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SURVEY ON MILITARY OPERATIONS OF FUZZY SET THEORY AND ITS APPLICATIONS*

Muhammet DEVECİ[1]
Yusuf KUVVETLİ[2]
İbrahim Zeki AKYURT[3]

[1]National Defence University, Turkish Naval Academy, Department of Industrial Engineering, Istanbul, Turkey,
muhammetdeveci@gmail.com; ORCID: 0000-0002-3712-976X

[2]Cukurova University, Department of Industrial Engineering, Adana, Turkey,
ykuvvetli@cu.edu.tr; ORCID: 0000-0002-9817-1371

[3]Turkish Airlines Flight Academy, Aydın, Turkey,
zekiakyurt@gmail.com; ORCID: 0000-0003-4817-5267

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ABSTRACT

In recent years, with the increase in investment in countries' military expenditures, the importance of decision-making problems in the military has gradually increased. There are various uncertainties such as vagueness, imprecise or ambiguous in decision-making problems. Decision making is the process of evaluating environmental factors from a holistic perspective, identifying problems and possible alternatives, making a systematic assessment, and identifying the most appropriate decision among them. The aim of this study is to investigate various types of fuzzy set that can be used in military problems under uncertainty. In addition, a detailed literature review of fuzzy decision making problems applied in the military are conducted.

Keywords: *Military Applications, Fuzzy Sets, Multi-Criteria Decision Making (MCDM), New Trends and Applications.*

BULANIK KÜMELER TEORİSİ VE UYGULAMALARININ ASKERİ OPERASYONLAR ÜZERİNE ARAŞTIRMALARI

ÖZ

Son yıllarda ülkelerin askeri harcamalarına yapılan yatırımların artmasıyla birlikte ordudaki karar alma sorunlarının önemi giderek artmıştır. Karar verme problemlerinde muğlaklık, kesin olmayan veya iki anlamlı gibi çeşitli belirsizlikler vardır. Karar verme, çevresel faktörleri bütüncül bir bakış açısıyla değerlendirme, sorunları ve olası alternatifleri belirleme, sistematik bir değerlendirme yapma ve aralarında en uygun kararı belirleme sürecidir. Bu çalışmanın amacı, belirsizlik altındaki askeri problemlerde kullanılabilecek çeşitli bulanık set türlerini incelemektir. Ayrıca, orduda uygulanan bulanık karar verme problemlerinin ayrıntılı bir literatür taraması yapılmaktadır.

Anahtar Kelimeler: *Askeri Uygulamalar, Bulanık Kümeler, Çok-Kriterli Karar Verme, Yeni Eğilimler ve Uygulamalar.*

1. INTRODUCTION

Military systems have complex structures and decision mechanisms for different types of real-world problems observed in air, navy and land forces around world armed forces. Weapon systems selection, resource location and allocation, arms transfers and management of military logistics are some examples for these problems. To handle such problems, a number of operations research and management science (OR/MS) tools such as multi-criteria decision making (MCDM), simulation, mathematical programming are commonly utilized by both academicians and practitioners. Studies conducted in military context generally have a significant impact on the performance of the organization since management of military organizations requires large amounts of money and resource (Tozan & Karatas, 2018; Karatas, Yakıcı, & Razi, 2019). Today, as well as the economic power, military force of a country is of great importance. Technological changes and increasing competition in the process of continuous development are triggering countries to increase their military spending in the world reached $1.8 trillion, with the USA, China, Saudi Arabia, India and France, respectively having the highest budget for defence. In addition, total world military spending increased by 2.6 percent in 2018 to $1.8 trillion (SIPRI, 2019). Most of the countries making these expenditures were shaken by the crisis in 2008. In the face of this situation, the importance of countries using their military defence costs and limited resources on priority strategies has increased. Table 1 presents countries with highest military expenditure in 2018 (SIPRI Military Expenditure Database, 2018).

Table 1. Top 15 military spenders in 2018 (US$ billion).

Countries	In current 2018 US$ Billion
USA	$649
Canada	$21.6
UK	$50
France	$63.8
Germany	$49.5
Italy	$27.8
Turkey	$19.0
Russia	$61.4
Brazil	$27.8
Saudi Arabia	$67.6
India	$66.5
China	$250
South Korea	$43.1
Japan	$46.6
Australia	$26.7

Fundamental military problems can be divided into two parts as problems related to planning of resources and operational problems (Wilkins & Desimone, 1993). Each problem has its own structure, characteristics and objective. Since the First World War, OR/MS techniques have been used in military for both planning and other operational problems (Karatas, Razi, & Gunal, 2017). These techniques have enormous impact on planning and managing military operations. Recent advances and developments in optimization techniques have enabled solving larger and more complex problems, resulting in high quality solutions in less computation times. Due to the great contribution of these modelling approaches, military services have saved money and operational research has led to the establishment of the department in many military units. However, operations research techniques are insufficient to completely solve the problems of managers in various decision environments. Some problems restrict the use of operations research techniques such as the difficulty of providing data in accordance with the established model, taking the parametric values used in decision-making, taking even long-time solutions on the computer, and not allowing the use of non-numerical or linguistic information.

In recent years, fuzzy MCDM approaches have been introduced for various decision-making (DM) problems. The DM process can be categorized as three classes according to the decision environment. These are decision-making under certainty, under risk, and under uncertainty. The focus of this study is to give a brief overview of fuzzy set applications and approaches for military decisions faced under uncertainty. The main contributions of this study are: *(i)* investigating the most commonly used fuzzy sets in literature, *(ii)* examining new trends in fuzzy sets theory, and *(iii)* provide a brief literature review of fuzzy set applications in military problems.

The following section presents an overview of fuzzy set theory and fuzzy sets extensions. Review of literature on military applications that implement fuzzy sets are given in the next section. This section focuses on the literature review and around military applications in an uncertain environment. Final section discusses our results and summarizes our findings.

2. FUZZY SET THEORY AND FUZZY SETS EXTENSION

The fundamentals of fuzzy sets were introduced by Zadeh (1965) to capture uncertainty and vagueness in knowledge. There have been published various fuzzy sets based studies in the literature that deal with uncertainty. Since 1965, fuzzy sets have been successfully implemented in various real-world applications to model uncertainty.

Fuzzy set, fuzzy logic and fuzzy system concepts provide a specific solution by processing linguistic information which is also provided by experts. While each linguistic knowledge corresponds to a fuzzy set, membership degree functions in these fuzzy sets can be decided by making personal preferences. Additionally, due to their capability of providing intermediate values (between mathematical expressions) fuzzy set theory is useful for modelling uncertainty and vagueness (Tozan, Karatas, & Vayvay, 2018). The main purpose of fuzzy logic systems that allow the modeling of daily spoken language is to consider how to solve this language by using this linguistic information (Karaköse & Akın, 2004; Karatas 2017; Karatas &

Akman, 2014; Karatas, 2020). Different types of fuzzy sets and their extensions are explained below:

2.1. Type-1 Fuzzy Sets

The concept of Type-1 fuzzy set theory was introduced by Zadeh (1965). A Type-1 fuzzy set is a generalization of a crisp set and single variable which is two-dimensional. It is presented on a universe of discourse *X*.

$$A = \{(x, \mu_A(x)) | x \in X\}$$

Where $\mu_A(x)$ denotes the degree of membership of the element x ∈ X to the set A. $\mu_A = X \to [0,1]$ that is a continuous grade.

When X is continuous, A can be represented as follows:

$$A = \int_x \mu_A(x)/x$$

The integral sign does not show integration, it shows the collection of all points $x \in X$. When X is discrete, A can be represented as follows:

$$A = \sum_x \mu_A(x)/x$$

The summation sign does not Show arithmetic addition, it shows the collection of all points $x \in X$. It represents the set theoretic operation of union.

2.2. Type-2 Fuzzy Sets (T2FSs)

There are two types of T2FSs which consist of general and interval T2FSs, respectively. The concept of general Type-2 fuzzy sets was originally defined by Zadeh in 1975 (Zadeh, 1975) as a generalized and extended traditional fuzzy sets to cope with uncertainty of membership function that is fuzzy and three-dimensional, which provide additional design degrees of freedom (Mendel et al., 2006).

Type-2 fuzzy sets denote $\tilde{A}$ that belongs to *X* universal set. It can be defined as follows (Mendel et al., 2006):

$$\tilde{A} = \{((x, u), \mu_{\tilde{A}}(x, u)) | \forall_x \epsilon X, \quad \forall_u \epsilon J_x \subseteq [0,1], \quad 0 \leq \mu_{\tilde{A}}(x, u) \leq 1)\}$$

Where J_x denotes [0, 1].

2.3. Interval Type-2 Fuzzy Sets (IT2FSs)

IT2FS developed by Mendel et al. (2006). IT2FS is a simplified version of Type-2 fuzzy set and characterized by two membership functions. An IT2FS allows us to incorporate the uncertainty about the membership function into the fuzzy set theory (Figueroa García, 2015; Deveci, Akyurt, & Yavuz, 2018; Eyoh, John, De Maere, & Kayacan, 2018; Naim & Hagras, 2012).

IT2FS $\tilde{A}$ are shown in Figure 1. The footprint of uncertainty (FOU) can be illustrated in Figure 1 as the blue shaded region (Mendel et al., 2006). Some basic definitions are presented as follows (Mendel et al., 2006; Lee & Chen, 2008):

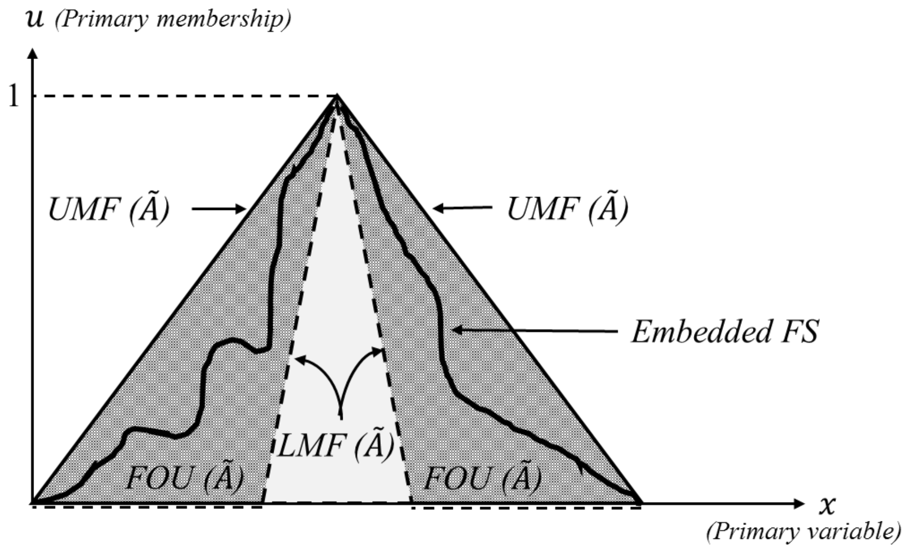

Figure 1. Interval Type-2 fuzzy set $\tilde{A}$ (lower and upper membership function can be denoted as LMF and UMF, respectively).

An IT2FS $\tilde{A}$ can also be stated as follows (Mendel & John, 2002):

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}\,(x,u)/(x,u)$$

Let $\mu_{\tilde{A}}$ states a Type-2 membership function. $\tilde{A}$ Type-2 fuzzy set can be defined as follows (Mendel et al., 2006):

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} 1/(x,u),$$

Where $J_x \subseteq [0,1]$.

2.4. Interval-Valued Fuzzy Sets (IVFs)

The concept of IVFs which has interval-valued membership function is introduced by Sambuc in 1975 (Bustince Sola et al., 2014). In the theory of fuzzy sets, it is often difficult to express an expert's opinion exactly on an interval [0, 1]. Therefore, it is more appropriate to show this degree of accuracy with an interval. Sambuc (1975) and Grattan (1976) stated that it is not enough to present a linguistic expression in the form of fuzzy sets. An IVSs A expressed on $(-\infty, +\infty)$ is represented as follows (Ashtiani, Haghighirad, Makui, & Ali Montazer, 2009; Stanujkic, Zavadskas, Brauers, & Karabasevic, 2015).

$$A = \{(x, [x, [\mu_A^L(x), \mu_A^U(x)]])\}$$

$$\mu_A^L(x), \mu_A^U(x): X \rightarrow [0,1] \quad \forall x \in X, \quad \mu_A^L(x) \leq \mu_A^U(x)$$

$$\mu_A(x) = [\mu_A^L(x), \mu_A^U(x)]$$

Where $\mu_A^L(x)$ and $\mu_A^U(x)$ indicate the lower and upper limit of membership degree, respectively.

2.5. Intuitionistic Fuzzy Sets (IFSs)

The concept of IFS is presented Atanassov (1986), which are characterized by member and non-member function. IFS are an extended version of fuzzy set. An intuitionistic fuzzy set A in X is of the form (Atanassov, 1986; Afful-Dadzie, Oplatkova, & Prieto, 2017):

$$A = \{(x, \mu_A(x), v_A(x))|x \in X \ \rightarrow [0,1]\}$$

Where $\mu_A(x)$ and $v_A(x)$ are denoted as the degree of membership function and degree of non-membership function, respectively. These membership functions are restricted by $0 \ \leq \ \mu_A \ (x) \ + \ v_A \ (x) \ \leq \ 1.$

And the intuitionistic index (called hesitation) can also be added by Atanassov, given as follows:

$$\pi_A \ (x) = 1 - \mu_A \ (x) + v_A \ (x)$$

Where $0 \leq \pi_A \ (x) \leq 1$ for each $x \in X$ and $\pi_A \ (x) + \mu_A \ (x) + v_A \ (x) = 1.$

2.6. Hesitant Fuzzy Sets

Hesitant fuzzy sets (HFS) ware proposed by Torra (2010) as a new extension of fuzzy sets to allow the membership degree of an element to be assigned. HFSs are powerful tools for managing simultaneous sources of uncertainty and can better describe the situation (Zhang, Ju, & Liu, 2017). H is the set of all HFSs that can be expressed mathematically in the following form (Xia & Xu, 2011):

$$H = \{\langle x, h_H(x)\rangle|x \in X\}$$

Where $h_H(x)$ is a set of hesitant fuzzy elements in $[0, 1]$.

2.7. Pythagorean Fuzzy Sets (PFSs)

PFS is a generalized concept of intuitionistic fuzzy sets and introduced by Yager & Abbasov (2013). PFS is a new tool to handle the uncertainty considering the membership grades (Yager, 2016).

$$P = \{\langle x, P(\mu_P(x), v_P(x))\rangle|x \in X\}$$

Where $\mu_P(x), v_P(x) \colon \rightarrow [0,1]$ and $\mu_P^2(x) + v_P^2(x) \leq 1$ for each $x \in X$. $\mu_P(x)$ and $v_P(x)$ are the membership and the non-membership degree of the element x to X in P, respectively. The hesitant degree $\left(\pi_P(x)\right)$ of $x \in X$ is defined as follows:

$$\pi_P(x) = \sqrt{1 - \mu_P^2(x) - v_P^2(x)}$$

2.8. Neutrosophic Fuzzy Sets

Neutrosophic sets (NSs) and logic which presented by Smarandache (1999) are the generalized the intuitionistic fuzzy sets. It allows cooperation on indeterminacy, hesitation and/or uncertainty. A NS can be characterized by a truth, an indeterminacy, and a falsity membership function. In neutrosophic sets, all three measures are independent and each one can be independently equal to 1 and therefore the sum of these grades can be added up to 3 (Smarandache, 1999; Haibin, Smarandache, Zhang, & Sunderraman, 2010).

$$A = \left\{\langle x, \left(T_A(x), I_A(x), F_A(x)\right)\rangle | x \in X\right\}$$

Where $T_A(x)$, $I_A(x)$ and $F_A(x)$ denote the truth, indeterminacy and falsity-membership function.

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$$

2.9. Spherical Fuzzy Sets

Spherical fuzzy set (SFS) is one of the extensions of intuitionistic fuzzy sets introduced by Kutlu & Kahraman (2019). A SFS can be defined as follow:

$$0 \leq \mu_A^2(x) + v_A^2(x) + \pi_A^2(x) \leq 1 \quad \forall x \in X$$

Where $\mu_P(x), v_P(x)$ and $\pi_P(x)$ denote the degree of membership, non-membership and hesitancy of x, respectively.

3. LITERATURE REVIEW ON MILITARY APPLICATIONS

Researchers have been attracted to consider fuzzy sets related problems in the previous works. Different decisions related with the military operations such as weapon selection, arms transfers, vehicle selection problems have been studied on the literature. Table 2 presents some studies on fuzzy MCDM based military applications.

The weapon selection problem has been proposed for different conditions and decision in the military operations. A weapon selection problem is a complex structure which includes multi-factors in multi-level with multi-hierarchy (Cheng, 1999). The fuzzy sets are suitable for this aim. In the Cheng's (1999) study, three missile systems are evaluated by considering tactics, technology, maintenance, economy and advancement criteria. Li, Huang, & Chen, (2010) proposed a new methodology that considers heterogeneous multi attribute group decision problems by measuring differences between positive and negative ideal solutions via weighted Minkowski distance. They illustrated the proposed approach with solving mission weapon system selection problem for three alternatives with the criteria that are airspace, time, target, missile, anti-jamming performance, shooting availability, maneuverability, man-machine design, C3 function, reliability engineering design, plan expense, expansibility, development schedule and risk. Yang, Wang, Xu, & Li (2014) proposed a fuzzy multi attribute decision making approach for weapon selection problem. The entropy method is applied to evaluate the weights of the criteria and the fuzzified TOPSIS method ranks the order of weapon systems. A case study conducted with seven criteria and three alternatives for missile system selection. Wang, Wang, Xu, & Ni (2014) proposed also weapon selection problem by extending fuzzy measures with dual hesitant fuzzy sets.

Aircrafts are vital for air-force and the selection of the aircrafts is one of the most complex and important problem in the air-forces. Sánchez-Lozano, Serna, & Dolón-Payán (2015) consider an aircraft selection problem in the Spanish Air-Force. They focused both technical and other criteria such as service, human and flying factors together and criteria weighted with analytical hierarchy process. Then, the best aircraft for training is evaluated by TOPSIS method.

Another vehicle selection problem arises in unmanned aerial vehicle assessment and selection domain. As an example, Lin & Hung (2011) proposed a new approach of fuzzy weighting average algorithm for unmanned aerial vehicle selection problem and developed a user-interface for this approach. Mission flexibility, operational suitability and operational assessment are selected as a main criterion and two-level hierarchical criteria are used for this aim. David, Octavio, David, & Victor (2015) considered autopilots in unmanned aerial vehicles which provides autonomously movements to aircraft. In this study, authors developed a fuzzy based system. The interested reader is also recommended Yakıcı, Karatas, & Yılmaz (2019), Yılmaz, Yakıcı, & Karatas (2019) and Karatas et al. (2019) for a review of UAV planning problems observed in defence and military.

Some management issues also need some decision-making such as human resources decisions. Moon, Lee, & Lim (2010) considers a human resource management topic in the military which is promotion ranking system in Korea. This new system screens the progress on promotion of the candidates with the service rating, multi-area aptitude, growth potential and innovativeness criteria. Fuzzy sets did not apply only MCDM problems in military applications but also other decision-making problems. Bean, Joubert, & Luhandjula (2016) proposed a single item inventory management problem with fuzzy-stochastic multi-objective modeling for considering uncertainties. The proposed approach compared with the classical inventory control approach for different scenarios. Braathen & Sendstad (2004) proposed a simulation game model which is based on fuzzy logic constraint satisfaction problem for a military headquarters' decision process. The application is two-fold as the games for air and land operations. Proposed model enables lots of different actions in the system. Similar management problems such as conflict analysis are studied in the Sutoyo, Mungad, Hamid, & Herawan's study (2016) which is based on soft-set theory. Lu & Wang (2011) proposed a MCDM problem for industrial cooperation program strategies. By using an alternative approach for classical systems, Taiwan's industrial cooperation program offset's is evaluated by policy, ability, economy and environment criteria. Analytical hierarchy process with fuzzy sets is applied for this aim.

There are lots of operational issues and missions in the military and it makes vital to operational decisions and operational decision-making capability in the military systems. Khanmohammaddi, Dagli, & Esfahlani (2012) predicted the irregular human behavior during the stressful missions by using fuzzy inference approach. The data obtained from the facial, movement, environmental and deliberating factors are evaluated with fuzzy inference systems to predict potential irregular behaviors in order to avoid failure on different military missions. Palaniappan, Zein-Sabatto, & Sekmen (2001) studied a multi-objective optimization problem with genetic algorithm for war resource allocation decisions. Multi-objective nature of problems is generally faced on the optimization decisions and in that study, authors proposed a genetic algorithm approach for handling this multi-objective problem. Fuzzy logic systems were used for setting genetic algorithm.

Another problem on the military decision making domain is related to arms transfers. Sanjian (2003) proposed a fuzzy system based models for arms transfers and their impacts of United States and Union of Soviet Socialist Republics. Some decisions were fuzzified such as political relationship, arms import and transfers and the relations were summarized. Juan, Huapu, Xu, Xianfeng, & Huijun (2014) considered the military path selection problem with data envelopment analysis and multi-objective fuzzy decision-making approach. Travel time, risks, response capability and costs were selected as parameters on the decisions.

Fuzzy set qualitative comparative analysis was applied for explaining the effects of NATO's military campaign in Libya (Haesebrouck, 2017). The study integrates burden sharing with balancing threats, politics, and domestic constraints.

The information systems in the military operations are coming crucial with the digitalization. Therefore, software systems related problems are arisen. Fuzzy sets are very important tool in also software coding. In the study of Schenker & Khoshgoftaar (1998), fuzzy case-based reasoning was proposed for fault identifying of a software metrics. The case study was conducted from a military command, control and communication system in order to show the proposed approach's performance. Arulkumaran & Gnanamurthy (2019) proposed a fuzzy-based attack detection system on the mobile adhoc network. The study specifically focused on black hole attacks with considering certificate, energy, packet and trust criteria.

Navy forces have some decisions in the sub-marine systems. Son, Park, & Joo (2014) focused an underwater sub-marine application of fuzzy sets. In the study, authors proposed a fuzzy *c*-means clustering algorithm for detecting a target by using sonar system. Radar signals processing have one of the main applications in the military operations. Salim, Abdel-Aty-Zohdy, & Zohdy (2010) proposed a new approach called as hyper-fuzzy model that ensures to incorporation between variables with historical learning and output predictions in the radar processing.

Table 2. Some studies on military problems using MCDM approaches.

Author(s)	Year	Main-criteria	Sub-criteria	Number of alternatives	MCDM problem	Fuzzy numbers	TOPSIS	AHP	FWA	Integral	DEA
Cheng	1999	5	23	3	Evaluating weapon system	Yes					
Li et al.	2010	14	39	3	Weapon system selection	Yes	x				
Moon et al.	2010	4	-	10	Appraisal and promotion ranking system	Yes					
Lin & Hung	2011	3	16	4	Evaluating unmanned aerial vehicle	Yes			x		
Lu & Wang	2011	4	20	6	Evaluating ICP Transactions types	Yes		x		x	
Yang et al.	2014	-	-	-	Weapon system selection	Yes	x				
Wang et al.	2014	-	-	-	Evaluating weapon system	Yes	x				
Juan	2014	3	-	5	Military transport path selection	Yes					x
Sánchez-Lozano et al.	2015	12	-	5	Evaluating military training aircrafts	Yes	x	x			
Sánchez-Lozano and Rodriguez	2020	-	13	4	Training aircraft selection	Yes		x			

4. CONCLUSION

This study attempts to fill this gap in the military problems that include MCDM applications using fuzzy sets in the current literature. Researchers tend to use different MCDM methods for different problems. There are lots of different MCDM approaches which are used for different aims. Therefore, different fuzzified methods can be used for different problems such as weapon selection problem, resource allocation problems and resource transferring problems.

Data management and cyber security is becoming vital not only the industry but also in the military. Therefore, software systems and cyber security related works is getting valuable for this area. Furthermore, fuzzy sets are easily converted to a decision support system by developing the user-interface can be found in some studies. The computational complexity of the related problems is easily handled with this approach and the systems may change to an expert system by using artificial intelligence methods including fuzzy sets, meta-heuristics (genetic algorithms i.e.), artificial neural networks. Meta-heuristics approaches are preferred to solve complex and combinatorial search problems by many researchers as well as practitioners. These approaches especially are classified with considering their characteristics such as the nature of the search process and use of memory.

Fuzzy sets and meta-heuristic approaches can also be used together in military problems. Firstly, fuzzy sets for importance rates of alternatives can be investigated as in the same manner as proposed for importance of criteria. Secondly, genetic algorithm can be used to see how it provides quality solutions where a large enough population is set. Finally, the problem can be solved using one of the multi-objective optimization algorithms defining a second objective. There are some weaknesses despite the advantages of using fuzzy logic. For example, fuzzy logic depends very much on the expert experience of the rules used in control problem. Since membership functions are found by trial, there may be a waste of time. So it can be less reliable than it is heuristic approach. Therefore, there may be some criticisms for its use in military problems. In addition, the membership functions can be improved and configured in order to handle uncertainty within the fuzzy sets using well-known meta-heuristics.

There are many different fuzzy sets considered in different military applications. The advantages of fuzzy sets are capable of modeling potentially different types of uncertainty and vagueness. It is also expected that the usage of various types of fuzzy sets such as fermatean fuzzy sets, nonstationary fuzzy sets, picture fuzzy sets, Q-rung orthopair fuzzy sets (q-ROFs) or rough sets introduced in recent years are widely used in the military problems. In addition, different MCDM approaches such as AHP, TOPSIS, VIKOR, TODIM, WASPAS, DEMATEL, COPRAS and so on can be used for military decision making problems.

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