Avrupa Bilim ve Teknoloji Dergisi Özel Sayı, S. 87-93, Ağustos 2020 © Telif hakkı EJOSAT'a aittir **Arastırma Makalesi** 



European Journal of Science and Technology Special Issue, pp. 87-93, August 2020 Copyright © 2020 EJOSAT <u>Research Article</u>

# Artificial Neural Network-Based Adaptive PID Controller Design for Vertical Takeoff and Landing Model

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(Bu yayın 26-27 Haziran 2020 tarihinde HORA-2020 kongresinde sözlü olarak sunulmuştur.)

(DOI: 10.31590/ejosat.779085)

ATIF/REFERENCE: Taşören, A. E., Gökçen, A., Soydemir, M. U., Şahin, S. (2020). Artificial Neural Network-Based Adaptive PID Controller Design for Vertical Takeoff and Landing Model. *European Journal of Science and Technology*, (Special Issue), 87-93.

#### Abstract

This study presents an artificial neural network (ANN) based adaptive proportional integral derivative (PID) controller algorithm which is developed to control the pitch angle of the vertical takeoff and landing (VTOL) system model. To find appropriate conventional PID controller parameters, either step response or vibration method might be used in a way of the widely used approach. They provide constant values for the conventional PID controller parameters for a closed-loop system however it is obvious that the unsatisfied tracking error performances in the closed-loop system might be addressed as a problem to be optimized. To overcome this problem, the parameters of the proposed adaptive PID controller might be determined with an ANN constructed feedforward multilayer perceptron. The proposed controller algorithm possesses the gradient descent with momentum update rule for the adaptiveness of the obtained PID parameters. The proposed adaptive PID controller algorithm is tested for the pitch angle of the VTOL system model in the MATLAB/Simulink environment in terms of the sinusoidal and step signals as the desired outputs. The obtained results are compared to the conventional PID controller whose parameters are tuned by Simulink PID tuner application in terms of mean square error, integral absolute error, the settling time, and the percentage overshoot.

Keywords: Adaptive PID controller, Artificial Neural Networks, VTOL system.

# Dikey Kalkış ve İniş Sistemi Modeli için Yapay Sinir Ağı Tabanlı Uyarlanır PID Kontrolör Tasarımı

#### Öz

Çalışmada dikey kalkış ve iniş (VTOL) sistem modelinin eğim açısını kontrol etmek için geliştirilen yapay sinir ağı (YSA) tabanlı uyarlanabilir oransal integral türev (PID) denetleyici algoritması tasarlanmıştır. Uygun geleneksel PID denetleyici parametrelerinin hesaplanılmasında, basamak yanıtı ve titreşim metodu yaygın olarak kullanılan tekniklerdir. Bu yöntemler kapalı döngü sistemi içinde geleneksel PID kontrolör için sabit parametreler bulurlar, ancak kapalı döngü sistemindeki yetersiz izleme hatası performansının en iyi hale getirilmesi mevcut bir problem olarak gösterilebilir. Bu sorunun üstesinden gelmek için, önerilen uyarlanabilir PID denetleyicinin parametreleri bir ileri beslemeli çok katmanlı algılayıcı YSA yardımı ile sağlanabilir. Önerilen kontrolör algoritması, elde edilen uyarlanabilir PID denetleyici parametrelerinin uyarlanabilirliği için devinirlik güncellemeli gradyan inişi yöntemini kullanmaktadır. Önerilen uyarlanabilir PID denetleyici algoritması, MATLAB / Simulink ortamında VTOL sistem modelinin yunuslama açısı için arzu edilen çıkışlar sinüs ve birim basamak sinyalleri anlamında test edilmiştir. Elde edilen sonuçlar, parametreleri Simulink PID ayarlayıcı uygulaması ile hesaplanmış geleneksel PID denetleyici ile ortalama kare hata, integral mutlak hata, oturma süresi ve aşım yüzdesi açısından karşılaştırılır.

Anahtar Kelimeler: Uyarlanabilir PID kontrolör, Yapay Sinir Ağları, VTOL sistemi.

### 1. Introduction

The proportional-integral-derivative (PID) controller promotes an efficient solution for real-world control systems because of being an easy way of implementation for designing and controlling the controlled plant. So, the PID controller has still significant popularity

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for academic researches and industrial applications such as level, flow, temperature, position, and speed control (Astrom and Hagglund, 1995; Ogata, 2010; Taşören et al., 2018; Varseveld and Bone 1997; Yamamoto et al., 2009). The determination of the suitable PID controller parameters might be achieved with the tuning algorithms such as Ziegler Nichols' step response and vibration methods. These algorithms are useful to find suitable parameters because of in a way of the simple approach; however, they provide fixed values for the controller parameters, and corresponding their tracking error performances are generally being limited in the closed-loop control systems (Astrom and Hagglund, 1995; Ogata, 2010; Skogestad, 2003; Taşören et al., 2018). In the literature, the design of adaptive PID controller schemes are developed with different approaches in terms of self-tuning algorithms (Clarke and Gawthrop, 1975; Vega et al., 1991; Wittenmark and Astrom, 1980; Yamamoto and Shah, 2004; Yamamoto et al., 2009). Clarke and Gawthrop (1975) showed that a general minimum variance could be used for the adaptive PID controller design approach. Wittenmark and Astrom (1980) developed the pole placement based auto-tuning PID controller scheme applications for the first time. Vega et al. (1991) developed the continuous-time autoregressive moving-average model-based general minimum variance controller to regulate the adaptive PID controller parameters in a way of self-tuning. Yamamoto and Shah (2004) proposed the Nyquist array method model based on the multivariate self-tuning PID controller algorithm against the plant having nonlinearities and its time-varying parameters via in online mode.

With advances in applications of artificial neural networks (ANN), ANN-based controllers possessing adaptation and generalization properties give useful opportunities for controlling both linear and nonlinear dynamical systems (Kumar et al., 2014; Kumar et al., 2016; Mosaad and Salem, 2014; Muruganandam and Madheswaran, 2013, Nohooji, 2020). Muruganandam and Madheswaran (2013) developed a hybrid PID controller with an ANN for enhancing the performance of DC series motor fed by a buck converter. Mosaad and Salem (2014) reported a sophisticated adaptive PID controller design for the load frequency control of power systems via a neuro-fuzzy inference system and an ANN modified genetic algorithm. Kumar et al. (2014) studied on a conventional PID controller providing robustness and adaptiveness via an ANN composed of recurrent neural networks. The proposed controller might be tuned for a permanent magnet synchronous motor position control problem in an online manner. Kumar et al. (2016) proposed an adaptive ANN-based PID controller for online control of a second-order and a dc motor system. Nohooji (2020) proposed an adaptive ANN-based PID controller design for a robot manipulator under the unknown exact dynamical model and disturbance effects.

In this study, the developed ANN-based adaptive PID control scheme is designed for controlling the pitch angle of the vertical takeoff and landing (VTOL) system model (Quanser, 2011). The constant values for the PID controller parameters determined via step response from the VTOL system model do not meet expectations on tracking error performances in the closed-loop system exactly. To improve the tracking error performance, the proposed adaptive PID controller is implemented with an ANN whose algorithm possessing the gradient descent with momentum update rule. It promotes the adaptiveness property for the obtained updated PID controller parameters. The proposed adaptive PID controller algorithm is examined with the desired outputs such as sinusoidal and step signals for the pitch angle of the VTOL model in the MATLAB/Simulink environment. The obtained results of the proposed adaptive PID controller whose parameters are tuned by Simulink PID tuner application in terms of mean square error (MSE), integral absolute error (IAE), the settling time and the percentage overshoot.

The rest of the paper is represented in the following sections. In Section 2, the description of the VTOL setup is given briefly and the proposed adaptive PID controller is presented. In Section 3, the simulation results are given, and conclusions and recommendations are presented in Section 4.

### 2. Material and Method

#### 2.1. VTOL System Model

Of nonlinear plant models in the control systems field, unmanned air vehicles (UAV) such as plane, quadrotor, helicopter plants have been recently used as a benchmark nonlinear UAV plants and their models to evaluate the tracking error performances of the developed candidate adaptive and/or robust controllers (Dydek et al, 2013; Ma et al, 2019). The VTOL system is widely used as a benchmark UAV plant and model in the control laboratory of the department of electrical and electronics engineering. VTOL system consists of a body rod whose one side attached to a variable speed fan called a propeller actuator, whereas the other side of the rod is attached to a counterweight (Quanser, 2011). It can mimic the behaves of the helicopters, and planes in terms of the pitch angle. The transfer function of the VTOL system is given in Eq. 1 where Y(s) is the controlled output called the pitch angle variable, and U(s) is the input voltage applied from the controlled voltage variable of the VTOL system. The free-body diagram of the VTOL system model is given in Fig. 1. The parameters, symbols, values, and units of the considered VTOL system model are given in Table 1 (Quanser, 2011).

$$\frac{Y(s)}{U(s)} = \frac{3.11}{s^2 + 0.576s + 10.7} \tag{1}$$

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Fig. 1. The Diagram of the VTOL System Model

Parameter	Symbol	Value	Unit
Equilibrium Current	I <sub>eq</sub>	1.0	A
Torque-thrust Constant	K <sub>t</sub>	0.0226	(Nm)/A
Moment of Inertia	J	0.0035	kgm <sup>2</sup>
Viscous Damping	В	0.002	(Nms)/rad
Natural Frequency	$\omega_n$	2.52	rad
Stiffness	K	0.022	(Nm)/rad
Measured Torque-thrust Constant	K <sub>tid</sub>	0.01	(Nm)/A
Measured Viscous Damping	B <sub>id</sub>	0.006	(Nms)/rad
Measured Stiffness	K <sub>id</sub>	0.015	(Nm)/rad
Length of the setup	$L_h$	0.3	m

Table 1. VTOL System Model Parameters and Their Values

#### 2.2. The Proposed Adaptive PID Control with ANN

The mathematical expression of the conventional PID controller is given in the following form (Eq.2). Herein, the parameters values of the PID controller might be determined as constant values by using the Ziegler Nichols's first method (Ogata, 2010).

$$u(t) = K_p e_t(t) + K_i \int e_t(t) dt + K_d \frac{de_t(t)}{dt}$$
(2)

where  $K_p$  is the proportional gain,  $K_i$  is the integral gain,  $K_d$  is the derivative gain,  $e_t(t)$  is the error signal of the closed-loop system, and u(t) is the controller output signal.

ANN ensure a learning process for input-output data of a system via considered learning error criteria. In the training stage, a multilayer perceptron ANN was used as a feedforward architecture in the manner of offline mode. The MLP has a single hidden layer with 20 neurons and it is given as a block diagram in Fig. 3. Herein, mathematical expression of the MLP-ANN and its learning error criteria are given in Eq.3



Fig. 2. MLP-ANN Architecture with Hidden Layer

$$\hat{y} = W_{out}^{T} tanh(W_{in}^{T} X + b_{in}) + b_{out}$$

$$E = \frac{1}{2} \sum (e)^{2} = \frac{1}{2} \sum (y - \hat{y})^{2}$$
(3)

e-ISSN: 2148-2683

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where X is the input vectors of the ANN, y'stands for  $W_{in}^T X + b_{in}$ , y'' is the output after the activation function,  $\hat{y}$  is the output of the ANN, e is the error signal, E is the mean square error used as a cost function of the learning error,  $b_{in}$ ,  $b_{out}$  and  $W_{in}$ ,  $W_{out}$  are Bayes and weights of the ANN, respectively.

The proposed adaptive PID control algorithm is based on the MLP-ANN trained model where the Jacobian matrix of the VTOL system model might be computed at each control-loop iteration and its block diagram is given in Fig. 3. The algorithm strategy includes the Jacobian matrix to find the gradient vector consisting of the PID controller parameters as  $K_p$ ,  $K_i$  and  $K_d$  (Chen and Huang, 2004). In the proposed algorithm, they are determined with the extension of the chain rule of the backpropagation algorithm in given Eq. 4 by using Eq. 2 and Eq. 3.



Fig. 3. The Proposed Adaptive PID Controller Scheme

$$\frac{dy}{du} \approx \frac{d\hat{y}}{dx} = W_{in}^T W_{out}^T \frac{dy''}{dy'}$$

$$\frac{dE}{dK_{p,i,d}} = \frac{dE}{de} \frac{de}{d\hat{y}} \frac{d\hat{y}}{dX} \frac{du}{dK_{p,i,d}}$$
(4)

where  $\frac{dy}{du} \cong \frac{d\hat{y}}{dx}$  is the Jacobian matrix of the ANN,  $\frac{dy''}{dy'} = 1 - tanh^2(y')$  is the derivative of the activation function with respect to its input variable,  $\frac{dE}{dK_{p,i,d}}$  are the derivatives of the cost function with respect to the PID parameters from Eq.3,  $\frac{dE}{de} = e$  is the derivative of the cost function with respect to the error from Eq.3 where  $\frac{de}{d\hat{y}}$  is determined as -1, and  $\frac{du}{dK_{p,i,d}}$  are the derivatives of the control signal u(t) with respect to the PID parameters. They might be assumed as  $\frac{du}{dK_p} = e_t(t), \frac{du}{dK_i} = \int_0^\infty e_t(t)dt$ , and  $\frac{du}{dK_d} = \frac{de_t(t)}{dt}$  where  $e_t(t)$  is the closed-loop system error from Eq.2.

In this study, to update the adaptive PID parameters, they are computed with a gradient descent method with the momentum term borrowed from (Yu et al., 1995) at each iteration is given in the following form (Eq.5) and the pseudo-code of the proposed algorithm is given in Fig. 4.

$$K_{p,i,d}(n) = K_{p,i,d}(n-1) - \alpha V K_{p,i,d}(n)$$

$$V K_{p,i,d}(n) = \beta \nabla_{K_{p,i,d}}(n-1) + (1-\beta) \nabla_{K_{p,i,d}}(n)$$
(5)

where  $K_{p,i,d}$  represents the PID controller parameters,  $\nabla_{K_{p,i,d}} = \frac{du}{dK_{p,i,d}}$  is computed from Eq.4, *n* represents the iteration number,  $\alpha$  is the learning rate, and  $\beta$  is the momentum term.

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Train: A	NN to compute $d\hat{y}/dX$ .
End trair	ning of ANN.
Start: ini	tialize $\alpha$ , $\beta$ , $K_{p,i,d}$ , $oldK_{p,i,d}$ , $old\nabla K_{p,i,d}$ .
While()	
Read:	actual pitch angle
Comp	ute error difference between the desired and actual pitch angle
	$d\hat{y}/dX$ is called from the trained ANN.
	$\nabla K_{p,i,d}$ by using Eq.4
	$VK_{p,i,d}$ by using Eq.4
	$K_{p,i,d}$ by using Eq.5
Updat	e: $VK_{p,i,d}$ as $old \nabla K_{p,i,d}$ .
	$K_{p,i,d}$ as $oldK_{p,i,d}$ .
Comp	<b>ute</b> controller output $u(t)$ by using Eq.2.
Contr	ol VTOL system output $y(t)$ in Eq.1.
End	

#### 3. Results and Discussion

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The proposed adaptive PID controller algorithm is implemented in the software in the loop of the real-time simulation model with MATLAB/Simulink simulation software environment (Sahin et al., 2016). The prepared codes of the proposed algorithm and VTOL system model are run with a personal computer having i7 microprocessor, 16 GB of RAM, and Windows 10 operating system. The sampling time is chosen as 1ms and the duration of each simulation study is 10s.

In the simulation studies, the MLP-ANN model of the VTOL system model is firstly trained in terms of the dy/du term given in Eq.4 of the VTOL system model, and its block diagram is given in Fig. 3 where the input vector of the MLP-ANN is chosen as  $X = \{y, y(t-1), u, u(t-1)\}$  with time delay operator as  $q^{-1}$ . Then, in given Fig. 4, the developed adaptive PID control algorithm is tested on the VTOL system model with the obtained controller parameters as  $K_p$ ,  $K_i$  and  $K_d$  which are updated with a gradient descent method with the momentum term given in the Eq.5. As for the simulation results, to observe the performance results for the proposed adaptive PID controller is given in time-wave forms in Fig. 5a. The time evolutions of the proposed adaptive PID controller parameters and the control signal are depicted 5b and 5c, respectively.



Fig. 5. The Responses of the Proposed Adaptive PID and the Conventional PID Controller for (a) the Sinusoidal Signal with 1Hz, (b) Time Evolutions of the Proposed Adaptive PID Controller Parameters, and (c) Time Evolution of the Control Signal u(t)

The responses of the sinusoidal and step signals of the desired VTOL pitch angle are depicted in time-wave forms in Fig. 6a and 6b, respectively. The fixed gain parameters of the conventional PID controller parameters tuned by the MATLAB PID tuner application function are determined as  $K_p = 29.599$ ,  $K_i = 34.108$  and  $K_d = 4.607$ . The performances of the proposed adaptive PID controller are compared to the performances of the conventional PID controller in terms of MSE and IAE for the two different desired outputs of the pitch angles such as sinusoidal and step signals. The obtained results are given in Table 2 where the results of MSE values are nearly identical values. However, the IAE values of the proposed adaptive PID controller are found as 0.3169 and 0.02096 for the sinusoidal and step signals of the desired outputs, respectively, and they are less than IAE values of the conventional PID controller.



Fig. 6. The Responses of the Proposed Adaptive PID and the Conventional PID Controller for the Desired Outputs (a) Sinusoidal Signal with 1Hz and (b) Step Signal

Table 2. Comparison of the Results of the Controller Performances in terms of Sinusoidal and Step Signal as Desired Outputs

	Sinusoidal signal with 1Hz		Step signal	
Method	MSE	IAE	MSE	IAE
Conventional PID	$4.231 \times 10^{-3}$	0.5857	$2.543 \times 10^{-4}$	0.04334
Proposed Adaptive PID	$1.233 \times 10^{-3}$	0.3169	$1.620 \times 10^{-4}$	0.02096

The performances of the proposed adaptive PID controller are compared to the performances of the conventional PID controller in terms of settling time and percentage overshoot for the desired output of the pitch angles as a step signal. The obtained results are given in Table 3 where the results of settling time and percentage overshoot values of the proposed adaptive PID controller are found as 0.48 and 0.25, respectively. The obtained results are better performances and dramatically less than the conventional PID controllers' values.

Table 3. Comparison of the Settling Time and Percentage Overshoot Results for the Step Signal as Desired Output

Method	Settling Time (s)	Percentage Overshoot (%)
Conventional PID	1.29	18.6
Proposed Adaptive PID	0.48	0.25

## 4. Conclusions and Recommendations

In this study, the proposed adaptive PID controller is developed by using MLP-ANN and adaptation rule in order to control the pitch angle of the VTOL system model on the MATLAB/Simulink simulation environment. The adaptation rule of the PID controller parameters is provided with a backpropagation method with gradient descent having a momentum term. The proposed adaptive PID controller algorithm is tested for the pitch angle in terms of the sinusoidal and step signals as the desired outputs. The obtained results are compared to the conventional PID controller whose parameters are tuned by Simulink PID tuner application in terms of MSE, IAE, the settling time, and the percentage overshoot. The tracking error performances of the proposed adaptive PID and the conventional PID controller are closely the same values in terms of MSE values which are less than  $4.231 \times 10^{-3}$  for both sinusoidal and step signals of the desired outputs. However, the IAE values of the proposed adaptive PID controller are less than the IAE values of the conventional PID controller. They are determined as 0.3169 and 0.02096 for the sinusoidal and step signals of the desired outputs, respectively. As for comparing the settling time and percentage overshoot, the results of the proposed adaptive PID controller are found as 0.48 and 0.25, respectively. The obtained results are better performances than the conventional PID controller are found as 0.48 and 0.25, respectively. The obtained results are better performances than the conventional PID controllers' values. In future directions, a data-dependent ANN model might be extended for the adaptive control algorithm which might be a pattern recognition and reinforcement learning in online mode for the VTOL system model.

# 4. Acknowledge

This work is supported by the Scientific and Technical Research Council of Turkey (TUBITAK) under 2209A – Research Project Support Programme for Undergraduate Students with project number 1919B011901769.

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