



Thermo Hydraulic and Economic Optimization of Double Pipe Heat Exchangers

Çift Borulu Isı Değiştiricilerinin Termo Hidrolik ve Ekonomik Optimizasyonu

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Abstract

A multi variable thermo economic and hydraulic optimization analysis is presented yielding simple algebraic formula for estimating the optimum area of counter flow tubular heat exchangers of both fluids unmixed type which are applied in industrial applications. An economic analysis method is used in the present study, together with the thermal and fluidic analyses of such heat exchangers, for thermal, economic and hydraulic optimization. The validity of the optimization formulations was checked with realistic applications.

Keywords: Counter flow heat exchanger, Optimization, Thermo economics, Thermo hydraulic, Unmixed type

Öz

Endüstriyel uygulamalarda kullanılan karşı akışlı, boru tipi, iki akışkanın karışmadığı tür ısı değiştiricilerinin optimum alan hesabı için basit cebirsel formül veren çok değişkenli termo ekonomik ve hidrolik optimizasyon analizi sunulmaktadır. Bu çalışmada, termal, ekonomik ve hidrolik optimizasyon için bu tür ısı değiştiricilerinin termal ve akışkan analizleri ile birlikte bir ekonomik analiz yöntemi kullanılmıştır. Optimizasyon formülasyonlarının geçerliliği gerçekçi uygulamalarla kontrol edilmiştir.

Anahtar Kelimeler: Zıt akışlı ısı değiştirici, Optimizasyon, Termo ekonomik, Termo hidrolik, Karışimsız tip


1. Introduction

Economics of heat exchanger operation is vitally significant. So, optimum operating temperatures for a tubular type heat exchanger as shown in Figure 1 is highly important in order to have maximum overall life cycle savings for these heat exchangers. As pipe diameter increases first cost of the heat exchanger also increases but operating cost decreases reversely. Optimum point at which maximum profit occurs must be detected so. The optimum values of effectiveness and diameter must be calculated at which minimum cost and so maximum savings occur for plate type heat exchangers that are widely applied for industrial applications for that reason. There exists many parameters for optimizing such heat exchangers in a thermo economical manner. Fixing and, so eliminating all of these thermal and economical parameters depending on the certainty of operating characteristics of applications due to design requirements and the most

efficient operating condition of the heat exchangers can determine the optimum sizing for those heat exchangers. It is known that the effectiveness of the heat exchanger is directly related to its size which affects with its initial cost whereas continuous operating cost of them depends upon inner diameter of tube. A thermo economic feasibility study is necessary together with hydraulic performance of it before installing the heat exchanging systems. The basic topic of the present work depends upon this idea. A new thermo economic optimization technique is realized and presented for this purpose. Original formulae is developed for calculating the optimum effectiveness and diameter at which the net maximum total life cycle savings occur. A thorough search of the current literature showed that there was no previous study on optimizing the net life cycle savings of a typical counter flow tubular heat exchanger in detail. A well known and practical method, P_1 - P_2 method, which was offered by (Duffie and Beckman, 1980), is used for optimizing the size and operating conditions of heat exchanger, and original interesting results are presented. Variable parameters used in formulating the optimization problem are listed as technical life of the heat exchanger,

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first cost of the heat exchanger per unit heat transfer area, annual interest rate, present net price of energy, annual energy price escalation rate, annual average operating time, ratio of minimum heat capacity rate into maximum heat capacity rate, design values of the difference of maximum and minimum temperatures of hot and cold fluids for single fluid heat exchanger, overall heat transfer coefficient of the heat exchanger, resale value and the ratio of annual maintenance and operation cost to the first original cost. Optimum effectiveness and diameter of tubular counter flow heat exchanger and optimum value of net life cycle savings can be calculated easily in a few minutes with the help of practical formulae developed in this study. A thorough search of the present literature showed that there were several studies about the heat exchangers (Vojtech et. al. 2011, Chung et al. 2002, Grazzini and Rinaldi 2001, Cornelissen and Hirs 1999, Georgiadis 1998, Şahin 1997, Edwards and Matavosian 1982, Wang et. al. 1999, Ibarra et. al. 2013, Kandilli and Koçlu 2011, Wu et. al. 2014). All of these studies are not directly related to the present work. Original formulae is developed and presented finally.

2. Mathematical Formulation

The net savings of a counter flow heat exchanger as shown in Figure 1 can be calculated by using the cost data (Burmeister 1998, Stoecker 1989) as:

$$S = P_1.C_E.Q.H - P_1.H.C_{El} \cdot \frac{\Delta P.V}{\square_p \cdot \square_m} - P_2.C_A.A_{HX} \quad (1)$$

Where

$$Q = (\dot{m}.C_p)_{\min} \cdot \square \cdot \Delta T_{\max} = \varepsilon \cdot \rho \cdot \dot{V} \cdot C_p \cdot \Delta T_{\max} \quad (2)$$

Pressure drop in a tube can be calculated by:

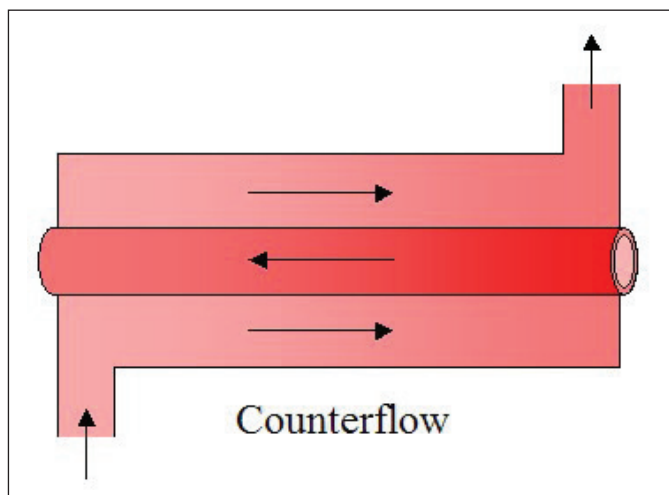


Figure 1. Counter flow double pipe heat exchanger.

$$\Delta P = \frac{\rho.V^2}{2}K = \frac{K.\rho}{2} \cdot V^2 = \frac{K.\rho}{2} \cdot \frac{V^2.16}{\pi^2.D^4} = \frac{8.K.\rho.V^2}{\pi^2.D^4} \quad (3)$$

The number of transfer units for equal heat capacity rates of counter current type heat exchanger that is the best operating condition is:

$$NTU = \frac{U.A_{HX}}{m.C_p} = \left(\frac{\varepsilon}{1-\varepsilon}\right) \Rightarrow A_{HX} = \frac{m.C_p}{U} \cdot \left(\frac{\varepsilon}{1-\varepsilon}\right) = \frac{\dot{V}.C_p.\rho}{U} \left(\frac{\varepsilon}{1-\varepsilon}\right) \quad (4)$$

The overall heat transfer is approximated to h as:

$$U \cong h = Nu \cdot \frac{k}{D} = Nu \cdot \frac{k \cdot \sqrt{V} \cdot \sqrt{\pi}}{\sqrt{4.V}} \quad (5)$$

Nusselt number can be evaluated from well-known Dittus Boelter equation:

$$Nu = 0.023.Re^{0.8} \cdot Pr^{0.4} = 0.023 \cdot \frac{V^{0.8}.D^{0.8}}{v^{0.8}} \cdot Pr^{0.4} \quad (6)$$

and it can be rewritten as follows.

$$Nu = 0.023 \cdot \frac{4^{0.8} \cdot \dot{V}^{0.8} \cdot D^{0.8}}{\pi^{0.8} \cdot D^{1.6} \cdot v^{0.8}} Pr^{0.4} \quad (7)$$

and convective and approximated overall heat transfer coefficient is determined by:

$$h \cong \frac{0.023 \cdot \dot{V}^{0.8} \cdot Pr^{0.4} \cdot k \cdot 4^{0.8}}{\pi^{0.8} \cdot D^{1.8} \cdot v^{0.8}} \cong U \quad (8)$$

Net savings function takes the form as in Equation (9).

$$S = P_1.C_E \cdot \square \cdot \dot{V} \cdot H.C_p \cdot \Delta T_{\max} \cdot \square - \frac{P_1.C_{El}}{\square_p \cdot \square_m} \frac{H8K.\rho.V^{0.2}.C_p}{4^{0.8} \cdot 0.023.Pr^{0.4}.k} \left(\frac{\varepsilon}{1-\varepsilon}\right) \cdot \pi^{0.8} \cdot D^{1.8} \cdot v^{0.8} \quad (9)$$

It can be simplified as:

$$S = a \cdot \square - \frac{b}{D^4} - c.D^{1.8} \cdot \left(\frac{\varepsilon}{1-\varepsilon}\right) \quad (10)$$

Savings function is derived wrt effectiveness and diameter separately as:

$$\frac{\partial S}{\partial \varepsilon} = a - cD_{opt}^{1.8} \left(\frac{1(1-\varepsilon_{opt}) + \varepsilon_{opt}}{(1-\varepsilon_{opt})^2}\right) = 0 \quad (11)$$

or

$$a - c.D_{opt}^{1.8} \cdot \frac{1}{(1-\varepsilon_{opt})^2} = 0 \quad (12)$$

and

$$\frac{\partial S}{\partial D} = \frac{4.b}{D_{opt}^5} - c \cdot \left(\frac{\varepsilon_{opt}}{1-\varepsilon_{opt}}\right) \cdot D_{opt}^{0.8} \cdot 1.8 = 0 \quad (13)$$

or

$$1.8.c \cdot \left(\frac{\varepsilon}{1-\varepsilon}\right) \cdot D_{opt}^{0.8} = \frac{4.b}{D_{opt}^5} \quad (14)$$

Equation (13) is now:

$$D_{opt}^{5.8} = \frac{4.b.(1 - \epsilon_{opt})}{1.8.c.\epsilon_{opt}} \quad (15)$$

and Equation (14) is:

$$\epsilon_{opt} = 1 - \sqrt{\frac{c}{a}} . D_{opt}^{0.9} \quad (16)$$

Simultaneous solution of Equations (15) and (16) yields to get:

$$D_{opt}^{5.8} = \frac{\sqrt[4.b]{\frac{c}{a}} . D_{opt}^{0.9}}{1.8.c.(1 - \sqrt{\frac{c}{a}} . D_{opt}^{0.9})} \quad (17)$$

or alternatively:

$$1.8.c.D_{opt}^{5.8} - 1.8.c.\sqrt{\frac{c}{a}} . D_{opt}^{6.7} - 4.b.\sqrt{\frac{c}{a}} . D_{opt}^{0.9} = 0 \quad (18)$$

The second derivatives are negative which indicate a local minimum.

$$\frac{\partial^2 S}{\partial^2 \epsilon} = \frac{d(-2 + 2\epsilon_{opt})}{(1 - \epsilon_{opt})^4} < 0 \text{ since } \epsilon_{opt} < 1 \quad (19)$$

and also:

$$\frac{\partial^2 S}{\partial^2 D} = -20.b.D_{opt}^{-6} - c.\left(\frac{\epsilon}{1 - \epsilon}\right).1.8.D_{opt}^{-0.2}.0.8 < 0 \quad (20)$$

Critical value of diameter can be found by setting S into zero as:

$$a.\epsilon_{opt} - c.D_{crit}^{1.8}.\left(\frac{\epsilon_{opt}}{1 - \epsilon_{opt}}\right) = \frac{b}{D_{crit}^4} \quad (21)$$

Yielding to:

$$\frac{b}{D_{crit}^4} + c.D_{crit}^{1.8}.\left(\frac{\epsilon_{opt}}{1 - \epsilon_{opt}}\right) = a.\epsilon_{opt} \quad (22)$$

and finally:

$$D_{crit} \cong \left[\frac{a.(1 - \epsilon_{opt})}{c} \right]^{1/1.8} \quad (23)$$

Likewise:

$$\epsilon_{crit} \cong 1 - \frac{c.D_{opt}^{1.8}}{a} \quad (24)$$

Payback period can be calculated by equalizing net savings, S, into zero as:

$$a.\epsilon_{opt} - \frac{b}{D_{opt}^4} - c.D_{opt}^{1.8}.\left(\frac{\epsilon_{opt}}{1 - \epsilon_{opt}}\right) = 0 \quad (25)$$

Which yields:

$$P_1.d.\epsilon_{opt} - \frac{P_1.e}{D^4} = c.D_{opt}^{1.8}.\left(\frac{\epsilon_{opt}}{1 - \epsilon_{opt}}\right) \quad (26)$$

P_1 and P_2 are defined in (Duffie et al. 1980) as:

$$i = f \text{ then } P_1 = \frac{N}{1 + i} \quad (27)$$

Then follows:

$$\frac{N_p}{1 + i} = \frac{c.D_{opt}^{1.8}.\left(\frac{\epsilon_{opt}}{1 - \epsilon_{opt}}\right)}{\left(d.\epsilon_{opt} - \frac{e}{D_{opt}^4}\right)} \quad (28)$$

and so:

$$N_p = (1 + i) \left[\frac{c.D_{opt}^{1.8}.\left(\frac{\epsilon_{opt}}{1 - \epsilon_{opt}}\right)}{d.\epsilon_{opt} - \frac{e}{D_{opt}^4}} \right] \quad (29)$$

$$\text{For } i \neq f \quad P_1 = \frac{1}{f - i} \left(1 - \left(\frac{1 + i}{1 + f} \right)^N \right) \quad (30)$$

$$P_1 = \frac{1}{f - i} \left(1 - \left[\frac{1 + i}{1 + f} \right]^N \right) = \frac{cD^{1.8}.\left(\frac{\epsilon}{1 - \epsilon}\right)}{d\epsilon - \frac{e}{D^4}} \quad (31)$$

$$N_p = \frac{\ln \left[1 - \frac{c.D^{1.8}.\left(\frac{\epsilon}{1 - \epsilon}\right).(f - i)}{d.\epsilon - \frac{e}{D^4}} \right]}{\ln \left(\frac{1 + i}{1 + f} \right)} \quad (32)$$

and

$$P_2 = 1 + P_1.M_s - R_v.(1 + f)^{-N} \quad (33)$$

For identical flow rate for same fluid on both sides we get the diameter of outer concentric tube is as in the following.

$$\frac{\pi}{4}(D_0^2 - D_i^2) = \frac{\pi}{4}D_i^2 \Rightarrow D_0 = \sqrt{2D_i} \quad (34)$$

The values of constants a, b, c, d, and e are:

$$a = P_1.C_E.\square.\dot{V}.H.C_p.\Delta T_{max} \quad (35)$$

$$b = \frac{P_1.C_{El} . 8.K.\rho.H.\dot{V}^3}{\square_p.\square_m.\pi^2} \quad (36)$$

$$c = \frac{P_2.C_A.\rho.\dot{V}^{0.2}.C_p}{4^{0.8}.0.023.Pr^{0.4}.k} . \pi^{0.8} . v^{0.8} \quad (37)$$

$$d = C_E.\square.\dot{V}.C_p.\Delta T_{max} \quad (38)$$

$$e = \frac{C_{El} . 8.k.\rho.\dot{V}^3}{\square_p.\square_m.\pi^2} \quad (39)$$

3. Results and Discussion

For a typical counter flow double pipe heat exchanger it is assumed that: $i=f=0.1$, $N=10$ yr, $C_E=10^{-6} \frac{\$}{Whr}$, $H=3000 \frac{hr}{yr}$, $C_A=500 \frac{\$}{m^2}$, $C_p=4187 \frac{J}{kgK}$, $DT_{max}=100^\circ C$, $\dot{V}=0.001 \frac{m^3}{s}$, $K=10$, $v=10^{-6} \frac{m}{s}$, $P_r=5$, $r=1000 \text{kg/m}^3$, $k=0.7 \frac{W}{mK}$, $M_s=0$, $R_v=0$, $C_{El}=10^{-4} \frac{\$}{Whr}$, $n_p=n_m=0.9$, $P_2=1$. Optimum diameter is calculated as 0.015 m by means of Equation (18). And optimum outer shell diameter is from Equation (34), whereas optimum effectiveness is calculated as 0.898 by Equation (16) as for this sample. On the other hand, it is calculated

that $D_{crit} \cong 0.0537m$ by the help of Equation (23), critical effectiveness is determined as 0.997 by using Equation (24) and payback is about 1.06 years that is found by Equation (29). Variation of savings for various values of diameter and effectiveness are shown in Figures 2 and 3. Maximum net overall life cycle savings of the heat exchanger are obtained as 8.669,64 for variable tube diameter as 0,015 and constant

effectiveness as shown in Figure 2 and also maximum net overall life cycle savings of the heat exchanger are obtained as 8.669,64 for constant tube diameter as 0,015 and variable effectiveness as shown in Figure 3. Multi variable variation of savings is available in Figure 4. It is clearly seen in Figure 4 that net overall life cycle saving increases with increasing

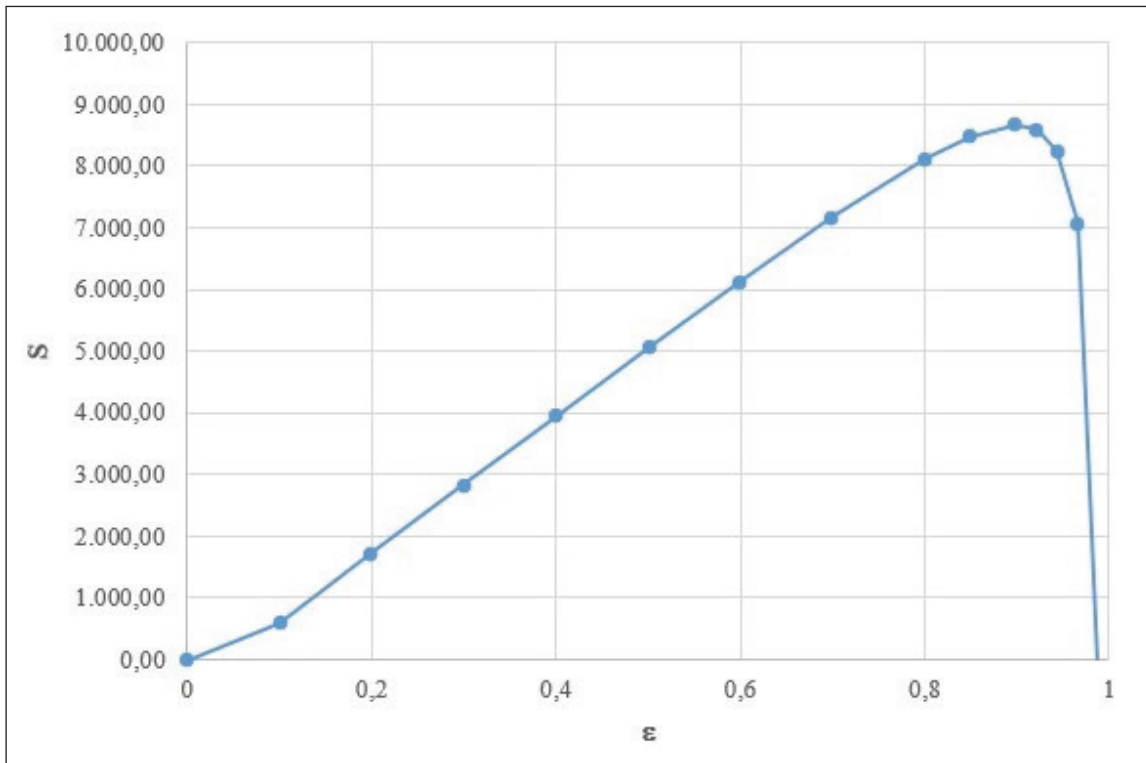


Figure 2. Variation of Net overall life cycle savings of the heat exchanger as a function of diameter of tube for constant effectiveness as $\epsilon=0,898$.

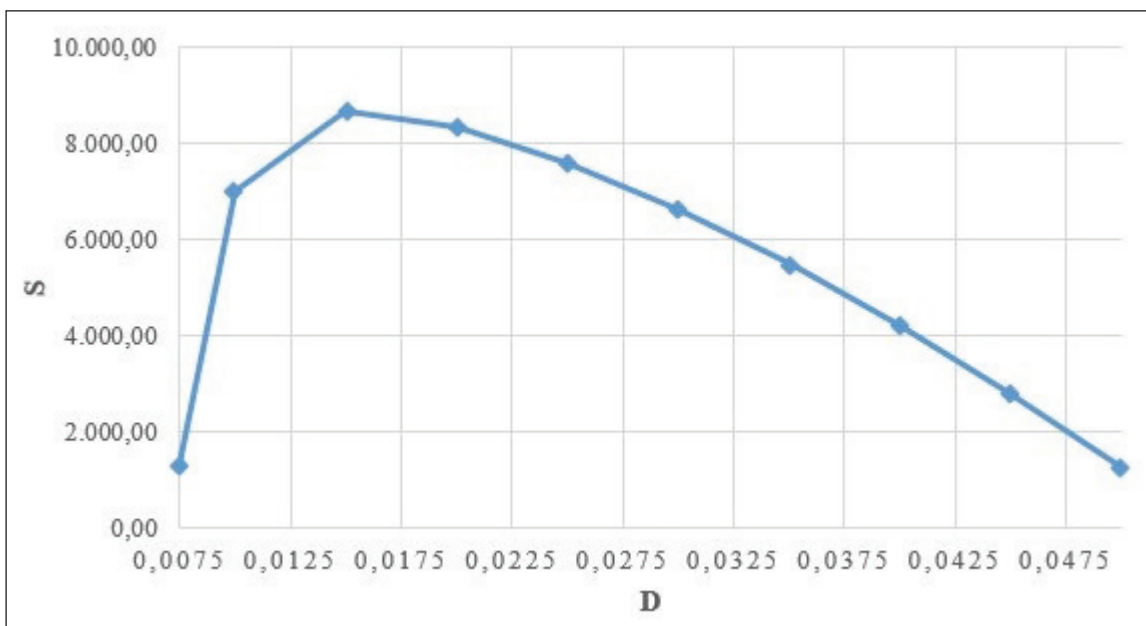


Figure 3. Variation of Net overall life cycle savings of the heat exchanger as a function of Effectiveness of tubular heat exchanger for constant tube diameter as $D=0,898m$.

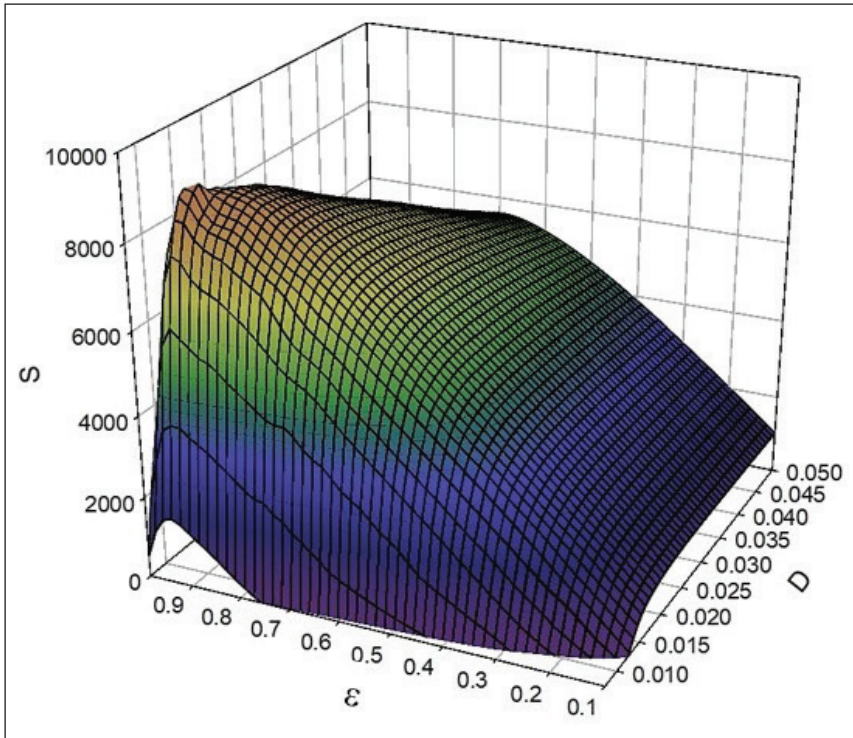


Figure 4. Variation of Net overall life cycle savings of the heat exchanger as a function of diameter of tube and Effectiveness of tubular heat exchanger.

effectiveness and decreases with increasing tube diameter. As it can be seen from all of these figures that net savings value begins to decrease after optimum diameter and effectiveness values.

4. Conclusion

It can be deduced that there exists always a local optimum diameter and effectiveness value in recovering waste heat by means of tubular counter current heat exchanger applications for the best operating conditions. Excessive heat transfer area will not be cost effective beyond the optimum values in spite of a greater heat transfer recovery potential. It is clear that there exist good hydraulic, thermal, and economic performances all together at the optimum point for these heat exchangers. This type of heat exchangers must be designed close to this optimum point. The present formulae may seem to be helpful for plate type heat exchanger designers and manufacturers when using for waste heat recovery by considering all of the system performance parameters together.

5. Nomenclature

- a Constant depending on values of fixed operating parameters as defined in Equation (35), (\$),
- A_{HX} Area of heat exchanger, (m^2),

- b Constant depending on values of fixed operating parameters as defined in Equation (36), (m^4),
- c Constant depending on values of fixed operating parameters as defined in Equation (37), ($\$/m^{1.8}$),
- C_A Area dependent first cost of the heat exchanger, ($\$/m^2$),
- C_E Cost of energy saved, ($\$/W hr$),
- C_P Specific heat of circulating fluid, [$J/(kg K)$]
- d Constant depending on values of fixed operating parameters as defined in Equation (38), ($\$/yr$),
- D Diameter of the tube, (m),
- D_o Diameter of outer concentric tube, (m),
- D_{opt} Optimum diameter, (m),
- D_{crit} Critical diameter, (m),
- e Constant depending on values of fixed operating parameters as defined in Equation (38), (m^4/yr),
- f Market discount rate in fraction,
- h Thermal convective heat transfer coefficient, [$W/(m^2K)$],
- H Annual time of operation, (h/yr),

i	Energy price escalation rate in fraction,
k	Thermal conductivity of fluid, [W/(mK)],
K	Overall flow resistance coefficient,
m	Mass rate of flow of circulating fluid, (kg/s)
M_s	Ratio of annual maintenance and operation cost into first original cost,
N	Technical life of plate flow heat exchanger, (yr),
NTU	Number of transfer units,
Nu	Nusselt number,
P_1	Ratio of the life cycle energy cost savings to the first year energy cost savings, (yr),
P_2	Ratio of the life cycle expenditures incurred because of the additional capital investment to the initial investment,
Pr	Prandtl number,
Q	Heat transfer, (W),
R_v	Ratio of resale value into the first original cost,
S	Net overall life cycle savings of the heat exchanger, (\$),
U	Overall heat transfer coefficient, [W/(m ² K)],
V	Velocity of fluid, (m/s),
\dot{V}	Volume flowrate, (m ³ /s),
DT_{max}	Maximum temperature difference between hot and cold fluid inlet in plate type heat exchangers, (C),
e	Effectiveness of tubular heat exchanger,
e_{opt}	Optimum effectiveness of heat exchanger,
ν	Kinematic viscosity of fluid, (m ² /s),
ρ	Density of the fluid, (kg/m ³).

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