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Q-soft Translation of Q-soft Subgroups

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Abstract — In this study, we introduce the concept Q-soft translations of Q-soft subgroups. Next we investigate the properties of them and we prove that every Q-soft translation of Q-soft subgroup is also Q-soft subgroup. Finally, we consider them under homomorphism and anti-homomorphism of Q-soft subgroups and Q-soft normal subgroups.

Keywords — *Q-soft subsets, group theory, Q-soft subgroups, Q-soft normal subgroups, Q-soft translation, homomorphism*

Introduction

In mathematics and abstract algebra, group theory studies the algebraic structures known as groups. The concept of a group is central to abstract algebra and other well-known algebraic structures, such as rings, fields, and vector spaces, can all be seen as groups endowed with additional operations and axioms. Groups recur throughout mathematics, and the methods of group theory have influenced many parts of algebra. Linear algebraic groups and Lie groups are two branches of group theory that have experienced advances and have become subject areas in their own right. The concept of soft sets was first formulated by Molodtsov [1] as a completely new mathematical tool for solving problems dealing with uncertainties. After then Maji et al. [2] defined the operations of soft sets. The operations of soft sets have also been studied by Ali et al. [3], Çağman et al. [4], Çağman [5], and Sezgin and Atagün [6] in detail. Some researchers have applied soft sets theory to many different areas such as decision making [7–9], algebras [10–13] using these operations. The author investigated soft Lie ideals and anti soft Lie ideals and extension of Q-soft ideals in semigroups [14, 15]. In [16, 17], the authors introduced the concept of Q-soft subgroups and Q-soft normal subgroups and discussed the characterisations them under homomorphism and anti-homomorphism. In this paper, we define Q-soft translations of Q-soft subgroups and we show some properties of them. Next we prove that every Q-soft translation of Q-soft subgroup is also Q-soft subgroup. Also we obtain between Q-soft translation of Q-soft subgroup of group G and subgroup of group G. Later we prove that soft image and soft pre-image of Q-soft translation of Q-soft subgroup under group homomorphism is also Q-soft subgroup. Finally, we prove that soft image and soft pre-image of Q-soft translation of Q-soft normal subgroup under group homomorphism is also Q-soft subgroup.

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preliminaries

In this section we recall some of the fundamental concepts and definition, which are necessary for this paper. For details we refer reders to [1,15–18]. Throughout this work, Q is a non-empty set, U refers to an initial universe set, E is a set of parameters and $P(U)$ is the power set of U.

Definition 2.1. For any subset A of E, a Q-soft subset $f_{A\times Q}$ over U is a set, defined by a function $f_{A\times Q}$, representing a mapping $f_{A\times Q} : E \times Q \to P(U)$, such that $f_{A\times Q}(x, q) = \emptyset$ if $x \notin A$. A soft set over U can also be represented by the set of ordered pairs $f_{A\times Q} = \{((x, q), f_{A\times Q}(x, q)) \mid (x, q) \in E \times Q, f_{A\times Q}(x, q) \in P(U)\}$. Note that the set of all Q-soft subsets over U will be denoted by $QS(U)$. From here on, "soft set" will be used without over U.

Definition 2.2. Let $f_{A\times Q}, f_{B\times Q} \in QS(U)$. Then,

i. $f_{A\times Q}$ is called an empty Q-soft subset, denoted by $\Phi_{A\times Q}$, if $f_{A\times Q}(x, q) = \emptyset$ for all $(x, q) \in E \times Q$.

ii. $f_{A\times Q}$ is called a $A\times Q$-universal soft set, denoted by $f_{A\tilde{\times}Q}$, if $f_{A\times Q}(x, q) = U$ for all $(x, q) \in A\times Q$.

iii. $f_{A\times Q}$ is called a universal Q-soft subset, denoted by $f_{E\tilde{\times}Q}$, if $f_{A\times Q}(x, q) = U$ for all $(x, q) \in E\times Q$.

iv. The set $Im(f_{A\times Q}) = \{f_{A\times Q}(x, q) : (x, q) \in A \times Q\}$ is called image of $f_{A\times Q}$ and if $A \times Q = E \times Q$, then $Im(f_{E\times Q})$ is called image of $E \times Q$ under $f_{A\times Q}$.

v. $f_{A\times Q}$ is a Q-soft subset of $f_{B\times Q}$, denoted by $f_{A\times Q}\tilde{\subseteq} f_{B\times Q}$, if $f_{A\times Q}(x, q) \subseteq f_{B\times Q}(x, q)$ for all $(x, q) \in E \times Q$.

vi. $f_{A\times Q}$ and $f_{B\times Q}$ are soft equal, denoted by $f_{A\times Q} = f_{B\times Q}$, if and only if $f_{A\times Q}(x, q) = f_{B\times Q}(x, q)$ for all $(x, q) \in E \times Q$.

vii. The set $(f_{A\times Q}\tilde{\cup} f_{B\times Q})(x, q) = f_{A\times Q}(x, q) \cup f_{B\times Q}(x, q)$ for all $(x, q) \in E \times Q$ is called union of $f_{A\times Q}$ and $f_{B\times Q}$.

viii. The set $(f_{A\times Q}\tilde{\cap} f_{B\times Q})(x, q) = f_{A\times Q}(x, q) \cap f_{B\times Q}(x, q)$ for all $(x, q) \in E \times Q$ is called intersection of $f_{A\times Q}$ and $f_{B\times Q}$.

Example 2.3. Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ be an initial universe set and $E = \{x_1, x_2, x_3, x_4, x_5\}$ be a set of parameters. Let $Q = \{q\}$, $A = \{x_1, x_2\}$, $B = \{x_2, x_3\}$, $C = \{x_4\}$, $D = \{x_5\}$, and $F = \{x_1, x_2, x_3\}$. Define

$$f_{A\times Q}(x, q) = \begin{cases} \{u_1, u_2, u_3\}, & \text{if } x = x_1 \\ \{u_1, u_5\}, & \text{if } x = x_2 \end{cases}$$

$$f_{B\times Q}(x, q) = \begin{cases} \{u_1, u_2\}, & \text{if } x = x_2 \\ \{u_2, u_4\}, & \text{if } x = x_3 \end{cases}$$

$$f_{F\times Q}(x, q) = \begin{cases} \{u_1, u_2, u_3, u_4\}, & \text{if } x = x_1 \\ \{u_1, u_2, u_5\}, & \text{if } x = x_2 \\ \{u_2, u_4\}, & \text{if } x = x_3 \end{cases}$$

$f_{C\times Q}(x_4, q) = U$ and $f_{D\times Q}(x_5, q) = \{\}$. Then we will have $f_{C\times Q} = f_{C\tilde{\times}Q}$ and $f_{D\times Q} = \Phi_{D\times Q}$. Note that the definition of classical subset is not valid for the soft subset. For example, $f_{A\times Q}\tilde{\subseteq} f_{F\times Q}$ does not imply that every element of $f_{A\times Q}$ is an element of $f_{F\times Q}$. Thus $f_{A\times Q}\tilde{\subseteq} f_{F\times Q}$ but $f_{A\times Q} \nsubseteq f_{F\times Q}$ as classical subset.

Definition 2.4. Let $\varphi : A \to B$ be a function and $f_{A\times Q}, f_{B\times Q} \in QS(U)$. Then soft image $\varphi(f_{A\times Q})$ of $f_{A\times Q}$ under φ is defined by

$$\varphi(f_{A\times Q})(y, q) = \begin{cases} \cup\{f_{A\times Q}(x, q) \mid (x, q) \in A \times Q, \varphi(x) = y\}, & \text{if } \varphi^{-1}(y) \neq \emptyset \\ \emptyset, & \text{if } \varphi^{-1}(y) = \emptyset \end{cases}$$

and soft pre-image (or soft inverse image) of $f_{B\times Q}$ under φ is $\varphi^{-1}(f_{B\times Q})(x, q) = f_{B\times Q}(\varphi(x), q)$ for all $(x, q) \in A \times Q$.

Definition 2.5. Let $(G,.),(H,.)$ be any two groups. The function $f : G \to H$ is called a homomorphism (anti-homomorphism) if $f(xy) = f(x)f(y)(f(xy) = f(y)f(x))$, for all $x, y \in G$.

Proposition 2.6. Let G be a group. Let H be a non-empty subset of G. The following are equivalent:

i. H is a subgroup of G.

ii. $x, y \in H$ implies $xy^{-1} \in H$ for all x, y.

Definition 2.7. Let $(G,.)$ be a group and $f_{G\times Q} \in QS(U)$. Then, $f_{G\times Q}$ is called a Q-soft subgroup over U if $f_{G\times Q}(xy,q) \supseteq f_{G\times Q}(x,q) \cap f_{G\times Q}(y,q)$ and $f_G(x^{-1},q) = f_{G\times Q}(x,q)$ for all $x,y \in G, q \in Q$. Throughout this paper, G denotes an arbitrary group with identity element e_G and the set of all Q-soft subgroup with parameter set G over U will be denoted by $S_{G\times Q}(U)$.

Example 2.8. Let $G = \{1,-1\}$ be a group and $U = \{u_1,u_2,u_3,u_4\}, Q = \{q\}$. Let $f_{G\times Q} = \{((1,q),\{u_1,u_2\}),((-1,q),\{u_1,u_3\})\}$, then $f_{G\times Q} \in S_{G\times Q}(U)$.

Proposition 2.9. $f_{G\times Q} \in S_{G\times Q}(U)$ if and only if $f_{G\times Q}(xy^{-1},q) \supseteq f_{G\times Q}(x,q) \cap f_{G\times Q}(y,q)$ for all $x,y \in G, q \in Q$.

Definition 2.10. Let $f_{G\times Q} \in S_{G\times Q}(U)$ then $f_{G\times Q}$ is said to be a Q-soft normal subgroup of G if $f_{G\times Q}(xy,q) = f_{G\times Q}(yx,q)$, for all $x,y \in G$ and $q \in Q$. Throughout this paper, G denotes an arbitrary group and the set of all Q-soft normal subgroup with parameter set G over U will be denoted by $NS_{G\times Q}(U)$.

Example 2.11. Let $U = \{u_1,u_2,u_3\}$ be an initial universe set and $(\mathbb{R},+)$ be an additive real group. Define $f_{\mathbb{R}\times Q} : \mathbb{R}\times Q \to P(U)$ as

$$f_{\mathbb{R}\times Q}(x,q) = \begin{cases} \{u_1,u_2\}, & \text{if } x \in \mathbb{R}^{\geq 0} \\ \{u_3\}, & \text{if } x \in \mathbb{R}^{<0} \end{cases}$$

then $f_{\mathbb{R}\times Q} \in NS_{\mathbb{R}\times Q}(U)$.

Q-soft Translation

Definition 3.1. Let $f_{G\times Q} \in QS(U)$ and $\alpha \in P(U)$. Then $T = T_\alpha^{f_{G\times Q}} : G \times Q \to P(U)$ is called a soft translation of $f_{G\times Q}$ if $T(x,q) = f_{G\times Q}(x,q) \cup \alpha$, for all $x \in G, q \in Q$ and $\alpha \in P(U)$.

Proposition 3.2. If T is a Q-soft translation of a Q-soft subgroup $f_{G\times Q}$ of a group G, then $T(x^{-1},q) = T(x,q)$ and $T(e,q) \supseteq T(x,q)$ for all $x \in G$ and $q \in Q$.

Proof. Let $x \in G$ and $q \in Q$. Then,

$$\begin{aligned} T(x,q) = f_{G\times Q}(x,q) \cup \alpha &= f_{G\times Q}((x^{-1})^{-1},q) \cup \alpha \\ &\supseteq f_{G\times Q}(x^{-1},q) \cup \alpha \\ &= T(x^{-1},q) \\ &= f_{G\times Q}(x^{-1},q) \cup \alpha \\ &\supseteq f_{G\times Q}(x,q) \cup \alpha \\ &= T(x,q) \end{aligned}$$

Then, $T(x^{-1},q) = T(x,q)$. Also

$$\begin{aligned} T(e,q) = f_{G\times Q}(e,q) \cup \alpha &= f_{G\times Q}(xx^{-1},q) \cup \alpha \\ &\supseteq (f_{G\times Q}(x,q) \cap f_{G\times Q}(x^{-1},q)) \cup \alpha \\ &= f_{G\times Q}(x,q) \cup \alpha \\ &= T(x,q) \end{aligned}$$

Thus, $T(e,q) \supseteq T(x,q)$. □

Proposition 3.3. Let T is a Q-soft translation of a Q-soft subgroup $f_{G\times Q}$ of a group G. If $T(xy^{-1},q) = T(e,q)$, then $T(x,q) = T(y,q)$ for all $x,y \in G$ and $q \in Q$.

PROOF. Let $x,y \in G$ and $q \in Q$. Then,

$$\begin{aligned} T(x,q) = f_{G\times Q}(x,q) \cup \alpha &= f_{G\times Q}(xy^{-1}y,q) \cup \alpha \\ &\supseteq (f_{G\times Q}(xy^{-1},q) \cap f_{G\times Q}(y,q)) \cup \alpha \\ &= (f_{G\times Q}(xy^{-1},q) \cup \alpha) \cap (f_{G\times Q}(y,q) \cup \alpha) \\ &= T(xy^{-1},q) \cap T(y,q) \\ &= T(e,q) \cap T(y,q) \\ &= T(y,q) \\ &= f_{G\times Q}(y,q) \cup \alpha \\ &= f_{G\times Q}(yx^{-1}x,q) \cup \alpha \\ &\supseteq (f_{G\times Q}(yx^{-1},q) \cap f_{G\times Q}(x,q)) \cup \alpha \\ &= (f_{G\times Q}(yx^{-1},q) \cup \alpha) \cap (f_{G\times Q}(x,q) \cup \alpha) \\ &= T(yx^{-1},q) \cap T(x,q) \\ &= T(e,q) \cap T(x,q) \\ &= T(x,q) \end{aligned}$$

Therefore, $T(x,q) = T(y,q)$. □

Proposition 3.4. If T is a Q-soft translation of a Q-soft subgroup $f_{G\times Q}$ of a group G, then T is a Q-soft subgroup of G.

PROOF. Let $x,y \in G$ and $q \in Q$. Then,

$$\begin{aligned} T(xy^{-1},q) = f_{G\times Q}(xy^{-1},q) \cup \alpha &\supseteq (f_{G\times Q}(x,q) \cap f_{G\times Q}(y^{-1},q)) \cup \alpha \\ &\supseteq (f_{G\times Q}(x,q) \cap f_{G\times Q}(y,q)) \cup \alpha \\ &= f_{G\times Q}(x,q) \cup \alpha) \cap (f_{G\times Q}(y,q) \cup \alpha) \\ &= T(x,q) \cap T(y,q) \end{aligned}$$

Therefore, by Proposition 2.9 we get that T is a Q-soft subgroup of G. □

Proposition 3.5. If T is a Q-soft translation of a Q-soft subgroup $f_{G\times Q}$ of a group G, then $H = \{x \in G : T(x,q) = T(e,q)\}$ is a subgroup of G.

PROOF. Let $x,y \in H$ and $q \in Q$. As $T(x^{-1},q) = T(x,q) = T(e,q)$ so $x^{-1} \in H$. Now

$$\begin{aligned} T(xy^{-1},q) \supseteq T(x,q) \cap T(y,q) = T(e,q) \cap T(e,q) &= T(e,q) \\ &= T((xy^{-1})(xy^{-1})^{-1},q) \\ &\supseteq T(xy^{-1},q) \cap T(xy^{-1},q) \\ &= T(xy^{-1},q) \end{aligned}$$

Thus, $T(xy^{-1},q) = T(e,q)$ and then $xy^{-1} \in H$. Therefore, Proposition 2.6 will give us that H is a subgroup of G. □

Proposition 3.6. Let T is a Q-soft translation of a Q-soft subgroup $f_{G\times Q}$ of a group G and $T(xy^{-1},q) = M^*$ such that for all $\alpha \in P(U)$ we have that $\alpha \subset M^*$. Then $T(x,q) = T(y,q)$ for all $x,y \in G$ and $q \in Q$.

PROOF. Let $x, y \in G$ and $q \in Q$. Now

$$\begin{aligned}T(x,q) = T(xy^{-1}y,q) \supseteq T(xy^{-1},q) \cap T(y,q) &= M^* \cap T(y,q)\\ &= T(y,q)\\ &= T(y^{-1},q)\\ &= T(x^{-1}xy^{-1},q)\\ &\supseteq T(x^{-1},q) \cap T(xy^{-1},q)\\ &= T(x,q) \cap T(xy^{-1},q)\\ &= T(x,q) \cap M^*\\ &= T(x,q)\end{aligned}$$

Therefore, $T(x,q) = T(y,q)$. □

Proposition 3.7. Let $\varphi : G \to H$ be a group epihomomorphism and $f_{G\times Q} \in S_{G\times Q}(U)$. If $T_\alpha^{f_{G\times Q}}$ be a Q-soft translation of $f_{G\times Q}$, then $\varphi(T_\alpha^{f_{G\times Q}}) \in S_{H\times Q}(U)$

PROOF. Let $h_1, h_2 \in H$ and $q \in Q$ then

$$\begin{aligned}\varphi(T_\alpha^{f_{G\times Q}})(h_1h_2^{-1},q) &= \cup\{T_\alpha^{f_{G\times Q}}(g_1g_2^{-1},q) \mid g_1,g_2 \in G, \varphi(g_1g_2^{-1}) = h_1h_2^{-1}\}\\ &= \cup\{T_\alpha^{f_{G\times Q}}(g_1g_2^{-1},q) \mid g_1,g_2 \in G, \varphi(g_1)\varphi(g_2^{-1}) = h_1h_2^{-1}\}\\ &= \cup\{T_\alpha^{f_{G\times Q}}(g_1g_2^{-1},q) \mid g_1,g_2 \in G, \varphi(g_1) = h_1, \varphi(g_2^{-1}) = h_2^{-1}\}\\ &= \cup\{T_\alpha^{f_{G\times Q}}(g_1g_2^{-1},q) \mid g_1,g_2 \in G, \varphi(g_1) = h_1, \varphi^{-1}(g_2) = h_2^{-1}\}\\ &= \cup\{f_{G\times Q}(g_1g_2^{-1},q) \cup \alpha \mid g_1,g_2 \in G, \varphi(g_1) = h_1, \varphi(g_2) = h_2\}\\ &\supseteq \cup\{[f_{G\times Q}(g_1,q) \cap f_{G\times Q}(g_2,q)] \cup \alpha \mid g_1,g_2 \in G, \varphi(g_1) = h_1, \varphi(g_2) = h_2\}\\ &= \cup\{[f_{G\times Q}(g_1,q) \cup \alpha] \cap [f_{G\times Q}(g_2,q) \cup \alpha] \mid g_1,g_2 \in G, \varphi(g_1) = h_1, \varphi(g_2) = h_2\}\\ &= [\cup\{f_{G\times Q}(g_1,q) \cup \alpha \mid g_1 \in G, \varphi(g_1) = h_1,\}]\\ &\quad \cap[\cup\{f_{G\times Q}(g_2,q) \cup \alpha \mid g_2 \in G, \varphi(g_2) = h_2,\}]\\ &= [\cup\{T_\alpha^{f_{G\times Q}}(g_1,q) \mid g_1 \in G, \varphi(g_1) = h_1,\}]\\ &\quad \cap[\cup\{T_\alpha^{f_{G\times Q}}(g_2,q) \mid g_2 \in G, \varphi(g_2) = h_2,\}]\\ &= \varphi(T_\alpha^{f_{G\times Q}})(h_1,q) \cap \varphi(T_\alpha^{f_{G\times Q}})(h_2,q)\end{aligned}$$

Then, by Proposition 2.9 we get that $\varphi(T_\alpha^{f_{G\times Q}}) \in S_{H\times Q}(U)$. □

Proposition 3.8. Let $\varphi : G \to H$ be a group homomorphism and $f_{H\times Q} \in S_{H\times Q}(U)$. If $T_\alpha^{f_{H\times Q}}$ be a Q-soft translation of $f_{H\times Q}$, then $\varphi^{-1}(T_\alpha^{f_{G\times Q}}) \in S_{G\times Q}(U)$.

PROOF. Let $g_1, g_2 \in G$ and $q \in Q$. Then,

$$\begin{aligned}\varphi^{-1}(T_\alpha^{f_{G\times Q}})(g_1g_2^{-1},q) &= T_\alpha^{f_{G\times Q}}(\varphi(g_1g_2^{-1}),q)\\ &= T_\alpha^{f_{G\times Q}}(\varphi(g_1)\varphi(g_2^{-1}),q)\\ &= T_\alpha^{f_{G\times Q}}(\varphi(g_1)\varphi^{-1}(g_2),q)\\ &= f_{G\times Q}(\varphi(g_1)\varphi^{-1}(g_2),q) \cup \alpha\\ &\supseteq [f_{G\times Q}(\varphi(g_1),q) \cap f_{G\times Q}(\varphi(g_2),q)] \cup \alpha\\ &= [f_{G\times Q}(\varphi(g_1),q) \cup \alpha] \cap [f_{G\times Q}(\varphi(g_2),q) \cup \alpha]\\ &= T_\alpha^{f_{G\times Q}}(\varphi(g_1),q) \cap T_\alpha^{f_{G\times Q}}(\varphi(g_2),q)\\ &= \varphi^{-1}(T_\alpha^{f_{G\times Q}})(g_1,q) \cap \varphi^{-1}(T_\alpha^{f_{G\times Q}})(g_2,q)\end{aligned}$$

Thus, $\varphi^{-1}(T_\alpha^{f_{G\times Q}})(g_1g_2^{-1},q) \supseteq \varphi^{-1}(T_\alpha^{f_{G\times Q}})(g_1,q) \cap \varphi^{-1}(T_\alpha^{f_{G\times Q}})(g_2,q)$. Now Proposition 2.9 gets us that $\varphi^{-1}(T_\alpha^{f_{G\times Q}}) \in S_{G\times Q}(U)$ □

Proposition 3.9. Let $\varphi : G \to H$ be a group anti- epihomomorphism and $f_{G\times Q} \in S_{G\times Q}(U)$. If $T_\alpha^{f_{G\times Q}}$ be a Q-soft translation of $f_{G\times Q}$, then $\varphi(T_\alpha^{f_{G\times Q}}) \in S_{H\times Q}(U)$.

Proof. Let $h_1, h_2 \in H$ and $q \in Q$ then

$$\begin{aligned}
\varphi(T_\alpha^{f_{G\times Q}})(h_1h_2^{-1},q) &= \cup\{T_\alpha^{f_{G\times Q}}(g_1g_2^{-1},q) \mid g_1,g_2 \in G, \varphi(g_1g_2^{-1}) = h_1h_2^{-1}\} \\
&= \cup\{T_\alpha^{f_{G\times Q}}(g_1g_2^{-1},q) \mid g_1,g_2 \in G, \varphi(g_2^{-1})\varphi(g_1) = h_1h_2^{-1}\} \\
&= \cup\{T_\alpha^{f_{G\times Q}}(g_1g_2^{-1},q) \mid g_1,g_2 \in G, \varphi(g_2^{-1}) = h_2^{-1}, \varphi(g_1) = h_1\} \\
&= \cup\{T_\alpha^{f_{G\times Q}}(g_1g_2^{-1},q) \mid g_1,g_2 \in G, \varphi^{-1}(g_2) = h_2^{-1}, \varphi(g_1) = h_1\} \\
&= \cup\{f_{G\times Q}(g_1g_2^{-1},q) \cup \alpha \mid g_1,g_2 \in G, \varphi(g_2) = h_2, \varphi(g_1) = h_1\} \\
&\supseteq \cup\{[f_{G\times Q}(g_1,q) \cap f_{G\times Q}(g_2,q)] \cup \alpha \mid g_1,g_2 \in G, \varphi(g_2) = h_2, \varphi(g_1) = h_1\} \\
&= \cup\{[f_{G\times Q}(g_1,q) \cup \alpha] \cap [f_{G\times Q}(g_2,q) \cup \alpha] \mid g_1,g_2 \in G, \varphi(g_2) = h_2, \varphi(g_1) = h_1\} \\
&= [\cup\{f_{G\times Q}(g_1,q) \cup \alpha \mid g_1 \in G, \varphi(g_1) = h_1, \}] \\
&\quad \cap[\cup\{f_{G\times Q}(g_2,q) \cup \alpha \mid g_2 \in G, \varphi(g_2) = h_2, \}] \\
&= [\cup\{T_\alpha^{f_{G\times Q}}(g_1,q) \mid g_1 \in G, \varphi(g_1) = h_1, \}] \\
&\quad \cap[\cup\{T_\alpha^{f_{G\times Q}}(g_2,q) \mid g_2 \in G, \varphi(g_2) = h_2, \}] \\
&= \varphi(T_\alpha^{f_{G\times Q}})(h_1,q) \cap \varphi(T_\alpha^{f_{G\times Q}})(h_2,q)
\end{aligned}$$

Then, by Proposition 2.9 we get that $\varphi(T_\alpha^{f_{G\times Q}}) \in S_{H\times Q}(U)$. □

Proposition 3.10. Let $\varphi : G \to H$ be a group anti-homomorphism and $f_{H\times Q} \in S_{H\times Q}(U)$. If $T_\alpha^{f_{H\times Q}}$ be a Q-soft translation of $f_{H\times Q}$, then $\varphi^{-1}(T_\alpha^{f_{G\times Q}}) \in S_{G\times Q}(U)$.

Proof. Let $g_1, g_2 \in G$ and $q \in Q$. Then,

$$\begin{aligned}
\varphi^{-1}(T_\alpha^{f_{G\times Q}})(g_1g_2^{-1},q) &= T_\alpha^{f_{G\times Q}}(\varphi(g_1g_2^{-1}),q) \\
&= T_\alpha^{f_{G\times Q}}(\varphi(g_2^{-1})\varphi(g_1),q) \\
&= T_\alpha^{f_{G\times Q}}(\varphi^{-1}(g_2)\varphi(g_1),q) \\
&= f_{G\times Q}(\varphi^{-1}(g_2)\varphi(g_1),q) \cup \alpha \\
&\supseteq [f_{G\times Q}(\varphi(g_1),q) \cap f_{G\times Q}(\varphi(g_2),q)] \cup \alpha \\
&= [f_{G\times Q}(\varphi(g_1),q) \cup \alpha] \cap [f_{G\times Q}(\varphi(g_2),q) \cup \alpha] \\
&= T_\alpha^{f_{G\times Q}}(\varphi(g_1),q) \cap T_\alpha^{f_{G\times Q}}(\varphi(g_2),q) \\
&= \varphi^{-1}(T_\alpha^{f_{G\times Q}})(g_1,q) \cap \varphi^{-1}(T_\alpha^{f_{G\times Q}})(g_2,q)
\end{aligned}$$

Thus, $\varphi^{-1}(T_\alpha^{f_{G\times Q}})(g_1g_2^{-1},q) \supseteq \varphi^{-1}(T_\alpha^{f_{G\times Q}})(g_1,q) \cap \varphi^{-1}(T_\alpha^{f_{G\times Q}})(g_2,q)$ and so by Proposition 2.9 we have $\varphi^{-1}(T_\alpha^{f_{G\times Q}}) \in S_{G\times Q}(U)$. □

Proposition 3.11. Let $\varphi : G \to H$ be a group epihomomorphism and $f_{G\times Q} \in NS_{G\times Q}(U)$. If $T_\alpha^{f_{G\times Q}}$ be a Q-soft translation of $f_{G\times Q}$, then $\varphi(T_\alpha^{f_{G\times Q}}) \in NS_{H\times Q}(U)$

Proof. Let $h_1, h_2 \in H$ and $q \in Q$ then

$$\begin{aligned}\varphi(T_\alpha^{f_{G\times Q}})(h_1h_2,q) &= \cup\{T_\alpha^{f_{G\times Q}}(g_1g_2,q) \mid g_1,g_2 \in G, \varphi(g_1g_2)=h_1h_2\}\\ &= \cup\{T_\alpha^{f_{G\times Q}}(g_1g_2,q) \mid g_1,g_2 \in G, \varphi(g_1)\varphi(g_2)=h_1h_2\}\\ &= \cup\{T_\alpha^{f_{G\times Q}}(g_1g_2,q) \mid g_1,g_2 \in G, \varphi(g_1)=h_1, \varphi(g_2)=h_2\}\\ &= \cup\{f_{G\times Q}(g_1g_2,q)\cup\alpha \mid g_1,g_2 \in G, \varphi(g_1)=h_1, \varphi(g_2)=h_2\}\\ &= \cup\{f_{G\times Q}(g_2g_1,q)\cup\alpha \mid g_1,g_2 \in G, \varphi(g_1)=h_1, \varphi(g_2)=h_2\}\\ &= \varphi(T_\alpha^{f_{G\times Q}})(h_2h_1,q)\end{aligned}$$

Then, $\varphi(T_\alpha^{f_{G\times Q}}) \in NS_{H\times Q}(U)$. □

Proposition 3.12. Let $\varphi : G \to H$ be a group homomorphism and $f_{H\times Q} \in NS_{H\times Q}(U)$. If $T_\alpha^{f_{H\times Q}}$ be a Q-soft translation of $f_{H\times Q}$, then $\varphi^{-1}(T_\alpha^{f_{H\times Q}}) \in NS_{G\times Q}(U)$.

Proof. Let $g_1, g_2 \in G$ and $q \in Q$. Then,

$$\begin{aligned}\varphi^{-1}(T_\alpha^{f_{H\times Q}})(g_1g_2,q) &= T_\alpha^{f_{H\times Q}}(\varphi(g_1g_2),q)\\ &= T_\alpha^{f_{H\times Q}}(\varphi(g_1)\varphi(g_2),q)\\ &= f_{H\times Q}(\varphi(g_1)\varphi(g_2),q)\cup\alpha\\ &= f_{H\times Q}(\varphi(g_2)\varphi(g_1),q)\cup\alpha\\ &= T_\alpha^{f_{H\times Q}}(\varphi(g_2)\varphi(g_1),q)\\ &= T_\alpha^{f_{H\times Q}}(\varphi(g_2g_1),q)\\ &= \varphi^{-1}(T_\alpha^{f_{H\times Q}})(g_2g_1,q)\end{aligned}$$

Thus, $\varphi^{-1}(T_\alpha^{f_{H\times Q}})(g_1g_2,q) = \varphi^{-1}(T_\alpha^{f_{H\times Q}})(g_2g_1,q)$ and so $\varphi^{-1}(T_\alpha^{f_{H\times Q}}) \in NS_{G\times Q}(U)$. □

Proposition 3.13. Let $\varphi : G \to H$ be a group anti epihomomorphism and $f_{G\times Q} \in NS_{G\times Q}(U)$. If $T_\alpha^{f_{G\times Q}}$ be a Q-soft translation of $f_{G\times Q}$, then $\varphi(T_\alpha^{f_{G\times Q}}) \in NS_{H\times Q}(U)$

Proof. Let $h_1, h_2 \in H$ and $q \in Q$ then

$$\begin{aligned}\varphi(T_\alpha^{f_{G\times Q}})(h_1h_2,q) &= \cup\{T_\alpha^{f_{G\times Q}}(g_1g_2,q) \mid g_1,g_2 \in G, \varphi(g_1g_2)=h_1h_2\}\\ &= \cup\{T_\alpha^{f_{G\times Q}}(g_1g_2,q) \mid g_1,g_2 \in G, \varphi(g_2)\varphi(g_1)=h_1h_2\}\\ &= \cup\{T_\alpha^{f_{G\times Q}}(g_1g_2,q) \mid g_1,g_2 \in G, \varphi(g_2)=h_2, \varphi(g_1)=h_1\}\\ &= \cup\{f_{G\times Q}(g_1g_2,q)\cup\alpha \mid g_1,g_2 \in G, \varphi(g_2)=h_2, \varphi(g_1)=h_1\}\\ &= \cup\{f_{G\times Q}(g_2g_1,q)\cup\alpha \mid g_1,g_2 \in G, \varphi(g_2)=h_2, \varphi(g_1)=h_1\}\\ &= \varphi(T_\alpha^{f_{G\times Q}})(h_2h_1,q)\end{aligned}$$

Then, $\varphi(T_\alpha^{f_{G\times Q}}) \in NS_{H\times Q}(U)$. □

Proposition 3.14. Let $\varphi : G \to H$ be a group anti-homomorphism and $f_{H\times Q} \in NS_{H\times Q}(U)$. If $T_\alpha^{f_{H\times Q}}$ be a Q-soft translation of $f_{H\times Q}$, then $\varphi^{-1}(T_\alpha^{f_{H\times Q}}) \in NS_{G\times Q}(U)$.

Proof. Let $g_1, g_2 \in G$ and $q \in Q$. Then,

$$\begin{aligned}\varphi^{-1}(T_\alpha^{f_{H\times Q}})(g_1g_2, q) &= T_\alpha^{f_{H\times Q}}(\varphi(g_1g_2), q)\\ &= T_\alpha^{f_{H\times Q}}(\varphi(g_2)\varphi(g_1), q)\\ &= f_{H\times Q}(\varphi(g_2)\varphi(g_1), q) \cup \alpha\\ &= f_{H\times Q}(\varphi(g_1)\varphi(g_2), q) \cup \alpha\\ &= T_\alpha^{f_{H\times Q}}(\varphi(g_1)\varphi(g_2), q)\\ &= T_\alpha^{f_{H\times Q}}(\varphi(g_2g_1), q)\\ &= \varphi^{-1}(T_\alpha^{f_{H\times Q}})(g_2g_1, q)\end{aligned}$$

Thus, $\varphi^{-1}(T_\alpha^{f_{H\times Q}})(g_1g_2, q) = \varphi^{-1}(T_\alpha^{f_{H\times Q}})(g_2g_1, q)$ and so $\varphi^{-1}(T_\alpha^{f_{H\times Q}}) \in NS_{G\times Q}(U)$. □

Conclusion

In this paper, we defined the concept Q-soft translations of Q-soft subgroups and investigated the properties of them and showed that every Q-soft translation of Q-soft subgroup is also Q-soft subgroup. Also, we considered them under homomorphism and anti-homomorphism of Q-soft subgroups and Q-soft normal subgroups. Now one can define the isomorphisms of them and it is can be as open problem.

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