Karaelmas Science and Engineering Journal
Journal home page: http://fbd.beun.edu.tr
DOI: 10.7212%2Fzkufbd.v8i1.816

Research Article
Received / Geliş tarihi : 29.12.2016
Accepted / Kabul tarihi : 27.11.2017

Completely Equiprime Ideals of Near-Ring Modules

Yakın Halka Modüllerinin Tam e-asal İdealleri

Funda Taşdemir*

Bozok University, Department of Mathematics, Yozgat, Turkey

Abstract

In this study, the concept of completely equiprime N-ideal (ideal of near-ring modules) is introduced. Also the interconnections of completely equiprime, equiprime and completely prime N-ideals are considered. It is proved that if P is a completely equiprime ideal of an N-group (near-ring module) Γ, then $(P: \Gamma)$ is a completely equiprime ideal of a near-ring N. The converse relation does not hold in general, however some additional conditions for holding the converse are provided. The connection between the concepts of completely equiprime N-ideal and IFP N-ideal is also observed.

2010 AMS-Mathematical Subject Classification Number: 16Y30,16D25.

Keywords: Completely equiprime N-ideal, Completely prime N-ideal, Equiprime N-ideal, IFP N-ideal

Öz

Bu çalışmada, tam e-asal N-ideal (yakın halka modülünün ideali) kavramı tanımlanmıştır. Aynı zamanda, tam e-asal, e-asal ve tam asal N-idealler arasındaki ilişkiler ele alınmıştır. Bir Γ N-grubunun (yakın halka modülü) P ideali tam e-asal olduğunda $(P: \Gamma)$ nın da N yakın halkasının tam e-asal ideali olduğu ispatlanmıştır. Bu gerektirmenin tersi genelde sağlanmazken, bazı ek şartlar ile tersinin doğruluğu sağlanmıştır. Ayrıca, tam e-asal N-ideal ve IFP N-ideal kavramları arasındaki ilişki de incelenmiştir.

2010 AMS-Konu Sınıflandırması: 16Y30,16D25.

Anahtar Kelimeler: Tam e-asal N-ideal, Tam asal N-ideal, E-asal N-ideal, IFP N-ideal

1. Introduction

Several definition of primeness have been proposed for near-rings, all of which generalize primeness for rings. Holcombe (1970) defined three types of primeness which are 0-prime, 1-prime and 2-prime. The concepts of type 1 and type 2 prime ideals have been introduced by Ramakotaiah and Rao (1979). Then, type 1 and type 2 prime ideals were called as 3-prime ideal and completely prime (c-prime) ideal, respectively (Groenewald 1991). Booth et al. (1990) defined a different generalization of prime rings, called equiprime (e-prime). Veldsman (1992) has examined equiprimeness in depth. Juglal et al. (2010) generalized the various notions of primeness (0-, 1-, 2-, 3-, c-primeness) that were defined in a near-ring to the near-ring module. Tasdemir et al. (2011) added to these five types by introducing equiprime N-ideals. Then they have studied 3-primeness and c-primeness in details (Taşdemir et al. 2013).

In this study, the concept of completely equiprime N-group is generalized to ideals of N-groups, called completely equiprime N-ideals. Also the interconnections of completely equiprime, equiprime and completely prime N-ideals are considered. Furthermore, some relationships between a completely equiprime N-ideal P and the ideal $(P: \Gamma)$ are obtained. The relation between the concepts of completely equiprime N-ideal and IFP N-ideal is also observed.

2. Preliminaries

Throughout this study N stands for a right near-ring. In Pilz's book (1983), for all undefined concepts concerning near-rings are given. For further information in near-rings of prime ideals could be found from Atagün (2010) and Dheena et al. (2004).

*Corresponding author: funda.tasdemir@bozok.edu.tr

Funda Taşdemir orcid.org/0000-0002-0349-3686

N is said to be a right permutable (RP) near-ring if $abc = acb$ for all $a,b,c \in N$ (Birkenmeier and Heatherly 1989). N_d is defined as the set $\{d \in N: d(a+b) = da + db, \forall a,b \in N\}$ (Pilz 1983).

If $a \in N \setminus I$, $ax - ay \in I$ implies $x-y \in I$, for all $x,y \in N$ then $I \triangleleft N$ is called a completely equiprime ideal (Booth and Groenewald 1992).

Γ is called a completely equiprime N-group if $a\gamma_1 = a\gamma_2$ implies $\gamma_1 = \gamma_2$ for $a \in N \setminus (0_\Gamma:\Gamma)_N$ and $\gamma_1,\gamma_2 \in \Gamma$ (Booth and Groenewald 1992). If there exists $\gamma \in \Gamma$ such that $N\gamma = \Gamma$, then Γ is called monogenic.

From now on, N will denote a zero-symmetric near-ring, Γ an N-group and P an N-ideal of Γ such that $N\Gamma \nsubseteq P$.

Definition 1 (Juglal et al. 2010) *Let $P \triangleleft_N \Gamma$. Then P is called:*

- 3-prime if $aN\gamma \subseteq P$ implies that $a\Gamma \subseteq P$ or $\gamma \in P$ for $a \in N$ and $\gamma \in \Gamma$.
- c-prime if $a\gamma \in P$ implies that $a\Gamma \subseteq P$ or $\gamma \in P$ for $a \in N$ and $\gamma \in \Gamma$.

Definition 2 (Taşdemir et al. 2011) *If $an\gamma_1 - an\gamma_2 \in P$ for all $n \in N$ implies that $a\Gamma \subseteq P$ or $\gamma_1-\gamma_2 \in P$ for $a \in N$ and $\gamma_1,\gamma_2 \in \Gamma$ then P is called an equiprime N-ideal.*

Γ is called a *v-prime N-group* if 0 is a v-prime N-ideal of Γ ($v=3,c,e$).

Proposition 1 (Juglal et al. 2010) *Let $P \triangleleft_N \Gamma$. Then the following are equivalent:*

- *P is 3-prime and $(P:\gamma) \triangleleft N$ for every $\gamma \in \Gamma \setminus P$.*
- *$N\Gamma \nsubseteq P$ and $(P:\gamma) = (P:\Gamma)$ for every $\gamma \in \Gamma \setminus P$.*
- *P is c-prime.*

Proposition 2 (Juglal et al. 2010, Taşdemir et al. 2011) *If $P \triangleleft_N \Gamma$ is c-prime(equiprime), then $(P:\Gamma) \triangleleft N$ is c-prime(equiprime).*

Proposition 3 (Taşdemir et al. 2011) *If N is RP, Γ is monogenic and $(P:\Gamma) \triangleleft N$ is equiprime, then $P \triangleleft_N \Gamma$ is equiprime.*

Corollary 1 (Taşdemir et al. 2011) *If N is RP and P is equiprime then P is c-prime.*

3. Completely Equiprime N-ideals

Definition 3 P is called a completely equiprime N-ideal if $a\gamma_1 - a\gamma_2 \in P$ implies that $a\Gamma \subseteq P$ or $\gamma_1-\gamma_2 \in P$ for $a \in N$ and $\gamma_1,\gamma_2 \in \Gamma$.

Γ is called a completely equiprime N-group if $N\Gamma \neq 0_\Gamma$ and 0 is a completely equiprime N-ideal of Γ.

From now on, in order to simplify, completely equiprime and equiprime are denoted by c-e-prime and e-prime, respectively.

Example 1 *Let $N = \{0,1,2,3,4,5,6,7\}$ with addition and multiplication defined by Table 1* (Pilz 1983).

Let $\Gamma = N^N$ as an N-group with the natural operation. It is clear that $P = \{0,2,5,7\}$ is an N-ideal of Γ. Also, P is c-e-prime.

Proposition 4 *Let $N \neq I \triangleleft N$. Then the following are equivalent:*

- I is c-e-prime (a)
- There is a c-e-prime N-group Γ with $I = (0_\Gamma : \Gamma)_N$ (b)

Proof. (a)⇒(b) Suppose that I is a c-e-prime ideal of N. Consider the N-group $\Gamma = N/I$ with the natural operations. Let $i \in I$ and $x \in N$. Then $ix \in I$. Hence, $i(x+I)=ix+I=I$, whence $I \subseteq (0_\Gamma:\Gamma)_N$. Now assume that $x \in (0_\Gamma:\Gamma)_N$. Then $xn+I=x(n+I)=I$ for all $n \in N$, whence $xN \subseteq I$. It follows that $xNx \subseteq I$. Since I is a c-e-prime ideal of N, it is also an e-prime ideal (Booth and Groenewald 1992) and therefore 3-prime. This implies that $x \in I$. Hence, $I=(0_\Gamma:\Gamma)_N$. It needs to be shown that Γ is a c-e-prime N-group. Suppose that $a \in N$, $a \notin (0_\Gamma:\Gamma)_N = I$ and that $x,y \in N$, $x+I \neq y+I$. Then $x-y \notin I$. Since I is a c-e-prime ideal, ax-

Table 1. Addition and multiplication tables of N.

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	0	5	6	7	4
2	2	3	0	1	6	7	4	5
3	3	0	1	2	7	4	5	6
4	4	7	6	5	0	3	2	1
5	5	4	7	6	1	0	3	2
6	6	5	4	7	2	1	0	3
7	7	6	5	4	3	2	1	0

.	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2	0	2	0	0	0	0
3	0	3	2	1	4	5	6	7
4	0	4	2	6	4	0	6	2
5	0	5	0	5	0	5	0	5
6	0	6	2	4	4	0	6	2
7	0	7	0	7	0	5	0	5

$ay \notin I$, whence $a(x+I) \neq a(y+I)$. Thus, Γ is a c-e-prime N-group and $I=(0_\Gamma:\Gamma)_N$.

(b) ⇒ (a) Suppose that Γ is a c-e-prime N-group and that $I=(0_\Gamma:\Gamma)_N$. Let $a,x,y \in N$, $a \notin I$, $x-y \notin I$. Then there exists $\gamma \in \Gamma$ such that $(x-y)\gamma \neq 0_\Gamma$, i.e. $x\gamma \neq y\gamma$. Since Γ is a c-e-prime N-group, this implies $ax\gamma \neq ay\gamma$ whence $(ax-ay)\gamma \neq 0_\Gamma$. It follows that $(ax-ay) \notin (0_\Gamma:\Gamma)_N = I$. Hence, I is a c-e-prime ideal of N.

Corollary 2 *Let $0 \neq N$ be a near-ring. Then the following are equivalent:*

- N is c-e-prime.
- There exists a faithful c-e-prime N-group Γ.

Proof. If I is replaced by the 0 ideal, then the result follows from Proposition 4.

Corollary 3 *Let $N \neq I \triangleleft N$. Then N/I is a c-e-prime near-ring iff N/I is a c-e-prime N-group.*

Proof. Let N/I be a c-e-prime near-ring. Then I is a c-e-prime ideal of N. Hence $\Gamma = N/I$ is a c-e-prime N-group by Proposition 4. Conversely, if N/I is a c-e-prime N-group, then $I = (0:N/I)$ is a c-e-prime ideal of N by Proposition 4. Hence I is a c-e-prime ideal of N and so N/I is a c-e-prime near-ring.

The following propositions show the interconnections of c-e-prime, c-prime and e-prime N-ideals.

Proposition 5 *If P is c-e-prime, then P is c-prime.*

Proof. Suppose that P is c-e-prime and $a\gamma \in P$ for $a \in N$ and $\gamma \in \Gamma$. This implies that $a\gamma = a\gamma - a0_\Gamma \in P$. Hence, $a\Gamma \subseteq P$ or $\gamma = \gamma - 0_\Gamma \in P$ since P is c-e-prime. Therefore, P is c-prime.

The following example illustrates that the converse of Proposition 5 does not hold true.

Example 2 *Let $(N,+)$ be the Klein's four group with multiplication table as given in Table 2.*

Table 2. Multiplication table of N.

.	0	a	b	c
0	0	0	0	0
a	0	a	a	a
b	0	b	b	b
c	0	c	c	c

Let $\Gamma = N^N$ as an N-group with the natural operation. Then, $P = 0$ is c-prime but not c-e-prime.

However, the following observation is obtained.

Proposition 6 *Let N be a RP near-ring, Γ be a monogenic N-group and $P \triangleleft_N \Gamma$ such that $N_d \backslash (P:\Gamma) \neq \phi$. Then P is c-e-prime iff P is c-prime.*

Proof. Let P be a c-prime N-ideal and $a \in N, \gamma_1,\gamma_2 \in \Gamma, a\gamma_1 - a\gamma_2 \in P$. It needs to be shown that $a\Gamma \subseteq P$ or $\gamma_1 - \gamma_2 \in P$. Since Γ is monogenic, there exists $\gamma_0 \in \Gamma$ such that $N\gamma_0 = \Gamma$. Then, $\gamma_1 = x\gamma_0$ and $\gamma_2 = y\gamma_0$ for some $x,y \in N$. Hence, $(ax-ay)\gamma_0 = ax\gamma_0 - ay\gamma_0 = a\gamma_1 - a\gamma_2 \in P$. Since, P is c-prime $(ax-ay)\Gamma \subseteq P$ or $\gamma_0 \in P$. If $\gamma_0 \in P$, then $\Gamma = N\gamma_0 \subseteq NP \subseteq P$. But this is a contradiction with $N\Gamma \nsubseteq P$. Then, $(ax-ay)\Gamma \subseteq P$ which implies that $(ax-ay) \in (P:\Gamma)$. Since $(P:\Gamma)$ is an ideal of N, $n_d(ax-ay) \in (P:\Gamma)$ for an $n_d \in N_d \backslash (P:\Gamma)$. Then, $n_d(ax-ay) = n_d ax - n_d ay = n_d xa - n_d ya(x-y)a \in (P:\Gamma)$ because N is RP. Furthermore, $(P:\Gamma)$ is a c-prime ideal of N since P is a c-prime N-ideal by Proposition 2. Hence, $(x-y) \in (P:\Gamma)$ or $a \in (P:\Gamma)$ since $n_d \notin (P:\Gamma)$. If $(x-y) \in (P:\Gamma)$, then $(x-y)\gamma_0 = x\gamma_0 - y\gamma_0 = \gamma_1 - \gamma_2 \in P$. If $a \in (P:\Gamma)$, then $a\Gamma \subseteq P$. Thus, P is c-e-prime. On the other hand, the converse follows from Proposition 5.

Example 3 *Consider the near-ring $(N,+,.)$ in Example 2. $P = 0$ is c-prime but not c-e-prime since $N_d \backslash (P:\Gamma) = \phi$.*

Proposition 7 *If P is c-e-prime, then P is e-prime.*

Proof. Suppose P is c-e-prime and $a \in N, \gamma_1,\gamma_2 \in \Gamma, an\gamma_1 - an\gamma_2 \in P$ for all $n \in N$. Since Γ is an N-group, then $an\gamma_1 - an\gamma_2 = a\gamma_3 - a\gamma_4 \in P$, where $\gamma_3 = n\gamma_1$ and $\gamma_4 = n\gamma_2$. Hence, $a\Gamma \subseteq P$ or $\gamma_3 - \gamma_4 \in P$, since P is c-e-prime. If $a\Gamma \subseteq P$, then the proof is completed. If $\gamma_3 - \gamma_4 \in P$, then $\gamma_3 - \gamma_4 = n\gamma_1 - n\gamma_2 \in P$ and again since P is c-e-prime, hence $n\Gamma \subseteq P$ or $\gamma_1 - \gamma_2 \in P$. If $n\Gamma \subseteq P$, then this is a contradiction with $N\Gamma \nsubseteq P$. Then $\gamma_1 - \gamma_2 \in P$. Therefore, P is e-prime.

Proposition 8 *Let N be a RP near-ring, Γ be a monogenic N-group and $P \triangleleft_N \Gamma$. Then P is e-prime iff P is c-e-prime.*

Proof. Suppose that P is e-prime. Let $a \in N, \gamma_1,\gamma_2 \in \Gamma, a\gamma_1 - a\gamma_2 \in P$. It needs to be shown that $a\Gamma \subseteq P$ or $\gamma_1 - \gamma_2 \in P$. Suppose $a\Gamma \nsubseteq P$ and $\gamma_1 - \gamma_2 \notin P$. Since Γ is monogenic, $N\gamma_0 = \Gamma$ for $\gamma_0 \in \Gamma \backslash P$ (If $\gamma_0 \in P$, then $\Gamma = N\gamma_0 \subseteq NP \subseteq P$. Then $\Gamma = P$. But this is a contradiction with $N\Gamma \nsubseteq P$. Then, $\gamma_1 = x\gamma_0$ and $\gamma_2 = y\gamma_0$ for some $x,y \in N$. If $\gamma_1 - \gamma_2 \notin P$, then $\gamma_1 - \gamma_2 = x\gamma_0 - y\gamma_0 = (x-y)\gamma_0 \notin P$

. Hence, $(x-y)\notin(P{:}\gamma_0)\supseteq(P{:}\Gamma)$. Moreover, if P is an e-prime N-ideal, then $(P{:}\Gamma)$ is an e-prime ideal of N by Proposition 2. Hence, $amx-amy\notin(P{:}\Gamma)$ for some $m\in N$ since $a,(x-y)\notin(P{:}\Gamma)$. So, $(ax-ay)m=axm-aym=amx-amy\notin(P{:}\Gamma)$, since *N* is RP. Furthermore, since $(P{:}\Gamma)$ is an ideal of *N*, this implies that $(ax-ay)\notin(P{:}\Gamma)$. Under the given assumptions, if P is an e-prime N-ideal it is also c-prime by Corollary 1. In addition, $(P{:}\gamma)=(P{:}\Gamma)$ for every $\gamma\in\Gamma\backslash P$ by Proposition 1. Hence, $(ax-ay)\notin(P{:}\gamma_0)$, whence $(ax-ay)\gamma_0\notin P$. Then, $a\gamma_1-a\gamma_2=ax\gamma_0-ay\gamma_0=(ax-ay)\gamma_0\notin P$. But this is a contradiction with the assumption. Hence, $a\Gamma\subseteq P$ or $\gamma_1-\gamma_2\in P$ and therefore it follows that *P* is c-e-prime. The converse follows from Proposition 7.

Corollary 4 *Let* $P\lhd_N\Gamma$. *Then P is c-e-prime* $\Rightarrow P$ *is e-prime* $\Rightarrow P$ *is 3-prime* $\Rightarrow P$ *is 2-prime* $\Rightarrow P$ *is 0-prime.*

Corollary 5 *Let* Γ *be an N-group. Then* Γ *is c-e-prime* $\Rightarrow\Gamma$ *is e-prime* $\Rightarrow\Gamma$ *is 3-prime* $\Rightarrow\Gamma$ *is 2-prime* $\Rightarrow\Gamma$ *is 0-prime.*

If $P\lhd_N\Gamma$, then $(P{:}\Gamma)\lhd N$. With the following two propositions the relationships between the c-e-primeness of P and that of $(P{:}\Gamma)$ are investigated.

Proposition 9 *If* $P\lhd_N\Gamma$ *is c-e-prime, then* $(P{:}\Gamma)\lhd N$ *is c-e-prime.*

Proof. Let $a\in N\backslash(P{:}\Gamma)$ and $x,y\in N$ such that $ax-ay\in(P{:}\Gamma)$. Then, $a\gamma_1-a\gamma_2=ax\gamma-ay\gamma=(ax-ay)\gamma\in P$, where $\gamma_1=x\gamma$ and $\gamma_2=y\gamma$, since Γ is an *N*-group. Then, $\gamma_1-\gamma_2\in P$ because *P* is c-e-prime. If $\gamma_1-\gamma_2\in P$, then $(x-y)\gamma=x\gamma-y\gamma=\gamma_1-\gamma_2\in P$ for all $\gamma\in\Gamma$. Hence, $(x-y)\in(P{:}\Gamma)$. So, $(P{:}\Gamma)$ is c-e-prime.

Corollary 6 *If* Γ *is a c-e-prime N-group, then* $(0{:}\Gamma)$ *is a c-e-prime ideal of N.*

Proof. If *P* is replaced by the 0 ideal, then the result follows from Proposition 9.

The following proposition shows that the converse holds when *N* is RP and Γ is monogenic.

Proposition 10 *Let N be a RP near-ring,* Γ *be a monogenic N-group and P be an N-ideal of* Γ. *Then* $P\lhd_N\Gamma$ *is c-e-prime iff* $(P{:}\Gamma)\lhd N$ *is c-e-prime.*

Proof. Let $(P{:}\Gamma)$ is c-e-prime. Then, $(P{:}\Gamma)$ is also e-prime (Booth and Groenewald 1992). Furthermore, under the given assumptions, *P* is an e-prime *N*-ideal of Γ by Proposition 3. Then, *P* is also c-e-prime by Proposition 8. On the other hand, if P is c-e-prime, then $(P{:}\Gamma)$ is c-e-prime by Proposition 9.

Proposition 11 *If* Γ *is a c-e-prime N-group and P be an N-ideal of* Γ, *then P is a c-e-prime N-group.*

Proof. Let $a\in N\backslash(0{:}P)$ and $x,y\in P$ such that *ax = ay*. It is obvious that $(0{:}\Gamma)\subseteq(0{:}P)$. Then $a\in N\backslash(0{:}\Gamma)$. Since $x,y\in\Gamma$ and Γ is c-e-prime, then *x = y*. Hence, *P* is a c-e-prime *N*-group.

4. Completely Equiprime *N*-ideals and IFP *N*-ideals

In this section, some relations between the concepts of c-e-prime *N*-ideal and IFP *N*-ideal are provided. For this purpose, firstly the following definition should be given.

Definition 4 (Taşdemir et al. 2013) *For* $a\in N$ *and* $\gamma\in\Gamma$, *if* $a\gamma\in P$ *implies* $an\gamma\in P$ *for all* $n\in N$, *then P is called an IFP N-ideal.* Γ *is called an IFP N-group if* $N\Gamma\neq 0_\Gamma$ *and 0 is an IFP N-ideal of* Γ.

The Main Theorem

Theorem 1 *If P is a c-e-prime N-ideal, then P is an IFP N-ideal.*

The following propositions are necessary to prove the Main Theorem.

Proposition 12 *Let P be an N-ideal of* Γ. *Then P is a c-e-prime N-ideal iff* Γ/P *is a c-e-prime N-group.*

Proof. Since P is an *N*-ideal of $\Gamma,\Gamma/P$ is an *N*-group with the natural operation. Then, it needs to be shown that Γ/P is c-e-prime. Since $N\Gamma\nsubseteq P$, there exists $a\in N$ and $\gamma\in\Gamma$ such that $a\gamma\notin P$ i.e. $a(\gamma+P)=a\gamma+P\neq P$. Thus $N(\Gamma/P)\neq 0_{\Gamma/P}$. Let $a\in N,\gamma_1+P,\gamma_2+P\in\Gamma/P$ and $a(\gamma_1+P)=a(\gamma_2+P)$. Then, $a\gamma_1-a\gamma_2\in P$ which implies that $a\Gamma\subseteq P$ or $\gamma_1-\gamma_2\in P$ since P is c-e-prime. If $a\Gamma\subseteq P$, then $a\gamma\in P$ for all $\gamma\in\Gamma$. Hence $a(\gamma+P)=P$ for all $\gamma\in\Gamma$. Therefore $a(\Gamma/P)=P$. If $\gamma_1-\gamma_2\in P$, then $\gamma_1+P=\gamma_2+P$. So Γ/P is a c-e-prime *N*-group. Conversely, suppose that Γ/P is a c-e-prime N-group. Then $0_{\Gamma/P}=P$ is a c-e-prime *N*-ideal of Γ/P. Let $a\in N$, $\gamma_1,\gamma_2\in\Gamma$ and $a\gamma_1-a\gamma_2\in P$. Hence $a(\gamma_1+P)=a(\gamma_2+P)$. Thus, $a(\Gamma/P)=P$ or $\gamma_1+P=\gamma_2+P$ since Γ/P is a c-e-prime *N*-group. Finally, $a\Gamma\subseteq P$ or $\gamma_1-\gamma_2\in P$ which implies that *P* is a c-e-prime *N*-ideal of Γ.

Proposition 13 *If* Γ *is a c-e-prime N-group, then* Γ *is an IFP N-group.*

Proof. Let $a\gamma=0_\Gamma$ for $a\in N,\gamma\in\Gamma$. Then $a\gamma=a0_\Gamma=0_\Gamma$ because N is zero-symmetric. Since Γ is c-e-prime, $a\Gamma=0_\Gamma$ or $\gamma=0_\Gamma$. If $a\Gamma=0_\Gamma$ then $a\in(0_\Gamma{:}\Gamma)$. Hence,

$an\gamma = a\gamma' = 0_\Gamma$ for all $n \in N$, since Γ is an *N*-group. If $\gamma = 0_\Gamma$, then $an\gamma = an0_\Gamma = 0_\Gamma$ for all $n \in N$. Therefore, Γ is an IFP *N*-group.

Proposition 14 *Let P be an N-ideal of* Γ. *Then P is an IFP N-ideal iff* Γ/P *is an IFP N-group.*

Proof. Suppose P is an IFP *N*-ideal of Γ. Then Γ/P is an *N*-group with the natural operation. Let $a \in N, \gamma + P \in \Gamma/P$ and $a(\gamma + P) = 0_{\Gamma/P}$. It needs to be shown that $an(\gamma + P) = 0_{\Gamma/P}$ for all $n \in N$. Then $a\gamma \in P$, since $a(\gamma + P) = P$. Then $an\gamma \in P$ for all $n \in N$ because P is IFP. Hence $an(\gamma + P) = P$ for all $n \in N$ whence Γ/P is an IFP *N*-group. For the converse, let Γ/P is an IFP *N*-group. Then $0_{\Gamma/P} = P$ is an IFP *N*-ideal of Γ/P. Let $a\gamma \in P$ for $a \in N$ and $\gamma \in \Gamma$. This implies that $a(\gamma + P) = P$. Since Γ/P is an IFP *N*-group, then $an(\gamma + P) = P$ for all $n \in N$ which implies that $an\gamma \in P$. Thus, P is an IFP *N*-ideal of Γ.

Now, the Main Theorem of this section could be proven.

Proof of the Main Theorem. Assume P is c-e-prime. Then, Γ/P is a c-e-prime *N*-group by Proposition 12. Hence Γ/P is also an IFP *N*-group by Proposition 13. Finally, P is an IFP *N*-ideal of Γ by Proposition 14.

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