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Research Article

Analysis of Parallel Resonance Circuit Consisting of a Capacitor Modelled Using Conformal Fractional Order Derivative Using Simulink

Abdulmajıd Abdullah Hamed Alhaj Mohammed[1] *, Kutsal Kandemir[1], Reşat Mutlu[1]

[1]Electronics and Communication Engineering Department, Tekirdağ Namık Kemal University, Çorlu Engineering Faculty, Tekirdağ, Turkey

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Abstract: Fractional-order circuit elements are becoming an essential part of modelling electronics and mechanic systems in the last few decades. Recently, conformable fractional derivate (CFD) has been introduced in literature. It is easier to understand and use than the other Fractional-order derivatives and it is already being used to model circuit components. CFD has already been used to model supercapacitors in literature. It is important to know how circuits behave for different types of current and voltage waveforms. In this paper, a parallel circuit which consists of a resistor, an inductor and a CFD capacitor fed by a current source has been examined for DC and sinusoidal signals using Simulink™ toolbox of Matlab™ since it was not possible to find analytical solutions of the circuit. Simulink provides an easier way to examine such circuits.

Keywords: *Fractional Order Derivative, Conformable Fractional Derivate. Circuit Analysis, Circuit Theory, Circuit Modeling.*

Simulink Kullanarak Uyumlu Kesirli Dereceli Türev ile Modellenen Bir Kondansatörden Oluşan Paralel Rezonans Devresinin Analizi

Özet: Kesirli derceli devre elemanları, son birkaç on yılda, elektronik ve mekanik sistemlerin modellenmesinin önemli bir parçası haline gelmeye başlamıştır. Son zamanlarda, uyumlu kesirli türev (CFD) literatürde ortaya çıkmıştır. Anlaşılması ve kullanılması diğer kesirli dereceli türevlerden daha kolaydır ve devre bileşenlerini modellemek için çoktan kullanılmaktadır. CFD literatürde süper kondansatörleri modellemek için hâlihazırda kullanılmıştır. Devrelerin farklı akım ve gerilim dalga biçimleri için nasıl davrandığını bilmek önemlidir. Bu makalede, bir akım kaynağı tarafından beslenen bir direnç, bir indüktör ve bir CFD kapasitörün paralel bağlanmasından oluşan devre, devrenin analitik çözümünü bulmak mümkün olmadığından dolayı, Matlab™'in Simulink™ alt programı ile DC ve sinüzoidal sinyaller kullanılarak incelenmiştir. Simulink bu devreleri incelemek için kolay bir yol sağlamaktadır.

Anahtar kelimeler: *Kesirli dereceli Türev, Uyumlu Kesirli Türev, Devre Analizi, Devre Teorisi, Devre Modellemesi.*

* Corresponding author.
E-mail address: 1140605607@nku.edu.tr (A. Alhaj)

1. Introduction

Fractional derivative is a derivative of any arbitrary order which is harder to use than ordinary derivative [1-5]. In recent years, the Fractional Calculus has found application areas in many engineering fields [4-5]. The fractional order circuit elements are especially useful to model biological systems [6-7]. Realization of fractional order elements have become also an important research area [8-11]. Such circuits may present interesting oscillation phenomenon [12]. Oscillation of impulsive conformable fractional differential equations has also been inspected in [13]. They are also useful for batter modeling [14]. A new simple fractional derivative called “the conformable fractional derivative” depending on the familiar limit definition of the derivative of a function and that break with other definitions has been suggested in [15]. The CFD is developed more in [16]. A conformal fractional derivative is actually not a fractional derivative that it is simply a first derivative multiplied by an additional simple factor depending on the independent variable. This new definition is a natural extension of the classical derivative and it has also the advantage of being different from other fractional differentials. Due to these properties, the conformal fractional derivative has become a hot research area. The conformal derivative has also the advantage of being physically interpretable compared to the other types of fractional derivatives [17]. A review on usage of Fractional derivatives in electrical circuit theory can be found in [18]. Supercapacitors have been modeled using fractional order models in [19-22]. The CFD has been used for modeling and analyzing an electric circuit containing a supercapacitor in [23]. Some fractional electrical circuit with sinusoidal inputs has been analyzed using Caputo and conformable derivative definitions have been done in [24-25]. A Liouville-Caputo sense approach has been used to obtain analytical solutions of electrical circuits described by fractional conformable derivatives in [26]. Other electrical circuits described by fractional conformable derivative have been examined in [27]. A CFD capacitor has been analyzed for common waveforms in [28]. It is important to analyze new circuit elements for different current and voltage waveforms so that they can find usage at full potential. An R-L-C parallel circuit is commonly thought in electric circuit theory due to its many applications. Such a circuit can be modified with a CFD capacitor. To the best of our knowledge, such a circuit has not been analyzed in the literature yet. We have tried to solve such a circuit analytically by hand and by a symbolic math program to no avail. In this paper, such a circuit fed by a current source has been analyzed for DC and sinusoidal waveforms using SimulinkTM toolbox of MatlabTM. The discussions are provided in the conclusion section.

The paper is organized as follows. The CFD capacitor model is given in the second section. Its examination of the parallel circuit for DC and AC signals are made using Simulink in the third section. The paper is finalized with the conclusion and discussion section.

2. Conformal Fractional Derivative and CFD Capacitor Constitutional Law

More information about CFD can be found in conformable fractional calculus [7]. Conformal Fractional Derivative (CFD) is described as

$$\frac{d^{\alpha} f(t)}{dt^{\alpha}} = f^{'}(t)t^{1-\alpha} = \frac{df(t)}{dt}t^{1-\alpha} \quad (1)$$

If a capacitor can be modeled using CFD, its constitutional law can be expressed as

$$i_C(t) = C_{\alpha}\frac{d^{\alpha} v_C(t)}{dt^{\alpha}} = C_{\alpha} v_C^{'}(t)t^{1-\alpha} = C_{\alpha}\frac{dv_C(t)}{dt}t^{1-\alpha} \quad (2)$$

where , and are CFD capacitor current, CFD capacitor voltage and CFD capacitor coefficient..

3. Examination of the parallel resistor-CFD Capacitor circuit for different waveforms Using Simulink

The parallel R-L-C circuit is shown in Figure 1.

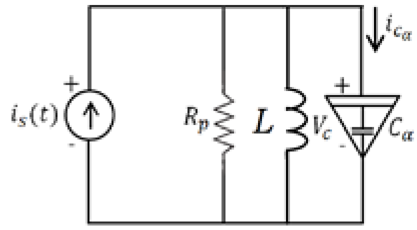


Figure 1: The parallel circuit with a CFD capacitor.

$$i_C(t) = C_{\alpha}\frac{d^{\alpha} v_C(t)}{dt^{\alpha}} \quad (3)$$

$$i_R(t) = \frac{v_R(t)}{R} = \frac{v_C(t)}{R} \quad (4)$$

$$v_L(t) = L\frac{di_L(t)}{dt} \quad (5)$$

$$i_S(t) = i_C(t) + i_R(t) + i_L(t) \quad (6)$$

$$i_S(t) = C_{\alpha}\frac{d^{\alpha} v_C(t)}{dt^{\alpha}} + \frac{v_C(t)}{R} + i_L(t) \quad (7)$$

$$i_S(t) = C_{\alpha}\frac{d^{\alpha} v_L(t)}{dt^{\alpha}} + \frac{v_L(t)}{R} + i_L(t) \quad (8)$$

$$i_S(t) = C_{\alpha}\frac{d^{\alpha}}{dt^{\alpha}}\left(L\frac{di_L(t)}{dt}\right) + \frac{L}{R}\frac{di_L(t)}{dt} + i_L(t) \quad (9)$$

$$i_S(t) = C_{\alpha}L\frac{d^2 i_L(t)}{dt^2}t^{1-\alpha} + \frac{L}{R}\frac{di_L(t)}{dt} + i_L(t) \quad (10)$$

4. Simulation Results of the Circuit

In this section, the system is simulated using Simulink.

4.1. Solution for a Constant Current Source

For a constant source , Eq. (10) turns into

$$I_{dc} = C_\alpha L \frac{d^2 i_L(t)}{dt^2} t^{1-\alpha} + \frac{L}{R}\frac{di_L(t)}{dt} + i_L(t) \qquad (11)$$

The equation () has been tried to be solved by us and a symbolic differential equation solver to not avail. That's why the system is chosen to be simulated numerically. The Simulink block diagram of the circuit described by equation (11) for a constant current source is shown in Figure 2. The circuit parameters given in Table 1 is used in simulations. The simulation results for a constant current is given in Figure 3. After quite a long time, after the whole source current will pass only from the short-circuited inductor while the current of the capacitor and voltage of the circuit nearly will equal zero.

Table 1. Parameters of the parallel circuit

Resistance of the Resistor	R	1Ω
Inductance of the inductor	L	1H
The capacitance coefficient	C_α	$1F/s^{0.5}$
The power of the CFD capacitor	α	0.5
Constant current source Magnitude	I_{dc}	1A

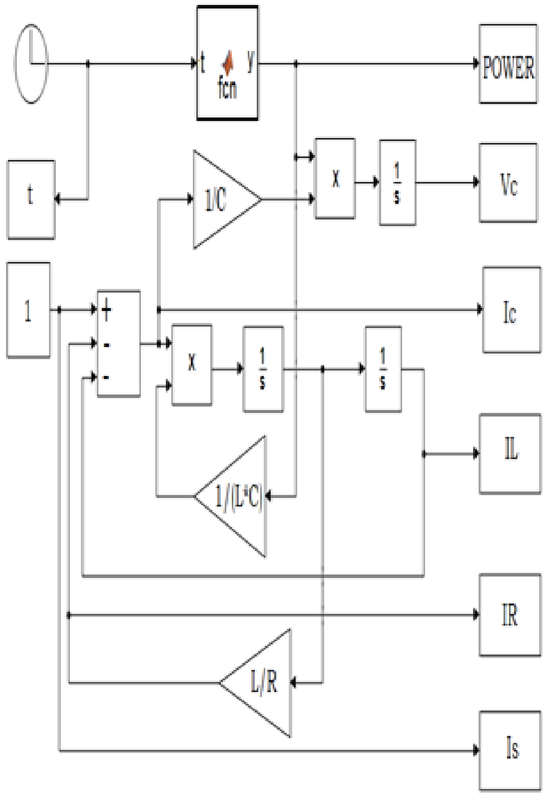
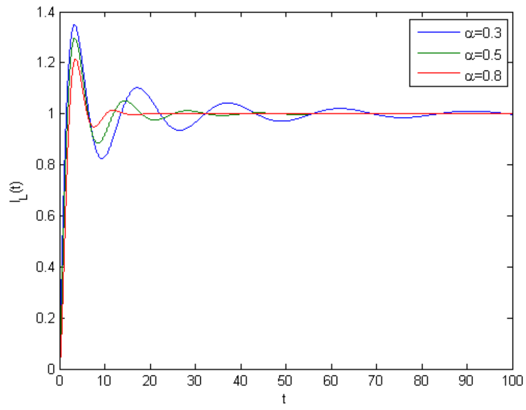
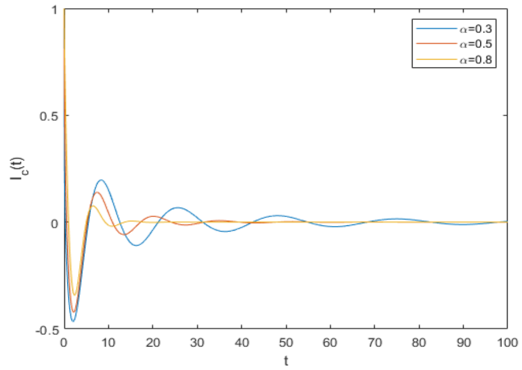


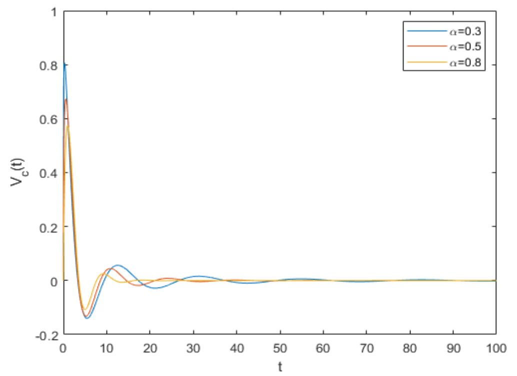
Figure 2. Simulink block diagram of the parallel circuit excited with a constant current source.



(a)



(b)



(c)

Figure 3. a) The inductor current, b) the CFD Capacitor current and c) the CFD Capacitor voltage vs time when the parallel circuit is fed by a constant current source simulated for three different alpha values.

4.2 Solution for a Sinusoidal Current Source

For a sinusoidal current source, :

Eq. (10) turns into

$$I_m \cos(\omega t + \varphi) = C_\alpha L \frac{d^2 i_L(t)}{dt^2} t^{1-\alpha} + \frac{L}{R}\frac{di_L(t)}{dt} + i_L(t) \qquad (12)$$

The equation (12) has been tried to be solved by us and a symbolic differential equation solver to not avail. That's why the system is chosen to be simulated numerically. The Simulink block diagram of the circuit described by equation (12) for a constant current source is shown in Figure 4. The circuit parameters given in Table 2 is used in simulations. The simulation results for a sinusoidal current in Figure 5.

For different values, the system has different transient phenomenon and different damping behavior as shown in Figure 5. When increases the sytem starts behaing almost a LTI R-L-C circuit. At low values, the circuit waveforms have envelopes which resembles one in AM modulation circuits.

Table 2. Parameters of the parallel circuit

Resistance of the Resistor	R	1Ω
Inductance of the inductor	L	1H
The capacitance coefficient	C_α	1F/ $s^{0.5}$
The power of the CFD capacitor	α	0.5
Sinusoidal Source Magnitude	I_m	1A
Sinusoidal Source Frequency	f	1/2 π Hz

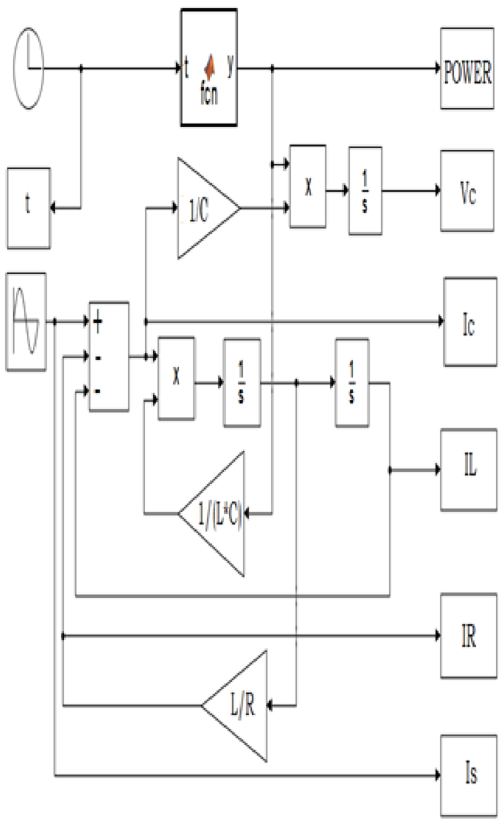
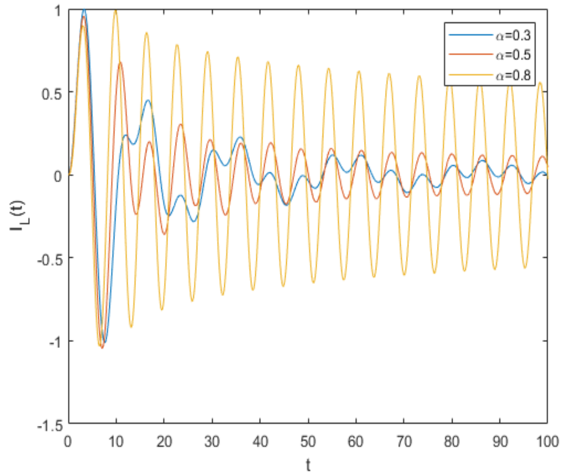
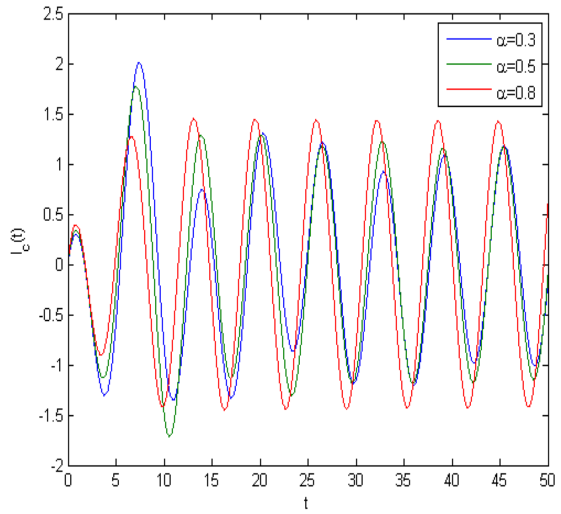


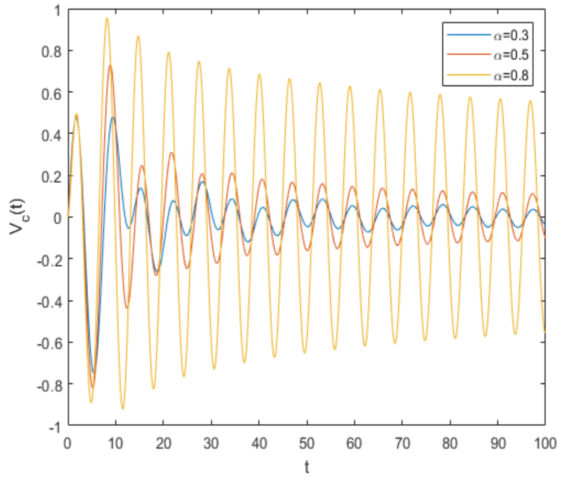
Figure 4. Simulink block diagram of the parallel circuit excited with a sinusoidal current source.



(a)



(b)



(c)

Figure 5. a) The inductor current, b) the CFD Capacitor current and c) the CFD Capacitor voltage vs time when the parallel circuit is fed by the sinusoidal current source simulated for three different alpha values.

5. Conclusions

Some capacitors can be modelled using fractional order derivatives. Every circuit element should be examined for the basic waveforms such as DC and sinusoidal signals. In this study, conformal fractional derivative is used to model a capacitor and then connected in parallel with a resistor and an inductor, the circuit is simulated with a DC and sinusoidal current source. The analytical solutions of the circuit for DC and sinusoidal sources could not have been done. That's why Simulink is used to simulate the circuit. The numerical solutions of the circuit waveforms for different sources are given. It is found that the Simulink lets the circuit examined parametrically easily. Only one of them is found exactly solvable and a useful tool for analyzing circuits with (a) CFD capacitor(s).

References

[1] Podlubny, I. (1998). Fractional differential equations: an introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications. Elsevier.

[2] Yang, X. J. (2019). General fractional derivatives: theory, methods and applications. Chapman and Hall/CRC.

[3] Ross, B. (1977). The development of fractional calculus 1695–1900. Historia Mathematica, 4(1), 75-89.

[4] Kilbas, A.A.; Srivastava, H.M.; Trujillo, J.J. Theory and Applications of Fractional Differential Equations; Elsevier: Amsterdam, The Netherlands, 2006.

[5] Babiarz, A., Czornik, A., Klamka, J., & Niezabitowski, M. (2017). Theory and applications of non-integer order systems. Lecture Notes Electrical Engineering, 407.

[6] Freeborn, T. J. (2013). A survey of fractional-order circuit models for biology and biomedicine. IEEE Journal on emerging and selected topics in circuits and systems, 3(3), 416-424.

[7] Moreles, M. A., & Lainez, R. (2016). Mathematical modelling of fractional order circuits. arXiv preprint arXiv:1602.03541.

[8] Adhikary, A., Khanra, M., Pal, J., & Biswas, K. (2017). Realization of fractional order elements. Inae Letters, 2(2), 41-47.

[9] Tsirimokou, G., Kartci, A., Koton, J., Herencsar, N., & Psychalinos, C. (2018). Comparative study of discrete component realizations of fractional-order capacitor and inductor active emulators. Journal of Circuits, Systems and Computers, 27(11), 1850170.

[10] Sotner, R., Jerabek, J., Kartci, A., Domansky, O., Herencsar, N., Kledrowetz, V., ... & Yeroglu, C. (2019). Electronically reconfigurable two-path fractional-order PI/D controller employing constant phase blocks based on bilinear segments using CMOS modified current differencing unit. Microelectronics Journal, 86, 114-129.

[11] Podlubny, I., Petráš, I., Vinagre, B. M., O'leary, P., & Dorčák, Ľ. (2002). Analogue realizations of fractional-order controllers. Nonlinear dynamics, 29(1-4), 281-296.

[12] Alagoz, B. B., & Alisoy, H. Z. (2014). On the

Harmonic Oscillation of High-order Linear Time Invariant Systems. Scientific Committee.

[13] Tariboon, J., & Ntouyas, S. K. (2016). Oscillation of impulsive conformable fractional differential equations. Open Mathematics, 14(1), 497-508.

[14] Alagöz, B. B., & Alisoy, H. Estimation of Reduced Order Equivalent Circuit Model Parameters of Batteries from Noisy Current and Voltage Measurements. Balkan Journal of Electrical and Computer Engineering, 6(4), 224-231.

[15] Khalil, R.; al Horani, M.; Yousef, A.; Sababheh, M. A new definition of fractional derivatuive. J. Comput. Appl. Math. 2014, 264, 65–70.

[16] Abdeljawad, T. (2015). On conformable fractional calculus. Journal of computational and Applied Mathematics, 279, 57-66.

[17] Zhao, D., & Luo, M. (2017). General conformable fractional derivative and its physical interpretation. Calcolo, 54(3), 903-917.

[18] Sikora, R. (2017). Fractional derivatives in electrical circuit theory–critical remarks. Archives of Electrical Engineering, 66(1), 155-163.

[19] Lewandowski, M., & Orzyłowski, M. (2017). Fractional-order models: The case study of the supercapacitor capacitance measurement. Bulletin of the Polish Academy of Sciences Technical Sciences, 65(4), 449-457.

[20] Kopka, R. (2017). Estimation of supercapacitor energy storage based on fractional differential equations. Nanoscale research letters, 12(1), 636. Data alaınan makale

[21] Freeborn, T. J., Elwakil, A. S., & Allagui, A. (2018, May). Supercapacitor fractional-order model discharging from polynomial time-varying currents. In 2018 IEEE International Symposium on Circuits and Systems (ISCAS) (pp. 1-5). IEEE.

[22] Freeborn, T. J., Maundy, B., & Elwakil, A. S. (2013). Measurement of supercapacitor fractional-order model parameters from voltage-excited step response. IEEE Journal on Emerging and Selected Topics in Circuits and Systems, 3(3), 367-376.

[23] Piotrowska, E. (2018, October). Analysis the conformable fractional derivative and Caputo definitions in the action of an electric circuit containing a supercapacitor. In Photonics Applications in Astronomy, Communications, Industry, and High-Energy Physics Experiments 2018 (Vol. 10808, p. 108081T). International Society for Optics and Photonics.

[24] Piotrowska, E., & Rogowski, K. (2017, October). Analysis of fractional electrical circuit using Caputo and conformable derivative definitions. In Conference on Non-integer Order Calculus and Its Applications (pp. 183-194). Springer, Cham.

[25] Piotrowska, Ewa. "Analysis of fractional electrical circuit with sinusoidal input signal using Caputo and conformable derivative definitions." Poznan University of Technology Academic Journals. Electrical Engineering (2019).

[26] Morales-Delgado, V. F., Gómez-Aguilar, J. F., &

Taneco-Hernandez, M. A. (2018). Analytical solutions of electrical circuits described by fractional conformable derivatives in Liouville-Caputo sense. AEU-International Journal of Electronics and Communications, 85, 108-117.

[27] Martínez, L., Rosales, J. J., Carreño, C. A., & Lozano, J. M. (2018). Electrical circuits described by fractional conformable derivative. International Journal of Circuit Theory and Applications, 46(5), 1091-1100.

[28] PALAZ U., MUTLU R., Analysis of a Capacitor Modelled with Conformable Fractional Derivative Under DC and Sinusoidal Signals. Submitted to Sakarya University Journal of Science.