



# Stabilization of Inverted Pendulum System Using Fuzzy Cognitive Map Based PD Controllers

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## Abstract

Soft computing techniques are frequently used in modeling and control applications of nonlinear systems. However, the fuzzy cognitive map method, which is one of the soft computing techniques, is rarely used in control applications as a main controller. In this study, a fuzzy cognitive map based PD controller structure is introduced and used for the stabilization of an inverted pendulum system which is a nonlinear, unstable, and under-actuated system. The proposed controller has two inputs which are the error and the change of error. In the proposed PD controller structure, the crisp input values are fuzzified to be handled in a fuzzy cognitive map process. Then, causal relationships between fuzzified inputs and a control output are defined by using weight parameters. Finally, the crisp control output value which will be applied to the system is obtained by using an activation function. The types of membership functions used for the fuzzification process and the activation function determine the nonlinear characteristics of the proposed fuzzy cognitive map based PD controller. The proposed controller has three tuning parameters which are one output gain and two weight parameters. To show the effectiveness and robustness of the proposed fuzzy cognitive map based PD controller, simulation studies are performed on an inverted pendulum system. Additionally, the performance of the proposed controller is compared with a PD controller. All controller parameters are determined by using a genetic algorithm. Comparison results indicate that the proposed fuzzy cognitive map based PD controller shows better control performance than the classical PD controller.

**Keywords:** Fuzzy cognitive map, Membership function, Nonlinear system, Inverted pendulum, PD control, Stabilization.

## Bulanık Bilişsel Harita Tabanlı PD Kontrolörler Kullanılarak Ters Sarkaç Sisteminin Stabilizasyonu

### Öz

Yumuşak hesaplama yöntemleri doğrusal olmayan sistemlerin modellenmesi ve kontrolünde sıklıkla kullanılan yöntemlerdir. Buna karşın, yumuşak hesaplama yöntemlerinden biri olan bulanık bilişsel harita yöntemi, kontrol uygulamalarında ana kontrolör olarak nadiren kullanılmaktadır. Bu çalışmada bulanık bilişsel harita tabanlı PD kontrolör yapısı önerilmiş ve bu yöntem doğrusal olmayan, kararsız ve eksik tahrikli bir sistem olan ters sarkaç sisteminin stabilizasyonu için kullanılmıştır. Önerilen kontrolör hata ve hatanın değişimi olmak üzere iki adet giriş değişkenine sahiptir. Önerilen PD kontrolör yapısında, kesin değerlere sahip girişler bulanık bilişsel harita hesaplama sürecinde değerlendirilebilmek için bulanıklaştırılmaktadır. Ardından bulanıklaştırılmış bu girişler ile kontrolör çıkışı arasındaki nedensel ilişkiler ağırlıklandırma parametreleri kullanılarak tanımlanmaktadır. Son olarak ise bir aktivasyon fonksiyonu kullanarak sisteme uygulanacak kesin çıkış değeri elde edilmektedir. Bulanıklaştırma işlemi için kullanılan üyelik fonksiyonlarının yapıları ve ayrıca aktivasyon fonksiyonunun yapısı, önerilen bulanık bilişsel harita tabanlı PD kontrolörün doğrusal olmayan karakteristiğini belirlemektedir. Önerilen kontrolör bir çıkış kazancı ve iki ağırlık parametresi olmak üzere üç adet ayar parametresine sahiptir. Önerilen bulanık bilişsel harita tabanlı PD kontrolörün etkinliğini ve dayanıklılığını göstermek için ters sarkaç sistemi üzerinden benzetim çalışmaları gerçekleştirilmiştir. Ayrıca önerilen PD kontrolörün performansı geleneksel yapıdaki PD kontrolör performansı ile karşılaştırılmıştır. Tüm kontrolör parametreleri genetik algoritma kullanılarak belirlenmiştir.

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Karşılaştırma sonuçları göstermiştir ki önerilen bilişsel harita tabanlı PD kontrolör geleneksel PD kontrolörden daha iyi bir kontrol performans göstermektedir.

**Anahtar Kelimeler:** Bulanık bilişsel harita, Üyelik fonksiyonu, Doğrusal olmayan sistem, Ters sarkaç, PD kontrol, Stabilizasyon.

## 1. Introduction

Modeling and control of nonlinear systems are more challenging tasks than the modeling and control of linear systems. Soft computing methods can provide effective solutions compared to classical modeling and control methods [1-6]. The fuzzy cognitive map (FCM) method is one of the soft computing techniques and developed by Kosko in 1986 as a fuzzy counterpart of the classical cognitive map method [7]. The FCM approach is effectively used in many areas such as engineering, economy, health, security, and management since it has the potential to express complex systems in an easy and understandable manner [8-15].

From the control engineering point of view, researchers have commonly utilized the FCM approach in supervisor controller design [16-18]. However, there are few studies in literature that show the effectiveness of using the FCM approach in the main controller design. In [19], the controllers designed based on the FCM approach are successfully used in a visual-based control application of a quadrotor. In [20], embedded dynamic fuzzy cognitive maps are used for control of an industrial mixer and satisfactory performance results are obtained compared with a fuzzy logic controller. In [21], the fuzzy cognitive map control is applied in controlling the temperature and humidity of a room. The obtained results show that FCM controller provides higher energy saving than a PID controller. In [22], a controller based on the fuzzy cognitive map approach is used for control of an industrial heatex process. In [23], three different PI-type FCM controllers are proposed to control nonlinear systems and the effective control performances of the proposed controllers are shown through simulation studies by comparing with a classical PI controller and a fuzzy PI controller.

The inverted pendulum is a nonlinear, unstable, under-actuated, and non-minimum phase system [24]. These characteristics make the control of these systems more challenging. Additionally, many complex systems, such as cranes, rockets, humanoid robots, two-wheeled self-balancing transporters, can be approximated by inverted pendulum systems. Thanks to these properties, the inverted pendulum system is widely used as a benchmark for the validation and performance comparison of control strategies. In literature, various classical and modern control methods are successfully applied in the control of inverted pendulum systems [25] such as PID control [26], linear quadratic regulator control [27], sliding mode control [28-29] Lyapunov based control [30] predictive control [31-32] fractional order PID control [33], fuzzy logic control [34-35], and neural network control [36].

In this study, a new control approach based on the fuzzy cognitive map method is proposed and applied in the control of the inverted pendulum system. The proposed controller is in a PD controller form and it has two inputs (error and derivative of error) and one control output. By using the FCM approach, causal relationships between controller inputs and the output are modeled. To apply the FCM concept, the crisp input values are firstly fuzzified by using membership functions. Then, by utilizing weight parameters, the causal relationships between fuzzified inputs and the control output are specified. Finally, an activation function is used to determine the crisp control output value which will be applied to the system. The nonlinear characteristic of the proposed FCM based PD controller directly depends on the types of chosen membership functions and the activation function. Therefore, by using appropriate membership functions and the activation function, the nonlinear characteristic of the controller can easily be increased. There are three tuning parameters in the proposed controller model. These are two weight parameters of causal relationships and one output gain. The effectiveness and robustness of the proposed FCM based PD controller are shown by simulation results. A classical PD controller is used in the simulation studies for the comparison purpose. The genetic algorithm is chosen for the determination of controller parameters and the integral time absolute error criterion is used for the performance criterion. Comparison results show that the proposed fuzzy cognitive map based PD controller outperforms the classical PD controller.

The paper is organized as follows. In Section 2, the inverted pendulum system is given. In Section 3, the proposed FCM based PD controller is introduced. In Section 4, simulation studies are given. Finally, in Section 5, the results and future studies are provided.

## 2. Inverted Pendulum System

The inverted pendulum system is shown in Figure 1. This system consists of a motor-driven cart that can move only horizontally and an inverted pendulum mounted on the cart. Therefore, it has two degrees of freedom of motion. In this system, it is aimed to control the position of the cart by stabilizing the pendulum in the upright position. This system is a nonlinear, non-minimum phase, under-actuated, and unstable system. Therefore, control of this system is a challenging task. This system is commonly used as a classical benchmark system for the validation, evaluation, and comparison of control methods.

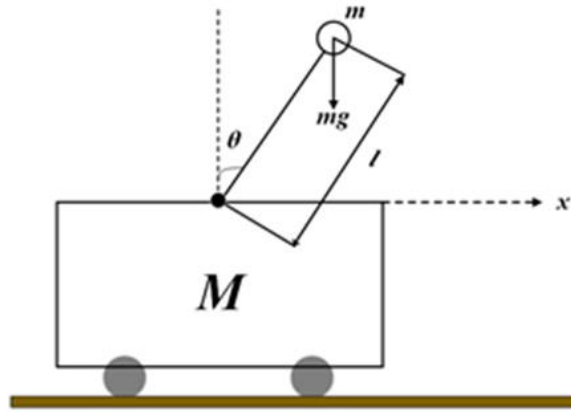


Figure 1. Inverted pendulum system on a cart

The nonlinear mathematical model of the inverted pendulum system is as follows [27]

$$\ddot{x} = \frac{u + ml(\sin \theta)\dot{\theta}^2 - mg \cos \theta \sin \theta}{M + m - m \cos^2 \theta} \quad (1)$$

$$\ddot{\theta} = \frac{u \cos \theta - (M + m)g \sin \theta + ml(\cos \theta \sin \theta)\dot{\theta}^2}{ml \cos^2 \theta - (M + m)l} \quad (2)$$

The definition of system states and parameters are shown in Table 1.

Table 1. States and Parameters of Inverted Pendulum System

State/Parameter	Description
$u$	Control force
$\dot{\theta}, \ddot{\theta}$	Angular velocity and angular acceleration
$x, \dot{x}, \ddot{x}$	Cart position, velocity, and acceleration
$M$	Cart Mass
$m$	Pendulum mass
$l$	Pendulum length

The corresponding system parameter values are given in Table 2.

Table 2. System Parameter Values

Symbol	Description	Value	Unit
$Ref - x$	Reference position	0.1	m
$Ref - \theta$	Reference angle	0	Radian
$M$	Cart Mass	2.4	kg
$m$	Pendulum mass	0.23	kg
$l$	Pendulum length	0.36	m
$g$	Gravity	9.81	kg/m <sup>2</sup>

### 3. Proposed Fuzzy Cognitive Map Based PD Controller

In this section, a new nonlinear PD controller based on FCM is introduced. The structure of the proposed controller model is shown in Figure 2 . This model has two inputs as the error ( $e$ ) and the error change ( $de$ ). The fuzzification process is applied to each input. In this way, crisp input values are converted to fuzzy values to be handled in the FCM process. The weight parameters,  $W_p$  and  $W_d$ , define the causal relationships between inputs and the output. The control signal crisp value which will be applied to the system is determined by using an activation function. Additionally, an output gain  $K$  is used to amplify the control signal level. Therefore, there are three tuning parameters in the proposed controller. In this model, types of membership functions and the activation function determine the nonlinear characteristic of the proposed PD controller.

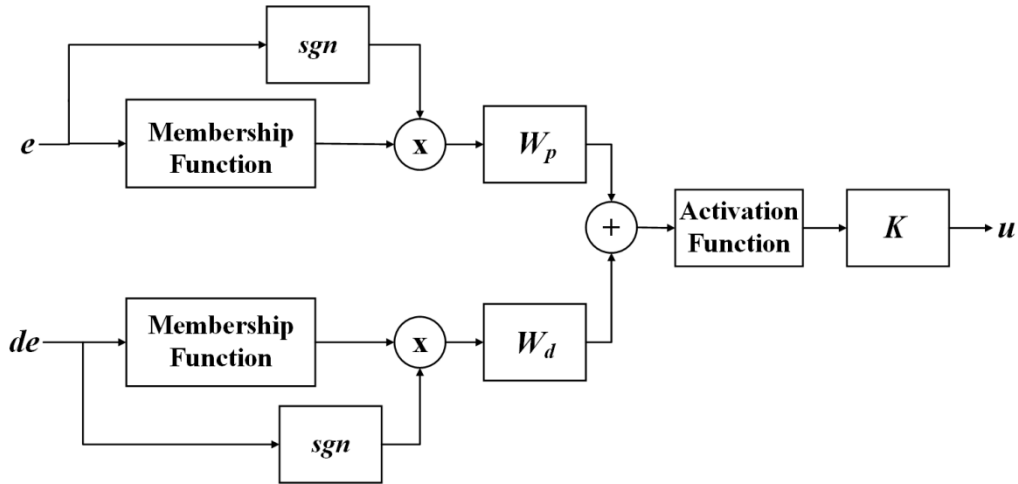


Figure 2. FCM based PD controller structure

The mathematical definition of the control law is as follows

$$u = f(\text{sgn}(e)\mu(e)W_p + \text{sgn}(de)\mu(de)W_d)K \tag{3}$$

where  $\mu$  is the membership function,  $f(.)$  is the activation function, and  $\text{sgn}(\cdot)$  indicates the sign function. The sign function is included in the model since fuzzified inputs possess only positive values in the interval of [0 1].

### 4. Simulation Studies

In simulation studies, the inverted pendulum system given in Section 2 is used. The control system model for the stabilization of the inverted pendulum system is given in Figure 3. Here, two FCM based PD controllers are used for the control of the pendulum angle and the cart position.

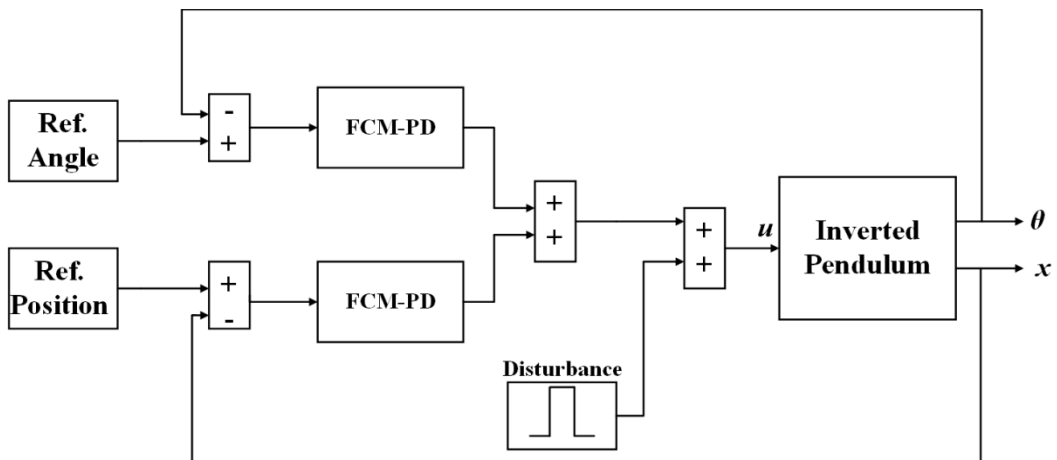


Figure 3. Control system model

In the simulations, the sampling time is chosen as 0.01 seconds. The desired cart position is 0.1m. For the pendulum angle controller, the membership functions of error and error change inputs are chosen as shown in Figure 4 by considering corresponding ranges of input variables.

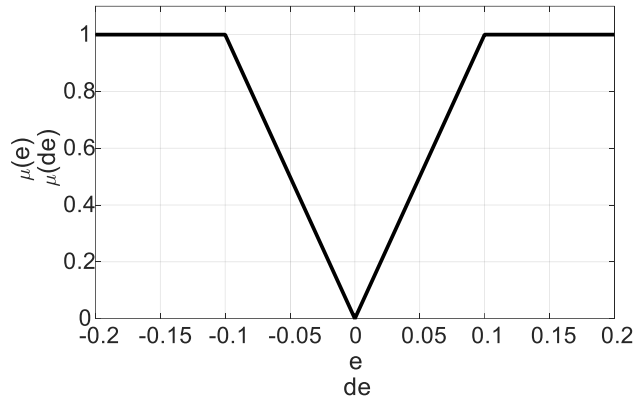


Figure 4. Membership functions used for the pendulum angle controller

The error membership function of the cart position controller is chosen as shown in Figure 5. Here, in order to show the effectiveness and design flexibility of the proposed FCM based PD controller, different type of membership function is used for the error change variable as shown in Figure 6. This nonlinear membership function is obtained by applying the hedge operator with the value of 0.85 to the membership function given in Figure 5 as  $\mu(de) = (\mu(e))^{0.85}$  [37].

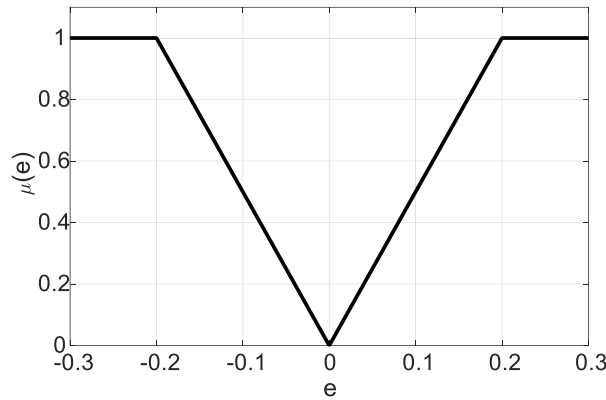


Figure 5. Error membership function used for the car position controller

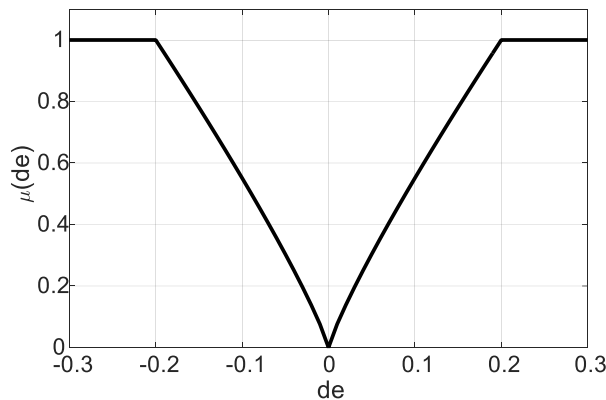


Figure 6. Error change membership function used for the car position controller

The activation function is chosen as  $f(x) = x$  for both controllers. The genetic algorithm is used in order to determine the appropriate parameter values of the FCM based PD controllers. In the optimization process, the population size and the number of generations are chosen as 50 and 100, respectively. The performance index for the optimization process is chosen as follows

$$Performance\ index = ITAE_{angle} + ITAE_{position} \quad (4)$$

Here  $ITAE_{angle}$  and  $ITAE_{position}$  denote the integral time absolute error values of the pendulum angle and the cart position, respectively. The obtained parameter values are shown in Table 3.

Table 3. Determined Parameter Values of FCM-PD Controllers

FCM-PD	Parameter	Value
Pendulum angle controller	$W_p$	-0.914
	$W_d$	-0.196
	$K$	14.55
Cart position controller	$W_p$	-0.804
	$W_d$	-0.396
	$K$	10.68

For performance comparison purpose, a classical PD controller is used in the simulation studies. The parameter values of the classical PD controller are determined by using the genetic algorithm with the same configuration. The determined parameter values are given in Table 4.

Table 4. Parameter Values of Classical PD Controllers

PD	Parameter	Value
Pendulum angle controller	$K_p$	-58.66
	$K_d$	-8.33
Cart position controller	$K_p$	-7.99
	$K_d$	-7.29

The performance comparison results are shown in Figure 7 and Table 5 for the nominal condition. The settling time criterion is chosen as the time that cart position response settles within  $\pm 5\%$  of its final values.

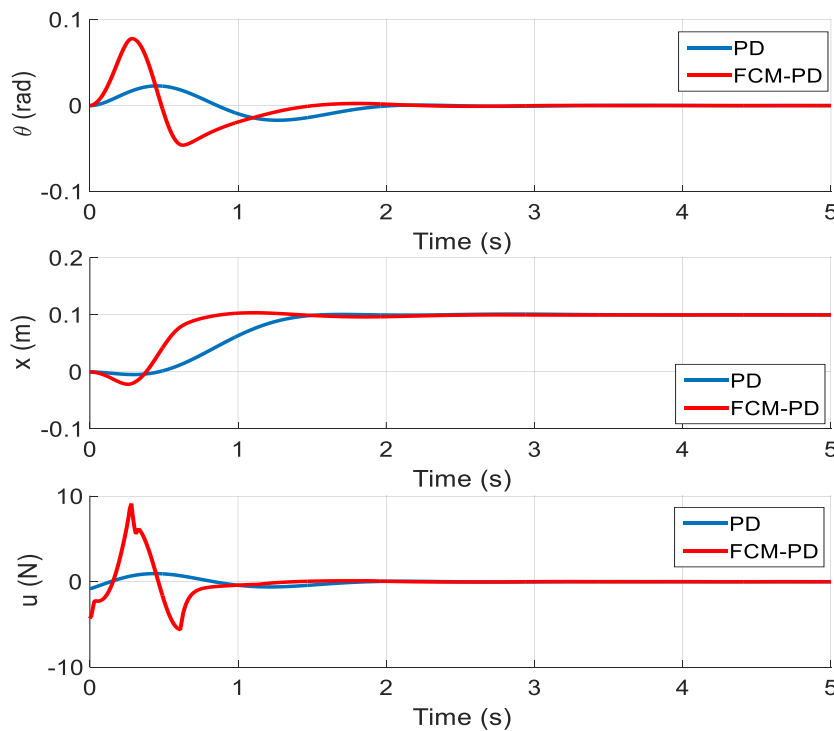


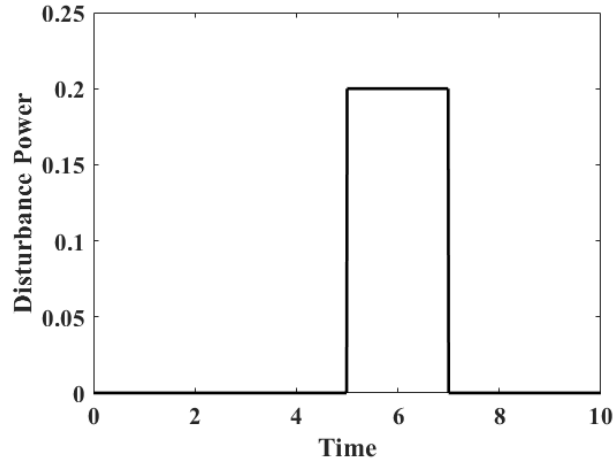
Figure 7. System responses for nominal condition

Table 5. Performance Comparison Results for Nominal Condition

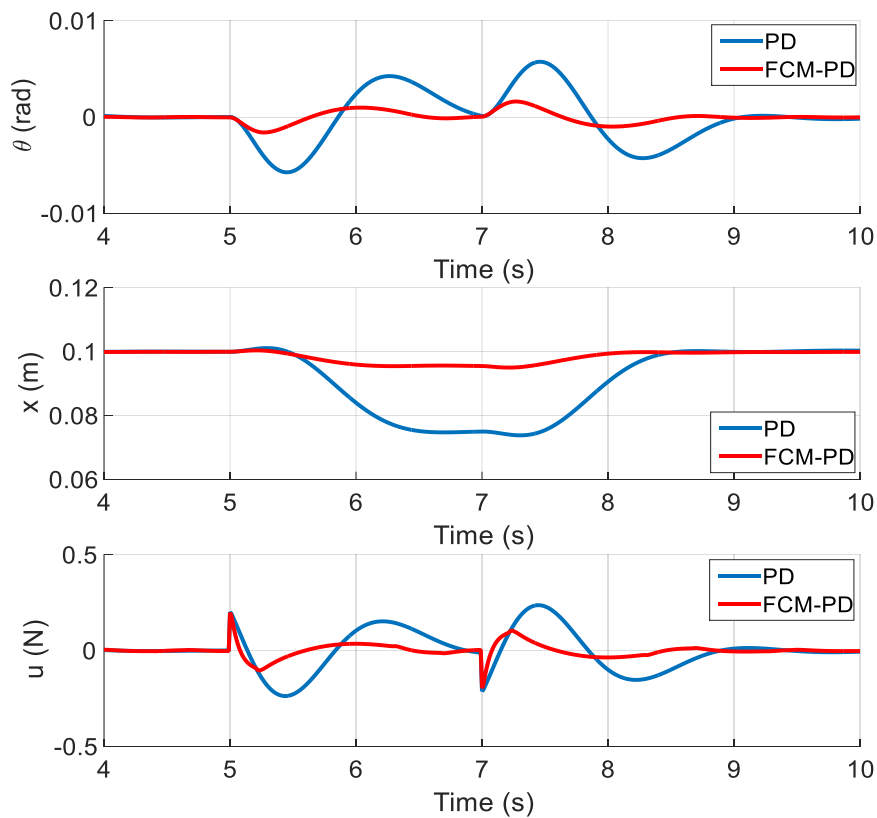
Controller	Settling Time (s)	ITAE <sub>angle</sub>	ITAE <sub>position</sub>	ITAE Total
PD Controller	1.37	0.0227	0.0495	0.0722
FCM-PD Controller	0.78	0.0261	0.0228	0.0489

As it is seen from the performance comparison results, the proposed FCM based PD controllers outperform the classical PD controllers by providing very low settling time as 0.78 seconds. On the other hand, approximately twice longer settling time value, 1.37 seconds, is obtained by using classical PD controllers.

To evaluate the robustness performance of the proposed FCM based PD controller and the classical PD controller, the disturbance with 0.2N amplitude given in Figure 8 is applied to control signals in the steady-state condition. The obtained pendulum angle and cart position responses and the corresponding control signals are given in Figure 9 for the steady-state time interval of [4 10] seconds.



*Figure 8. Control signal disturbance [N]*



*Figure 9. System responses with control signal disturbance*

As it is seen from Figure 9, the proposed FCM based PD controllers show higher robustness than the classical PD controllers in the presence of the input disturbance.

## 4. Conclusions

In this study, the fuzzy cognitive map based PD controller structure is introduced and used for the stabilization of an inverted pendulum system. The results are compared with the classical PD controller. Comparison results show the effectiveness and robustness of the proposed FCM based PD controller.

The proposed FCM based PD controller structure provides several advantages over the classical PD controller. The FCM based PD controller is a nonlinear controller and its nonlinear characteristics can easily be increased by using different types of membership functions and the activation function. Therefore, the proposed controller can provide higher control performance, especially for nonlinear systems. Additionally, the proposed FCM based PD controllers can easily be replaced with the existing PD controllers since both controllers have the same base form. In this way, the control performance can be improved further especially in the nonlinear system control applications.

In future studies, the effects of using different types of membership functions and activation functions on controller performance will be evaluated on a practical application.

## References

- [1] Er, M., Deng, C., & Wang, N. (2018). A Novel Fuzzy Logic Control Method for Multi-Agent Systems with Actuator Faults. 2018 IEEE International Conference On Fuzzy Systems (FUZZ-IEEE), 1-7.
- [2] Amuthameena, S., & Monisa, S. (2017). Design of fuzzy logic controller for a non-linear system. 2017 IEEE International Conference On Electrical, Instrumentation And Communication Engineering (ICEICE), 1-7.
- [3] Azizi, M., Ejlali, R., Mousavi Ghasemi, S., & Talatahari, S. (2019). Upgraded Whale Optimization Algorithm for fuzzy logic based vibration control of nonlinear steel structure. *Engineering Structures*, 192, 53-70.
- [4] Zhang, X., Ma, X., Zhu, H., & Liu, H. (2017). Neural network controller design for uncertain nonlinear systems based on backstepping control algorithm. 2017 29Th Chinese Control And Decision Conference (CCDC), 1623-1627.
- [5] Zhang, Y., Tao, G., & Chen, M. (2016). Adaptive Neural Network Based Control of Noncanonical Nonlinear Systems. *IEEE Transactions On Neural Networks And Learning Systems*, 27(9), 1864-1877.
- [6] Slama, S., Errachdi, A., & Benrejeb, M. (2018). Model reference adaptive control for MIMO nonlinear systems using RBF neural networks. 2018 International Conference On Advanced Systems And Electric Technologies (IC\_ASET), 346-351.
- [7] Kosko, B. (1986). Fuzzy cognitive maps. *International Journal Of Man-Machine Studies*, 24(1), 65-75.
- [8] Papageorgiou, E., & Salmeron, J. (2013). A Review of Fuzzy Cognitive Maps Research During the Last Decade. *IEEE Transactions On Fuzzy Systems*, 21(1), 66-79.
- [9] Arruda, L., Mendonca, M., Neves, F., Chrun, I., & Papageorgiou, E. (2018). Artificial Life Environment Modeled by Dynamic Fuzzy Cognitive Maps. *IEEE Transactions On Cognitive And Developmental Systems*, 10(1), 88-101.
- [10] Salmeron, J. (2009). Supporting Decision Makers with Fuzzy Cognitive Maps. *Research-Technology Management*, 52(3), 53-59.
- [11] Najafi, A., Amirkhani, A., Papageorgiou, E., & Mosavi, M. (2017). Medical decision making based on fuzzy cognitive map and a generalization linguistic weighted power mean for computing with words. 2017 IEEE International Conference On Fuzzy Systems (FUZZ-IEEE), 1-6.
- [12] Yaman, D., & Polat, S. (2009). A fuzzy cognitive map approach for effect-based operations: An illustrative case. *Information Sciences*, 179(4), 382-403.
- [13] Hajek, P., & Prochazka, O. (2016). Interval-valued fuzzy cognitive maps for supporting business decisions. 2016 IEEE International Conference On Fuzzy Systems (FUZZ-IEEE), 531-536.
- [14] Sudhagar, C. (2019). Role of Fuzzy Cognitive Maps in Smart Education System. 2019 4Th MEC International Conference On Big Data And Smart City (ICBDSC), 1-6.
- [15] Papageorgiou, E. (2011). Review study on fuzzy cognitive maps and their applications during the last decade. 2011 IEEE International Conference On Fuzzy Systems (FUZZ-IEEE 2011).
- [16] Groumpos, P., & Stylios, C. (2000). Modelling supervisory control systems using fuzzy cognitive maps. *Chaos, Solitons & Fractals*, 11(1-3), 329-336.
- [17] Stylios, C., & Groumpos, P. (2004). Modeling Complex Systems Using Fuzzy Cognitive Maps. *IEEE Transactions On Systems, Man, And Cybernetics - Part A: Systems And Humans*, 34(1), 155-162.
- [18] Stylios, C., & Groumpos, P. (1999). Fuzzy Cognitive Maps: a model for intelligent supervisory control systems. *Computers In Industry*, 39(3), 229-238.
- [19] Amirkhani, A., Shirzadeh, M., Papageorgiou, E., & Mosavi, M. (2016). Visual-based quadrotor control by means of fuzzy cognitive maps. *ISA Transactions*, 60, 128-142.



- [20] Mendonça, M., Neves, F., de Arruda, L., Chrun, I., & Papageorgiou, E. (2016). Embedded Dynamic Fuzzy Cognitive Maps for Controller in Industrial Mixer. *Intelligent Decision Technologies* 2016, 57, 251-261.
- [21] Farinaz, B., Abdul, R., Khairulmizam, S., & Hossein, E. (2017). Energy saving by applying the fuzzy cognitive map control in controlling the temperature and humidity of room. *International Journal Of Physical Sciences*, 12(1), 13-23.
- [22] de Souza, L., Prieto Soares, P., Mendonca, M., Mourhir, A., & Papageorgiou, E. (2018). Fuzzy Cognitive Maps and Fuzzy Logic applied in industrial processes control. *2018 IEEE International Conference On Fuzzy Systems (FUZZ-IEEE)*, 1-8.
- [23] Denizci, A., Karadeniz, S., & Ulu, C. Fuzzy Cognitive Map Based PI Controller Design. *International Conference On Intelligent And Fuzzy Systems (INFUS 2020)*, in press.
- [24] Bettayeb, M., Boussalem, C., Mansouri, R., & Al-Saggaf, U. (2014). Stabilization of an inverted pendulum-cart system by fractional PI-state feedback. *ISA Transactions*, 53(2), 508-516.
- [25] Boubaker, O. (2013). The Inverted Pendulum Benchmark in Nonlinear Control Theory: A Survey. *International Journal Of Advanced Robotic Systems*, 10(5), 1-9.
- [26] Chang, W., Hwang, R., & Hsieh, J. (2002). A self-tuning PID control for a class of nonlinear systems based on the Lyapunov approach. *Journal Of Process Control*, 12(2), 233-242.
- [27] Prasad, L., Tyagi, B., & Gupta, H. (2014). Optimal Control of Nonlinear Inverted Pendulum System Using PID Controller and LQR: Performance Analysis Without and With Disturbance Input. *International Journal Of Automation And Computing*, 11(6), 661-670.
- [28] Irfan, S., Mehmood, A., Razzaq, M., & Iqbal, J. (2018). Advanced sliding mode control techniques for Inverted Pendulum: Modelling and simulation. *Engineering Science And Technology, An International Journal*, 21(4), 753-759.
- [29] Mun-Soo Park, & Dongkyoung Chwa. (2009). Swing-Up and Stabilization Control of Inverted-Pendulum Systems via Coupled Sliding-Mode Control Method. *IEEE Transactions On Industrial Electronics*, 56(9), 3541-3555.
- [30] Siuka, A., & Schöberl, M. (2009). Applications of energy based control methods for the inverted pendulum on a cart. *Robotics And Autonomous Systems*, 57(10), 1012-1017.
- [31] Messikh, L., Guechi, E., & Benloucif, M. (2017). Critically damped stabilization of inverted-pendulum systems using continuous-time cascade linear model predictive control. *Journal Of The Franklin Institute*, 354(16), 7241-7265.
- [32] Gawthrop, P., & Wang, L. (2006). Intermittent predictive control of an inverted pendulum. *Control Engineering Practice*, 14(11), 1347-1356.
- [33] Mishra, S., & Chandra, D. (2014). Stabilization and Tracking Control of Inverted Pendulum Using Fractional Order PID Controllers. *Journal Of Engineering*, 2014, 1-9.
- [34] Roose, A., Yahya, S., & Al-Rizzo, H. (2017). Fuzzy-logic control of an inverted pendulum on a cart. *Computers & Electrical Engineering*, 61, 31-47.
- [35] Jianqiang Yi, Yubazaki, N., & Hirota, K. (1999). Upswing and stabilization control of inverted pendulum and cart system by the SIRMs dynamically connected fuzzy inference model. *FUZZ-IEEE'99. 1999 IEEE International Fuzzy Systems. Conference Proceedings (Cat. No.99CH36315)*, 400-405.
- [36] Upadhyay, D., Tarun, N., & Nayak, T. (2013). ANN based intelligent controller for inverted pendulum system. *2013 Students Conference On Engineering And Systems (SCES)*, 1-6.
- [37] Zadeh, L. (1972). A Fuzzy-Set-Theoretic Interpretation of Linguistic Hedges. *Journal Of Cybernetics*, 2(3), 4-34.