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Authors: Songül ÖZÜM

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On the Loci of Relaxation Time and Magnetic Dispersion Maxima in the Mean-Field Ising Model

Songül ÖZÜM[*1]

Abstract

Based on the phenomenological approach, loci of relaxation time and magnetic dispersion maxima near the critical regime in a spin-1/2 mean-field Ising model were performed. The shift in temperature (T) of relaxation time (τ) maximum was detected and its behavior near the second-order transition points are presented at different magnetic field values (h) and different lattice coordination numbers (q). An expression for the dynamic (or complex) susceptibility ($\chi = \chi_1 - i\chi_2$) is also derived. The temperature dependence of the magnetic dispersion (χ_1) and magnetic absorption (χ_2) factors have been studied near the critical regime. It is found that the maximum of χ_1 as a function of frequency (ω) and kinetic coefficient (L) obeying an approximately exponential increases and decreases in T- ω and T-L planes near the critical region.

Keywords: Ising model, mean-field approximation, phenomenological approach, relaxation time, magnetic dispersion maxima

1. INTRODUCTION

The study of relaxation phenomena (RP) has attracted much attention in many areas of condensed matter and statistical physics. Recent efforts on the RP in many different systems are devoted to either experimental [1-5] or theoretical [6-9] basis. Besides above works, it is mostly known that the RP in different Ising systems are one of the most actively studied problems in statistical physics and encountered in different areas of physics [10-29]. Similarly, the magnetic responses of Ising systems have long time been a subject of interest because of their potential applications as: spin glasses [30], cobalt-based alloys [31], magneto-optical devices [32], magnetic properties of magnetic fluids [33]. To achieve this aim, the authors constructed different types of Ising systems such as spin-1/2 Ising ferromagnet [34], Ising antiferromagnet [35], kinetic Ising model [36], spin-1/2 Ising system by using Monte Carlo simulations [37], an Ising system using the Glauber dynamics [38]. The static and dynamic properties of the magnetic responses of Ising systems have been investigated so far using a variety of techniques such as mean field approximation [27, 39, 40], Onsager's theory of irreversible thermodynamics [41]. However, the dynamical magnetic response properties have not been studied in detail, e.g., the loci of relaxation time and magnetic dispersion maxima in a spin-1/2 mean-field Ising model.

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[*] Corresponding Author: songulozum@hitit.edu.tr
[1] Hitit University, Turkey, ORCID: https://orcid.org/0000-0003-2123-5856

In this paper, we would like to investigate the loci of relaxation time and magnetic dispersion maxima near the critical point in a spin-1/2 mean-field Ising model in the presence of oscillating external magnetic field. Since then, we describe the model and give static properties in Section 2. Then, in Section 3, we derived the kinetic (or rate) equations and relaxation time under the phenomenological approach. The complex (magnetic) susceptibility is obtain and magnetic dispersion and absorption factors are calculated with the solution of rate equations in same section. In Sec. 4, we present and discuss the calculated results. Section 5 includes the summary and some concluding remarks related with the topic.

2. THE MODEL AND ANALYSIS FOR EQUILIBRIUM STATE UNDER THE MEAN-FIELD APPROXIMATION

The spin-1/2 Ising model can be described through the Hamiltonian (in the presence of an external magnetic field h)

$$\mathrm{H} = -J\sum_{<ij>} S_i S_j - h\sum_i S_i \,, \tag{1}$$

where J is the bilinear coupling between the spins at sites i and j. q is the coordination number of the lattice (i.e. the number of nearest neighbours). Letting m and N be magnetization and the total number of Ising spins, Gibbs function $G(G = E - TS - hm)$ may be written in the Curie-Weiss approximation

$$\begin{aligned} G(m,h,T) &= -\frac{1}{2}NJqm^2 \\ &+ NkT\left[\begin{matrix}\left(\frac{1+m}{2}\right)\ln\left(\frac{1+m}{2}\right) \\ +\left(\frac{1-m}{2}\right)\ln\left(\frac{1-m}{2}\right)\end{matrix}\right], \\ &- hm \end{aligned} \tag{2}$$

where k and T are the Boltzmann factor and temperature, respectively. Also, the second derivative of G is

$$\frac{\partial^2 G}{\partial m^2} = -NJq + \frac{NkT}{1-m^2}\,, \tag{3}$$

and we write the critical temperature (T_C) by $T_C = Jq$. The magnetic field h is given by

$$h = \frac{\partial G}{\partial m} = -NJqm + \frac{1}{2}NkT\ln\frac{1+m}{1-m}\,. \tag{4}$$

The self-consistent equation has been obtained using Eq. (4) as

$$m = \tanh(\beta(Jqm + h))\,. \tag{5}$$

Thermal variations of m for different lattice coordination numbers ($q = 4, 6$) in the case of $L = 0.01$ and $J = 1$ are plotted for the system which undergoes a second-order phase transition (SOPT) in Figure 1. The dotted lines in the figure show T_C. From this figure one can see that m decrease to zero continuously from their values at $T_C = 4$ for $q = 4$ and $T_C = 6$ for $q = 6$ as the temperature increases; hence a SOPT occurs.

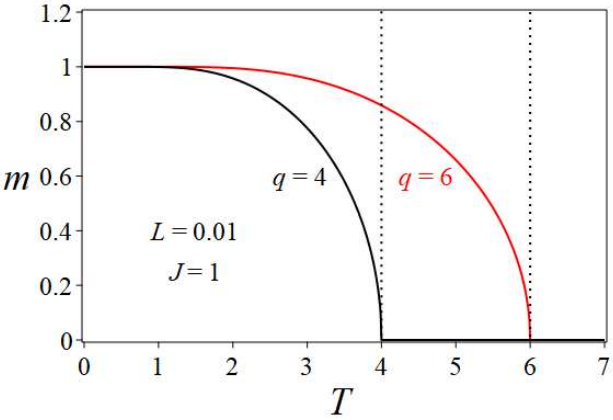

Figure 1 Temperature (T) dependence of m at various lattice coordination numbers (q) for $L = 0.01$ and $J = 1$

3. DERIVATIONS OF KINETIC EQUATION, RELAXATION TIME AND DYNAMIC SUSCEPTIBILITY

In this section, a mean-field approximation is used for the magnetic Gibbs free-energy production and a force and a current are defined. Then, the rate equation for the long-range order parameter (LROP or magnetization, m) is obtained within linear response theory. By solving these equations relaxation time (τ) is calculated for temperatures near the SOPT. A good reference for the description of relaxation properties of the Ising model is [27], whose notation is used here.

For a kinetic spin-1/2 Ising system, we define the $m(t)$ and for a nonequilibrium state, the τ towards equilibrium is written

$$\dot{m} = -\frac{m - m_0}{\tau}. \qquad (6)$$

Where τ characterizes the rate at which the LROP m approaches the equilibrium (m_0). Eq. (6) is the simplest equation of irreversible thermodynamics [42] and can also be written as follows

$$\dot{m} = LX, \qquad (7)$$

where L is the rate constant (or kinetic coefficient) and X is the generalized force conjugate to the current $\dot{m}$ by differentiating ΔG with respect to $m - m_0$:

$$X = \frac{d(\Delta G)}{d(m - m_0)}, \qquad (8)$$

with

$$\Delta G = \frac{1}{2}\begin{bmatrix} \phi_{mm}(m - m_0)^2 + 2\phi_{mh}(m - m_0)(h - h_0) \\ + \phi_{hh}(h - h_0)^2 + \phi_h(h - h_0) \end{bmatrix}. \qquad (9)$$

In Eq. (9), the coefficients are expressed:

$$\phi_{mm} = \left(\frac{\partial^2 G}{\partial m^2}\right)_{eq}, \quad \phi_{mh} = \left(\frac{\partial^2 G}{\partial m \partial h}\right)_{eq}, \quad \phi_{hh} = \left(\frac{\partial^2 G}{\partial h^2}\right)_{eq}$$

$$\phi_h = \left(\frac{\partial G}{\partial h}\right)_{eq}. \qquad (10)$$

The kinetic equation is found using Eqs. (8)-(10) in the Eq. (7):

$$\dot{m} = L\phi_{mm}(m - m_0) + L\phi_{mh}(h - h_0). \qquad (11)$$

One can introduce the rate equation when $h = 0$, i.e., $h - h_0$ to find the τ for the single RP. Eq. (11) can be written

$$\dot{m} = L\phi_{mm}(m - m_0). \qquad (12)$$

If we had assumed a solution form with $m - m_0 = \exp(-t/\tau)$ for Eq. (12), we find

$$\frac{1}{\tau} = L\phi_{mm}. \qquad (13)$$

Using Eq. (10), one obtaines the relaxation time

$$\tau = -\frac{1 - m_0{}^2}{NL(-Jq + Jqm_0{}^2 + kT)}. \qquad (14)$$

The spin system is stimulated by a time dependent small external magnetic field $h(t) = h_1 e^{i\omega t}$ oscillating at an angular frequency ω. The quantities will oscillate at the same ω in the steady-state: therefore

$$m(t) - m_0 = m_1 e^{i\omega t}. \qquad (15)$$

Substituting Eq. (9) into the rate equation Eq. (7) we obtain as following equation:

$$i\omega m_1 e^{i\omega t} = L\phi_{mm} m_1 e^{i\omega t} + L\phi_{mh} h_1 e^{i\omega t}. \qquad (16)$$

Solving Eq. (16) for m_1 / h_1 yields as follows

$$\frac{m_1}{h_1} = \frac{L\phi_{mh}}{i\omega - L\phi_{mm}}. \qquad (17)$$

We will use Eq. (17) to obtain the complex (magnetic) susceptibility $\chi(\omega)$. The Ising system induced magnetization is written as

$$m(t)-m_\infty=\mathrm{Re}(m_1e^{i\omega t}). \tag{18}$$

m_∞ is the magnetization induced by a h oscillating at ω. In addition, $\chi(\omega)$ is given

$$m(t)-m_\infty=\mathrm{Re}\left[\chi(\omega)h_1e^{i\omega t}\right], \tag{19}$$

in which $\chi(\omega)=\chi_1(\omega)-i\chi_2(\omega)$ is the dynamic susceptibility. Real $\chi_1(\omega)$ and imaginary $\chi_2(\omega)$ parts of $\chi(\omega)$ are magnetic dispersion and absorption factors, respectively. Eq. (16) can be given

$$\chi(\omega)=\frac{m_1}{h_1}. \tag{20}$$

The magnetic dispersion and absorption factors become

$$\chi_1(\omega)=\frac{\phi_{mm}L^2}{\phi_{mm}{}^2L^2+\omega^2}=L\phi_{mh}\frac{\tau}{1+\omega^2\tau^2}. \tag{21}$$

$$\chi_2(\omega)=\frac{L\omega}{\phi_{mm}{}^2L^2+\omega^2}=L\phi_{mh}\frac{\tau^2\omega}{1+\omega^2\tau^2}. \tag{22}$$

4. NUMERICAL RESULTS AND DISCUSSION

Firstly, we plot the relaxation time τ as a function of T at using different lattice structures (with $q=4,\,6$ corresponding to the square and simple cubic lattice structures, respectively) for the case $L=0.01$ and $J=1$ in Figure 2. In this figure, τ grows rapidly with increasing T and diverges as the T approaches the SOPT temperature. The curves shift towards higher temperatures with increasing q.

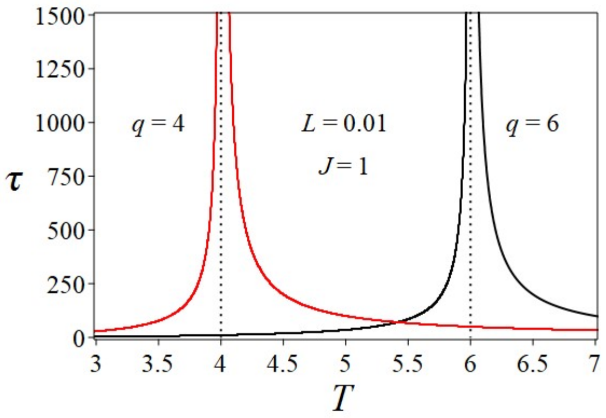

Figure 2 T dependence of τ at various q for $L=0.01$ and $J=1$

Thermal behaviours of τ are performed for four values of the external fields ($h=0,\ 0.03,\ 0.05,\ 0.1$) for $L=0.01$ and $q=6$ and for two lattice coordination numbers $q=4,6$ with $h=0.05$, $L=0.01$. The results are displayed in Figures 3(a) and 3(b), respectively. Figure 3(a) shows that τ (black-colored curve) grows rapidly with increasing T and diverges to infinity around T_C (as seen dotted line) when $h=0$. This result is a very good overall agreement with the relaxation phenomena around the Curie temperature belonging to the Bethe approximation in Barry's works [34]. On the other hand, for $h\neq 0$, maxima of the curves (or peaks) are observed in Figure 3(a). In particular, the maxima of these curves depend on the external field. One can see that with the increase of h ($h=0.03,\ 0.05$ and 0.1) the maxima become smaller and shift towards higher T. In Figure 3(b), for the sake of comparison in the case of different lattice structures ($q=4$, square lattice and $q=6$, simple cubic lattice), we have also calculated τ vs. T for this system with $h=0.05$. The peaks become smaller and shift towards higher T with increasing q. Also, we construct the plots of the maxima of τ that obtained from Figure 3(a) predicted for Ising model with $L=0.01$ and $J=1$ on the $h-T$ plane in Figure 3(c).

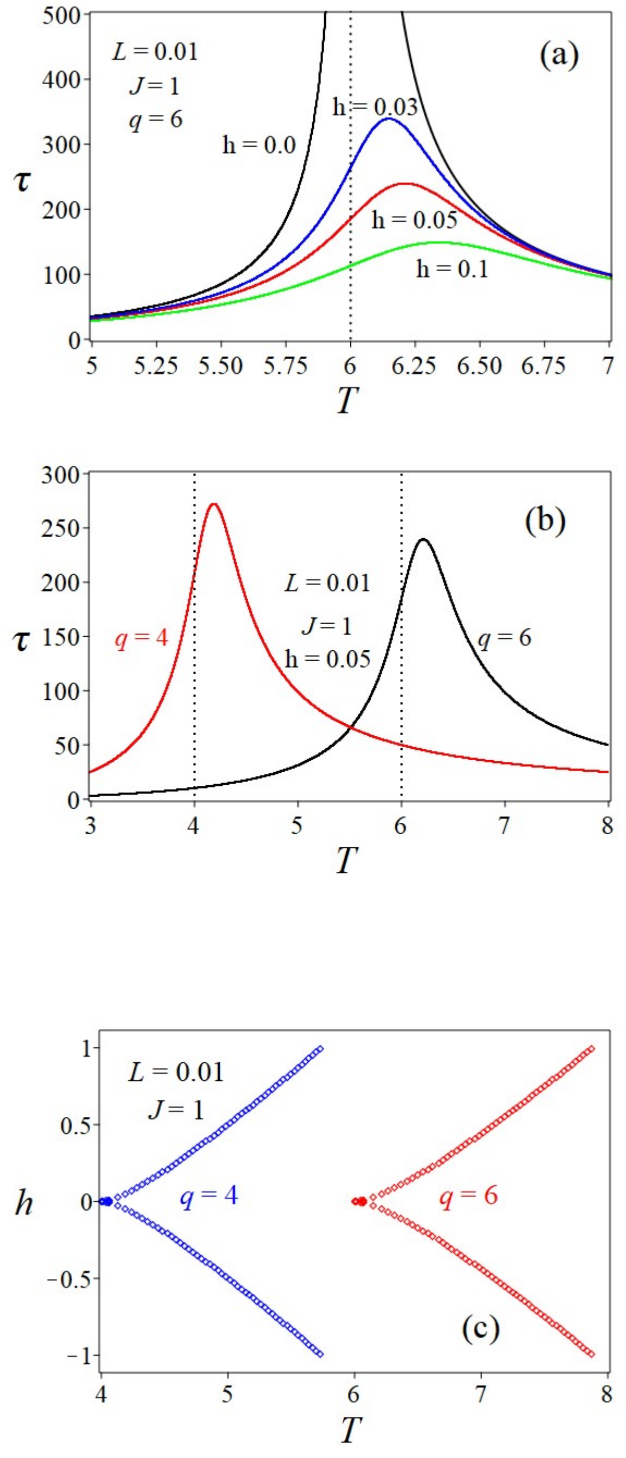

Figure 3 (a) T dependence of τ at various h for $L = 0.01$, $J = 1$ and $q = 6$. (b) T dependence of τ for various q with $L = 0.01$, $J = 1$ and $h = 0.05$. (c) Loci of maxima of τ (blue-colored square for $q = 4$) and (red-colored square for $q = 6$) with $L = 0.01$ and $J = 1$ on the $h - T$ plane

Figure 4 shows that the temperature behaviors of the χ_1 and χ_2 for the lower frequency regime $\omega\tau << 1$ for the case $L = 0.01$ and $J = 1$. Figures 4(a) and 4(b) show χ_1 and χ_2 increase with T and tend to infinity around the phase transition point. The χ_1 is independent of the ω, whereas χ_2 depends on ω. In these figures, dotted lines illustrate the T_C and the black-, blue- and red-colored curves are for $\omega = 2 \times 10^{-5}$, 4×10^{-5}, 6×10^{-5}, respectively. These results are in qualitative agreement with the obtained calculations by Barry and Harrington [34, 35, 41] and Gülpınar and co-workers [39, 40].

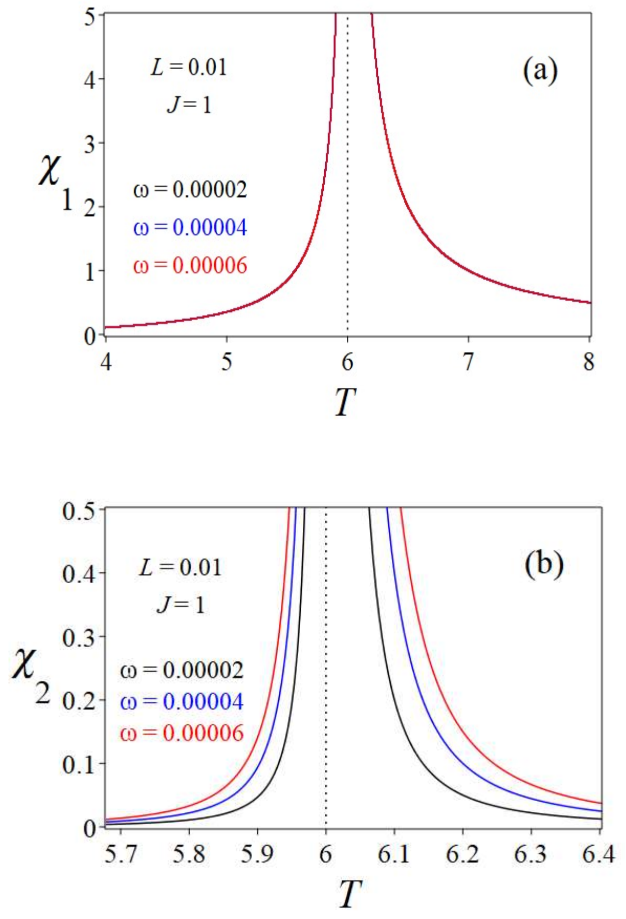

Figure 4 (a) χ_1 and (b) χ_2 as a function of the T for the low-frequency region $(\omega\tau << 1)$ when $L = 0.01$ and $J = 1$

The temperature behaviors of χ_1 and χ_2 are shown in the $\omega\tau >> 1$ for the case of $L = 0.01$ and $J = 1$ in Figures 5(a) and 5(b). In both figures, dotted lines represent the T_C. χ_1 has two local maxima in the FM and paramagnetic (PM) phase regions, as shown in Figure 5(a). In this figure, one can see that these maxima are ω - dependent. The maximum observed at a temperature in the FM phase decreases and shifts to lower T when ω increases. The peak found at a T in the PM phase decreases but shifts towards higher T. A local minimum or a sharp dip is seen at the T_C for magnetic dispersion χ_1. These results are also in coherent with other theoretical studies of dynamic susceptibility for well-known spin systems [34, 35, 39-47].

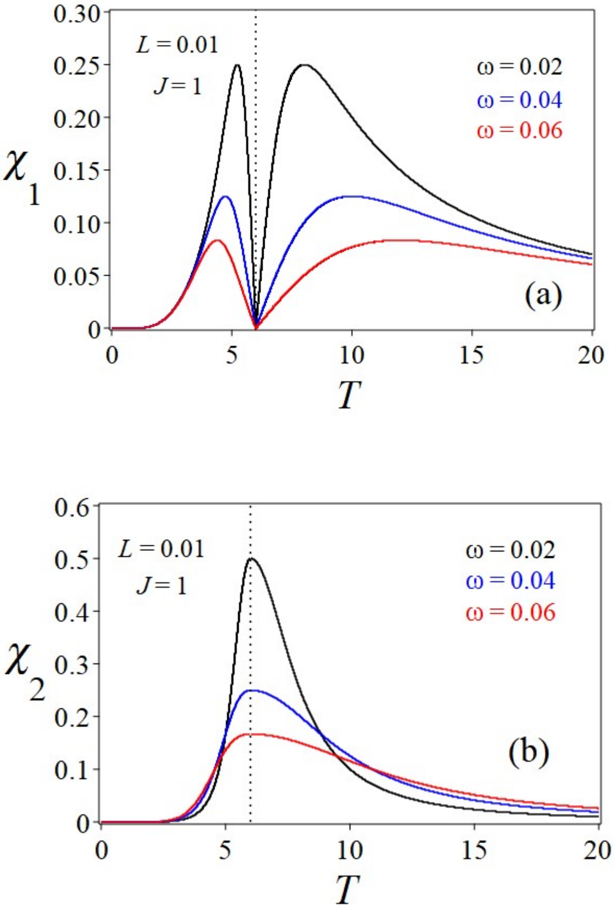

Figure 5 Same as Figure 4 but for the high-frequency region ($\omega\tau >> 1$)

To observe the effect of rate constant (or kinetic coefficient) L on χ_1 and χ_2 vs T curves, we have drawn, in Figure 6, for different values of L with $J = 1$ and $\omega = 2 \times 10^{-5}$ obeying the $\omega\tau << 1$ condition. The red-, blue- and green-colored curves in Figures 6(a) and 6(b) correspond to the cases $L = 0.03$, 0.02, 0.01, respectively. χ_1 in the FM and PM regions does not depend on the statistical rate parameter while χ_2 inversely proportional to L (Figure 6(b)). Although χ_1 is very similar to that in Figure 4(a), χ_2 is different from the case in Figure 4(b).

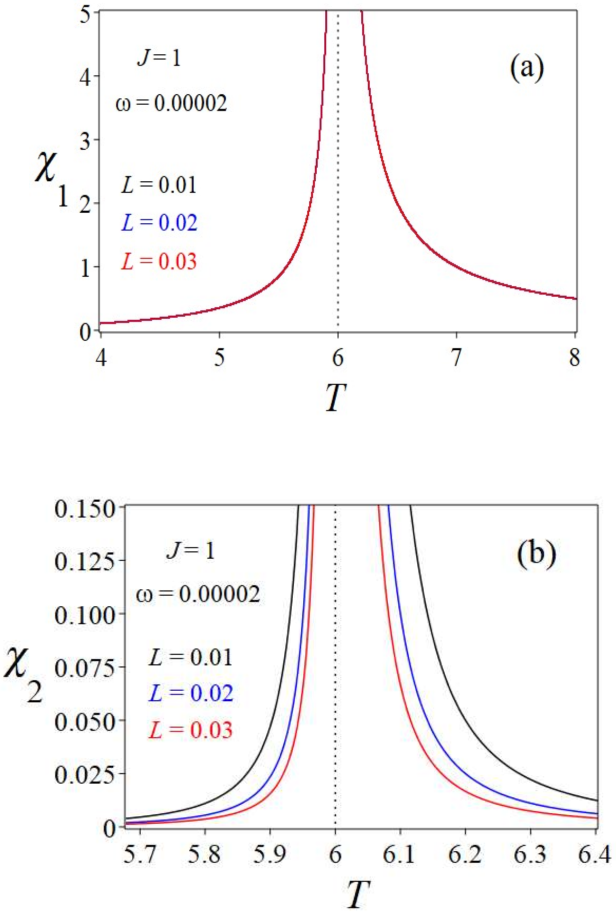

Figure 6 (a) χ_1 and (b) χ_2 as a function of the T for various values of L when $J = 1$ and $\omega = 2 \times 10^{-5}$

Figure 7 shows χ_1 and χ_2 vs T curves using $J = 1$ and different L values for $\omega\tau >> 1$. In these figures, the dotted lines refer to the T_C. Also, we have found that increasing values of L raises the peaks for χ_1 and χ_2. One can see in Figure 7(a), the heights and loci of the peaks obtained for χ_1 in the FM and PM phases depend on the L.

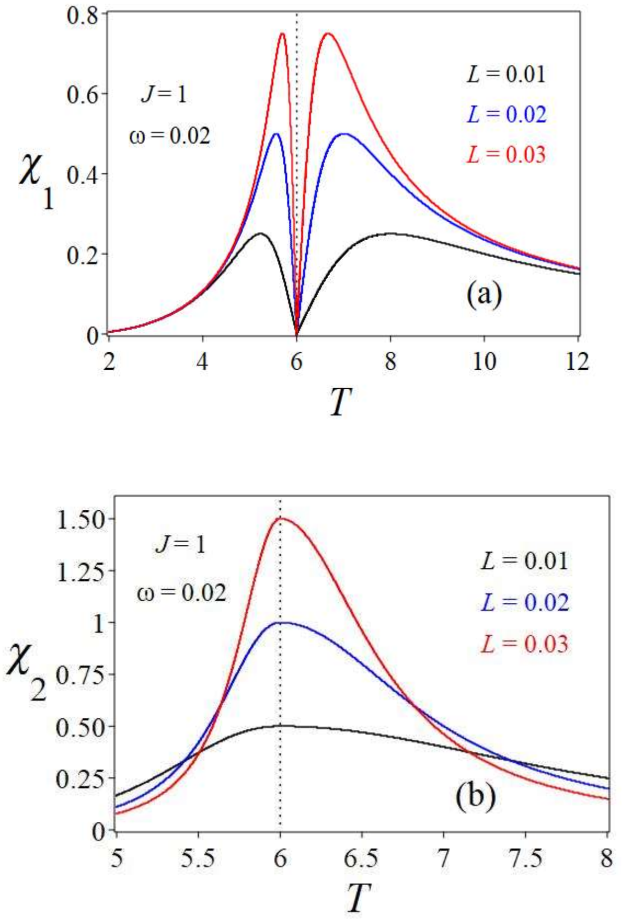

Figure 7 Same as Figure 6 but for $\omega = 0.02$

As the last figures, we represent the loci of magnetic dispersion maxima in T - ω and T - L planes in Figures 8(a) and 8(b). As seen in Figure 8(a), the temperatures of maxima observed in the PM phase (T_P^m) exponentialy increase (red-colored open diamonds) whereas the peak temperature in FM phase (T_F^m) exponentialy decrease (blue-colored open diamonds) with increasing frequency from 0 to 1. We also described, the exponential variation of the T with the L for $\omega = 0.02$ and $J = 1$ in Figure 8(b). It could be emphasized that the functional behaviours of $T_{F,P}^m$ obey second-order exponential form as $T_{F,P}^m = \pm A_1 \exp(\mp\omega / a_1) \pm A_2 \exp(\mp\omega / a_2) \pm B$. In Figure 8(a), for the blue-colored open diamonds we have determined the constants as $A_1 = 2.13$, $a_1 = 0.04$, $A_2 = -1.42$, $a_2 = 30.1$ and $B = 5.35$ while for the red-colored open diamonds these are obtained to be $A_1 = 36.9$, $a_1 = -0.27$, $A_2 = -34.9$, $a_2 = -1.80$, $B = -7.9$. Also, we can present the exponential variation of the temperatures $T_{F,P}^m$ with the L by $T_{F,P}^m = \pm A_1 \exp(\mp L / a_1) \pm A_2 \exp(\mp L / a_2) \pm B$ in Figure 8(b). As a result of our calculations, the fit to second-order exponential function gave values equal to $A_1 = 3.4$, $a_1 = 0.014$, $A_2 = 0.9$, $a_2 = 2.2$, $B = -5.35$ for the red diamonds and $A_1 = 1.0$, $a_1 = -0.02$, $A_2 = -0.01$, $a_2 = -0.02$, $B = 5.9$ for the blue diamonds in Figure 8(b). For Figures 8(a) and 8(b), converging to $T \approx 6$ (horizontal dotted lines) of both peak temperatures presents that χ_1 diverges to infinity is the continuous phase transition temperature. This result is the expected behaviour for Ising systems.

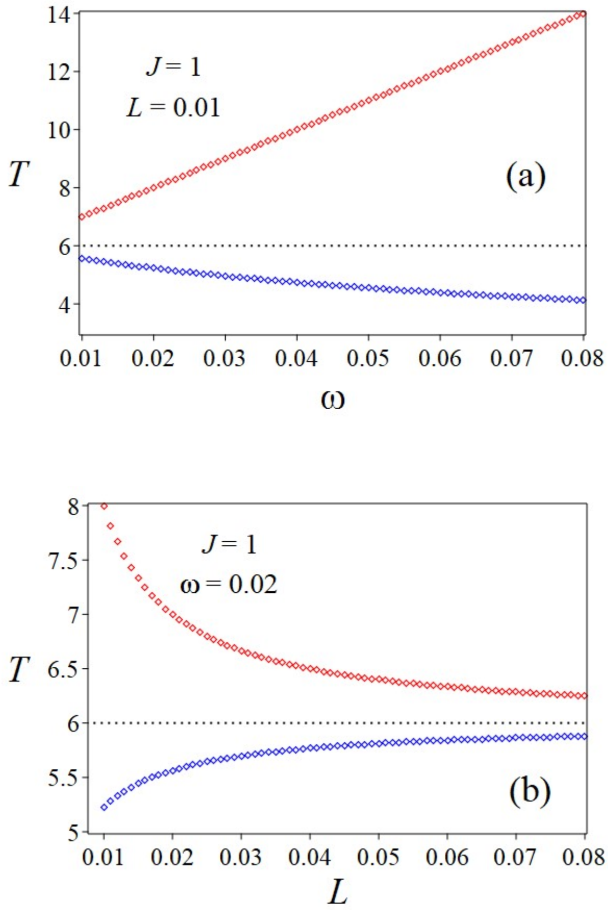

Figure 8 Loci of maxima of χ_1 (a) for $L = 0.01$ on the $T - \omega$ plane and (b) for $\omega = 0.02$ on the $T - L$ plane. $J = 1$

5. CONCLUSION

In this paper, we have investigated the loci of relaxation time and magnetic dispersion maxima in the mean-field spin-1/2 Ising model near the critical region. Firstly, having used LROP (magnetization) description, we obtained the simplest relaxation time (τ) based on phenomenological theory. Using different lattice coordination numbers (q) and magnetic field (h) values, temperature vs τ has been discussed. τ tends to infinity at $h = 0$ near the phase transitions while it shows a peak in the presence of h. The plots of the maxima of τ for different q values have been also investigated. The temperature dependence of the magnetic dispersion (χ_1) and absorption (χ_2) factors have been analyzed and illustrated in the case of $L = 0.01$ and $J = 1$ for low- and high-frequency regimes. χ_1 and χ_2 diverges to infinity at low-frequency regime as χ_1 has two frequency-dependent local maxima (or peaks) in the FM and PM phases. In order to observe the effect of rate constant L on the temperature dependence of χ_1 and χ_2, we have plotted the magnetic dispersion and absorption factors in the low- and high-frequency regimes. As a result of frequency and kinetic coefficient dependence of χ_1, we have shown loci of maxima of χ_1 with interesting features in T - ω and T - L planes. The study of dynamic response of a spin system in the presence of sinusoidally varying magnetic field is an important subject for all magnetic systems and their potential applications. It shoul be mentioned that the knowledge of dynamic susceptibility reveal the technological importance of a variety of physical phenomena such as nanocomposite particles for the design of magneto-optical devices.

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The Declaration of Conflict of Interest/ Common Interest

No conflict of interest or common interest has been declared by the author.

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The author declares that this document does not require an ethics committee approval or any special permission.

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