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TREND-CYCLE MODELING OF TURKISH INFLATION: DOES STOCHASTIC VOLATILITY PLAY A ROLE?

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ABSTRACT

Inflation dynamics in Turkey have been notoriously volatile for several decades. Starting in the 1970 decade, inflation rates frequently reached three digit levels, remained high until 2000 and decreased to relatively low levels after 2001 as a consequence of a series of reforms. Considering the volatile and alternating behavior of inflation dynamics in Turkey, the purpose of this work is to utilize different specifications, including time-varying trend and stochastic volatility setups, in order to test, to which extent stochastic volatility is relevant for modeling inflation in Turkey for the period 1955-2020. Our results suggest that inflation volatility is indeed time-varying and that the period 1980-2000 was characterized by increasing trend inflation and inflation volatility. Also, we show that specifications with stochastic volatility are preferred over those with constant volatility.

Key Words: *Inflation Dynamics, Stochastic Volatility, Turkish Economy.*

JEL Codes: *E31, E52, C51.*

TÜRKİYE ENFLASYONU İÇİN TREND-DÖNGÜ MODELLEMESİ: STOKASTİK OYNAKLIĞIN BİR ÖNEMİ VAR MI?

ÖZET

Türkiye'nin enflasyon dinamikleri, bilindiği üzere on yıllardır aşırı oynaklıklara tabi olmuştur. 1970'li yıllarda üç haneli seviyelere ulaşan enflasyon oranları, 2000 yılına kadar yüksek kalıp, bir dizi reformun neticesinde 2001'den sonra görece düşük seviyelere düşmüştür. Bu çalışmanın amacı, Türkiye'de hâkim olan enflasyon dinamiklerinin değişkenliği ve oynaklığını göz önünde bulundurarak, 1955-2020 seneleri için enflasyon modellemesi yapmaktır. Bu amaç için çeşitli zamanla değişen trend ve stokastik oynaklık modelleri kullanılmakta ve stokastik oynaklığın enflasyon modellemesi için ne ölçüde önemli olduğu araştırılmaktadır. Bulgularımız, enflasyon oynaklığının zamanla değişken olduğunu ve 1980-2000 döneminde trend enflasyonunun ve enflasyon oynaklığının arttığını göstermektedir. Ayrıca bulgularımız, stokastik oynaklık içeren modellerin, sabit oynaklık varsayımında bulunan modellere tercih edildiğini göstermektedir.

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1. INTRODUCTION

Macroeconomic dynamics in emerging economies have been subject to significant volatility over the past decades, owing to large external imbalances, budget deficits, political instability and other factors. Among these, inflation dynamics have prominently been subject to instabilities until the 2000 decade, when many emerging economies saw high and variable inflation rates. For several decades, Turkey was among the textbook examples of economies that exhibited high and variable inflation rates.

To provide a brief account of inflation dynamics in Turkey, inflation rates during the 1960 decade remained stable and averaged at around 6%. From the early 1970's onward however, Turkey's inflation rate became prominently high and volatile. In the period between 1970 and 1980, the effects of the two oil-price crises and domestic political fluctuations manifested themselves in volatile inflation rates that mostly hovered around two digit levels. Following a severe contraction in 1977 that came as a result of a debt crisis, and the oil price crisis of 1979 which forced policy makers to attempt to abandon the fixed exchange rate regime, there have been sharp devaluations in the Turkish lira against other major currencies, resulting in deterioration of pricing behavior. The effect of these events was a soaring of the inflation rate to three-digit levels for the first time in the first quarter of 1980. While a stabilization program that was enacted following the crisis was successful in decreasing inflation rates temporarily, it was not successful in stabilizing rates due to budget deficits and monetization of debt.

These instabilities continued throughout the 1990 decade, where political instability, external shocks, large external imbalances, budget deficits etc. contributed once more to inflation rates that averaged at around 80%. The last significant hike in the inflation rate occurred after the 2001 crisis that came as a result of previously built-up imbalances and was triggered by a public spat between politicians. The high inflation period that lasted for roughly three decades came to a halt after series of reforms were implemented following the severe financial crisis in 2001. As a result of these reforms, the central bank adopted implicit inflation targeting in 2001 and switched to full-fledged inflation targeting in 2006. During this period, inflation rates decreased to single digits for the first time after three decades remained relatively low thereafter. Finally, in the years after 2017, inflation rates increased to two-digit levels once more as a result of deteriorating external balances and a currency shock that ensued in 2018. Because it is not our aim to provide a very detailed account of Turkey's experience with inflation, we refer the reader to the literature that surveys the causes and effects of inflation in Turkey for further information (see among others Kibritçioğlu (2001), Ersel and Özatay (2008)).

Figure 1. Annual CPI Inflation Rate in Turkey 1960-2020

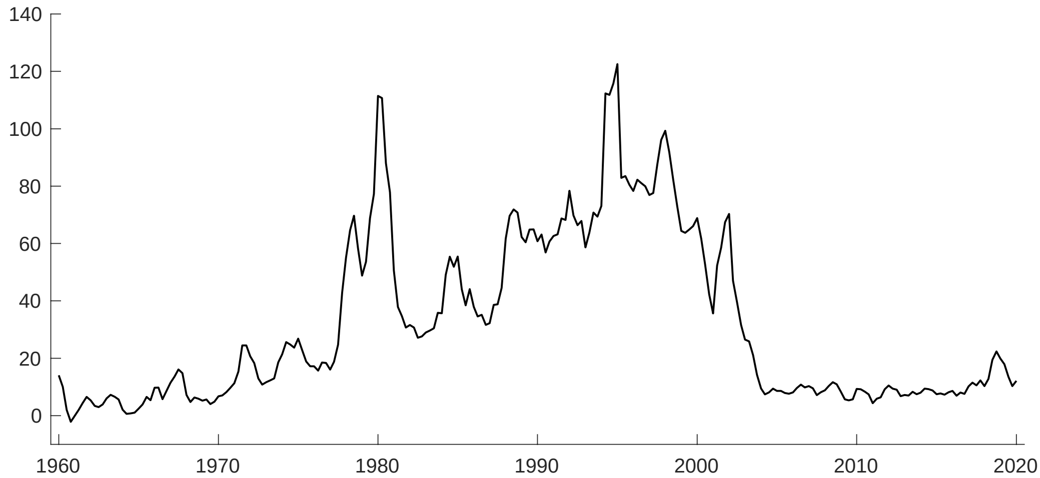


Table 1. Consumer Price Inflation (Year-on-Year Percentage Change) and Its Standard Deviation in Turkey

	1960-1970	1970-1980	1980-1990	1990-2000	2000-2010	2010-2019
Inflation rate	5.86%	26.42%	49.36%	77.49%	23.23%	9.84%
Standard dev.	4.29	18.65	21.44	16.41	20.95	4.02

Source: International Financial Statistics, IMF

Recently, especially after the 2008 global financial crisis, there has been an increased interest in analyzing inflation dynamics using various forms of trend inflation and stochastic volatility. Among these, Cogley and Sbordone (2008) and Ascari and Sbordone (2014) formulate trend inflation as a time-varying process, whereas authors such as Sims and Zha (2006), Stock and Watson (2007) or Primiceri (2006) argue that the inclusion of stochastic volatility is relevant for inflation modeling.

While these authors mostly focused on the US and other advanced economies, other authors modeled Turkish inflation using a batter of univariate and multivariate models. Among these, Öğünç et al. (2013) formulate the inflation process to include a time-varying trend component, whereas other studies such as Mandalinci (2017) show that the inclusion of stochastic volatility provides better out-of-sample forecasting results for modeling inflation rates in emerging market economies. Against this background, we analyze inflation as a process that consists of a trend and cycle component via different specifications that include both constant and stochastic volatility terms. Our findings indicate that a specification that includes both stochastic volatility terms for the trend as well as cycle components best accounts for inflation dynamics in Turkey. The paper is organized as follows: in section 2 we introduce the specifications and the data we use, in section 3 we present and discuss our results and in section 4, we conclude.

2. MODELS

The modeling of macroeconomic time series as processes that consist of trend and cycle parts was discussed extensively in the 1980's (see e.g. Nelson and Plosser (1982) and Harvey (1985)). More recently, authors introduced models in which the variance of the cyclical part is modeled as a stochastic process. Among these, Cogley and Sargent (2005) incorporated stochastic volatility into a VAR model, whereas Justiniano and Primiceri (2008) incorporated this feature into a DSGE framework. Both these works and subsequent empirical investigations reveal that incorporating stochastic volatility is important and generally results in more accurate estimation and forecasting quality.

In this work, we utilize three different specifications to model inflation dynamics in Turkey. The first specification (model 1) is an unobserved components model with constant volatility, which we compare to moving averages of the inflation rate. In the second specification (model 2), we use a stochastic volatility model which estimates inflation as a process that consists of a time-varying trend part and a transitory cycle part which contains stochastic volatility. In the third specification, we specify a time-varying trend with the addition of stochastic volatility for both transitory and trend parts of inflation. To summarize, we use the following three specifications:

M1: Contains a time-varying trend with constant volatility.

M2: Contains a time-varying trend and stochastic volatility for the transitory part.

M3: Contains a time-varying trend and stochastic volatility for the transitory part and trend inflation.

2.1. Model 1: Time-Varying Trend without Stochastic Volatility

The model consists of a trend part and cycle component similar to those used by e.g. Morley et al. (2003) or Stock and Watson (2007):

$$y_t = \tau_t + \varepsilon_t \quad (1)$$

$$\tau_t = \tau_{t-1} + u_t \quad (2)$$

where $y = (y_1, \ldots , y_T)^T$ is the vector of observables and $\tau = (\tau_1, \ldots, \tau_T)^T$ is the unobservable trend component; the error terms and their variances are formulated as $\varepsilon_t \sim N(0, \sigma^2)$ and $u_t \sim N(0, \omega^2)$. In this model, the cyclical term ε_t is not subject to non-linearity and hence can be estimated with a methodology such as the EM algorithm. The technical details of the algorithm are presented in the appendix.

2.2. Model 2: Time-Varying Trend with Stochastic Volatility

Although the model without non-linear volatility gives meaningful results, it may also be useful to formulate a model in which the volatility is subject to non-linearity and changes over time. In contrast to inflation rates elsewhere, inflation in Turkey has been very volatile and subject to changes for several decades. In the first stochastic volatility model, we consider a standard log-normal stochastic volatility model as utilized by Kim et al. (1998) and Chan (2013). The model consists of a constant trend part μ_t and a cycle part whose time-varying volatility is described by h_t:

$$y_t = \mu_t + \varepsilon_t^y \qquad \varepsilon_t^y \sim \mathcal{N}(0, e^{h_t}) \tag{3}$$

$$\mu_t = \mu_{t-1} + \varepsilon_t^\tau \qquad \varepsilon_t^\mu \sim \mathcal{N}(0, \omega_\mu^2) \tag{4}$$

$$h_t = \mu_h + \phi_h(h_{t-1} - \mu_h) + \varepsilon_t^h \qquad \varepsilon_t^h \sim \mathcal{N}(0, \omega_h^2) \tag{5}$$

where y_t is the time series to be modeled, its variance h_t represents the log-volatility term which follows an AR(1) process and $|\Phi_h| < 1$. One computational difficulty with estimating stochastic volatility models is that the volatility term is non-linear. A solution to this is the use an approximation such as the Gaussian mixture model (GMM) as introduced in Kim et al. (1998). The details of the GMM algorithm, which are used for both stochastic volatility models, are discussed in the appendix.

2.3. Model 3: Time-Varying Trend with Stochastic Volatility in Transitory Part and Trend

The last specification we use is similar to the second model, but is augmented to include a stochastic volatility term for the trend part as well. Specifically, it is formulated as follows:

$$y_t = \mu_t + \varepsilon_t^y \qquad \varepsilon_t^y \sim \mathcal{N}(0, e^{h_t}) \tag{6}$$

$$\mu_t = \mu_{t-1} + \varepsilon_t^\tau \qquad \varepsilon_t^\mu \sim \mathcal{N}(0, e^{g_t}) \tag{7}$$

$$h_t = \mu_h + \phi_h(h_{t-1} - \mu_h) + \varepsilon_t^h \qquad \varepsilon_t^h \sim \mathcal{N}(0, \omega_h^2) \tag{8}$$

$$g_t = \mu_g + \phi_g(g_{t-1} - \mu_g) + \varepsilon_t^g \qquad \varepsilon_t^g \sim \mathcal{N}(0, \omega_g^2) \tag{9}$$

where both variance terms h_t and g_t follow autoregressive processes of order 1 with

$|\Phi_h| < 1$ and $|\Phi_g| < 1$. This model, which allows for the inclusion of stochastic volatility for trend as well as cycle terms is again estimated using GMM.

2.4. Data

For the estimation of the two models we use quarterly Consumer Price Index (CPI) data for the period 1955Q1-2020Q1 that was obtained from the International Financial Statistics (IFS) database of the IMF. Inflation rate is calculated as the quarter-on-quarter percentage change of the CPI: $y_t = 400*\log(CPI_t - CPI_{t-1})$. In the next section, estimation results for the three specifications that were used will be discussed.

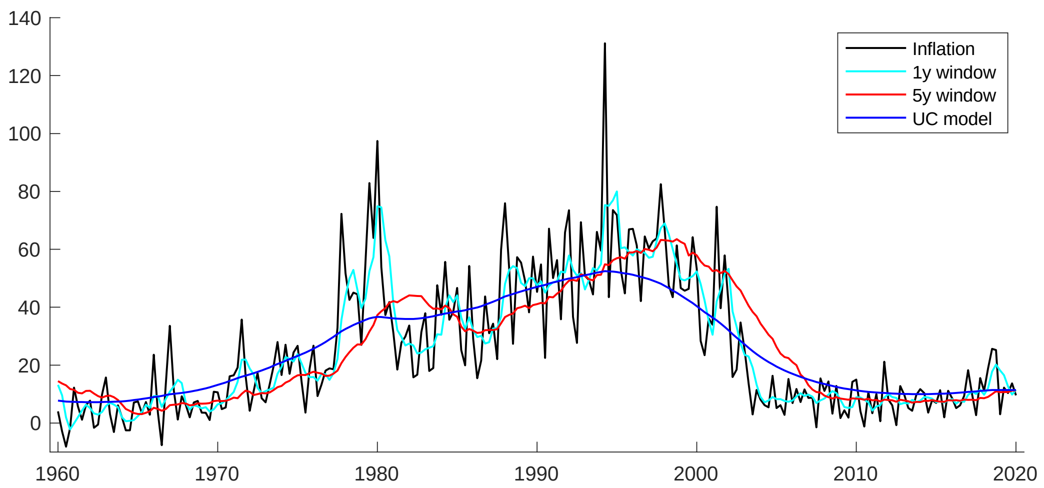
3. RESULTS

3.1. Model 1

With constant volatility, the first model consists of a trend and cycle component as described in equations 1.1. and 1.2. Inflation rates soared in Turkey starting with the 1970's and peaked in the beginning of 1980. The dramatic increase is reflected in the trend part, which increases during this period and decreases somewhat thereafter. However, the increase in trend inflation remains and peaks ultimately in 1995. This peak corresponds to the 1994 crisis after which inflation rates soared to three digit levels. Although inflation rates soared again during the 2001 financial crisis, trend inflation decreases after 1995 continually. Although there has been some increase in volatility after 2010, there is no considerable increase in trend inflation according to this model.

The model delivers some of the conventional wisdom surrounding developments regarding inflation in Turkey in the past several decades. Despite this, the model does not model volatility explicitly and hence is not entirely different from a rolling-window estimation. Below are plots of the 1-year and 5–year rolling window estimates of inflation for Turkey along with the inflation rate and the trend estimate as implied by the first model. As can be seen, the estimates provided with the rolling window model and the UC model are quite similar. A difference is that the "inflation trend" as provided by the rolling window estimation comes with a lag of several years after the one provided by the UC model.

Figure 2. Inflation, UC and Rolling Window Estimations



3.2. Model 2

Authors such as Stock and Watson (2008) or Chan (2013) establish that the inflation process can be modeled to contain stochastic volatility and that this approach can provide better forecasts in comparison to models that assume fixed parameter settings. Estimating the model represented by equations (3)-(5) results in the following graphs for the time-varying trend τ_t and stochastic volatility $\exp(h_t/2)$ (with 95% confidence intervals):

Figure 3. Time-Varying Trend τ_t

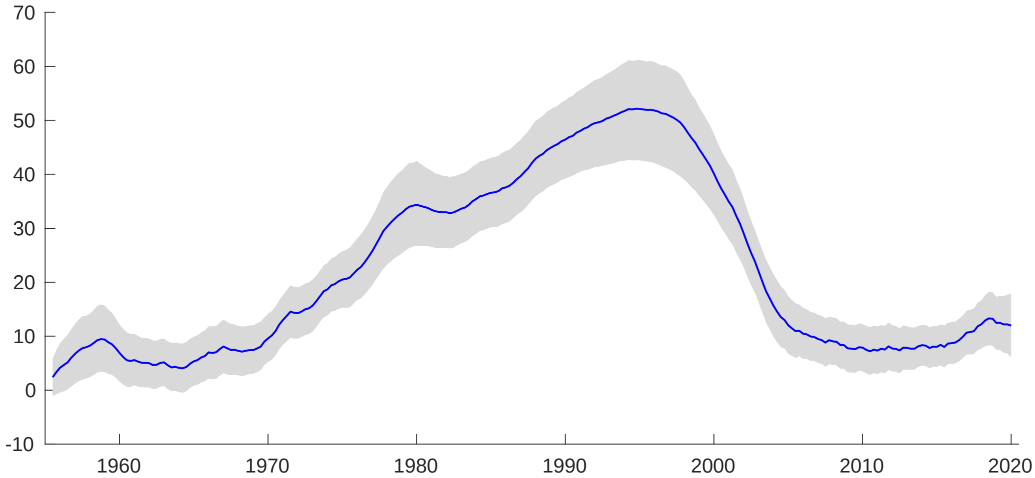
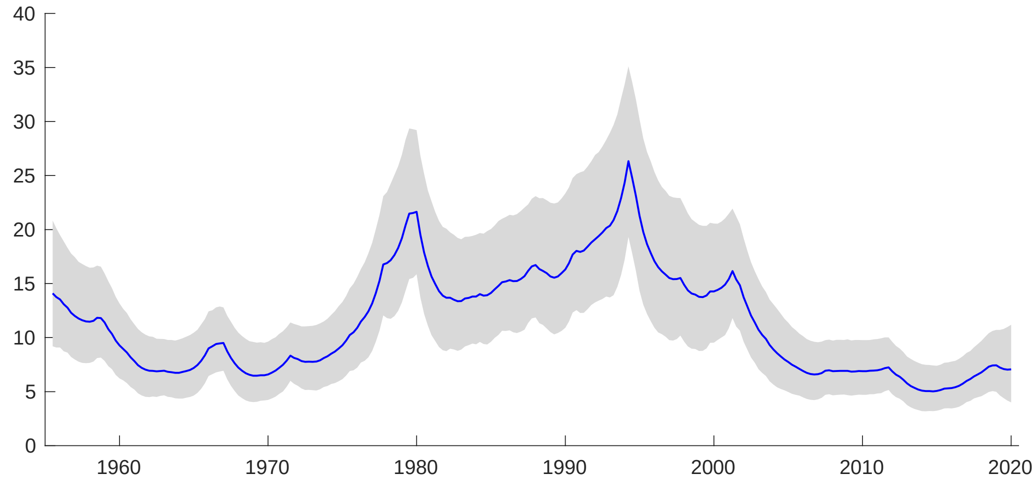


Figure 4. Time-Varying Volatility Exp(h_t/2)



The trend estimate of model 2, which is depicted in figure 3, is similar to the estimate of the first model: trend inflation starts increasing continuously until the end of the 1990 decade and starts declining thereafter. In contrast to model 1 however, model 2 indicates that there was an increase in trend inflation at the end of the 2010 decade, in line with significant increases in the inflation rate after 2018.

Figure 4 shows that the univariate inflation process exhibits significant time-variation, implying that the stochastic volatility model provides a richer picture of inflation dynamics in Turkey in contrast to model 1. Broadly speaking, there are episodes of high volatility before 2005 and a relatively calm period thereafter. Specifically, in the crisis periods of 1979-1980, 1994 and 2001, inflation volatility soars significantly and remains relatively high in the periods in-between. During these periods, Turkey experienced a myriad of crises, including debt, currency and political crises that resulted in turmoil in the domestic economy and in devaluations in the Turkish lira. After the 2001 crisis, volatility decreases to lower levels, in accordance with the significant decline in the inflation rate, while experiencing a subtle increase after 2018.

3.3. Model 3

In the time period that we investigate, monetary policy implementation in Turkey went through various regimes such as fixed exchange rate regime, different forms of managed floating regimes, inflation targeting, an unorthodox policy regime or combinations thereof[1].

Due to these substantial changes, one can expect that trend inflation may have experienced substantial volatility as well. In order to explore whether the inclusion of stochastic volatility for the trend process provides further useful information, we present in the following the results of estimating model 3.

Figure 5. Time-Varying Trend τ_t

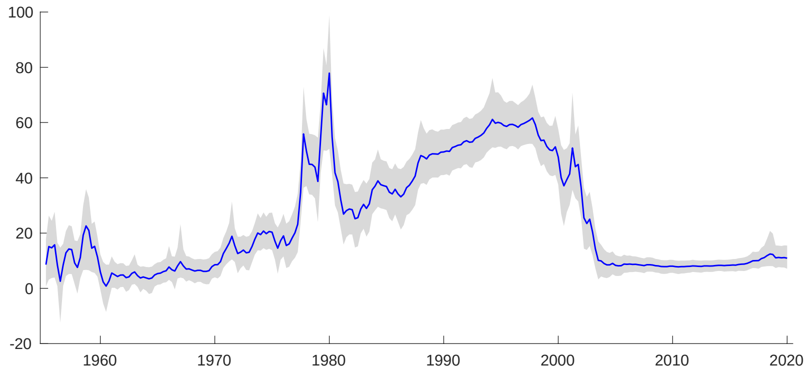
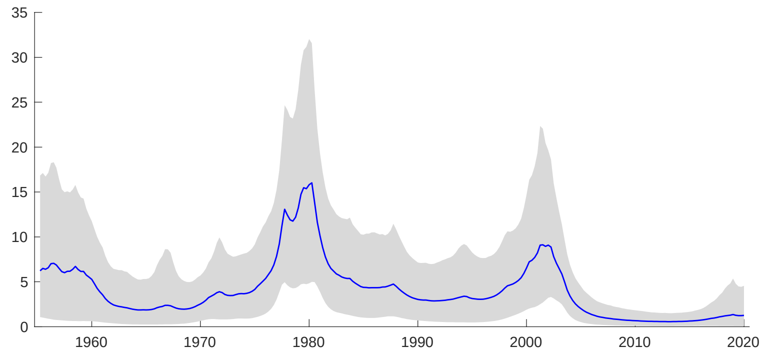
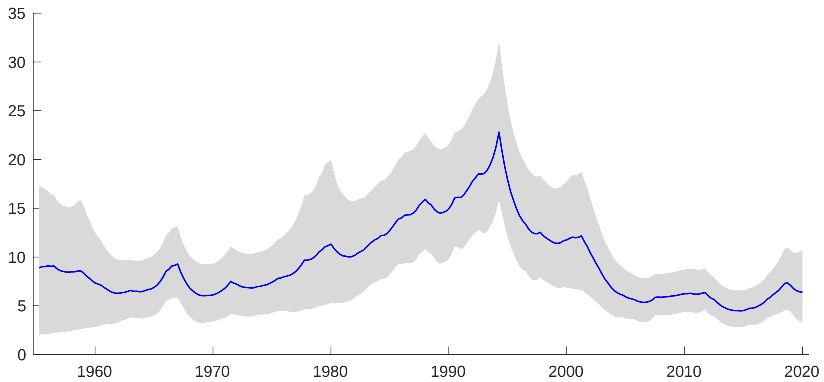


Figure 6. Time-Varying Volatility of Trend Exp(g_t/2)



[1] See e.g. Kara ve Öğünç (2008).

Figure 7. Time-Varying Volatility Exp(h_t/2)



The time-varying trend estimate of model 3, as depicted in figure 5, is similar to the estimate of model 2 and indicates that trend inflation soared until 2000 and decreased thereafter, with the most significant spike occurring at the end of the 1970 decade. The time-varying volatility component of trend τ_t (shown in figure 6) remains relatively stable throughout the estimation period but increases substantially after the 1977 debt crisis and the subsequently ensuing high inflation period, and during the 2001 crisis. Model 3's estimate of time-varying volatility of inflation, as depicted in figure 7, once more indicates that volatility increased after the 1970 decade and started declining with the 2000 decade. In contrast to model 2 however, volatility in the 1970 decade is relatively muted.

4. DISCUSSION

4.1. Model Comparison

Models 2 and 3, which allow the inflation process to be described by a stochastic volatility specification both indicate that there is significant variation in estimated inflation volatility, implying that the inclusion of stochastic volatility is indeed important.

In order to test, whether the inclusion of stochastic volatility is supported by a specification test, we compute the log Bayes factor. This test, which produces a positive number when it suggests that a model is preferred to other models, uses the Savage-Dickey density ratio[2].

Table 2. Specification Tests

log BF_1	**log BF_2**	**log BF_3**
31.2 (2.55)	70.8 (13.80)	233.6 (15.89)

Here, log BF_1 denotes the Bayes factor of including stochastic volatility in the transitory part of inflation, log BF_2 denotes the Bayes factor of including stochastic volatility in the trend equation, and BF_3 of having stochastic volatility in both the transitory and trend parts against the alternative of no

[2] For further technical details on the computation of the Bayes factor, we refer the reader to Chan (2018).

stochastic volatility. The test implies that all specifications that include stochastic volatility are supported against their alternatives and that inclusion of stochastic volatility for the trend equation is supported stronger than only having stochastic volatility in the transitory part. Also, the test suggests that including stochastic volatility for trend as well as for the transitory parts (model 3) is strongly supported against the alternative of a constant volatility model (model 1).

4.2. Turkey's Monetary Policy Experience

We gave an account of Turkey's monetary policy experience in section 1. It is the aim of this subsection to corroborate the evidence presented by the models we have utilized to historical events. Both models 2 and 3 imply that inflation trend as well as stochastic volatility started increasing after the 1970 decade and remained high until the 2000 decade. As mentioned earlier, this period was (generally) marked by a lack of a coherent monetary policy framework and by continuous macroeconomic imbalances such as high budget and current account deficits. As explained by Leigh and Rossi (2002), the pre-2001 period was also marked by high exchange rate pass-through, which in combination with high exchange rate volatility contributed to high inflation volatility. The adoption of implicit inflation targeting in 2001 and explicit inflation targeting in 2006 likely contributed to a decrease in trend inflation and in the volatility of both trend and transitory parts of inflation. This is in line with evidence provided by Gonçalves and Salles (2006), who show that inflation targeting contributed to lower inflation rates and inflation volatility in emerging economies. Overall, we believe that the stochastic volatility model provides a meaningful account of inflation dynamics of Turkish inflation within a univariate framework.

4. CONCLUSION

In this paper we used various unobserved components models with and without stochastic volatility to analyze inflation dynamics in Turkey. Because Turkey's inflation has been notoriously high and volatile for several decades, we utilized several specifications.

Among these, the estimation results show that the unobserved components model with constant volatility only provides the course of the trend component of the inflation series in Turkey but, due to the formulation of the inflation process, does not account for volatility in inflation data. Consequently, we estimated two additional models which include stochastic volatility for the transitory part of inflation for the trend equation. While there are subtle differences between the results of the different specifications, all models conclude that trend inflation in Turkey increased after the 1970 decade and decreased after the 2001 crisis. Also, models 2 and 3, which include stochastic volatility, imply that inflation volatility has been time varying and was especially high during the period 1980-2000. In contrast, volatility of trend inflation increased significantly during the 1977-1980 crisis period and the 2001 crisis and decreased after the adoption of inflation targeting and various reforms were implemented following 2001. Finally, inflation trend and volatilities of the transitory and trend parts of inflation all increased following the 2018 currency crisis. In order to test, which model is preferred by a specification test, we used the log Bayes ratio, which indicates that the model with stochastic volatilities in transitory and trend parts of inflation is strongly preferred to constant volatility specifications.

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5. APPENDIX

5.1. Moving Average Model: EM-Algorithm

The EM-algorithm is useful when latent variables are present in a model and is frequently used to compute the Maximum Likelihood estimate. The algorithm consists of the two steps, Expectation (M) and Maximization (M).

In the first step, the latent variable is estimated using the conditional expectation with model parameters and the observable data. After the conditional expectation is formed, the second step involves maximization of the likelihood function.

Specifically, given equations (1) and (2), the model is transformed such that it becomes

$$\tau = C^{-1}u \ \sim N(0, (C^T\Sigma^{-1}C^T)^{-1}) \qquad \text{(A1)}$$

where Σ^{-1} is a matrix with variances on the main diagonal. Using the conditional density, the log-likelihood function becomes

$$lnf\left(\tau \middle| y, \sigma_{i-1}^2, \omega^2\right) = -0.5(\tau^T C^T \Sigma^{-1} C + \sigma_{i-1}^2)\tau - \frac{2}{\sigma_{i-1}^2} y^T \tau + F \qquad \text{(A2)}$$

where F is a constant. We denote the term $(C^T\Sigma^{-1}C + \sigma^{-2}_{i-1}I)$ as T_i. The Expectation step then involves the calculation of the mean vector $\hat{\tau} = T^{-1}_i\, y/ \sigma^2_{i-1}$. Then, given $\hat{\tau}$ and T_i, the Maximization step updates the value of σ^2. These steps are repeated until there is no additional increase in the likelihood function.

5.2. Stochastic Volatility: Gaussian Mixture Model

With the presence of stochastic volatility, the standard EM algorithm that was described above cannot be used to estimate the stochastic volatility model. A feasible approach in such situations is the Gaussian mixture model (GMM). The underlying idea of many applications of GMM is that most non-standard (non-Gaussian) distributions can be approximated via a mix of Gaussian distributions.

In the case of the stochastic volatility model we consider, the measurement equation is first transformed such that it becomes linear in the volatility term:

$$y_t^* = h_t + \varepsilon_t^* \quad \text{(A3)}$$

where

$$y_t^* = ln((y - \tau)^2 + c) \quad \text{(A4)}$$

$$\varepsilon_t^* = ln\varepsilon_t^2 \quad \text{(A5)}$$

Equations (5) and (A3) now build a system of a linear state space model. But because the term ε_t* now has a chi-squared distribution and not a Gaussian distribution anymore, a GMM is used to approximate a Gaussian distribution. For this, we follow the procedure introduced in Kim, Shephard and Chib (1998) and use a seven component model. The mixture is given by

$$p(\varepsilon_t^*) \approx \sum_{i=1}^{7} q_i f_N\left(\varepsilon_t^* \middle| m_i, v_i^2\right) \quad \text{(A6)}$$

where f_N (ε_t*|m_i, v_i^2) is the pdf of a normally distributed variable with mean m_i, and variance v_i^2.

Finally, with the help of a mixture density indicator $s_t = i$, the algorithm detects, which component ε_t* is drawn from and $q_i = Pr(s_t=i)$. As a result, a linear state space model, comparable with the one used for the previous model can be used for the estimation of the stochastic volatility model. The Bayesian estimation is done by sampling for h from f(h|y*, s, ω^2) and s from f(s|y*,h) and ω^2 from f(ω^2|h).

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