

Elementary Teachers' Beliefs about Mathematical Problem Solving

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ABSTRACT

The purpose of this study is to investigate teachers' beliefs about mathematical problem solving. In this study, phenomenology was used to gain an insight about the elementary classroom teachers' beliefs about mathematical problem solving. To collect data, interviews were conducted with ten elementary teachers. The data were analyzed through content analysis. According to analysis, the nature of mathematical problem solving, the importance of mathematical problem solving, problem solving process in classroom environment, problem solving ability, teaching the correct procedure to solve problems, several ways of solving a problem, the importance of understanding, and the importance of problem posing are eight themes of the study.

Keywords: Problem solving, beliefs, elementary teachers



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INTRODUCTION

In mathematics education, various interpretations of "problem" and "problem solving" have been proposed over the years (Lester, 2013). Traditionally, problem has been defined as "a situation that confronts a person, that requires resolution, and for which the path to the solution is not immediately known" (Posamentier & Krulik, 1998, p. 1). Polya (1981) stated that:

Solving a problem means finding a way out of a difficulty, a way around an obstacle, attaining an aim which was not immediately attainable. Solving problems is the specific achievement of intelligence, and intelligence is the specific gift of mankind: problem solving can be regarded as the most characteristically human activity (p. ix).

Definition of problem solving refers to the process of applying existing knowledge to new situations (Ministry of National Education, 2013; Reys, Suydam, Lindquist & Smith, 1998). Understanding the difference between mathematical problem and exercise is also important. An exercise is "designed to check whether a student can correctly use a recently introduced term or symbol of the mathematical vocabulary" (Polya, 1953, p. 126). However, problem solving requires "much more than simple recall of facts or application of well-learned procedures" (Lester, 1994, p. 668).

Problem solving is one of the most important activities in human life (Jonassen, 2000). According to the National Council of Teachers of Mathematics (NCTM) (2000), "solving problems is not only a goal of learning mathematics but also a major means of doing so" (p. 52). Improving problem solving skill is one of the main goals of mathematics education (Posamentier & Krulik, 1998). So, problem solving has been a major component in the school mathematics curriculum of many countries around the world (Lesh & Zawojewski, 2007). In educational curriculum, problem solving is not only a skill to be taught, but also a teaching method and a goal for mental development (Brown, 2003). However, literature shows convincingly that many students in schools of today do not have sufficient knowledge and capabilities based on learning, thinking and problem solving skills (De Corte, 2003).

Problem solving skills of students are influenced by various factors such as cognitive factors, mathematical knowledge, age, attitudes, beliefs, and so on (Charles & Lester, 1982). Similarly, students' problem solving performance may be influenced by the teacher's instructional practices. On the other hand, the teacher's knowledge and beliefs play an important role in determining what happens during instruction (Carpenter & Fennema, 1991; Putnam, Heaton, Prawat & Remillard, 1992). Levin, He and Allen (2013) stated that in their study they observed teachers' beliefs in action in their classroom practices. In this respect, Wijaya, Van den Heuvel-Panhuizen and Doorman (2015) emphasize that teachers' beliefs may play an important role in students' problem solving because teachers' teaching practice is influenced by their thinking about mathematics teaching and learning.

Richardson (1996) describes beliefs as "psychologically-held understandings, premises or propositions about the world that are felt to be true" (p. 103). According to Kloosterman (2002), "a student's belief is something the student knows or feels that affects effort – in this case effort to learn mathematics" (p. 248). Results of some studies indicate that students believe there are always rules to follow in mathematics and think all problems can be solved by applying rules (e.g., Dossey, Mullis, Lindquist & Chambers, 1988). However, Charles and Lester (1982)





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emphasized that "the person has no readily available procedure for finding the solution" (p. 5). In addition, some students believe that "knowing why an answer is correct is as important as getting the correct answer" (Dossey et. al., 1988, p. 102). However, for some students it does not matter why the answer is correct because they think that "only geniuses are capable of discovering, creating, or really understanding mathematics" (Schoenfeld, 1988, p. 151). Schommer-Aikins, Duell and Hutter (2005) concluded that middle school students' beliefs might play a role in their problem solving performance. Students' beliefs about problem solving affect their learning and problem solving (Kloosterman & Stage, 1992). In this sense, investigating teachers' and students' beliefs about mathematical problem solving is important.

Because of the importance of belief about mathematical problem solving, many studies were conducted on this issue. Some studies on belief about mathematical problem solving aimed to investigate mathematical problem solving beliefs held by students and teachers. For example, Kayan and Çakıroğlu (2008) investigated pre-service elementary mathematics teachers' beliefs about mathematical problem solving. At the end of the study, they concluded that although pre-service teachers had positive beliefs about mathematical problem solving, they had some traditional beliefs about the importance of computational skills and solving problems following predetermined procedure. Similarly, Ford (1994) examined teachers' and students' mathematical problem solving as the application of computational skills.

In a paper about 25 years ago, Chapman (1997) pointed out that there was limited research on teaching problem solving from a teacher's perspective. In the current literature, when the studies focusing on beliefs about problem solving are examined, it can be seen that these studies were mainly conducted with students and prospective teachers (e.g., Callejo & Vila, 2009; Deringöl, 2018; Giovanni & Sangcap, 2010; Haciömeroğlu, 2011; Kayan & Çakıroğlu, 2008; Mason, 2003; Sağlam & Dost, 2014; Schommer-Aikins et al., 2005; Ünlü & Sarpkaya Aktaş, 2016). On the other hand, the studies dealing directly with the in-service teachers' beliefs are limited (e.g., Andrews & Xenofontos, 2015; Chen, Dooren & Verschaffel, 2011; Emenaker, 1996; Ford, 1994). Therefore, this study focuses on teachers' beliefs about problem solving.

The purpose of this study is to investigate teachers' beliefs about problem solving. More specifically, the study focused on the elementary teachers' beliefs about mathematical problem solving. For this purpose, the following research question was posed to guide the study: What are the beliefs of elementary teachers about mathematical problem solving?

METHOD

Research Design

In this study, qualitative research method was used to gain an insight about the elementary teachers' beliefs about mathematical problem solving. Qualitative research methods enable researchers to obtain insights, understandings and deep information about the issue under investigations (Patton, 1990). Phenomenology was used as main research design in this study. Marshall and Rossman (2006) stated that "phenomenology is the study of lived experiences and the ways we understand those experiences to develop a worldview" (p. 104). Since the purpose of the study is to explore elementary teachers' beliefs about mathematical problem solving,





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qualitative research methodology give us in-depth information about the phenomenon being investigated.

Participants

In this study, maximum variation sampling strategy for purposeful sampling was used to select the participants (Patton, 1990). The present study was conducted with ten elementary teachers (Teacher A, B, C, D, E, F, G, H, I, J) working in public elementary schools in Konya, Turkey at the end of the spring semester of 2016-2017 academic year. Teaching experience of the participating teachers ranged from 10 to 25 years. General information about participants is given in Table 1.

Participant	Gender	Years of teaching experience	Grade level taught
А	Male	18	3
В	Male	11	4
С	Male	20	2
D	Male	15	1
Е	Female	21	2
F	Female	15	4
G	Female	11	1
Н	Female	20	1
I	Female	25	3
J	Male	10	4

Table 1.	General	Information	about	Participants
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Data Collection Instrument

Phenomenology uses the interviews as the major data collection method (Yıldırım & Şimsek, 2006). Qualitative interviews can be employed either as the dominant strategy for data collection, or in conjunction with observation, document analysis, or other techniques (Bogdan & Biklen, 1998). In this study, the data were collected through semi-structured and in-depth interviews.

Interviews were conducted with ten elementary teachers. After the related literature and the reliable and valid instruments regarding beliefs about mathematical problem solving were reviewed, an interview schedule with 13 open-ended questions was developed by the researcher. The questions were asked to learn about elementary teachers' beliefs about mathematical problem solving. Seven interview questions were adapted from *Indiana Mathematics Belief Scales* (Kloosterman & Stage, 1992) and *Mathematics Belief Instrument* (Hart, 2002). Other questions were written specifically to identify teachers' beliefs about mathematical problem solving and problem solving process. Two field experts studying in the field of mathematics education were asked for opinions on this interview schedule. The interview schedule was restructured in accordance with the expert opinions. A pilot study was conducted prior to the actual study. After the pilot study was conducted, interview schedule was shaped again. The interviews were conducted by the researcher. Each interview lasted for approximately 25 minutes.

Data Analysis Procedures

Bogdan and Biklen (1998) define qualitative data analysis as "working with data, organizing them, breaking them into manageable units, synthesizing them, searching for patterns,





discovering what is important and what is to be learned, and deciding what you will tell others" (p. 157). For this study, the data were analyzed through content analysis.

Yıldırım and Şimşek (2006) stated that there are four stages to analyze a qualitative data. They explained these stages as: coding data, generating themes, organizing of codes and themes, defining and interpreting findings. After the data come from interviews were transcribed, the data were coded carefully. The codes were categorized to generate meaningful themes to identify teachers' beliefs about mathematical problem solving. Themes were then produced and labeled. Excerpts from the data were presented to describe and exemplify the themes in detail.

Trustworthiness

Researchers are responsible for establishing the trustworthiness of qualitative research through several strategies. To ensure this, Lincoln and Guba (1985, as cited in Marshall & Rossman, 2006) proposed four constructs: credibility, transferability, dependability and confirmability. In order to ensure credibility, the interview schedule with the open-ended interview questions was developed by consulting experts; and pilot study was conducted before the actual study. Similarly, prolonged engagement and member checks were implemented. Transferability has been guaranteed with maximum variation sampling and in-depth description of processes. To address dependability and confirmability, an audit trail was employed and findings of the current study were confirmed by a researcher (Erlandson, Harris, Skipper & Allen, 1993, as cited in Yıldırım & Şimşek, 2006).

FINDINGS

The findings of the data were presented under the following eight themes: the nature of mathematical problem solving, the importance of mathematical problem solving, problem solving process in classroom environment, problem solving ability, teaching the correct procedure to solve problems, several ways of solving a problem, the importance of understanding and the importance of problem posing.

The nature of mathematical problem solving

The elementary teachers have different points of view about the meaning of "problem solving". The results revealed that most of the teachers perceived problem solving as a process of arranging and interpreting existing knowledge. From their own words, Teacher G and Teacher H asserted that:

Problem solving is a process in which we arrange the given and our knowledge and then determine which operations to use according to the asked. (Teacher G)

Problem solving is the interpretation of mathematical knowledge. (Teacher H)

On the other hand, a few teachers perceived problem solving as performing arithmetic operations. To illustrate, Teacher A said that:

Problem solving means performing arithmetic operations on given numerical data.





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The importance of mathematical problem solving

The results of this study indicated that elementary teachers were aware of the importance of problem solving in mathematics lessons. Teachers stated that problem solving helped students to improve their self-confidence. They also stated that problem solving was important for understanding mathematics. According to them, problem solving is a skill that students will use throughout their lives. Similarly, teachers stated that problem solving was the main element of mathematics. They also said that problem solving helped students to develop reasoning skills. In this regard, Teacher G and F pointed out that:

Problem solving is the main element of mathematics because an individual who knows how to use mathematical knowledge has understood mathematics. In mathematics lessons, it is important to learn mathematical concepts and to use concepts learned. So, problem solving is mathematics itself and is the heart of mathematics lesson. (Teacher G)

Problem solving is an essential skill. Students improve their knowledge and gain selfconfidence through problem solving. Additionally, students develop skills to solve the problems they encounter in their everyday lives. (Teacher F)

The problem solving process in classroom environment

Teachers also mentioned their beliefs about how the problem solving process in classroom environment should be. Participating teachers reported that while solving problems, it is important that the problem is clearly stated and suitable for student level, and it vital to follow problem solving steps, understand the problem, relate problem to real life and concretize the problem. Teachers also think that they need to follow a process and use different problem solving strategies while solving problem. However, they had difficulty in specifying problem solving strategies. Only three teachers mentioned some problem solving strategies: making a drawing, intelligent guessing and testing, making an organized list, making a table, finding a pattern, and solving simpler analogous problem. Below an example is given of elementary teachers' opinions on the problem solving process in classroom environment.

In classroom environment, while solving problems it is important to follow problem solving steps. I think a certain process should be followed while solving the problem. Because of my students' grade level, I want my students to follow a certain process including reading and understanding the problem, expressing it in their own sentences, determining which arithmetic operation will be used and explaining why this operation have been chosen. (Teacher C)

Problem solving ability

The results revealed that teachers believed that their students' problem solving abilities could be improved. They believed that in order to improve their students' problem solving skills, they could follow several ways such as taking into account students' different ways of solving a problem, allowing time for reading-understanding and reasoning studies, associating problems with daily life, solving a lot of problems, asking fun questions that require reasoning. For instance, Teacher H said that:

I think problem solving skill is related to intelligence and ability to use intelligence. So, improving problem solving ability is possible. In order to improve my students' problem





solving abilities, I ask them fun questions that require reasoning. I think this is very useful.

Teacher I, C, and E also said that following an approach from simple to difficult was useful to improve students' problem solving skills. In addition, Teacher G expressed that understanding mathematical concepts, mathematical computational skills and following problem solving steps has played important roles in improving students' problem solving abilities.

Teaching the correct procedure to solve problems

The results revealed that six participating teachers believed that students had to be taught the correct procedures to solve mathematical problems. According to them, in this case, students can produce their own solutions to different problems. Similarly, they think that students have to be taught correct procedures to solve mathematical problems but students should never neglect to try different methods and techniques. According to them, learning correct procedures to solve problems allows you to use the time correctly. For example, Teacher A expressed that:

I think students have to be taught the correct procedure to solve the problems. So, students can produce their own solutions to different problems.

On the other hand, four of the participants disagreed that students had to be taught the correct procedure to solve the problems. According to them, there can be multiple solutions to answer a problem; and finding the right procedure by the students helps them develop their creativity. For instance, Teacher J said that:

I disagree that students have to be taught the correct procedure to solve the problems. Students should discover the way of solving on their own. So, this enables effective learning. Similarly, this helps students develop creativity.

Several ways of solving a problem

The results revealed that teachers thought that there was not only one correct way to solve any mathematics problem. Similarly, all of the teachers stated that knowing different ways of solving the same problem was useful for students. In addition, all teachers believed that students could develop different methods other than the teacher or the textbooks use. Accordingly, Teacher D asserted that:

There is not only one correct way to solve any mathematics problem. Knowing different ways of solving the same problem is certainly useful for students because different students may prefer different methods. That is, examining different solution methods for the same problem helps students understand the concept.

Similarly, they believed that their students could develop different ways while solving problems, as Teacher D said that:

Since each student's imagination, problem solving method and technique is different, they can develop different approaches to the problem.





On the other hand, one teacher stated that she believed students could develop different approaches while solving problems, but only a few students have achieved this. More specifically, Teacher F expressed that:

I think that our students can develop different ways while solving problems. However, only one or two students from each class have achieved this.

The importance of understanding

Most participants believed that if a person did not understand why an answer to a mathematics problem was correct, then they had not really solved the problem. In this regard, teachers stated that in addition to getting a right answer in mathematics, it was also important to understand why the answer was correct. For example, Teacher B said that:

If a person who finds the answer of the problem does not understand why the answer is correct, in such a case, they have not fully understood the problem. It is important to understand the problem and develop different ways of solving the problem.

On the other hand, one participant stated that in such a situation the person has solved the problem but has not understood. More specifically, Teacher I said that:

A person who finds the solution of the problem but does not understand why the solution is correct, I think, has solved the problem but has not understood. It does not really matter if you understand some mathematics problems as long as you get the right answer because understanding some problems requires a high level of mathematical knowledge.

The importance of problem posing

All teachers believed that teachers should do studies to improve their students' problem posing abilities. They emphasized that teachers should encourage students to write their own mathematical problems. Similarly, they think that posing a new mathematical problem is an indicator of a deeper understanding of the problem. For instance, Teacher B expressed that:

Problem posing is a crucial component of mathematics. Posing problem helps students look at the problem from a different perspective. Problem posing practices also provide deeper understanding of the problem and this increases students' self-confidence in mathematics.

RESULTS and DISCUSSION

A substantial body of research shows a relationship between teachers' beliefs and their teaching practices (e.g., Andrews & Xenofontos, 2015; Levin et al., 2013). "Teachers' beliefs, knowledge, judgments, thoughts, and decisions have a profound effect on the way they teach as well as on students' learning in their classroom" (Carpenter & Fennema, 1991, p. 3). Therefore, meaningful changes in instruction require essential changes in teachers' knowledge and beliefs (Putnam et al., 1992). For these reasons, in this study, elementary teachers' beliefs about problem solving were investigated.







A problem is typically defined as a situation that requires a solution and for which there is no immediately obvious way (Posamentier & Krulik, 1998). According to results of this study, most elementary teachers viewed problem solving as a process of arranging and interpreting existing knowledge. However, a few teachers perceived problem solving as performing arithmetic operations. Similarly, Ford (1994) indicated that both teachers and students view mathematical problem solving as the application of computational skills. However, teachers need to know that problem solving requires much more than performing arithmetic operations. In this respect, it is necessary to support teachers with in-service trainings about mathematical problem solving.

As the results indicated, teachers believed that problem solving helped students to improve their self-confidence, was helpful for understanding mathematics and for developing students' reasoning abilities, was a skill that students would use throughout their lives, and was the main element of mathematics. Therefore, it can be said that participating teachers have recognized problem solving as an integral part of the mathematics teaching and learning. In educational curriculum, problem solving is usually seen as a skill to be taught, as a teaching method and as a goal for mental development (Brown, 2003). According to NCTM (2000), solving problems is the major means of learning and doing mathematics. Therefore, it can be concluded from the results of the current study that elementary teachers were aware of the importance of problem solving in mathematics lessons.

The findings of this study demonstrated that teachers believed that it was possible to improve students' problem solving ability. Borko and Shavelson (1990) stated that teachers' teaching practices were affected by their beliefs about students' ability and learning. Accordingly, in the current study, elementary teachers stated that they used various ways in mathematics lessons to improve their students' problem solving skills.

As a result of this study, some teachers stated that students should not need to be taught the correct procedure to solve problems. They expressed that teachers should not directly tell or show students how to solve problems instead they should discover ways of solving on their own. However, the other participating teachers believed that students had to be taught the correct procedures to solve problems. They thought, in this case, students could produce their own solutions to different problems and use the time correctly. In this respect, in-service trainings about mathematical problem solving might help teachers to understand and develop mathematical problem solving knowledge for teaching.

Emenaker (1996) indicated that some preservice teachers believed that there was only one correct way to solve a particular problem. On the other hand, results of the current study showed that teachers were aware of the fact that a particular problem might have more than one solution. They also thought that knowing different ways of solving the same problem is useful for students and that students can explore different problem solving strategies on their own. That is to say, teachers were aware of the fact that there may be several ways of solving any problem and exploring these ways is important.

According to the findings, teachers believed that if a person did not understand why an answer to a mathematics problem was correct, then they had not really solved the problem. Thus, it can be concluded that elementary teachers in this study emphasized importance of understanding during the problem solving process. Similarly, Dossey et al. (1988) found that most of the eleventh-grade students believed that "knowing why an answer is correct is as important as getting the correct answer "(p. 102).





Problem posing is considered as a significant component of problem solving process (English, 2003). Similarly, results of this study indicated that teachers viewed problem posing as a crucial component of mathematics. Hence, it can be said that elementary teachers were aware of the importance of problem posing in problem solving process.

During the interviews elementary teachers also mentioned their beliefs about problem solving process in classroom environment. Results of the study showed that elementary teachers thought that during the instructional process, it was important that the problem is clearly stated and suitable for student level, and it vital to follow problem solving steps, understand the problem, relate problem to real life and concretize the problem. Similarly, they stated that during the problem solving process, it was useful to follow a certain process and use different problem solving strategies. However, they had difficulty in specifying these problem solving strategies. This result is consistent with the result of the Topbaş-Tat's (2018) study conducted with prospective elementary mathematics teachers. Investigating prospective teachers' opinions and problem solving processes before and after the problem solving instruction, the author concluded that before the problem solving instruction prospective teachers had limited knowledge and experience about problem solving process and strategies. On the other hand, the author found that problem solving strategies were learned and used for problem solving after the problem solving instruction. Similarly, Posamentier and Krulik (1998) emphasized that "in fact, it is often the teachers themselves who are not aware of the many problem-solving strategies that can be used to provide efficient and elegant solutions to many problems" (p. xv). Therefore, it can be concluded that elementary teachers' beliefs about usage of different problem solving strategies were positive, but their knowledge about these strategies were limited. In this respect, opportunities for teachers to learn problem solving strategies should be provided to increase their professional knowledge, skills and experiences in teaching problem solving.

REFERENCES

- Andrews, P., & Xenofontos, C. (2015). Analysing the relationship between the problem-solvingrelated beliefs, competence and teaching of three Cypriot primary teachers. *Journal of Mathematics Teacher Education*, 18, 299–325.
- Bogdan, R. C., & Biklen, S. K. (1998). *Qualitative research for education: An introduction to theory and methods*. Boston: Allyn and Bacon.
- Borko, H., & Shavelson, R. J. (1990). Teacher decision making. In B. F. Jones & L. Idol (Eds.), Dimensions of thinking and cognitive instruction (pp. 311–346). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.
- Brown, N.M. (2003). A Study of elementary teachers' abilities, attitudes, and beliefs about problem solving. *Dissertation Abstracts International*, 64(10), 3620.
- Callejo, M. L., & Vila, A. (2009). Approach to mathematical problem-solving and students' belief systems: Two case studies. *Educational Studies in Mathematics*, 72, 111–126.
- Carpenter, T. P., & Fennema, E. (1991). Research and cognitively guided instruction. In E. Fennema, T. P. Carpenter, & S. J. Lamon (Eds.), *Integrating research on teaching and learning mathematics* (pp. 1–16). Albany, NY: State University of New York Press.
- Chapman, O. (1997). Metaphors in the teaching of mathematical problem solving. *Educational Studies in Mathematics*, 32, 201–228.
- Charles, R., & Lester, F. (1982). *Teaching problem solving: What, why and how*. Palo Alto, CA: Dale Seymour Publications.





- Chen, L., Dooren, W. V., & Verschaffel, L. (2011). An investigation on Chinese teachers' realistic problem solving abilities and beliefs. *Journal of Mathematics Education*, 4(2), 80–96.
- De Corte, E. (2003). Transfer as the productive use of acquired knowledge, skills, and motivations. *Current Directions in Psychological Science*, *12*(4), 142–146.
- Deringöl, Y. (2018). Sınıf öğretmeni adaylarının matematik problemi çözmeye yönelik inançları ile problem kurma özyeterlik inançlarının incelenmesi [Examination of problem solving beliefs and problem posing self-efficacy beliefs of prospective classroom teachers]. *Türk Bilgisayar ve Matematik Eğitimi Dergisi*, 9(1), 31–53.
- Dossey, J. A., Mullis, I. V. S., Lindquist, M. M., & Chambers, D. L. (1988). *The Mathematics report card: Are we measuring up? Trends and achievement based on the 1986 national assessment.* Princeton, N.J.: Educational Testing Service.
- Emenaker, C. (1996). A problem-solving based mathematics course and elementary teachers' beliefs. *School Science and Mathematics*, 96(2), 75–84.
- English, L. D. (2003). Problem posing in elementary curriculum. In F. Lester, & R. Charles (Eds.), *Teaching mathematics through problem solving*. Reston, Virginia: National Council of Teachers of Mathematics.
- Ford, M. I. (1994). Teachers' beliefs about mathematical problem solving in the elementary school. *School Science and Mathematics*, 94(6), 314–322.
- Giovanni, P., & Sangcap, A. (2010). Mathematics-related beliefs of Filipino college students: Factors affecting mathematics and problem solving performance. *Procedia Social and Behavioral Sciences*, *8*, 465–475.
- Haciömeroğlu, G. (2011). Sınıf öğretmeni adaylarının matematiksel problem çözmeye ilişkin inançlarını yordamada epistemolojik inançlarının incelenmesi [Examining elementary preservice teachers' epistemological beliefs as predictors of beliefs about mathematical problem solving]. Buca Eğitim Fakültesi Dergisi, 30, 206–220.
- Hart, L. C. (2002). Preservice teachers' beliefs and practice after participating in an integrated content/methods course. *School Science and Mathematics*, 102(1), 4–14.
- Jonassen, D. H. (2000). Toward a design theory of problem solving. *Educational Technology Research and Development*, 48(4), 63–85.
- Kayan, F., & Çakıroğlu, E. (2008). İlköğretim matematik öğretmen adaylarının matematiksel problem çözmeye yönelik inançları [Preservice elementary mathematics teachers' mathematical problem solving beliefs]. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi*, 35, 218–226.
- Kloosterman, P. (2002). Beliefs about mathematics and mathematics learning in the secondary school: Measurement and implications for motivation. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 247–269). Dordrecht: Kluwer Academic Publishers.
- Kloosterman, P., & Stage, F. K. (1992). Measuring beliefs about mathematical problem solving. *School Science and Mathematics*, 92(3), 109–115.
- Lesh, R., & Zawojewski, J. S. (2007). Problem solving and modeling. In F. Lester, (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 763–804), Charlotte, NC: Information Age Publishing.
- Lester, F. K. (1994). Musings about mathematical problem-solving research: 1970-1994. *Journal for Research in Mathematics Education*, 25(6), 660–675.
- Lester, F. K. (2013). Thoughts about research on mathematical problem-solving instruction. *The Mathematics Enthusiast*, 10(1), 245–278.





- Levin, B. B., He, Y., & Allen, M. H. (2013). Teacher beliefs in action: A cross-sectional, longitudinal follow-up study of teachers' personal practical theories. *The Teacher Educator*, 48(3), 201-217.
- Marshall, C., & Rossman, G. B. (2006). Designing qualitative research. Thousand Oaks, CA: Sage.
- Mason, L. (2003). High school students' beliefs about maths, mathematical problem solving, and their achievement in maths: A cross-sectional study. *Educational Psychology*, 23(1), 73–85.
- Ministry of National Education (2013). Ortaokul matematik dersi (5, 6, 7 ve 8. sınıflar) öğretim programı. Ankara: Talim ve Terbiye Kurulu Başkanlığı.
- NCTM (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- Patton, M. Q. (1990). Qualitative evaluation and research methods. Newbury Park, CA: Sage.
- Polya, G. (1953). On teaching problem solving. In H. F. Fehr (Ed.), *The Learning of mathematics: Its theory and practice* (pp. 228–270). 21st yearbook of the NCTM. Reston, VA: NCTM.
- Polya, G. (1981). *Mathematical discovery: On understanding, learning and teaching problem solving.* New York: Wiley.
- Posamentier, A. S., & Krulik, S. (1998). *Problem-solving strategies for efficient and elegant solutions*. California: Corwin Press.
- Putnam, R. T., Heaton, R. M., Prawat, R. S., & Remillard, J. (1992). Teaching mathematics for understanding: Discussing case studies of four fifth-grade teachers. *The Elementary School Journal*, 93(2), 213–228.
- Reys, R., Suydam, M., Lindquist, M., & Smith, N. (1998). *Helping children learn mathematics*. Boston: Allyn and Bacon.
- Richardson, V. (1996). The role of attitudes and beliefs in learning to teach, In J. Sikula (Ed.) *Handbook of research on teacher education* (pp.102-119). New York: Macmillan.
- Sağlam, Y., & Dost, Ş. (2014). Pre service science and mathematics teachers' beliefs about mathematical problem solving. *Procedia Social and Behavioral Sciences*, 116, 303–306.
- Schoenfeld, A. (1988). When good teaching leads to bad results: The disasters of 'well-taught' mathematics courses. *Educational Psychologist*, 23(2), 145–166.
- Schommer-Aikins, M., Duell, O. K., & Hutter, R. (2005). Epistemological beliefs, mathematical problem-solving beliefs, and academic performance of middle school students. *The Elementary School Journal*, 105(3), 289–304.
- Topbaş-Tat, E. (2018). Problem solving instruction: Prospective mathematics teachers' opinions and problem solving processes. *International Journal of Eurasia Social Sciences*, 9(32), 960– 990.
- Ünlü, M., & Sarpkaya Aktaş, G. (2016). İlköğretim matematik öğretmen adaylarının problem kurma özyeterlik ve problem çözmeye yönelik inançları [Pre-service elementary mathematics teachers' self-efficacy beliefs about problem posing and beliefs about problem solving]. Abant İzzet Baysal Üniversitesi Eğitim Fakültesi Dergisi, 16(4), 2040–2059.
- Wijaya, A., Van den Heuvel-Panhuizen, M., & Doorman, M. (2015). Teachers' teaching practices and beliefs regarding context-based tasks and their relation with students' difficulties in solving these tasks. *Mathematics Education Research Journal*, 27(4), 637–662.
- Yıldırım, A., & Şimsek, H. (2006). *Sosyal bilimlerde nitel araştırma yöntemleri* [Qualitative research methods in social sciences]. Ankara: Seçkin yayıncılık.