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Some New Integral Inequalities for *n*-Times Differentiable Strongly Convex Functions

n-Kez Türevlenebilen Güçlü Konveks Fonksiyonlar İçin Bazı Yeni İntegral Eşitsizlikler

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Abstract

In this work, by using an integral identity together with both the Hölder and the Power-Mean integral inequality we establish several new inequalities for *n*-time differentiable strongly convex functions.

Keywords: Hölder Integral inequality, Power-Mean Integral inequality, Strongly convex function

Öz

Bu makalede, hem Hölder hem de Power-Mean integral eşitsizliği ile birlikte bir integral eşitliği kullanılarak *n*-kez türevlenebilen güçlü konveks fonksiyonlar için bir kaç yeni eşitsizlik bulunmuştur.

Anahtar Kelimeler: Hölder integral eşitsizliği, Power-Mean integral eşitsizliği, Güçlü konveks fonksiyon

1. Introduction

Definition 1.1. A function $f: I \subseteq \mathbb{R} \to \mathbb{R}$ is said to be convex if the inequality

$$f(tx+(1-t)y) \leq tf(x)+(1-t)f(y)$$

is valid for all $x,y \in I$ and $t \in [0,1]$. If this inequality reverses, then f is said to be concave on interval $I \neq \emptyset$. This definition is well known in the literature.

For some inequalities, generalizations and applications concerning convexity see (Jiang et al. 2012, Kırmacı et al. 2007, Özdemir and Kırmacı 2003, Hwang 2003, Jiang 2012, Kırmacı et al. 2007, Özdemir and Kırmacı 2003, Özdemir and Yıldız 2014, Pěcarić et al. 1992). Recently, in the literature there are so many papers about *n*-times differentiable functions on several kinds of convexities. In references (Bai et al. 2012, Cerone et al. 1999, Cerone et al. 2000, Dragomir and Pearce 2000, Hwang 2003, Jiang et al. 2012, Özdemir and Yıldız 2014, Wang et al. 2012, Maden et al. 2016), readers can find some results about this issue.

Definition 1.2. Let $I \subset \mathbb{R}$ be an interval and c be a positive number. A function $f: I \subset \mathbb{R} \to \mathbb{R}$ is called *strongly convex with modulus c* if

$$f(tx+(1-t)y) \leq tf(x)+(1-t)f(y)-ct(1-t)(x-y)^2$$

for all $x,y \in I$ and $t \in [0,1]$. Strongly convex functions were introduced by Polyak (Polyak 1966). They have useful properties in optimization theory. For instance, if f is strongly convex, then it is bounded from below, its level sets $\{x \in I: f(x) \leq \lambda\}$ are bounded for each λ and f has a unique minimum on every closed subinterval of I (Roberts and Varberg 1973). Since strong convexity is a strengthening of the notion of convexity, some properties of strongly convex functions are just "stronger versions" of known properties of convex functions. For instance, a function $f: I \to \mathbb{R}$ is strongly convex with modulus c if and only if for every $x_0 \in intI$ there exists an $a \in \mathbb{R}$ such that

$$f(x) \geq c(x-x_0)^2+a(x-x_0)+f(x_0), \quad x \in I$$

i.e. f has a quadratic support at x_0. For a twice differentiable f, f is strongly convex with modulus c if and only if $f'' \geq 2c$ (Roberts and Varberg 1973). The aim of this paper is to present some further properties of strongly convex functions

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which are counterparts of known results for convex functions. First, we characterize real functions defined on I which can be separated by a strongly convex function. As a consequence, we obtain a Hyers–Ulam stability result for strongly convex functions. Next, counterparts of the classical Jensen and Hermite–Hadamard inequalities are proved. Finally, it is shown that strong convexity is equivalent to generalized convexity in the sense of Beckenbach with respect to a certain two-parameter family. Many papers have been written by a number of mathematicians concerning inequalities for different classes of strongly convex functions see for instance the recent papers (Azócar et all. 2012, Cortez 2016, Angulo et al. 2011, Merentes and Nikodem 2010, Lara et al. 2015, Gera and Nikodem 2011, Nikodem and Páles 2011, Sarikaya and Yaldiz 2013) and the references within these papers.

Throughout this paper we will use the following notations and conventions. Let $J=[0,\infty)\subset\mathbb{R}=(-\infty,+\infty)$, and a, $b\in$ J with 0<a<b and $f\in L[a,b]$ and

$$A(a,b)=\frac{a+b}{2},$$
$$L_p(a,b)=\left(\frac{b^{p+1}-a^{p+1}}{(p+1)(b-a)}\right)^{\frac{1}{p}}, a\neq b, p\in\mathbb{R}, p\neq -1,0$$

be the arithmetic and generalized logarithmic mean for a, $b>0$ respectively.

2. Main Results

We will use the following Lemma (Maden et al. 2017) for we obtain the main results:

Lemma 2.1. Let $f:I\subseteq\mathbb{R}\to\mathbb{R}$ be n-times differentiable mapping on I° for $n\in\mathbb{N}$ and $f^{(n)}\in L[a,b]$, where $a,b\in I^{\circ}$ with $a<b$, we have the identity

$$\sum_{k=0}^{n-1}(-1)^k\left(\frac{f^{(k)}(b)b^{k+1}-f^{(k)}(a)a^{k+1}}{(k+1)!}\right)-\int_a^b f(x)dx=\frac{(-1)^{n+1}}{n!}\int_a^b x^n f^{(n)}(x)dx.$$

where an empty sum is understood to be nil.

Theorem 2.1. For $\forall n\in\mathbb{N}$; let $f:I\subset[0,\infty)\to\mathbb{R}$ be n-times differentiable function on I° and $a,b\in I^{\circ}$ with $a<b$. If $f^{(n)}\in L[a,b]$ and $|f^{(n)}|^q$ for $q>1$ is strongly convex on $[a,b]$, then the following inequality holds:

$$\left|\sum_{k=0}^{n-1}(-1)^k\left(\frac{f^{(k)}(b)b^{k+1}-f^{(k)}(a)a^{k+1}}{(k+1)!}\right)-\int_a^b f(x)dx\right|$$
$$\leq\frac{1}{n!}(b-a)L_{np}^{n}(a,b)\left[A\left(|f^{(n)}(a)|^q,|f^{(n)}(b)|^q\right)-c\frac{(b-a)^2}{6}\right]^{\frac{1}{q}}$$

Proof. If $|f^{(n)}|^q$ for $q>1$ is strongly convex on $[a,b]$, using Lemma1.1, the Hölder integral inequality and inequality

$$|f^{(n)}(x)|^q=\left|f^{(n)}\left(\frac{x-a}{b-a}b+\frac{b-x}{b-a}a\right)\right|^q$$
$$\leq\frac{x-a}{b-a}|f^{(n)}(b)|^q+\frac{b-x}{b-a}|f^{(n)}(a)|^q-c(b-x)(x-a),$$

we have

$$\left|\sum_{k=0}^{n-1}(-1)^k\left(\frac{f^{(k)}(b)b^{k+1}-f^{(k)}(a)a^{k+1}}{(k+1)!}\right)-\int_a^b f(x)dx\right|$$
$$\leq\frac{1}{n!}\int_a^b x^n|f^{(n)}(x)|dx$$
$$\leqslant\frac{1}{n!}\left(\int_a^b x^{np}dx\right)^{\frac{1}{p}}\left(\int_a^b|f^{(n)}(x)|^q dx\right)^{\frac{1}{q}}$$
$$=\frac{1}{n!}\left(\int_a^b x^{np}dx\right)^{\frac{1}{p}}\left(\int_a^b\left|f^{(n)}\left(\frac{x-a}{b-a}b+\frac{b-x}{b-a}a\right)\right|^q dx\right)^{\frac{1}{q}}$$
$$\leq\frac{1}{n!}\left(\int_a^b x^{np}dx\right)^{\frac{1}{p}}\left(\int_a^b\left[\begin{array}{l}\frac{x-a}{b-a}|f^{(n)}(b)|^q+\frac{b-x}{b-a}|f^{(n)}(a)|^q-\\ c(b-x)(x-a)\end{array}\right]dx\right)^{\frac{1}{q}}$$
$$=\frac{1}{n!}\left(\frac{x^{np+1}}{np+1}\Bigg|_a^b\right)^{\frac{1}{p}}\left(|f^{(n)}(b)|^q\left(\frac{x^2}{2(b-a)}-\frac{ax}{(b-a)}\right)\Bigg|_a^b\right.$$
$$\left.+|f^{(n)}(a)|^q\left(\frac{bx}{b-a}-\frac{x^2}{2(b-a)}\right)\Bigg|_a^b-c\left[-\frac{x^3}{3}+\frac{a+b}{2}x^2-abx\right]\Bigg|_a^b\right)^{\frac{1}{q}}$$
$$=\frac{1}{n!}\left(\frac{b^{np+1}-a^{np+1}}{np+1}\right)^{\frac{1}{p}}\left[\begin{array}{l}\left(\frac{b-a}{2}\right)|f^{(n)}(b)|^q+\left(\frac{b-a}{2}\right)|f^{(n)}(a)|^q-\\ c\frac{(b-a)^3}{6}\end{array}\right]^{\frac{1}{q}}$$
$$=\frac{1}{n!}(b-a)L_{np}^{n}(a,b)\left[A\left(|f^{(n)}(a)|^q,|f^{(n)}(b)|^q\right)-c\frac{(b-a)^2}{6}\right]^{\frac{1}{q}}$$

This completes the proof of theorem.

Corollary 2.1. Under the conditions Theorem 2.1 for $n = 1$ we have the following inequality:

$$\left|\frac{f(b)b-f(a)a}{b-a}-\frac{1}{b-a}\int_a^b f(x)dx\right|$$
$$\leq L_p(a,b)\left[A(|f'(a)|^q,|f'(b)|^q)-c\frac{(b-a)^2}{6}\right]^{\frac{1}{q}}$$

Theorem 2.2. For $\forall n\in\mathbb{N}$; let $f:I\subset[0,\infty)\to\mathbb{R}$ be n-times differentiable function on I° and a, $b\in I^{\circ}$ with $a<b$. If $f^{(n)}\in L[a,b]$ and $|f^{(n)}|^q$ for $q\geq1$ is strongly convex on $[a,b]$, then the following inequality holds:

$$\left|\sum_{k=0}^{n-1}(-1)^k\left(\frac{f^{(k)}(b)b^{k+1}-f^{(k)}(a)a^{k+1}}{(k+1)!}\right)-\int_a^b f(x)dx\right|$$

$$\le \frac{1}{n!}(b-a)^{1-\frac{1}{q}} L_n^{n\left(\frac{q-1}{q}\right)}(a,b)\left\{\left|f^{(n)}(b)\right|^q \begin{bmatrix} L_{n+1}^{n+1}(a,b)- \\ aL_n^n(a,b) \end{bmatrix}\right.$$
$$+\left|f^{(n)}(a)\right|^q\left[bL_n^n(a,b)-L_{n+1}^{n+1}(a,b)\right]$$
$$\left.+(b-a)\left[L_{n+2}^{n+2}(a,b)-(a+b)L_{n+1}^{n+1}(a,b)+abL_n^n(a,b)\right]\right\}^{\frac{1}{q}}$$

Proof. From Lemma1.1 and Power-mean integral inequality, we obtain

$$\left|\sum_{k=0}^{n-1}(-1)^k\left(\frac{f^{(k)}(b)b^{k+1}-f^{(k)}(a)a^{k+1}}{(k+1)!}\right)-\int_a^b f(x)dx\right|$$
$$\le \frac{1}{n!}\int_a^b x^n\left|f^{(n)}(x)\right|dx$$
$$\le \frac{1}{n!}\left(\int_a^b x^n dx\right)^{1-\frac{1}{q}}\left(\int_a^b x^n\left|f^{(n)}(x)\right|^q dx\right)^{\frac{1}{q}}$$
$$\le \frac{1}{n!}\left(\int_a^b x^n dx\right)^{1-\frac{1}{q}}\left(\int_a^b x^n\begin{bmatrix}\frac{x-a}{b-a}\left|f^{(n)}(b)\right|^q+\frac{b-x}{b-a}\left|f^{(n)}(a)\right|^q- \\ c(b-x)(x-a)\end{bmatrix}dx\right)^{\frac{1}{q}}$$
$$=\frac{1}{n!}\left(\frac{x^{n+1}}{n+1}\bigg|_a^b\right)^{1-\frac{1}{q}}\left[\begin{cases}\frac{\left|f^{(n)}(b)\right|^q}{b-a}\left[\frac{x^{n+2}}{n+2}-a\frac{x^{n+1}}{n+1}\right]+ \\ \frac{\left|f^{(n)}(a)\right|^q}{b-a}\left[b\frac{x^{n+1}}{n+1}-\frac{x^{n+2}}{n+2}\right]\end{cases}\right.$$
$$\left.+c\left[\frac{x^{n+3}}{n+3}-(a+b)\frac{x^{n+2}}{n+2}+ab\frac{x^{n+1}}{n+2}+ab\frac{x^{n+1}}{n+1}\right]\bigg|_a^b\right]^{\frac{1}{q}}$$
$$=\frac{1}{n!}(b-a)^{1-\frac{1}{q}}\left[\frac{b^{n+1}-a^{n+1}}{(n+1)(b-a)}\right]^{1-\frac{1}{q}}\left\{\left|f^{(n)}(b)\right|^q\begin{bmatrix}\frac{(b^{n+2}-a^{n+2})}{(n+2)(b-a)}- \\ a\frac{(b^{n+1}-a^{n+1})}{(n+1)(b-a)}\end{bmatrix}\right.$$
$$+\left|f^{(n)}(a)\right|^q\left[b\frac{(b^{n+1}-a^{n+1})}{(n+1)(b-a)}-\frac{(b^{n+2}-a^{n+2})}{(n+2)(b-a)}\right]$$
$$+c(b-a)\left[\left(\frac{b^{n+3}-a^{n+3}}{(n+3)(b-a)}\right)-(a+b)\left(\frac{b^{n+2}-a^{n+2}}{(n+2)(b-a)}\right)\right.$$
$$\left.\left.+ab\left(\frac{b^{n+1}-a^{n+1}}{(n+1)(b-a)}\right)\right]\right\}^{\frac{1}{q}}$$
$$=\frac{1}{n!}(b-a)^{1-\frac{1}{q}}L_n^{n\left(\frac{q-1}{q}\right)}(a,b)$$
$$\times\begin{cases}\left|f^{(n)}(b)\right|^q\left[L_{n+1}^{n+1}(a,b)-aL_n^n(a,b)\right]+ \\ \left|f^{(n)}(a)\right|^q\left[bL_n^n(a,b)-L_{n+1}^{n+1}(a,b)\right]\end{cases}$$
$$\left.+c(b-a)\left[L_{n+2}^{n+2}(a,b)-(a+b)L_{n+1}^{n+1}(a,b)+abL_n^n(a,b)\right]\right\}^{\frac{1}{q}}.$$

Corollary 2.2. Under the conditions Theorem 2.2 for n = 1 we have the following inequality:

$$\left|\frac{f(b)b-f(a)a}{b-a}-\frac{1}{b-a}\int_a^b f(x)dx\right|$$
$$\le\left(\frac{1}{6}\right)^{\frac{1}{q}}A^{1-\frac{1}{q}}(a,b)\begin{Bmatrix}(2b+a)\left|f'(b)\right|^q+(b+2a)\left|f'(a)\right|^q- \\ c\frac{(b-a)^2(b+a)}{2}\end{Bmatrix}^{\frac{1}{q}}$$

Corollary 2.3. Under the conditions Theorem 2.2 for q = 1 we have the following inequality:

$$\left|\sum_{k=0}^{n-1}(-1)^k\left(\frac{f^{(k)}(b)b^{k+1}-f^{(k)}(a)a^{k+1}}{(k+1)!}\right)-\int_a^b f(x)dx\right|$$
$$\le\frac{1}{n!}\begin{cases}\left|f^{(n)}(b)\right|\left[L_{n+1}^{n+1}(a,b)-aL_n^n(a,b)\right]+ \\ \left|f^{(n)}(a)\right|\left[bL_n^n(a,b)-L_{n+1}^{n+1}(a,b)\right]\end{cases}$$
$$\left.+c(b-a)\left[L_{n+2}^{n+2}(a,b)-(a+b)L_{n+1}^{n+1}(a,b)+abL_n^n(a,b)\right]\right\}.$$

Theorem 2.3. For $\forall n\in\mathbb{N}$; let $f:I\subset[0,\infty)\to\mathbb{R}$ be n-times differentiable function on I° and $a, b\in I^{\circ}$ with $a<b$. If $f^{(n)}\in L[a, b]$ and $|f^{(n)}|^q$ for $q>1$ is strongly convex on $[a, b]$, then the following inequality holds:

$$\left|\sum_{k=0}^{n-1}(-1)^k\left(\frac{f^{(k)}(b)b^{k+1}-f^{(k)}(a)a^{k+1}}{(k+1)!}\right)-\int_a^b f(x)dx\right|$$
$$\le\frac{1}{n!}(b-a)^{\frac{1}{p}}\begin{cases}\left|f^{(n)}(b)\right|^q\left(L_{nq+1}^{nq+1}-aL_{nq}^{nq}\right)+ \\ \left|f^{(n)}(a)\right|^q\left(bL_{nq}^{nq}-L_{nq+1}^{nq+1}\right)\end{cases}$$
$$\left.+c(b-a)\left[L_{nq+2}^{nq+2}-(a+b)L_{nq+1}^{nq+1}+abL_{nq}^{nq}\right]\right\}^{\frac{1}{q}}$$

Proof: If $|f^{(n)}|^q$ for $q>1$ is strongly convex on $[a, b]$, using Lemma1.1 and the Hölder integral inequality, we have the following inequality:

$$\left|\sum_{k=0}^{n-1}(-1)^k\left(\frac{f^{(k)}(b)b^{k+1}-f^{(k)}(a)a^{k+1}}{(k+1)!}\right)-\int_a^b f(x)dx\right|$$
$$\le\frac{1}{n!}\left(\int_a^b 1^p dx\right)^{\frac{1}{p}}\left(\int_a^b x^{nq}\left|f^{(n)}(x)\right|^q dx\right)^{\frac{1}{q}}$$
$$\le\frac{1}{n!}\left(\int_a^b 1.dx\right)^{\frac{1}{p}}\left(\int_a^b x^{nq}\begin{bmatrix}\frac{x-a}{b-a}\left|f^{(n)}(b)\right|^q+\frac{b-x}{b-a}\left|f^{(n)}(a)\right|^q- \\ c(b-x)(x-a)\end{bmatrix}dx\right)^{\frac{1}{q}}$$
$$=\frac{1}{n!}\left(x\bigg|_a^b\right)^{\frac{1}{p}}\left(\frac{\left|f^{(n)}(b)\right|^q}{b-a}\left(\frac{x^{nq+2}}{nq+2}-\frac{ax^{nq+1}}{nq+1}\right)\bigg|_a^b\right.$$
$$+\frac{\left|f^{(n)}(a)\right|^q}{b-a}\left(\frac{bx^{nq+1}}{nq+1}-\frac{x^{nq+2}}{nq+2}\right)\bigg|_a^b$$
$$\left.-c\left[-\frac{x^{nq+3}}{nq+3}+(a+b)\frac{x^{nq+2}}{nq+2}-ab\frac{x^{nq+1}}{nq+1}\right]\bigg|_a^b\right)^{\frac{1}{q}}$$
$$=\frac{1}{n!}(b-a)^{\frac{1}{p}}\left\{\left|f^{(n)}(b)\right|^q\left(\frac{b^{nq+2}-a^{nq+2}}{(nq+2)(b-a)}-a\frac{b^{nq+1}-a^{nq+1}}{(nq+1)(b-a)}\right)\right.$$
$$+\left|f^{(n)}(a)\right|^q\left(\frac{b^{nq+1}-a^{nq+1}}{(nq+1)(b-a)}-\frac{b^{nq+2}-a^{nq+2}}{(nq+2)(b-a)}\right)$$
$$-c(b-a)\left[-\frac{b^{nq+3}-a^{nq+3}}{(nq+3)(b-a)}+(a+b)\frac{b^{nq+2}-a^{nq+2}}{(nq+2)(b-a)}\right.$$
$$\left.\left.-ab\frac{b^{nq+1}-a^{nq+1}}{(nq+1)}\right]\right\}^{\frac{1}{q}}$$
$$=\frac{1}{n!}(b-a)^{\frac{1}{p}}\left\{\left|f^{(n)}(b)\right|^q\left(L_{nq+1}^{nq+1}-aL_{nq}^{nq}\right)\right.$$
$$+\left|f^{(n)}(a)\right|^q\left(bL_{nq}^{nq}-L_{nq+1}^{nq+1}\right)$$
$$\left.+c(b-a)\left[L_{nq+2}^{nq+2}-(a+b)L_{nq+1}^{nq+1}+abL_{nq}^{nq}\right]\right\}^{\frac{1}{q}}$$

Corollary 2.4. Under the conditions Theorem 2.3 for $n = 1$ we have the following inequality:

$$\left| \frac{f(b)b - f(a)a}{b-a} - \frac{1}{b-a} \int_a^b f(x)dx \right|$$
$$\leq \left\{ \begin{array}{l} \frac{|f'(b)|^q}{b-a}(L_{q+1}^{q+1} - aL_q^q) + \frac{|f'(a)|^q}{b-a}(bL_q^q - L_{q+1}^{q+1}) + \\ c[L_{q+2}^{q+2} - (a+b)L_{q+1}^{q+1} + abL_q^q] \end{array} \right\}^{\frac{1}{q}}$$

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