g_{p1} + \\
 &96a_{p2}b_{p1}c_{p2}g_{p1} - 192a_{p1}b_{p2}c_{p2}g_{p1} - \\
 &800a_{p2}c_{p1}c_{p2}g_{p1} + 800a_{p1}c_{p2}^2g_{p1} + 768a_{p2}d_{p1}d_{p2}g_{p1} - \\
 &768a_{p1}d_{p2}^2g_{p1} + 64a_{p2}^2e_{p1}g_{p1} - 64a_{p1}a_{p2}e_{p2}g_{p1} - \\
 &192a_{p2}^2f_{p1}g_{p1} + 192a_{p1}a_{p2}f_{p2}g_{p1} + 32a_{p2}b_{p1}^2g_{p2} - \\
 &32a_{p1}b_{p1}b_{p2}g_{p2} - 192a_{p2}b_{p1}c_{p1}g_{p2} + 96a_{p1}b_{p2}c_{p1}g_{p2} + \\
 &800a_{p2}c_{p1}^2g_{p2} + 96a_{p1}b_{p1}c_{p2}g_{p2} - 800a_{p1}c_{p1}c_{p2}g_{p2} - \\
 &768a_{p2}d_{p1}^2g_{p2} + 768a_{p1}d_{p1}d_{p2}g_{p2} - 64a_{p1}a_{p2}e_{p1}g_{p2} + \\
 &64a_{p1}^2e_{p2}g_{p2} + 192a_{p1}a_{p2}f_{p1}g_{p2} - 192a_{p1}^2f_{p2}g_{p2} \quad (24)
 \end{aligned}$$

$$\begin{aligned}
 w_6 = &-80a_{p2}b_{p1}b_{p2}e_{p1} + 80a_{p1}b_{p2}^2e_{p1} + \\
 &112a_{p2}b_{p2}c_{p1}e_{p1} + 112a_{p2}b_{p1}c_{p2}e_{p1} - \\
 &224a_{p1}b_{p2}c_{p2}e_{p1} - 208a_{p2}c_{p1}c_{p2}e_{p1} + 208a_{p1}c_{p2}^2e_{p1} + \\
 &128a_{p2}d_{p1}d_{p2}e_{p1} - 128a_{p1}d_{p2}^2e_{p1} + 80a_{p2}^2e_{p1}^2 + \\
 &80a_{p2}b_{p1}^2e_{p2} - 80a_{p1}b_{p1}b_{p2}e_{p2} - 224a_{p2}b_{p1}c_{p1}e_{p2} + \\
 &112a_{p1}b_{p2}c_{p1}e_{p2} + 208a_{p2}c_{p1}^2e_{p2} + 112a_{p1}b_{p1}c_{p2}e_{p2} - \\
 &208a_{p1}c_{p1}c_{p2}e_{p2} - 128a_{p2}d_{p1}^2e_{p2} + 128a_{p1}d_{p1}d_{p2}e_{p2} - \\
 &160a_{p1}a_{p2}e_{p1}e_{p2} + 80a_{p1}^2e_{p2}^2 + 112a_{p2}b_{p1}b_{p2}f_{p1} - \\
 &112a_{p1}b_{p2}^2f_{p1} - 208a_{p2}b_{p2}c_{p1}f_{p1} - 208a_{p2}b_{p1}c_{p2}f_{p1} + \\
 &416a_{p1}b_{p2}c_{p2}f_{p1} + 1008a_{p2}c_{p1}c_{p2}f_{p1} - 1008a_{p1}c_{p2}^2f_{p1} - \\
 &896a_{p2}d_{p1}d_{p2}f_{p1} + 896a_{p1}d_{p2}^2f_{p1} - 224a_{p2}^2e_{p1}f_{p1} + \\
 &224a_{p1}a_{p2}e_{p2}f_{p1} + 208a_{p2}^2f_{p1}^2 - 112a_{p2}b_{p1}^2f_{p2} + \\
 &112a_{p1}b_{p1}b_{p2}f_{p2} + 416a_{p2}b_{p1}c_{p1}f_{p2} - \\
 &208a_{p1}b_{p2}c_{p1}f_{p2} - 1008a_{p2}c_{p1}^2f_{p2} - 208a_{p1}b_{p1}c_{p2}f_{p2} + \\
 &1008a_{p1}c_{p1}c_{p2}f_{p2} + 896a_{p2}d_{p1}^2f_{p2} - 896a_{p1}d_{p1}d_{p2}f_{p2} + \\
 &224a_{p1}a_{p2}e_{p1}f_{p2} - 224a_{p1}^2e_{p2}f_{p2} - 416a_{p1}a_{p2}f_{p1}f_{p2} + \\
 &208a_{p2}^2f_{p2}^2 + 128a_{p2}b_{p2}d_{p1}g_{p1} - 896a_{p2}c_{p2}d_{p1}g_{p1} + \\
 &128a_{p2}b_{p1}d_{p2}g_{p1} - 256a_{p1}b_{p2}d_{p2}g_{p1} - \\
 &896a_{p2}c_{p1}d_{p2}g_{p1} + 1792a_{p1}c_{p2}d_{p2}g_{p1} - 128a_{p2}^2g_{p1}^2 - \\
 &256a_{p2}b_{p1}d_{p1}g_{p2} + 128a_{p1}b_{p2}d_{p1}g_{p2} + \\
 &1792a_{p2}c_{p1}d_{p1}g_{p2} - 896a_{p1}c_{p2}d_{p1}g_{p2} + \\
 &128a_{p1}b_{p1}d_{p2}g_{p2} - 896a_{p1}c_{p1}d_{p2}g_{p2} + \\
 &256a_{p1}a_{p2}g_{p1}g_{p2} - 128a_{p1}^2g_{p2}^2 \quad (25)
 \end{aligned}$$

APPENDIX B. Comparison of the absolute maximum error with available literature

Reference	Mechanism configuration	Function	Input Range	Maximum Error (in absolute value)
Our study	Slider-crank mechanism	$y = \sin(x)$	$0^\circ \leq x \leq 90^\circ$	Precision Point: -0.00014533 Sub-domain: 0.00025793 Galerkin: -0.00010383
		$y = \tan(x)$	$0^\circ \leq x \leq 45^\circ$	Precision Point: -0.00011431 Sub-domain: -0.00102642 Galerkin: -0.00053734
		$y = e^x$	$0 \leq x \leq 1$	Precision Point: 0.00027071 Sub-domain: -0.00069492 Galerkin: -0.00037269
		$y = \ln(x)$	$1 \leq x \leq 2$	Precision Point: 0.00025632 Sub-domain: -0.00080016 Galerkin: -0.00055926
Singh et al. 2005	Four-bar mechanism	$y = \sin(x)$	$0^\circ \leq x \leq 90^\circ$	-0.012573
		$y = \tan(x)$	$0^\circ \leq x \leq 45^\circ$	-0.004634
		$y = e^x$	$0 \leq x \leq 1$	-0.000489
		$y = \ln(x)$	$1 \leq x \leq 2$	0.002359
Akçalı and Dittrich 1989	Four-bar mechanism	$y = \sin(x)$	$0^\circ \leq x \leq 90^\circ$	-0.0630
		$y = \tan(x)$	$0^\circ \leq x \leq 45^\circ$	-0.0015
Jaiswal and Jawale 2018	Four-bar mechanism	$y = \sin(x)$	$0^\circ \leq x \leq 90^\circ$	0.002666
		$y = \tan(x)$	$0^\circ \leq x \leq 45^\circ$	0.000821
		$y = e^x$	$0 \leq x \leq 1$	0.000203
		$y = \ln(x)$	$1 \leq x \leq 2$	0.000046
Mehtar et al. 2015	Four-bar mechanism	$y = \tan(x)$	$0^\circ \leq x \leq 45^\circ$	0.0010
		$y = e^x$	$0 \leq x \leq 1$	0.00094218
		$y = \ln(x)$	$1 \leq x \leq 2$	0.000059580