



Analysis and Simulation of Fractional-Order Diabetes Model

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Abstract

In this article, we research the diabetes model and its consequences using the Caputo and Atangana Baleanu fractional derivatives. A deterministic mathematical model is corresponding to the fractional derivative of diabetes mellitus. The Laplace transformation is used for the diagnostic structure of the diabetes model. Picard-Lindelof's method shows the existence and uniqueness of the solution. Finally, numerical simulations are made to illustrate the effects of changing the fractional-order to obtain the theoretical results, and comparisons are made for the Caputo and Atangana Baleanu derivative. The results of the following work by controlling plasma glucose with the fractional-order model make it a suitable candidate for controlling human type 1 diabetes.

Keywords: Fractional order glucose insulin system, Stability, Picardas Lindelof approach, Fixed point theorem, ABC derivative.

1. Introduction

More than 8% of the adult people (age 20 to 79 years) are affected by diabetes mellitus in the present day [1]. According to a report that this number will increase to 55% within 20 years [1], and it will increase

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with the passage of time. This issue tells the importance of the treatment of diabetes and discovering better methodologies for diabetic patient lifestyle support. As the recurring pattern, bleeding edge in remedial science doesn't give a full treatment for the disease, the patients need to grasp a remarkable lifestyle with some degree of unprecedented treatment for each kind of diabetes; for specific patients, it is adequate to concentrate on the sustenance confirmation while some diabetic patients need subcutaneous insulin injections which help to reduce insulin deficiency. There are two types of diabetes, type 1 diabetes and type 2 diabetes. Type 1 diabetic patients have insufficiency of endogenous insulin and type 2 diabetes (T2D) described by partial inadequacy of endogenous insulin creation and obstruction. The following work focuses on the day by day life backing of type 1 and types 2 diabetic patients treated with insulin infusions. Long-acting (or basal) type of subcutaneous insulin have a day-long impact on diabetic patients, normally controlled only in one part of the day either at the first part of the day or at night, blood glucose level (BGL) is controlled by short-acting (or bolus) type. The little slip-ups in picking the correct portion of these infusions can prompt a basically low BGL, that is, hypoglycemia, which may introduce a moment health-related crisis, or a continued, too much increase BGL, is known as hyperglycemia, which brings about serious confusions over the long haul. Evaluating insulin needs before each dinner and different exercises, it is a day by day task for every diabetic patient. These choices are generally founded on understanding which is in some cases fairly wasteful practically speaking, bringing about taking high values of glycated hemoglobin (HbA1c) [2,3]. Perceptions legitimize creating blood glucose forecast calculations and way of life bolster applications that used to aid diabetic patients in completing their day to day existence [4,5].

The modeling of irresistible diseases is an instrument that has been utilized to think about the systems by which infections spread, to foresee the future course of an outbreak and to assess procedures to control a pandemic. In the mid-tenth, the law of mass activity to clarify epidemic behavior was applied by William Hamer and Ronald Ross. Compartmental models were at a peak in 1920. The Kermack McKendrick pandemic model and the Reed Frost scourge model (1928) both models portray the connection between contaminated, helpless and resistant people within the populace. The conduct of episodes fundamentally was the same as that saw in many recorded epidemics. It was also described by Kermack-McKendrick [8].

There are two sorts of epidemic models. "Stochastic" signifies being or having an arbitrary interchangeable. Assessment of probability distributions of potential results by taking into consideration irregular variety in one or more contributions after some time with the help of a device known as the stochastic model. Presentation, disease and different disease elements were opportunity varieties for the stochastic model. Deterministic or compartmental numerical models are regularly used for enormous populaces for example tuberculosis. Different subgroups or compartments made in the deterministic model and these subgroups speak a particular phase of the plague. Letters, for example, M, S, E, I, and R are regularly used to recognize various stages. These days, diabetes is worldwide a peaceful epidemic intensely expanding the charge of non-transmittable illnesses and by large invigorated by lessening the degrees of action. Mostly, two kinds of diabetes are examined: Type 1 diabetes, made 10 to 15 percent of the population and it mostly affected people beneath the age of 40. Then, Type II diabetes made 85 to 90 percent of the population. The spread of heftiness is common in all age groups, type 2 diabetes is more common in children as compared to type 1; see [9]. Different models describe diabetes and its consequences in [9-12]. Fragmentary expansions of scientific

models of whole number request speak to the regular reality in an exceptionally precise manner explained by different researchers and mathematicians with fractional order are given in [13-17]. Some epidemic model with fractional order techniques are given in [18,19].

In this article, the diabetes model is explained by Caputo and ABC derivative. Numerical simulations are made at different fractional order values. Uniqueness and stability analysis also verified with fixed point theorem and Picard Lindolof approach. At the end results and discussion of the system explained.

2. Preliminaries

Here, we survey the focal thoughts with respect to the Caputo and the Atangan-Baleanu fractional derivative.

Definition 1. For a function $f \in C^n$ of order $\zeta > 0$, the Caputo derivative is as follows:

$${}_C D_t^\zeta(f(t)) = I^{n-\zeta} D^n f(t) = \frac{1}{\Gamma(n-\zeta)} \int_0^t \frac{f^{(n)}(z)}{(t-z)^{\zeta+n-1}} dz,$$

in which $n-1 < \zeta < n \in \mathbb{N}$ and is defined for the absolute continuous functions, so that ${}_C D_t^\zeta(f(t))$ approaches to $f'(t)$ as $\zeta \rightarrow 1$.

Definition 2. Let for a function $z(t)$ in which $\zeta \in (0, 1)$, then we represent the integral of fractional order ζ is as,

$$I_t^\zeta(z(t)) = \frac{2(1-\zeta)}{(2-\zeta)\kappa(\zeta)} g(t) + \frac{2\zeta}{(2-\zeta)\kappa(\zeta)} \int_0^t z(s) ds, t \geq 0. \tag{1}$$

Remark 2. In the above equation (1), The remainder of the non-integer-type Caputo function integral with order $\zeta \in (0, 1)$ is a mean in z with order integral 1. In this way, it requires,

$$\frac{2}{2\kappa(\zeta) - \zeta\kappa(\zeta)} = 1 \tag{2}$$

implies that $\kappa = \frac{2}{2-\zeta}$, $\zeta \in (0, 1)$. A new Caputo derivative with $\zeta \in (0, 1)$ is suggested on the basis of equation (2) and represented by

$$D_t^\zeta(z(t)) = \frac{1}{1-\zeta} \int_a^t z'(x) \exp[-\zeta \frac{t-x}{1-\zeta}] dx. \tag{3}$$

3. Nonlinear Fractional Order Differential System

3.1. Liouville-Caputo sense

The Modified Analytical Homotopy Method (MHATM) was recommended in [20]. The strategy is a trial structure dependent on the technique for joining homotopy analysis and the transformation of Laplace with polynomial homotopy. The essential strides of this technique are depicted as seeking after:

Step 1. In this step, we should look at the condition below

$$D_t^\kappa \{g(j, t)\} + \tau[j]g(j, t) + \wedge[h]g(j, t) = \eta(j, t), \quad t > 0, \quad j \in \mathbb{R}, \quad 0 < \kappa \leq 1, \tag{4}$$

here $\tau[j]$ is a linear operator bounded to j and the non-linear operator $\wedge[j]$ in j is Lipschitz continuous and satisfying $|\wedge(g) - \wedge(\phi)| \leq \vartheta|g - \phi|$, where $\theta > 0$ and $\eta(j, t)$ is a continuous function. It is possible to treat the boundary and initial conditions equally.

Step 2. We obtain the following equation of m -th order deformation by applying the methodology proposed in [21]

$$g_m(j, t) = (X_m + \hbar)g_{m-1} - \hbar(1 - X_m)\sum_{b=0}^{h-1} t^b g^{(b-1)}(0) + \hbar\mathcal{L}^{-1}\left(\frac{1}{s^\kappa}\mathcal{L}(\tau_{m-1}[j]g_{m-1}(j) + \sum_{k=0}^{m-1} P_k(g_0, g_1, \dots, g_m) - \Psi(j, t))\right), \tag{5}$$

Where the transformation of Laplace is applied in Caputo sense and P_k is the polynomial homotopy defined by Odibat in [22].

Step 3. Regarding homotopy polynomials, the non-linear term $\wedge[j]g(j, t)$ is extended as

$$\wedge[g(j, t)] = \wedge\left(\sum_{k=0}^{m-1} g_m(j, t)\right) = \sum_{m=0}^{\infty} P_m g^m$$

Step 4. Extending the non-linear term in equation (5) as a progression of homotopy polynomials, we can calculate the various $g_m(j, t)$ solutions for $m > 1$ and equation (4) is the summary of an infinite series which generally quickly joins the exact solutions

$$g(j, t) = \sum_{\infty}^{m=0} g_m(j, t).$$

Consider the mathematical model [23] in which D reflects the number of non-complicated diabetes at time t , C corresponds to the number of complicated diabetes and the incidence of diabetes mellitus I. Several parameters have been taken and a mathematical model is developed based on these parameter values. The model shall be defined as:

$$\begin{aligned} \frac{dD}{dt} &= I - (\lambda + \mu)D + \gamma C \\ \frac{dC}{dt} &= I + \lambda D - (\gamma + \mu + \nu + \delta)C, \end{aligned} \tag{6}$$

where I reflects the occurrence of mellitus diabetes. μ refers the rate of natural mortality, λ refers to the probability of a diabetic spreading a disease, γ relates to the rate of healing complications, ν corresponds to the rate at which complicated diabetic patients convert critically disabled patients, and δ denotesthe complicated mortality rate. $N(t) = C(t) + D(t)$ shows the size of diabetics at the time t . We take $N(t) = C(t) + D(t)$, and thus the equation (6) becomes:

$$\begin{aligned} \frac{dC}{dt} &= -(\lambda + \theta)C + \lambda N, \eta > 0 \\ \frac{dN}{dt} &= I - (\nu + \delta)C - \mu N \end{aligned} \tag{7}$$

In equation (7), $\theta = \gamma + \mu + \nu + \delta$ in which initial conditions are as follows:

$$C(0) = C_0, N(0) = N_0. \tag{8}$$

Solution. We also applied the Laplace transform to the system’s first formula (7).

$$s^\kappa \bar{C}(s) - s^{\kappa-1} C(0) = \mathcal{L}\{-(\lambda + \theta)C + \lambda N\}$$

The initial conditions are taken and the above equation is simplified

$$\bar{C}(s) = \frac{C(0)}{s} + \mathcal{L}\{-(\lambda + \theta)C + \lambda N\} \tag{9}$$

Now applying the inverse Laplace transformation to equation (9) and acquire:

$$C(t) = C_0 + \mathcal{L}^{-1} \frac{1}{s^\kappa} \mathcal{L}\{-(\lambda + \theta)C + \lambda N\}$$

For the other equations shown in equation (7), we get

$$N(t) = N_0 + \frac{It^\kappa}{\Gamma\kappa + 1} + \mathcal{L}^{-1} \frac{1}{s^\kappa} \mathcal{L}\{-(\nu + \delta)C\mu N\}$$

In this step, we take a linear form operator, so that

$$[\phi_h(\mathbf{t}; \mathbf{p})] = \mathcal{L}[\phi_h(t; p)], \quad h = 1, 2$$

with the following property $(\mathbf{e}) = 0$ in which e is constant. First of all we explain the system below

$$N[\phi_1(t; p)] = \mathcal{L}[\phi_1(t; p)] - C_0 + \frac{1}{s^\kappa} \mathcal{L}\{-\lambda\phi_1 - \theta\phi_1 + \lambda\phi_2\}$$

$$N[\phi_2(t; p)] = \mathcal{L}[\phi_2(t; p)] - N_0 + \frac{1}{s^\kappa} \mathcal{L}\{-\nu\phi_1 - \delta\phi_1\mu\phi_2\}$$

The equation of so-called zero-order deformation is shown as

$$(1 - p)[\phi_h(\mathbf{t}; \mathbf{p}) - \mathbf{u}_0(\mathbf{t})] = p\hbar[\phi_h(t; p)], \quad h = 1, 2,$$

when $p = 0$ and $p = 1$, we acquire

$$\phi_h(t; 0) = u_0(t), \quad \phi_h(t; 1) = u(i), \quad h = 1, 2,$$

Where the deformation equations of the mth-order are given

$$\mathcal{L}\{C_m(t) - P_m C_{m-1}(t)\} = \hbar S_m(C_{m-1}^\rightarrow, t)$$

$$\mathcal{L}\{N_m(t) - P_m N_{m-1}(t)\} = \hbar S_m(N_{m-1}^\rightarrow, t) \tag{10}$$

Transforming the inverse Laplace into the equation (10) We’ve got this

$$C_m(t) = P_m C_{m-1}(t) + \hbar S_m(C_{m-1}^\rightarrow, t)$$

$$N_m(t) = P_m N_{m-1}(t) + \hbar S_m(N_{m-1}^\rightarrow, t)$$

where

$$S_m(C_{m-1}^\rightarrow, t) = \mathcal{L}[C_{m-1}(t)] + (1 - P_m)(C_0 - \frac{1}{s^\kappa} \mathcal{L}\{-\lambda C_{m-1} - \theta C_{m-1} + \lambda N_{m-1}\})$$

$$S_m(N_{m-1}^{\rightarrow}, t) = \mathcal{L}[N_{m-1}(t)] - (1 - P_m)(N_0 + I \frac{t^\kappa}{\Gamma\kappa + 1} + \frac{1}{s^\kappa} \mathcal{L}\{-\nu C_{m-1} - \delta C_{m-1} - \mu N_{m-1}\})$$

The solution for mth-order deformation equation (10) is the following

$$C_m(t) = (P_m + \hbar)C_{m-1} - \hbar(1 - P_m)(C_0) + \hbar \mathcal{L}^{-1}\{\frac{1}{s^\kappa} \mathcal{L}\{-\lambda C_{m-1} - \theta C_{m-1} + \lambda N_{m-1}\}\}$$

$$N_m(t) = (P_m + \hbar)N_{m-1} - \hbar(1 - P_m)(N_0 + I \frac{t^\kappa}{\Gamma\kappa + 1}) + \hbar \mathcal{L}^{-1}\{\frac{1}{s^\kappa} \mathcal{L}\{-\nu C_{m-1} - \delta C_{m-1} - \mu N_{m-1}\}\} \tag{11}$$

Finally, the solutions of the equation (7)

$$C(t) = C_0(t) + C_1(t) + C_2(t) + \dots = \sum_{m=0}^{\infty} C_m(t)$$

$$N(t) = N_0(t) + N_1(t) + N_2(t) + \dots = \sum_{m=0}^{\infty} N_m(t) \tag{12}$$

Through combining the Laplace transform (1) and its inverse, another model (7) solution can be obtained. The iterative scheme is given through

$$C_n(t) = C_0 + \mathcal{L}^{-1}\{\frac{1}{s^\kappa} \mathcal{L}\{-\lambda C_{n-1}(t) - \theta C_{n-1}(t) + \lambda N_{n-1}(t)\}(s)\}(t)$$

$$N_n(t) = N_0 + \mathcal{L}^{-1}\{\frac{1}{s^\kappa} \mathcal{L}\{I - \nu C_{n-1}(t) - \delta C_{n-1}(t) - \mu N_{n-1}(t)\}(s)\}(t) \tag{13}$$

where C_0 and N_0 are the initial conditions. If n tends to infinity, it is assumed that the solution is a limit

$$C(t) = \lim_{n \rightarrow \infty} C_{(n)}(t)$$

$$N(t) = \lim_{n \rightarrow \infty} N_{(n)}(t)$$

Theorem 3.1. The Equations recursive form. (12) is stable.

Proof: We're going to suppose the following. There are two positive constants G and H can be found so that for all $0 \leq t \leq T \leq \infty$,

$$\| C(t) \| < G; \| N(t) \| < H.$$

Here, we suppose a subset of $L_2((q, r)(0, T))$ which is defined as following

$$\tau = \{\kappa : (q, r)(0, T) \rightarrow \tau, \frac{1}{\Gamma(\kappa)} \int (t - \beta)^{(\kappa-1)} w(\beta) y(\beta) g \beta < \infty\}$$

The suppose operator is called ζ

$$\zeta(C, N) = -\lambda C(t) - \theta C(t) + \lambda N(t)$$

$$= I - \nu C(t) - \delta C(t) - \mu N(t)$$

Then

$$\begin{aligned} &< \zeta(C, N) - \zeta(C_1, N_1), (C - C_1, N - N_1) >, \\ &< -\lambda(C(t) - C_1(t)) - \theta(C(t) - C_1(t)) + \lambda(N(t) - N_1(t)), (C(t) - C_1(t)) > \\ &< I - \nu(C(t) - C_1(t)) - \delta(C(t) - C_1(t)) - \mu(N(t) - N_1(t)), (N(t) - N_1(t)) > \end{aligned}$$

in which,

$$C(t) \neq C_1(t); \quad N(t) \neq N_1(t)$$

Therefore, we have to apply the norm and the absolute value on both sides

$$\begin{aligned} &< \zeta(C, N) - \zeta(C_1, N_1), (C - C_1, N - N_1) >, \\ &< \{-\lambda - \theta + \frac{\lambda \| N(t) - N_1(t) \|}{\| C(t) - C_1(t) \|}\} \| C(t) - C_1(t) \|^2 \\ &< \{\frac{I}{\| N(t) - N_1(t) \|} - \nu \frac{\| C(t) - C_1(t) \|}{\| N(t) - N_1(t) \|} - \delta \frac{\| C(t) - C_1(t) \|}{\| N(t) - N_1(t) \|} - \mu\} \| N(t) - N_1(t) \|^2 \end{aligned}$$

where

$$\begin{aligned} &< \zeta(C, N) - \zeta(C_1, N_1), (C - C_1, N - N_1) >, \\ &< W \| C(t) - C_1(t) \|^2, \\ &< X \| N(t) - N_1(t) \|^2 \end{aligned} \tag{14}$$

with

$$\begin{aligned} W &= \{-\lambda - \theta + \frac{\lambda \| N(t) - N_1(t) \|}{\| C(t) - C_1(t) \|}\} \\ X &= \{\frac{I}{\| N(t) - N_1(t) \|} - \nu \frac{\| C(t) - C_1(t) \|}{\| N(t) - N_1(t) \|} - \delta \frac{\| C(t) - C_1(t) \|}{\| N(t) - N_1(t) \|} - \mu\} \end{aligned}$$

Additionally, if we find a non-null vector (C_1, N_1) using a certain routine as above, we get

$$\begin{aligned} &< \zeta(C, N) - \zeta(C_1, N_1), (C - C_1, N - N_1) >, \\ &< W \| C(t) - C_1(t) \| \| C(t) \| \\ &< X \| N(t) - N_1(t) \| \| N(t) \| \end{aligned} \tag{15}$$

We conclude from the results of equations (13) and (14) that the iterative method used is stable.

We can now propose a system solution given by equation (7) use the Adams Bash forth-Moulton predictor-corrector approach as follows [28]

$$\begin{aligned} C(t) &= \sum_{u=0}^{n-1} \delta_1^u \frac{t^u}{u!} + \frac{1}{\Gamma(\kappa)} \int_0^t (t - \psi)^{\kappa-1} [-\lambda C(\psi) - \theta C(\psi) + \lambda N(\psi)] du, \\ N(t) &= \sum_{u=0}^{n-1} \delta_2^u \frac{t^u}{u!} + \frac{1}{\Gamma(\kappa)} \int_0^t (t - \psi)^{\kappa-1} [I - \nu C(\psi) - \delta C(\psi) - \mu N(\psi)] du. \end{aligned} \tag{16}$$

4. Atangana-Baleanu sense

Using the Atangana-Baleanu fractional order derivative, we solve the following time-fractional model according to the methodology stated in [20].

$$\begin{aligned} {}_0^{ABC}D^\kappa C(t) &= -\lambda C - \theta C + \lambda N \\ {}_0^{ABC}D^\kappa N(t) &= I - \nu C - \delta C - \mu N \end{aligned} \tag{17}$$

with initial conditions are $C(0) = C_0 = 0, N(0) = N_0 = 0$

The Laplace transform is applied to the system’s first equation (17).

$$\frac{R(\kappa) s^\kappa \tilde{C}(s) - s^{\kappa-1} C(0)}{1 - \kappa s^\kappa + \frac{\kappa}{1-\kappa}} = \mathcal{L}\{-\lambda C(t) - \theta C(t) + \lambda N(t)\}$$

We take initial conditions and simplify the above equation

$$\tilde{C}(s) = \frac{C(0)}{s} + \frac{(1 - \kappa)s^\kappa + \kappa}{R(\kappa)s^\kappa} \mathcal{L}\{-\lambda C(t) - \theta C(t) + \lambda N(t)\}, \tag{18}$$

We apply the inverse Laplace transform to equation (18), we’ll get

$$C(t) = C_0 + \mathcal{L}^{-1}\left\{\frac{(1 - \kappa)s^\kappa + \kappa}{R(\kappa)s^\kappa} \mathcal{L}\{-\lambda C(t) - \theta C(t) + \lambda N(t)\}\right\},$$

Now, for the other equations shown in (17), we get

$$N(t) = N_0 + I \frac{\kappa}{R(\kappa)s(\kappa)} + \mathcal{L}^{-1}\left\{\frac{(1 - \kappa)s^\kappa + \kappa}{R(\kappa)s^\kappa} \mathcal{L}\{-\nu C(t) - \delta C(t) - \mu N(t)\}\right\},$$

In this case, we chose a linear operator of the type

$$[\phi_h(\mathbf{t}; \mathbf{p})] = \mathcal{L}[\phi_h(t; p)], \quad h = 1, 2. \tag{19}$$

with the property $(c) = 0$, in which c is constant. Next describe the model below

$$N[\phi_1(t; p)] = \mathcal{L}[\phi_1(t; p)] - C_0 + \frac{(1 - \kappa)s^\kappa + \kappa}{R(\kappa)s^\kappa} \mathcal{L}\{-\lambda\phi_1 - \theta\phi_1 + \lambda\phi_2\}$$

$$N[\phi_2(t; p)] = \mathcal{L}[\phi_2(t; p)] - N_0 + I \frac{\kappa}{R(\kappa)s(\kappa)} - \frac{(1 - \kappa)s^\kappa + \kappa}{R(\kappa)s^\kappa} \mathcal{L}\{\nu\phi_1 + \delta\phi_1\mu\phi_2\}$$

The so-called zero-order deformation equation is given in

$$(1 - p)[\phi_h(\mathbf{t}; \mathbf{p}) - \mathbf{u}_0(i)] = p\hbar N[\phi_h(t; p)], \quad h = 1, 2,$$

when $p = 0$ and $p = 1$, we have

$$\phi_h(t; 0) = u_0(t), \quad \phi_h(t; 1) = u(i), \quad h = 1, 2,$$

Where the equations of the m th-order deformation are given

$$\mathcal{L}\{C_m(t) - P_m C_{m-1}(t)\} = \hbar S_m(C_{m-1}^\rightarrow, t)$$

$$\mathcal{L}\{N_m(t) - P_m N_{m-1}(t)\} = \hbar S_m(N_{m-1}^\rightarrow, t) \tag{20}$$

Using the inverse Laplace transform to the equation (20) We've got this

$$C_m(t) = P_m C_{m-1}(t) + \hbar S_m(C_{m-1}^\rightarrow, t)$$

$$N_m(t) = P_m N_{m-1}(t) + \hbar S_m(N_{m-1}^\rightarrow, t)$$

where

$$S_m(C_{m-1}^\rightarrow, t) = \mathcal{L}[C_{m-1}(t)] - (1 - P_m)C_0 + \frac{(1 - \kappa)s^\kappa + \kappa}{R(\kappa)s^\kappa} \mathcal{L}\{-\lambda C_{m-1} - \theta C_{m-1} + \lambda N_{m-1}\}$$

$$S_m(N_{m-1}^\rightarrow, t) = \mathcal{L}[N_{m-1}(t)] - (1 - P_m)(N_0 + \frac{(1 - \kappa)}{R(\kappa)} + I \frac{\kappa t^\kappa}{\Gamma(\kappa + 1)}) + \frac{(1 - \kappa)s^\kappa + \kappa}{R(\kappa)s^\kappa} \mathcal{L}\{-\nu C_{m-1} - \delta C_{m-1} + \mu N_{m-1}\}$$
(21)

The mth-order deformation equation solution (20) is specified as

$$C_m(t) = (P_m + \hbar)C_{m-1} - \hbar(1 - P_m)C_0 + \hbar \mathcal{L}^{-1}\{\frac{(1 - \kappa)s^\kappa + \kappa}{R(\kappa)s^\kappa} \mathcal{L}[-\lambda C_{m-1} - \theta C_{m-1} + \lambda N_{m-1}]\}$$

$$N_m(t) = (P_m + \hbar)N_{m-1} - \hbar(1 - P_m)(N_0 + \frac{(1 - \kappa)}{R(\kappa)} + I \frac{t^\kappa}{R(\kappa)\Gamma\kappa + 1}) - \hbar \mathcal{L}^{-1}\{\frac{(1 - \kappa)s^\kappa + \kappa}{R(\kappa)s^\kappa} \mathcal{L}[-\nu C_{m-1} - \delta C_{m-1} + \mu N_{m-1}]\}$$
(22)

Finally, the solutions of the equation (7)

$$C(t) = C_0(t) + C_1(t) + C_2(t) + \dots = \sum_{m=0}^{\infty} C_m(t)$$

$$N(t) = N_0(t) + N_1(t) + N_2(t) + \dots = \sum_{m=0}^{\infty} N_m(t)$$
(23)

System (17) is similar to the Volterra form in the Atangana-Baleanu sense. The accompanying following iterative scheme converges to the exact solution which carries the limit to a high estimation of n

$$C_{n+1}(t) = \frac{1 - \kappa}{R(\kappa)} \{-\lambda C_n(t) - \theta C_n(t) + \lambda N_n(t)\} + \frac{\kappa}{R(\kappa)\Gamma(\kappa)} \int_0^t (t - \psi)^{\kappa-1} \{-\lambda C_n(\psi) - \theta C_n(\psi) + \lambda N_n(\psi)\} d\psi,$$

$$N_{n+1}(t) = \frac{1 - \kappa}{R(\kappa)} \{1 - by_n(t) - x_n(t)x_n(t)\} + \frac{\kappa}{R(\kappa)\Gamma(\kappa)} \int_0^t (t - \psi)^{\kappa-1} \{I - \nu C_n(\psi) - \delta C_n(\psi) + \mu N_n(\psi)\} d\psi,$$

Theorem 3.2. Utilizing the Picard-Lindelof approach, we exhibit the presence of the solution.

Proof. The following operator is considered as

$$\begin{aligned} \zeta_1(t, \varsigma) &= -\lambda C(t) - \theta C(t) + \lambda N(t) \\ \zeta_2(t, \varsigma) &= I - \nu C(t) - \delta C(t) + \mu N(t) \end{aligned} \tag{24}$$

where $\zeta_1(t, \varsigma)$ and $\zeta_2(t, \varsigma)$ are contraction respect to ϑ, ρ and v for the first and second functions, respectively. Let

$$\begin{aligned} \Delta_1 &= \sup \| \gamma_{\epsilon, k_1} \zeta_1(t, \varsigma) \|; \\ \Delta_2 &= \sup \| \gamma_{\epsilon, k_2} \zeta_2(t, \varsigma) \|; \end{aligned}$$

where,

$$\begin{aligned} \gamma_{\epsilon, k_1} &= |t - a, t + a| \times [\vartheta - k_1, \vartheta + k_1] = \epsilon_1 \times k_1 \\ \gamma_{\epsilon, k_2} &= |t - a, t + a| \times [\vartheta - k_2, \vartheta + k_2] = \epsilon_1 \times k_2 \end{aligned}$$

Considering the Picardas operator, we have

$$\vartheta : \gamma(\epsilon_1, k_1, k_2) \rightarrow \gamma(\epsilon_1, k_1, k_2) \tag{25}$$

defined as follows

$$\vartheta \Delta(t) = \Delta_0(t) \Delta(t, \Delta(t)) \frac{1 - \kappa}{R(\kappa)} + \frac{\kappa}{R(\kappa)\Gamma(\kappa)} \int_0^t (t - \psi)^{\kappa-1} \Xi(\psi, \Delta(\psi)) d\psi,$$

where

$$\Delta(t) = \{G(t), C(t), N(t)\} = \{g_1, g_2, g_3\}, \text{ and } \Xi(t, \Delta(t)) = \{\zeta_1(t, \vartheta(t)), \zeta_2(t, \vartheta(t)), \zeta_1(t, \vartheta(t))\}.$$

Now we presume that all solutions are bound in a certain amount of time

$$\begin{aligned} \|\Delta(t)\|_{\infty} &\leq \max\{k_1, k_2, k_3\}, \\ \|\Delta(t) - \Delta_0(t)\| &= \|\Xi(t, \Delta(t)) \frac{1 - \kappa}{R(\kappa)} + \frac{\kappa}{R(\kappa)\Gamma(\kappa)} \int_0^t (t - \psi)^{\kappa-1} \Delta(\psi, \Delta(\psi)) d\psi \| \\ &\leq \frac{1 - \kappa}{R(\kappa)} \|\Xi(i, \Delta(i))\| + \frac{\kappa}{R(\kappa)\Gamma(\kappa)} \int_0^t (t - \psi)^{\kappa-1} \|\Xi(\psi, \Delta(\psi))\| d\psi \\ &\leq \frac{1 - \kappa}{R(\kappa)} X = \max\{k_1, k_2, k_3\} + \frac{\kappa}{R(\kappa)} \zeta \vartheta^{\kappa} \leq \vartheta \zeta \leq k = \max\{k_1, k_2, k_3\} \end{aligned}$$

here we have

$$\vartheta < \frac{k}{\zeta}$$

We use the Banach space fixed point theorem alongside the metric, we acquire

$$\begin{aligned} \|\vartheta \Delta_1 - \vartheta \Delta_2\|_{\infty} &= \sup_{i \in \epsilon} |\Delta_1 - \Delta_2|, \\ \|\vartheta \Delta_1 - \vartheta \Delta_2\| &= \|\{\Xi(t, \Delta_1(t)) - \Xi(i, \Delta_2(i))\} \frac{1 - \kappa}{R(\kappa)} \\ &+ \frac{\kappa}{R(\kappa)\Gamma(\kappa)} \int_0^t (t - \psi)^{\kappa-1} \{\Xi(\psi, \Delta_1(t)) - \Xi(\psi, \Delta_2(t))\} d\psi \|, \end{aligned}$$

$$\begin{aligned} &\leq \frac{1-\kappa}{R(\kappa)} \|\Xi(\psi, \Delta_1(t)) - \Xi(\psi, \Delta_2(t))\| + \frac{\kappa}{R(\kappa)\Gamma(\kappa)} \int_0^t (t-\psi)^{\kappa-1} \{\|\Xi(\psi, \Delta_1(t)) - \Xi(\psi, \Delta_2(t))\|\} d\psi, \\ &\leq \frac{1-\kappa}{R(\kappa)} \omega \|\Delta_1(t) - \Delta_2(t)\| + \frac{\kappa\omega}{R(\kappa)\Gamma(\kappa)} \int_0^t (t-\psi)^{\kappa-1} \|\Delta_1(t) - \Delta_2(t)\| d\psi, \\ &\leq \left\{ \frac{1-\kappa}{R(\kappa)} \omega + \frac{\kappa\omega\vartheta^\kappa}{R(\kappa)\Gamma(\kappa)} \right\} \|\Delta_1(t) - \Delta_2(t)\|, \\ &\leq \vartheta\omega \|\Delta_1(t) - \Delta_2(t)\|, \end{aligned}$$

With ω less than 1. Since Δ is a contraction, we have that $\theta\omega < 1$, and the specified ϑ operator is also a contraction. We presume that a unique set of solutions is the process (23). The Atangana-Baleanu fractional integral numerical approximation [29] using the Adams-Moulton rule is given by

$$\psi_t^\kappa [g(t_{n+1})] = \frac{1-\kappa}{R(\kappa)} \frac{g(t_{n+1}) - g(t_n)}{2} + \frac{\kappa}{\Gamma(\kappa)} \sum_{k=0}^\infty \left[\frac{g(t_{k+1}) - g(t_k)}{2} \right] b_k^\kappa,$$

where $b_k^\kappa = (k+1)^{1-\kappa} - (k)^{1-\kappa}$. Using the above numerical scheme, we have

$$\begin{aligned} C_{n+1}(t) - C_n(t) &= C_0^n(t) + \left\{ \frac{1-\kappa}{R(\kappa)} \left[-\lambda \left(\frac{C_{n+1}(t) - C_n(t)}{2} \right) - \theta \left(\frac{C_{n+1}(t) - C_n(t)}{2} \right) + \lambda \left(\frac{N_{n+1}(t) - N_n(t)}{2} \right) \right] \right\} \\ &+ \frac{\kappa}{R(\kappa)} \sum_{k=0}^\infty (k+1)^{1-\kappa} \left[-\lambda \left(\frac{C_{k+1}(t) - C_k(t)}{2} \right) - \theta \left(\frac{C_{k+1}(t) - C_k(t)}{2} \right) + \lambda \left(\frac{N_{n+1}(t) - N_n(t)}{2} \right) \right] \\ N_{n+1}(t) - N_n(t) &= N_0^n(t) + \left\{ \frac{1-\kappa}{R(\kappa)} \left[I - \nu \left(\frac{C_{n+1}(t) - C_n(t)}{2} \right) - \delta \left(\frac{C_{n+1}(t) - C_n(t)}{2} \right) - \mu \left(\frac{N_{n+1}(t) - N_n(t)}{2} \right) \right] \right\} \\ &+ \frac{\kappa}{R(\kappa)} \sum_{k=0}^\infty (k+1)^{1-\kappa} \left[I - \nu \left(\frac{C_{k+1}(t) - C_k(t)}{2} \right) - \delta \left(\frac{C_{k+1}(t) - C_k(t)}{2} \right) - \mu \left(\frac{N_{k+1}(t) - N_k(t)}{2} \right) \right] \end{aligned}$$

5. Numerical Results and Discussion

Theoretical answer of the model of diabetes and its complications with the fractional derivative of Caputo and Atangana Baleanu was explained by ABC derivative. In this model, C corresponds to the number of complicated diabetes and $N(i) = C(i) + D(i)$ refers to the size of diabetics in which initial conditions are $C(0) = C_0$ and $N(0) = N_0$, while the parameter I reflects the presence of diabetes mellitus, μ refers to the rate of natural death, ν corresponds to the rate of conversion of complicated diabetic patient into disabled diabetic patients, λ refers to the probability of a diabetic spreading a disease, γ relates to the rate of healing complications, and δ denotes the complicated mortality rate with $I = 60,000$, $\delta = 0.05$, $\mu = 0.02$, $\lambda = 0.02$, $\gamma = 0.08$ and $\nu = 0.05$. By utilizing Caputo and ABC fractional derivative, the numerical results for various

fractional estimations of η are acquired. Figures 1 and 2 refer to the graphical solution with the Caputo derivative of diabetes and its complications. From figures 3 and 4, we use fractional-order derivative of ABC. In figures 5 the comparison of Caputo sense derivative and ABC derivative for the $C(t)$ is represented.

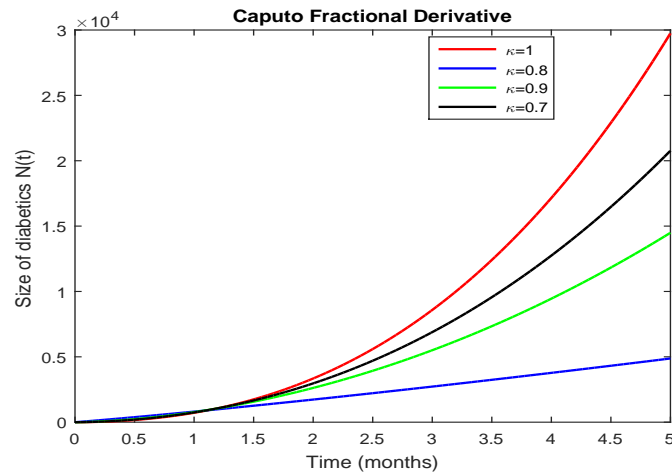


Figure 1: $C(t)$ diabetics having complications with Caputo fractional derivative

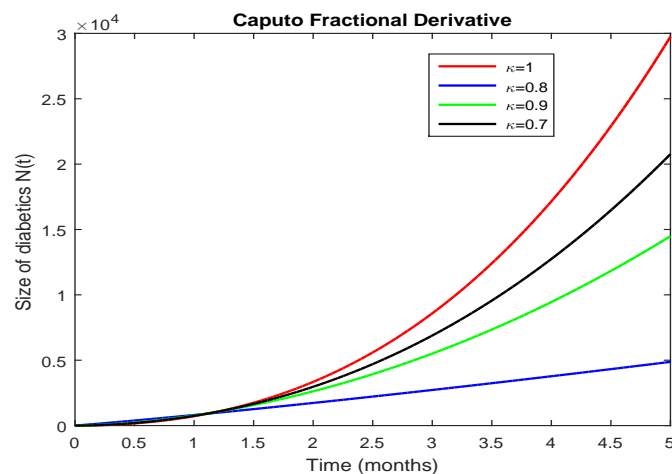


Figure 2: $N(t)$ size of diabetes with Caputo fractional derivative

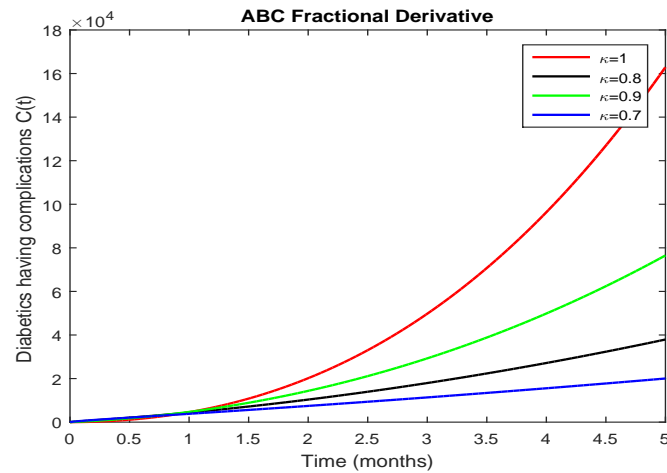


Figure 3: $C(t)$ diabetics having complications with ABC fractional derivative

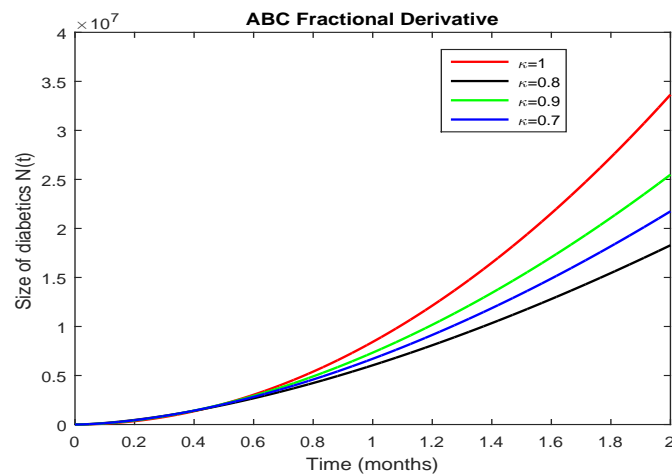


Figure 4: $N(t)$ size of diabetes with ABC fractional derivative

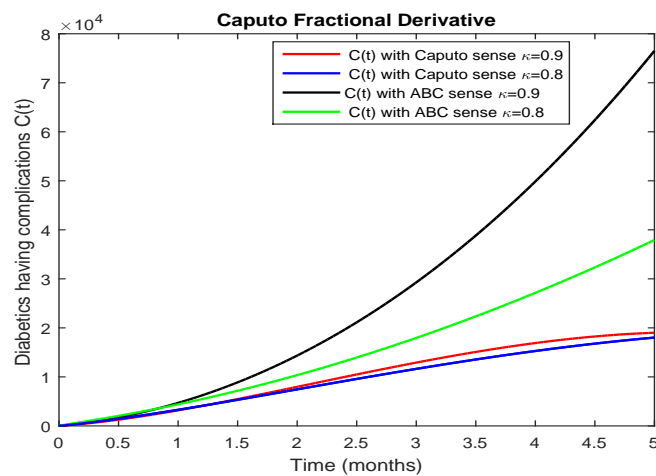


Figure 5: Comparison of $C(t)$ with ABC and Caputo fractional derivative

6. Conclusions

In this article, nonlinear fractional-order model with ABC derivative utilized for the treatment compartment of insulin is discussed. The premise of this fractional model is describe of non-singular exponentially diminishing kernels that shows up in the ABC derivation. Theoretical and numerical examination of the bio-medical glucose-insulin model are introduced. Fixed point theory and the Picardas Lindelof approach are aid to develop an investigation of answers. Effect generated by arbitrary order are overcome with the help of outcomes of numerical. Graphs are used to give information about the parameters of diabetes with complications and size of diabetes-related to time. ABC derivative gives continuous monitoring with finite time when contrasted with Caputo system in the human body. we see that the non-integer order of ABC fractional derivation reveals more engrossing attributes. The possibility of present research has significant results for diabetes restorative experts and its related confusions.

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