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Dynamic Response of an Euler-Bernoulli Beam Coupled with a Tuned Mass Damper under Moving Load Excitation

Mehmet Akif KOÇ^{*1}

Abstract

In this study, dynamic analysis of Euler-Bernoulli beam and Tuned Mass Damper (TMD) interaction problem under the effect of moving load was carried out by the mode superposition method. After the differential equations of TMD are derived by Lagrange method, beams and TMD motion equations are integrated and matrices belonging to the motion equation of the whole system are obtained. The motion equation of the system is solved in the time domain using the Newmark- β algorithm. The effect of TMD on damping vibrations has been examined in terms of parameters such as frequency, damping rate, mass ratio and moving load speed. In addition, the effect of TMD on Dynamic Amplification Factor (DAF) was examined. As a result, with the TMD application carried out in this study, approximately 14% to 24% improvement was achieved in beam deformations and accelerations.

Keywords: TMD, moving load, Newmark-Beta, DAF

1. INTRODUCTION

The damping of mechanical and structural vibrations is of great importance in the machine elements industry, in the fields of construction, automotive, aerospace and robotics. In the past few decades, researchers have made a lot of effort to reduce vibrations on engineering structures [1–3]. The most traditional method of damping vibrations is passive vibration damping techniques, and this technology has been applied in the literature quite a lot [1, 4–7]. Thanks to recent tremendous advances in digital signal processing, sensors and actuator technologies, active vibration control algorithms have been

quickly applied to different engineering problems [8, 9].

Beam type structures have many applications in engineering, especially robotics, mechanics, aviation and construction. The low damping of such structures and the increasing trend of designing especially lighter structures recently cause the beam type structures to vibrate at low mode frequencies. For this reason, one of the biggest challenges engineers face is to protect these types of structures from excessive vibrations and prevent them from being damaged.

The idea of protecting the main structure from excessive vibrations in control engineering is

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based on Tuned Mass Damper (TMD) technology, which is connected to this main structure with spring and damping elements, which absorbs vibration energy. The frequency of this secondary structure connected to the main structure is usually adjusted to the frequency of the primary structure. Thus, the main structure is protected from the destructive effect caused by excessive vibrations. Due to the simplicity and low cost of TMD technology, it has been possible to apply it in many engineering fields [10, 11].

A good mathematical model was needed to represent the physical model of the Beam-TMD in order to effectively implement TMD in beam type structures and to optimize the position, frequency and basic parameters of this secondary structure, which will be placed in the main structure. One of the most used methods for modeling beam-TMD interaction is the Finite Elements Method (FEM). In the study [12] FEM was used in the analysis of the TMD model used to reduce vibrations in the Timoshenko beam under harmonic and random excitation force. Dynamic Vibration absorber has been used to reduce vibrations in the Timoshenko beam under the effect of harmonic distributed load [13]. Wu [14] proposed using a dynamic vibration damper to the middle of the bridge to dampen the vibrations of the beam under the action of a moving load. After obtaining the equation of motion of the system, they used FEM to determine the beam dynamics. The simple model used to obtain the optimal resistance and damping ratio of the vibration absorber with Den Hartog's approach is given in [15]. Greco and Santini [16]

analyzed the beam with a rotary viscous damper placed on both ends under the effect of a moving load. Scientists have shown in their work that the performance of the damper depends on the speed of the moving load.

When we examine the literature, analytical methods, FEM, energy equations and series expansion methods are used to solve such problems. In this study, the mode superposition method was used to analyze the beam TMD interaction problem. In this way, it has been proven that the beam absorber interaction problems can be analyzed effectively with the presented method. In the study, a TMD was placed in the middle of the bridge beam to reduce vibrations in the Euler-Bernoulli beam. The dynamic response of the beam is analyzed for different moving load speeds and different mass ratios.

2. FORMULATION OF EULER-BERNOULLI BEAM COUPLED TO A DYNAMIC MASS DAMPER

In this section, initially the equations of motion in the differential form for an Euler-Bernoulli beam with simply supported boundary conditions and attached n TMDs, as illustrated in Figure 1, has been derived. The parameters x and P on the figure show the time-dependent position and force value of the moving load on the beam relative to the reference point taken from the left end of the bridge, respectively.

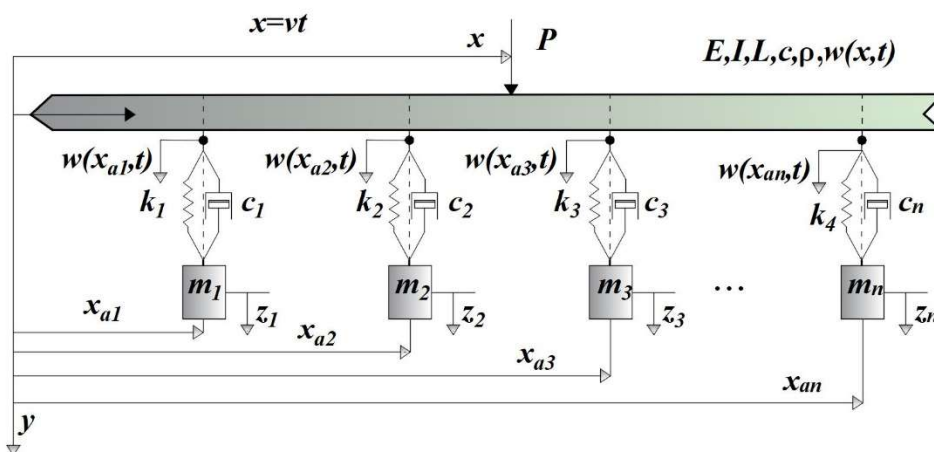


Figure 1. Euler-Bernoulli beam with a linear TMD traversed by a moving load.

2.1. Deriving Equation of Motion for TMD

The kinetic and potential energies for the TMDs shown in Figure 1 are written as follows:

$$E_k = \frac{1}{2} \left(\sum_{i=1}^n m_i \dot{z}_i^2 \right), (i=1, \dots, n) \tag{1a}$$

$$E_p = \frac{1}{2} \sum_{i=1}^n \left(k_i (z_i - w_i(x_{a,i}, t))^2 \right), (i=1, \dots, n) \tag{1b}$$

Similarly, the damping function is written as shown in Equation (2).

$$D = \frac{1}{2} \sum_{i=1}^n c_i (\dot{z}_i - \dot{w}_i(x_{a,i}, t))^2, (i=1, \dots, n) \tag{2}$$

The system's Lagrange function is written taking into account the difference between kinetic energy and potential energy. The Lagrange function is written as follows:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{z}_i} \right) - \frac{\partial L}{\partial z_i} + \frac{\partial D}{\partial \dot{z}_i} = Q_i \tag{3}$$

Using the Lagrange function specified in Equation (3), the motion equation of TMD is written as follows:

$$m_i \ddot{z}_i + \left\{ \begin{matrix} k_i (z_i - w_i(x_{a,i}, t)) + \\ c_i (\dot{z}_i - \dot{w}_i(x_{a,i}, t)) \end{matrix} \right\} = 0 \tag{4}$$

2.2. Deriving Equation of Motion for Beam

In the problem of interaction between moving load, Euler-Bernoulli beam and TMD, the following assumptions are made:

- The moving load is always in contact with the beam.
- Beam cross-sectional area is fixed and modeled in accordance with Euler-Bernoulli theorem.
- The speed of the moving load on the beam is constant and any accelerated movement is excluded from the scope of this study.

- Absorbers placed on the beam are considered to have linear characteristics.

With all these assumptions, according to the Euler-Bernoulli beam theory, the motion equation for the bridge beam over which the moving load passes is written as follows:

$$EI \frac{\partial^4 w(x,t)}{\partial x^4} + \rho \frac{\partial w^2(x,t)}{\partial t^2} + c_b \frac{\partial w(x,t)}{\partial t} = P_b(x,t) \tag{5}$$

Parameters E , I , ρ and c_b in Equation (5) are bridge beam elasticity module, bridge beam constant cross-sectional area, mass of unit length and damping coefficient, respectively. However, parameter $w(x,t)$ represents the transverse deformation of the bridge beam at position x at any time t . In Equation (5), as is customary, the prime symbol indicates the derivative with respect to the spatial variable, i.e. $w'(x,t) = dw/dx$, and the dot symbol is derivative with respect to the time coordinate, i.e. $\dot{w}(x,t) = dw/dt$. Parameter $P_b(x,t)$ is an external load function and is expressed as follows.

$$P_b(x,t) = - \sum_{k=1}^{N_v} f_k \delta(x-vt) + \left\{ k(z - w(x_{a,i}, t)) + c(\dot{z} - \dot{w}(x_{a,i}, t)) \right\} \tag{6}$$

The parameters k_i and c_i in Equation (6) are the spring and damping coefficients of the TMD which connected to the bridge, respectively. The parameter z_i parameter is the vertical displacement of the i th absorber connected to the beam. The parameter N_v in Equation (6) represents the number of moving loads on the beam. In this study, it is accepted that only one moving load passes over the beam. The expression $\delta(x-vt)$ in Equation (6) is the Dirac-delta function in the x direction, expressed as:

$$\int_a^b f_k \delta(x-vt) = \begin{cases} 1 & 0 \leq t \leq \frac{l}{v} \\ 0 & \text{else} \end{cases} \tag{7}$$

According to Galerkin's formulation, the transverse deformation of any point on the beam is expressed as follows.

$$w(x,t) = \sum_{i=1}^N \varphi_i(x) \eta_i(t) \tag{8}$$

The expressions $\varphi_i(x)$ and $\eta_i(t)$ in Equation (8) are the mode function and modal coordinate obtained with the boundary conditions of the beam, respectively. The parameter N in Equation (8) represents the number of modes for calculation of the bridge dynamic. The mode function for the simply supported beam is written as follows:

$$\varphi_i(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{i\pi x}{L}\right), \tag{9}$$

The orthogonality conditions between the mode functions given in Equation (9) are written as follows.

$$\int_0^L \varphi_n(x) \sum_{i=1}^{\infty} \varphi_i(x) \eta_i(t) dx = \begin{cases} 0 & i \neq n \\ \int_0^L \varphi_n^2(x) dx \eta_n(t) & i = n \end{cases} \tag{10}$$

The equation of motion given by equation (5) is written as follows with the expressions of Galerkin function, Equation (4) and orthogonality conditions given by Equation (10).

$$m_i \ddot{z}_i \varphi_n + \ddot{\eta}_n + 2\zeta_n \omega_n \dot{\eta}_n + \omega_n^2 \eta_n = -W_n \varphi_n \delta \tag{11}$$

The expression ζ_n in equation (11) is the damping ratio corresponding to the n th mode of the beam, and is expressed as follows depending on the damping coefficient of the beam.

$$\zeta_n = \frac{c}{2\rho\omega_n} \tag{12}$$

The parameter ω_n in Equation (12) represent n th natural frequency of beam and expressed as follows:

$$f_n = \frac{\omega_n}{2\pi} = \frac{n^2\pi}{2L^2} \left(\sqrt{\frac{EI}{\rho}} \right) \text{ (Hz.)} \tag{13}$$

2.3. Coupling Beam and TMD Equation of motions

The equation of motion given by equation (11) is written in matrix form as follows.

$$[M(t)]\{\ddot{Y}\} + [C(t)]\{\dot{Y}\} + [K(t)]\{Y\} = Q(t) \tag{14}$$

In equation (14), $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices of the beam-TMD system, respectively, written as follows:

$$[M(t)] = \begin{bmatrix} m_1 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ 0 & m_2 & 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & 0 & \ddots & \vdots & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & m_n & 0 & 0 & \dots & 0 \\ m_1\varphi_{11} & m_2\varphi_{21} & \dots & m_1\varphi_{n1} & 1 & 0 & \dots & 0 \\ m_1\varphi_{12} & m_2\varphi_{22} & \dots & m_1\varphi_{n2} & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \ddots & \vdots \\ m_1\varphi_{1n} & m_2\varphi_{2n} & \dots & m_1\varphi_{1n} & 0 & 0 & \dots & 1 \end{bmatrix} \tag{15a}$$

$$[C(t)] = \begin{bmatrix} c_1 & 0 & \dots & 0 & -c_1\varphi_{11} & -c_1\varphi_{12} & \dots & -c_1\varphi_{1n} \\ 0 & c_2 & 0 & 0 & -c_2\varphi_{21} & -c_2\varphi_{22} & \dots & -c_2\varphi_{2n} \\ \vdots & 0 & \ddots & \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & c_n & -c_n\varphi_{n1} & -c_n\varphi_{n2} & \dots & -c_n\varphi_{nn} \\ 0 & 0 & \dots & 0 & 2\zeta_1\omega_1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & 0 & 2\zeta_2\omega_2 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 2\zeta_n\omega_n \end{bmatrix} \tag{15b}$$

$$[K(t)] = \begin{bmatrix} k_1 & 0 & \dots & 0 & -k_1\varphi_{11} & -k_1\varphi_{12} & \dots & -k_1\varphi_{1n} \\ 0 & k_2 & 0 & 0 & -k_2\varphi_{21} & -k_2\varphi_{22} & \dots & -k_2\varphi_{2n} \\ \vdots & 0 & \ddots & \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & k_n & -k_n\varphi_{n1} & -k_n\varphi_{n2} & \dots & -k_n\varphi_{nn} \\ 0 & 0 & \dots & 0 & \omega_1^2 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & 0 & \omega_2^2 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & \omega_n^2 \end{bmatrix} \quad (15c)$$

Similarly, the terms $\{\ddot{Y}\}$, $\{\dot{Y}\}$ and $\{Y\}$ are acceleration, velocity, and displacement vectors respectively, which are expressed as follows:

$$\{\ddot{Y}\} = \{\ddot{y}_1, \ddot{y}_2, \dots, \ddot{y}_n, \ddot{\eta}_1, \ddot{\eta}_2, \dots, \ddot{\eta}_n\}^T \quad (16a)$$

$$\{\dot{Y}\} = \{\dot{y}_1, \dot{y}_2, \dots, \dot{y}_n, \dot{\eta}_1, \dot{\eta}_2, \dots, \dot{\eta}_n\}^T \quad (16b)$$

$$\{Y\} = \{y_1, y_2, \dots, y_n, \eta_1, \eta_2, \dots, \eta_n\}^T \quad (16c)$$

$$\{Q\} = \begin{Bmatrix} 0, 0, \dots, 0, \\ -W\varphi_{21}\delta, -W\varphi_{22}\delta, \dots, -W\varphi_{2n}\delta \end{Bmatrix}^T \quad (16d)$$

2.4. Numerically Solution Algorithm

In this study, the Newmark-β algorithm is used to solve the equation of motion given by Equation (14), and the solution algorithm is as follows.

- **Step 1:** Input bridge absorber parameters
- **Step 2:** Set the initial value $X_0=0, V_0=0, t=0$.
- **Step 3:** Calculate the location of the moving car
- **Step 4:** Create **M**, **C** and **K** matrix and **Q** force vector
- **Step 5:** Calculate acceleration with

$$A_0 = M^{-1}(Q - CV_0 - KX_0) \quad (17a)$$

$$X_1 = X_0 + V_0dt + 1/2A_0dt^2 \quad (17b)$$

$$V_1 = V_0 + A_0dt \quad (17c)$$

$$A_1 = M^{-1}(Q - CV_1 - KX_1) \quad (17d)$$

- **Step 6:** Calculate acceleration with

$$X_2 = X_0 + V_0dt + (-1/2)A_0dt^2 + 1/4A_1dt^2 \quad (17e)$$

$$V_2 = V_0 + (-1/2)A_0dt + 1/2A_1dt \quad (17f)$$

$$A_2 = M^{-1}(Q - CV_2 - KX_2) \quad (17g)$$

- **Step 7:** Determine $A_I=A_2, V_I=V_2, X_I=X_2$
- **Step 8:** if error is not small enough, go to step Step 6. If it is enough $t=t+dt$.
- **Step 9:**if $t_i \leq T$ go to Step 3

3. SIMULATION RESULTS

In this section, the beam and load parameters given in Table 1 are used to analyze the beam and TMD interaction under the effect of the moving load shown in Figure 1.

Table 1. Bridge and moving load parameters used in this study.

Bridge parameters	
Elastic modulus (E), (GPa)	207
Mass per unit length (ρ), (Ton)	20
Cross-sectional inertia moment (I), (m ⁴)	0.174
Beam length (L), (m)	100
Damping coefficient (c), (Ns/m)	1750
Number of modes (n)	4

3.1. Defining Mode Number, Mod Frequencies and Critical Speeds

Before starting numerical analysis, the number of beam modes to be used in the analyses was

determined. For this, first of all, simulation is done in different modes and the results are compared. Since the beam used in the study and the parameter values given in Table 1 is a large and bulky structure, it vibrates under the influence of low mode frequencies. Therefore, in this study, the first four modes of beam are taken into account in calculations.

The ratio of the bridge excitation frequency ω of the vehicle on the bridge to the natural frequency of the bridge $i.(i=1,2,\dots,n)$ is called the speed parameter and is expressed as in equation (18). If $\omega = \omega_i$, an event called resonance occurs, which has a negative effect for the bridge.

$$\alpha = \frac{\omega}{\omega_j} = \frac{\omega}{2\pi f_j} = \frac{vL}{j^2\pi} \left(\frac{\mu}{EI} \right)^{1/2} = \frac{v}{v_{kr}} \quad (18)$$

Table 2. Beam frequencies used in this study.

Mode Number	1	2	3	4
Frequency (Hz.)	0.2108	0.8432	1.8972	3.3728
Critical speed (m/s)	42.15	168.63	379.43	674.55

In Table 2, vibration frequencies of the first four modes of beam used in this study and critical speed values of these frequencies are given.

3.2. Determining TMD Parameters

In this section, basic parameters are determined for damped TMD placed at the midpoint of the beam. The first is the mass ratio and is determined as follows:

$$\mu = \frac{M_{TMD}}{M_{structure}}, \quad c_{cr,i} = 2m_i\omega_i, \quad (19)$$

$$\zeta_{TMD,i} = \frac{c_i}{c_{cr,i}}, \quad \omega_{d,i} = \omega_{n,i}\sqrt{1-\zeta_i^2}$$

Within equation (17), M_{TMD} represents the mass of the absorber placed in the central point of the beam, and $M_{structure}$ represents the total mass of the beam structure. Similarly, within equation (19), $c_{cr,i}$, $\zeta_{TMD,i}$, $\omega_{d,i}$ represent the critical damping value, damping rate and damped natural frequency of the absorber respectively. Table 3

shows the values used in the study for the first mode of the beam.

Table 3. Basic parameters for beam mode 1.

Mode Num	μ	$c_{cr,i}$	ζ_i	$\omega_{d,i}$	$x_{a,i}$
1	0.01	52956	0.35	1.24	L/2

In Figure 2, the effect of using TMD for the constant transition speed $v = 25 \text{ m / s}$ from the moving load beam on the transverse displacement at the midpoint of the beam is shown. The x axis is given the dimensionless position of the moving load on the bridge. The dimensionless position is obtained by dividing the moving load's time dependent position ($x=vt$) on the beam by the beam length (L). As shown in the figure, in the absence of TMD, the maximum deformation at the midpoint of the beam is 20.33 mm, while in the case of the TMD beam, this value is determined as 17.37 mm. As can be seen, 14.55% improvement was achieved in maximum beam mid-point displacement. This situation is explained by the fact that TMD, which is connected to the middle point of the beam and whose natural frequency is adjusted to the first mode frequency of the beam, absorbs the beam energy.

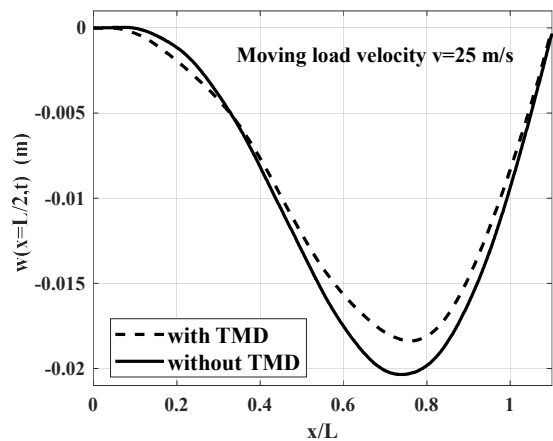


Figure 2. Beam midpoint transverse displacement.

In Figure 3, the comparison of the bridge mid-point transverse acceleration value with the case with TMD and the case without TMD is presented. As shown in the figure, in the absence of TMD, the bridge mid-point maximum transverse acceleration value is 0.025 m/s^2 , while in the case of TMD this value is obtained as 0.019

m/s². As can be seen, TMD bridge mid-point has a 24% reduction in maximum acceleration value of bridge mid-point as well as maximum displacement.

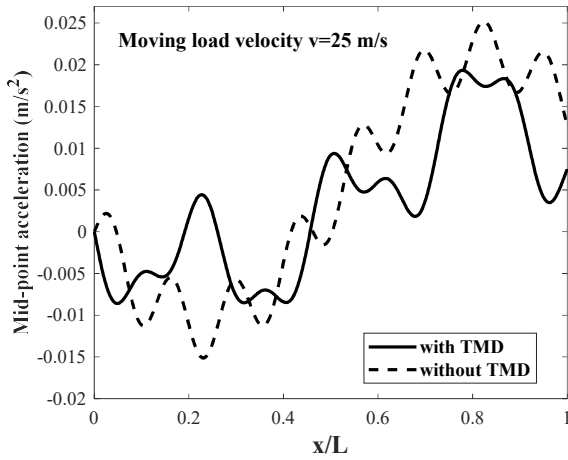


Figure 3. Beam midpoint transverse acceleration.

In Figure 4, the effect of the mass ratio given in Equation (19) on the maximum midpoint deformation of the beam is shown for different TMD damping rates. As seen in the figure, as the mass ratio increases, the maximum point deformations of the bridge midpoint decrease. Similarly, the increase in TMD damping ratio also caused this value to decrease.

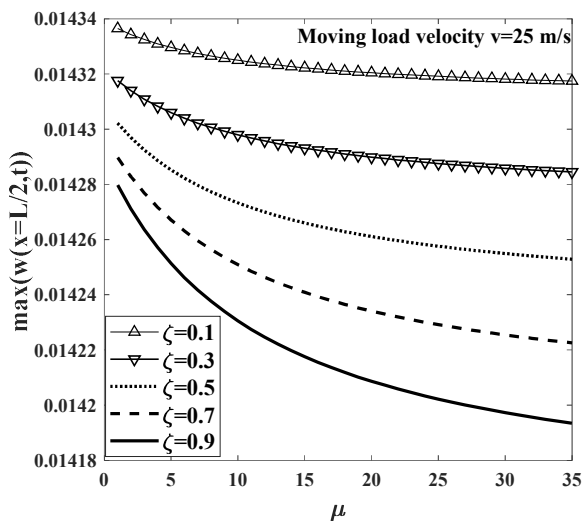


Figure 4. Maximum beam midpoint displacement for different TMD damping ratio ζ_{TMD} .

In Figure 5, the effect of the constant speed of the moving load over the beam on the Dynamic Amplification Factor (DAF) for different beam

damping ratios ($\zeta = 1\%$, $\zeta = 3\%$ and $\zeta = 5\%$.) is shown.

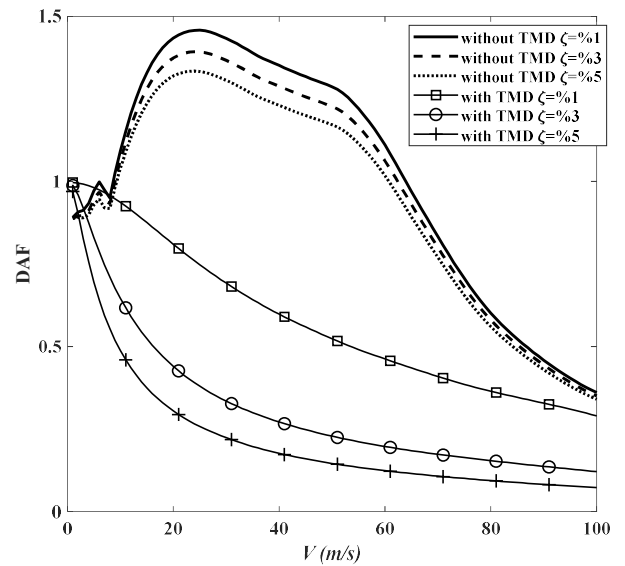


Figure 5. Beam DAF value for different beam damping ratio.

When the moving load passes on the DAF beam, the ratio of the maximum deformation ($R_d(x)$) occurring at the bridge midpoint to the static static deformation ($R_s(x)$) formed by placing the moving load on the bridge midpoint is as follows.

$$DAF = \frac{R_d(x)}{R_s(s)} \quad (20)$$

As can be seen in Figure 5, considering the 1%, 3% and 5% beam damping ratio values, the maximum DAF values were determined as 1.45, 1.39, 1.33, respectively, for the case where there was no TMD. As can be seen, the dynamic displacement value for the beam in the case of moving load is about 50% more than static collapse. This situation proves the importance of moving load problems. In case of TMD, the maximum DAF values for the same damping rates are below 1. Also, as shown in the figure, the maximum DAF value decreases as the beam damping rate increases.

4. CONCLUSIONS

In this study, a dynamic analysis of the simply supported Euler-Bernoulli beam under the effect

of moving load was carried out. To reduce the maximum deformations on the beam, TMD was applied and the following results were obtained

- TMD application has been shown to be effective in reducing maximum acceleration deformations in a beam. An improvement of about 14% at maximum deformation and about 24% at maximum acceleration were achieved.
- The frequency at which the vibration absorber is adjusted is very important in TMD application. In cumbersome structures such as TMD beam, damping is more effective if it is set to a frequency corresponding to one of the low mode frequencies.
- The absorption rate and the increase in mass ratio of the absorber make damping more effective. However, although it is theoretically possible to increase the mass ratio too much, it is not possible in practice.
- TMD application has proven to be very effective in improving maximum DAF values.

Research and Publication Ethics

This paper has been prepared within the scope of international research and publication ethics.

Ethics Committee Approval

This paper does not require any ethics committee permission or special permission.

Conflict of Interests

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this paper.

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