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Research Article

Effects of Prandtl Number and Magnetic Field on Mixed Convection Heat Transfer in an Inclined Lid-Driven Enclosure

Eğik Hareketli Kapalı Bir Bölge İçinde Karışık Konveksiyonla Isı Transferi Üzerinde Prandtl Sayısı ve Manyetik Alanın Etkileri

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Abstract

This paper analyzes heat transfer and fluid flow of laminar mixed convection in an inclined square lid-driven enclosure in the presence of magnetic field. Vertical sides of the enclosure moves upward direction when inclination angle is zero. Governing equations of flow and temperature in the form of stream function-vorticity formulation were solved numerically using the differential quadrature method. The study is based on Richardson number (from 0.01 to 100), Prandtl number (from 0.054 to 0.71), inclination angle of enclosure (from 0° to 180°), Hartmann number (from 0 to 100) and magnetic field directions (from 0° to 90°) respectively. It is shown by the results that, the flow and temperature field are affected significantly by the movement of side walls of the enclosure. For all values of Richardson and Prandtl numbers, Hartmann number is effective on heat transfer. Additionally, convective fluid flow decreases with increasing Hartmann number. The results showed that the movement of the side walls of the enclosure has a significant effect on the flow and temperature fields. Hartmann number is effective on heat transfer for all values of Richardson and Prandtl numbers and it decreases the convective fluid flow. Increasing the Prandtl number increase the average Nusselt number at the heated surface.

Keywords: DQ_method, Lid-driven enclosure, Magnetic field, Mixed convection, Prandtl number

Öz

Bu çalışmada, manyetik alanın etkisi altındaki hareketli duvarlara sahip eğik kare kapalı bir bölge içindeki laminer karışık konveksiyon akışı ve ısı transferi analiz edilmiştir. Eğim açısı sıfır olduğunda kapalı bölgenin dik yan duvarları yukarı doğru hareket etmektedir. Yönetici denklemler, diferansiyel kuadratür (DQ) yöntemi kullanılarak akım fonksiyonu, girdap ve sıcaklık için nümerik olarak çözülmüştür. Yönetici parametreler, Richardson sayısı (0.01- 100), Prandtl sayısı (0.054 -0.71), kapalı bölgenin eğim açısı (0°-180°), Hartmann sayısı (0-100) ve manyetik alanın yönü (0°-90°) aralıklarında alınmıştır. Sonuçlar göstermektedir ki, yan duvarların hareketi akış ve sıcaklık alanlarını etkilemektedir. Hartmann sayısı, tüm Richardson ve Prandtl sayıları için ısı transferi üzerinde önemli bir etkiye sahiptir ve konvektif akışkan akışını azaltmaktadır. Prandtl sayısı arttıkça, ısıtılmış yüzeylerde ortalama Nusselt sayısı artmaktadır.

Anahtar Kelimeler: DQ_Metot, Hareketli kapalı bölge, Manyetik alan, Karışık konveksiyon, Prandtl sayısı

1. Introduction

Flow fields and temperature distributions in lid-driven cavities are important for both thermal sciences and fluid mechanics due to its wide applications area such as cooling of electronic devices, geothermal applications, furnaces, air-solar collectors etc. Shankar and Despande (2000) showed an example for the lid-driven configuration. They indicated that in the short-dwell coater used to produce high-grade paper and photographic film, the structure of the field in the liquid pond can greatly influence the quality of the coating on the roll and the cavity is a rectangular parallelepiped in which the lid generates the motion. This phenomenon contains pure fluid motion. Ghia et al. (1982) performed a study to show the flow fields inside the lid-driven enclosure as a benchmark study. As per Sharif (2007) study, the resulting flow decreases under mixed convection regime, while a temperature gradient is applied, in case the shear driven and buoyancy effects are of comparable magnitude. When a temperature gradient is imposed such that the shear driven and buoyancy effects are of comparable magnitude then the resulting flow falls under mixed convection regime

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as indicated by Sharif (2007). In this case, the analysis becomes more complex. Investigation of opposing or aiding mechanism of the mixed convection is performed by Torrance et al. (1972), Iwatsu et al. (1993), Mohammad and Viskanta (1993). Oztop and Dagtekin (2004) studied this effect in double-sided lid-driven enclosure. They tested the three possibilities of the moving walls and found that direction of the moving wall is the most important parameter on flow and temperature fields especially at low Reynolds numbers.

Control of heat transfer is very important issue in both industrial heat transfer and other applications from the energy saving point of view. One way of this control parameter is application of magnetic field at different directions. It is important especially in crystal growth processes. In earlier studies, effects of magnetic field on natural convection heat transfer in enclosures with different configurations have been studied by Büyük (2006), Ece and Büyük (2006). Magnetohydrodynamic mixed convection also important for channel flows as studied by Chamkha (2002). However, Chamkha (1998) made a numerical analyze to obtain flow and temperature field of hydromagnetic combined convection flow in a lid-driven enclosure with left side moves up/down with internal heat generation or absorption. They solved the governing equations based on finite-volume approach along with the ADI scheme. They found that flow behavior and the heat transfer characteristics inside the cavity are strongly affected by the presence of the magnetic field. Significant reductions in the mean Nusselt number were produced for both aiding and opposing flow situations as the strength of the applied magnetic field was increased. Mixed convection flow under the presence of a magnetic field of arbitrary direction in an inclined square enclosure with isothermally heated and moving side walls was studied by Ece and Öğüt (2009) where stationary and adiabatic horizontal walls were taken into account. Ece and Öğüt (2009) studied mixed convection flow under the presence of a magnetic field of arbitrary direction in an inclined square enclosure with isothermally heated and moving side walls, and stationary and adiabatic horizontal walls was considered. The study Teamah and El-Maghlany (2010) is based on the mixed convection in a rectangular lid-driven cavity under the combined buoyancy effects of thermal and mass diffusion, where both upper and lower surfaces are being insulated and impermeable. The present study is concerned with the mixed convection in a rectangular lid-driven cavity under the combined buoyancy effects of thermal and mass diffusion. Both upper and lower surfaces are being insulated and impermeable. While, steady state laminar regime is considered, constant different temperatures and concentration are applied along the vertical walls of the enclosure. Constant different temperatures and concentration are imposed along the vertical walls of the enclosure, steady state laminar regime is considered. For analyzing the mixed convection flow and heat transfer in a lid-driven cavity with sinusoidal wavy bottom surface in presence of transverse magnetic field, the numerical simulation was studied by the Nasrin and Parvin (2011). It is derived by the results of study that, while the number of waves increasing, the average Nusselt number at the heated surface increases as well as the Reynolds number while decreases with increasing Hartmann number.

The main purpose of this study is to analyze the effects of Prandtl number and magnetic field on mixed convection in an inclined lid-driven enclosure under magnetic forces. Effects of following governing parameters on fluid flow and heat transfer, ie Richardson number, inclination angle of the enclosure, Prandtl number and inclination angle of magnetic forces has been studied.

2. Material and Methods

The physical model is shown in Fig. 1 with coordinate system and boundary conditions, schematically. Inclination angle of the enclosure with respect to coordinate system is also shown in Figure. It is a square inclined double sided lid-driven enclosure. Moving sides of the enclosure are isothermal and they have different temperature. They have constant speed as U. In the figure, $T_h > T_c$. Other two sides are kept adiabatic. Magnetic field of strength B is applied at an angle ϕ according to coordinate system.

3. Computational Details

3.1. Governing Equations

Governing equations are based steady, laminar, mixed convection flow under the presence of a magnetic field of arbitrary direction in an inclined square enclosure with isothermally heated and moving side walls, and stationary and adiabatic horizontal walls. It is considered that, dimensional coordinates with the x^*-axis measuring along the bottom wall and y^*-axis being normal to it along the left wall. Boundary conditions and the coordinate system are shown in Fig.1. The angle of inclination of the enclosure from horizontal in the counterclockwise direction is denoted by γ. Magnetic field of strength B is applied at an angle ϕ.

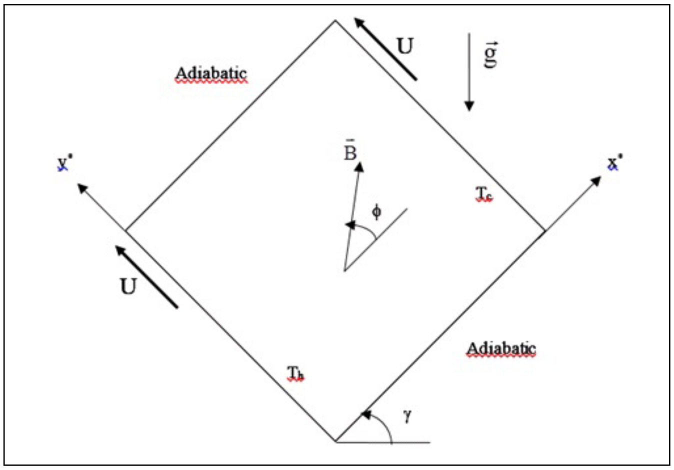

Figure 1. Schematic diagram of the physical system.

The top and the bottom walls are insulated and the fluid is isothermally heated and cooled by the left and the right side walls at uniform temperatures of T_h and T_c,respectively. The vertical walls are also allowed to move with equal and constant speeds in the same or opposite directions.

Dimensionless variables used in the analysis were defined as below,

$$x = \frac{x^*}{L}, y = \frac{y^*}{L}, \tag{1}$$

$$u = \frac{u^*}{U}, v = \frac{v^*}{U}, p = \frac{[p^* + \rho_0 g(x^* \sin\gamma\, y^* \cos\gamma)]}{\rho_0 U^2}, \quad \theta = \frac{T - T_c}{T_h - T_c}. \tag{2}$$

U is the speed of the side walls of the enclosure, g is the gravitational acceleration, u* and v* are the dimensional velocity components in the x* and y* directions respectively L is the dimensional length of the square enclosure, p* is the dimensional pressure and ρ_0 is the density of the fluid at temperature T_c. in the study

As per equations, a dimensionless stream function and vorticity were defined,

$$u = \frac{\partial\psi}{\partial y}, v = -\frac{\partial\psi}{\partial x}, \tag{3}$$

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}. \tag{4}$$

The governing equations under Boussinesq approximation in terms of the dimensionless variables are as follows

$$\omega = -\nabla^2\psi, \tag{5}$$

$$u\frac{\partial\omega}{\partial x} + v\frac{\partial\omega}{\partial y} = \frac{1}{Re}\left(\frac{\partial^2\omega}{\partial x^2} + \frac{\partial^2\omega}{\partial y^2}\right) + \frac{Gr}{Re^2}\left(\cos\gamma\frac{\partial\theta}{\partial x} - \sin\gamma\frac{\partial\theta}{\partial y}\right) + \frac{Ha^2}{Re}\left[\cos\phi\left(\sin\phi\frac{\partial u}{\partial x} - \cos\phi\frac{\partial v}{\partial x}\right) + \sin\phi\left(\sin\phi\frac{\partial u}{\partial y} - \cos\phi\frac{\partial v}{\partial y}\right)\right] \tag{6}$$

$$u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} = \frac{1}{RePr}\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right). \tag{7}$$

The Reynolds, Prandtl, Grashof and Hartman numbers are defined as follows

$$Re = \frac{UL\rho_0}{\mu}, Pr = \frac{\mu}{\rho_0\alpha}, Gr = \frac{\rho_0^2 g\beta L^3(T_h - T_c)}{\mu^2}, \quad Ha = LB\sqrt{\frac{\sigma}{\mu}}, \tag{8}$$

where μ is the viscosity, β is the coefficient of thermal expansion, α is the thermal diffusivity and σ is the electrical conductivity of the fluid respectively. Boundary conditions are

$$u(x,0) = 0, v(x,0) = 0, u(x,1) = 0, v(x,1) = 0, \tag{9}$$

$$u(0,y) = 0, v(0,y) = 1, u(1,y) = 0, v(1,y) = 1, \tag{10}$$

$$\theta(0,y) = 1, \theta(1,y) = 0, \left.\frac{\partial\theta}{\partial y}\right|_{y=0} = 0, \left.\frac{\partial\theta}{\partial y}\right|_{y=1} = 0. \tag{11}$$

The boundary condition given by Eqs. (9) and (10) may also be expressed in terms of the stream function.

$$\psi(x,0) = 0, \psi(x,1) = 0, \psi(0,y) = 0, \psi(1,y) = 0, \tag{12}$$

$$\left.\frac{\partial\psi}{\partial y}\right|_{y=0} = 0, \left.\frac{\partial\psi}{\partial y}\right|_{y=1} = 0, \left.\frac{\partial\psi}{\partial x}\right|_{x=0} = 1, \left.\frac{\partial\psi}{\partial x}\right|_{x=1} = 1. \tag{13}$$

Dimensionless local Nusselt number is defined as

$$Nu = -\left.\frac{\partial\theta}{\partial x}\right|_{x=0} \tag{14}$$

Average Nusselt number can be obtained by the integration of the local Nusselt number along the hot wall of the enclosure.

$$Nu_a = -\int_0^1 \left.\frac{\partial\theta}{\partial x}\right|_{x=0} dy \tag{15}$$

3.2. Numerical Technique

The dimensionless governing equations were solved for stream function, vorticity and temperature ratio using the differential quadrature method originally introduced by Bellman et al. (1972). The method is based on the assumption that, the unknown functions can be locally approximated as polynomials. The basis of the method is to assume that the unknown functions can be locally approximated as polynomials. The first and second order partial derivatives of a two-variable function F(x, y) at a point (x_i, y_j) can be approximated as

$$F_x(x_i, y_j) = \sum_{k=1}^{N} a_{ik} F(x_k, y_j), F_y(x_i, y_j) = \sum_{k=1}^{N} \bar{a}_{jk} F(x_i, y_k) \quad (16)$$

$$F_{xx}(x_i, y_j) = \sum_{k=1}^{N} b_{ik} F(x_k, y_j), F_{yy}(x_i, y_j) = \sum_{k=1}^{N} \bar{b}_{jk} F(x_i, y_k) . (17)$$

Here (N-1) is the degree of the polynomial behavior assumed and the weight functions are given by Shu (2000), Ece and Buyuk (2006) as follows

$$\omega|_{\eta=0} = \pm \frac{\partial^2 \psi}{\partial \eta^2}\bigg|_{\eta=0}, \quad (18)$$

where ς is the normal coordinate measured away from the wall and the differentiation was implemented using the equations for the second order derivatives given by Eqs. (17). When the absolute values of the difference between the successive solutions for stream function, vorticity and temperature ratio at each mesh point are less than 10^{-6}, iterations were finalized

3.3. Validation of the Code and Computation

Test for validation code of the case of Re=0 which corresponds to natural convection case has been carried on. For this reason, results of present study compared with the Davis (1983), Shu et al. (2003) results as given in Table 1. As can be shown from the table, the present code shows good agreement with the literature. Additionally, a series of grid tests on ψ_{max} values from 51x51 to 101x101 at different Hartmann numbers were studied. Results are shown in Table 2. The lowest iteration numbers from the table, namely 101x101 has been chosen.

Table 1. Comparison of the results for Re = 0, Ra = 10^4, Pr = 0.71, Ha = 0 and γ = 0°.

	$\|\psi_{max}\|$	$\|\psi^*_{max}\|$ (m²/s)
Present study	5.074	$1.071 10^{-4}$
Davis (1983)	5.071	$1.071\ 10^{-4}$
Shu et al. (2003)	5.084	$1.074\ 10^{-4}$

4. Results and Discussion

A complicated numerical study has been carried on to analyze the effects of Prandtl number, Richardson number, direction of magnetic field, and Hartmann number on mixed convection flow and thermal behavior in an inclined lid driven enclosure. Flow and temperature fields are also introduced via streamlines and isotherms. Governing parameters are taken in the range of Richardson number (from 0.01 to 100), Prandtl number (from 0.054 to 0.71), inclination angle of enclosure (from 0° to 180°), Hartmann number (from 0 to 100) and magnetic field directions (from 0° to 90°). The Grashof number was kept at a constant value of 10^4 for whole study.

To see the effects of Richardson number on flow and temperature distributions, streamlines and isotherms are plotted in Fig. 2 (a) to (d) for constant values as Pr = 0.71, Ha = 0 and γ = 0°. As can be shown from the figures, double circulation cells were formed in counterclockwise directions. Again figure shows that the right cell is dominant to the left due to strong convection flows form near the hot wall. This situation also obtained from the isotherms in Fig. 2 (a) on the right. Thus, the flow strength is very strong as can be shown from the stream function values on the figures. For Ri = 1 (mixed convection regime) the length of cell near the hot wall increases as shown in Fig. 2 (b) (on the left). Circle shaped main cell on the left of the streamline patterns of the Fig. 2 (b) becomes smaller with increasing Richardson number. Both isotherms and streamlines show the same behavior with natural convection of differentially heated square cavity (Fig. 2 (c)). To see the effect of Hartmann number on flow and temperature fields, the streamlines (on the left) and isotherms (on the right) are plotted in Fig. 3 for Pr = 0.71, Ri = 1 (mixed convection) and γ = 0°. Fig. 3 (a) shows the contours for Ha = 10. In this case, Hartmann number is not so effective on both flow field and temperature distribution due to small value. But Hartmann number makes significant effects for higher values on convection regime and it decreases the flow strength for lid-driven flow. Dimension of right cell also becomes smaller with the increasing of magnetic forces. For higher values

Table 2. Grid independence study for Ri=1 and γ = 0°.

Grid points			51x51		81x81		101x101	
Ha	Pr	φ	Iteration	ψ_{max}	Iteration	ψ_{max}	Iteration	ψ_{max}
0	0.71		12318	-0.118	8223	-0.118	3383	-0.118
100	0.71	0°	14034	-0.905	9678	-0.881	4352	-0.901

of Hartmann number namely Ha = 50 (Fig. 3 (b)) and Ha = 100 (Fig. 3 (c)), which strong magnetic force occurs, cells elongate through the vertical walls and they distribute symmetrically especially at the highest Hartmann number. In the same manner, isotherms are affected with the increasing of Hartmann number as illustrated in Fig. 3 (on the right). For the highest value of Hartmann number, isotherms are almost parallel to each other and isothermal walls due to weak convection regime of heat transfer. In other words, this situation indicates that the overall heat is transferred by conduction in the middle of the enclosure. Inclination effects of temperature and flow fields are presented in Fig. 4 (a) to (e) for different inclination angle from 30^o to 180^o. As shown from the Fig. 1 that when inclination angle is zero both sides of the enclosure move positive y-direction and temperature of the left side is higher than that of

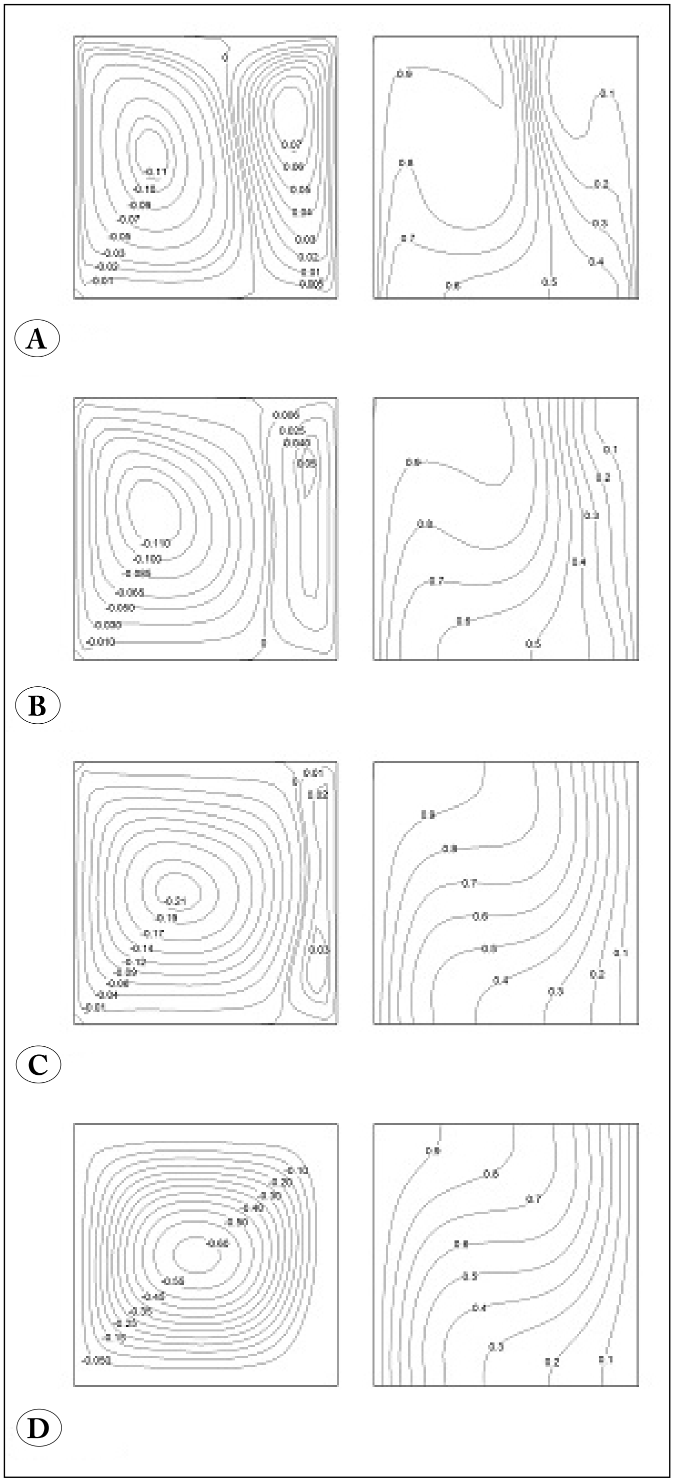

Figure 2. Streamline and isotherm for different Ri numbers at Pr= 0.71, Ha=0 and γ = 0^o, **A)** Ri = 0.1, **B)** Ri = 1, **C)** Ri = 10 and **D)** Ri = 100.

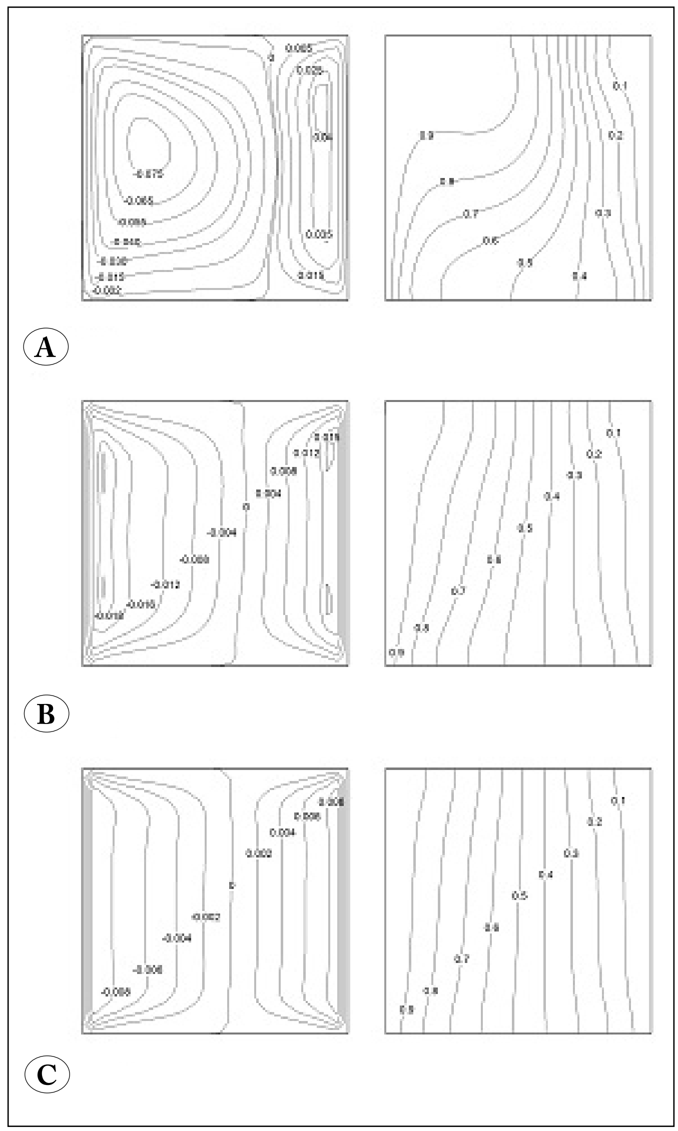

Figure 3. Streamline and isotherm for different Ha numbers at Pr = 0.71, Ri = 1 and γ = 0^o, **A)** Ha = 10, **B)** Ha = 50, **C)** Ha = 100.

right sides. Test results shows that inclination angle is not effective on both temperature distribution and flow field for Ri < 1 (forced convection). Thus, I showed only by mean Nusselt numbers for this case with Table 1. As can be given from the table, values of mean Nusselt numbers are almost same for Ri < 1 (natural convection). Thus, results for Ri = 10 were illustrated. As shown from Fig. 4 (a) and (b) three different cells were formed inside the cavity for $\gamma = 30^o$ and 60^o. Flow strength is decreased when inclination angle changes from 30^o to 60^o. For $\gamma = 90^o$ (the cavity with heated from the bottom wall), symmetric flow and temperature fields were formed. For higher values of inclination angle, flow direction of main and other cells completely changes from clockwise to counterclockwise when it compares with smaller inclination angle. For $\gamma = 180^o$, the third cell is almost disappeared. Fig. 5 (a) to (f) are plotted to show the effects of Prandtl number on flow and temperature fields in the presence of magnetic forces. Thus, the values for Pr = 0.1 and Pr = 1 at Ha = 50 and $\gamma = 0^o$ and compared with different values of Richardson number were examined. For Ri = 0.1, two circulation cells were formed and they elongate along the vertical wall due to magnetic forces. In this case, Pr number makes little effect on streamlines but isotherms cumulate at the middle part of the enclosure. When Pr = 0.1 (Fig. 5(c)) and Pr = 1 (Fig. 5 (d)) at Ri = 1 are compared, it is seen that conduction mode of heat transfer is dominant as can be shown from isotherms in Fig. 5(c). They parallel to each other. Multiple cells were formed for case of natural convection (Ri > 1) as shown in Fig. 5(e) and 5 (f). Higher flow strength was formed for higher Prandtl number. Length of double main cells becomes smaller with increasing of Prandtl number and convection heat transfer is more significant. As indicated above that angle of direction of the magnetic field with respect to the coordinate system is chosen as another governing parameter which affects the flow and temperature fields. Fig. 6 (a) to (c) are plotted for Pr = 0.054, Ha = 50, Ri = 1 and $\gamma = 0^o$ to see the effects of this parameter. The center of eddies is observed to be near the left and right walls when magnetic field is applied normal to the hot wall ($\phi = 0^o$). However, a big circulation cell were formed on the middle of the cavity due to hot wall and application of magnetic field to the bottom wall ($\phi = 90^o$). Intersection of eddies near the left and right wall become diagonal due to application of diagonal magnetic field. In all three cases, isotherms become almost parallel to the left and right vertical walls. It means that magnetic field is a more effective parameter on flow field than that of temperature distribution.

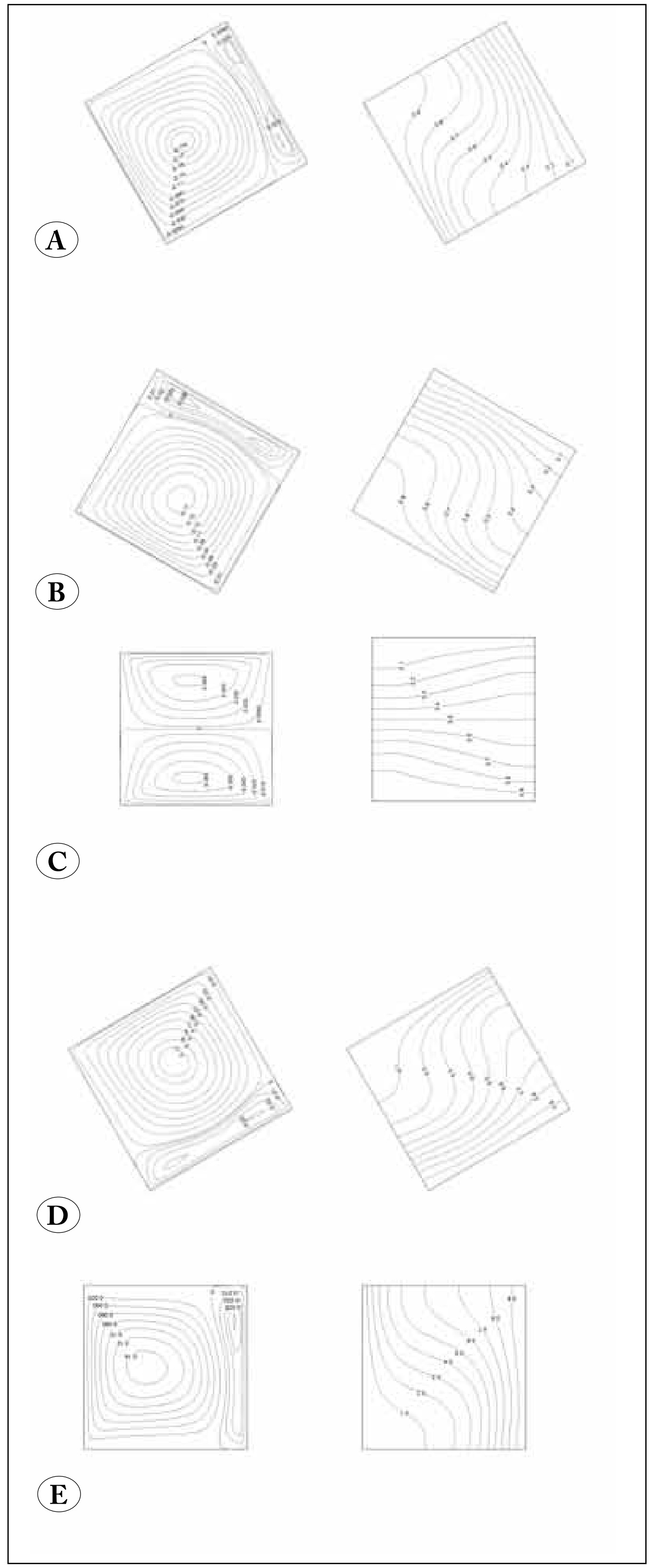

Figure 4. Streamline and isotherm for different inclination angle at Pr = 0.71, Ha = 10 and Ri = 10, **A)** $\gamma = 30°$, **B)** $\gamma = 60°$, **C)** $\gamma = 90°$, **D)** $\gamma = 120°$, **E)** $\gamma = 180°$.

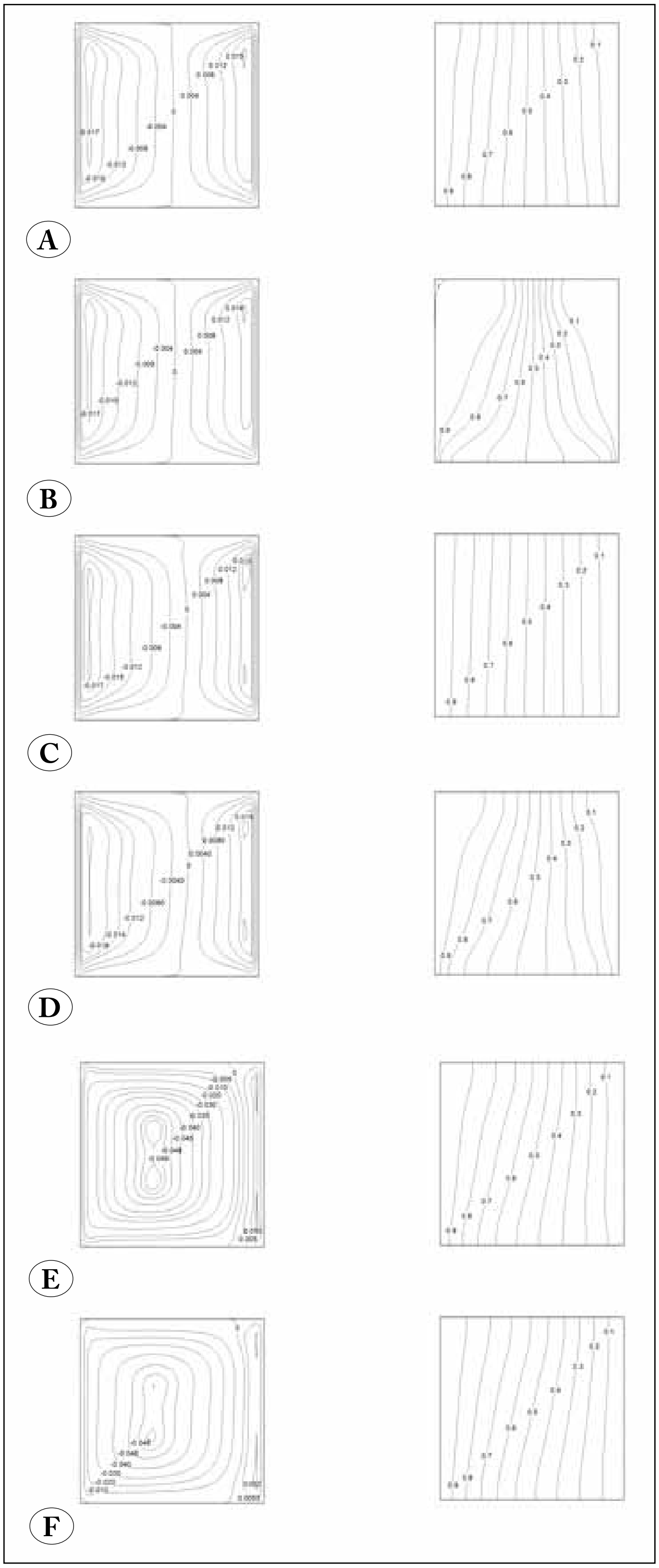

Figure 5. Streamline and isotherm for different Ri numbers at Ha = 50 and $\phi = 0°$, $\gamma = 0°$, **A)** Ri = 0.1, Pr = 0.1, **B)** Ri = 0.1, Pr = 1, **C)** Ri = 1, Pr = 0.1 **D)** Ri = 1, Pr = 1, **E)** Ri = 100, Pr = 0.1 and **F)** Ri = 100, Pr = 1.

Table 3 shows average Nusselt number for all governing parameters. As given in the table that average Nusselt number decreases with increasing of Richardson number. It means that flow friction becomes more effective with increasing of buoyancy effects. Similarly, it decreases with increasing of Hartmann number due to decreasing of convection heat transfer and flow strength. The inclination angle of enclosure affects the average Nusselt number. Mean Nusselt number reaches maximum value where inclination angle of enclosure is $\gamma = 90°$ and increases with increasing Prandtl number.

5. Conclusions

Analysis of mixed convection in inclined double-sided lid-driven cavity under magnetic forces were studied. Two-dimensional equations of conservation of mass, momentum and energy were solved using differential quadrature technique. The results showed that

- While magnetic field increases, strength of flow and heat transfer with convection decreases.

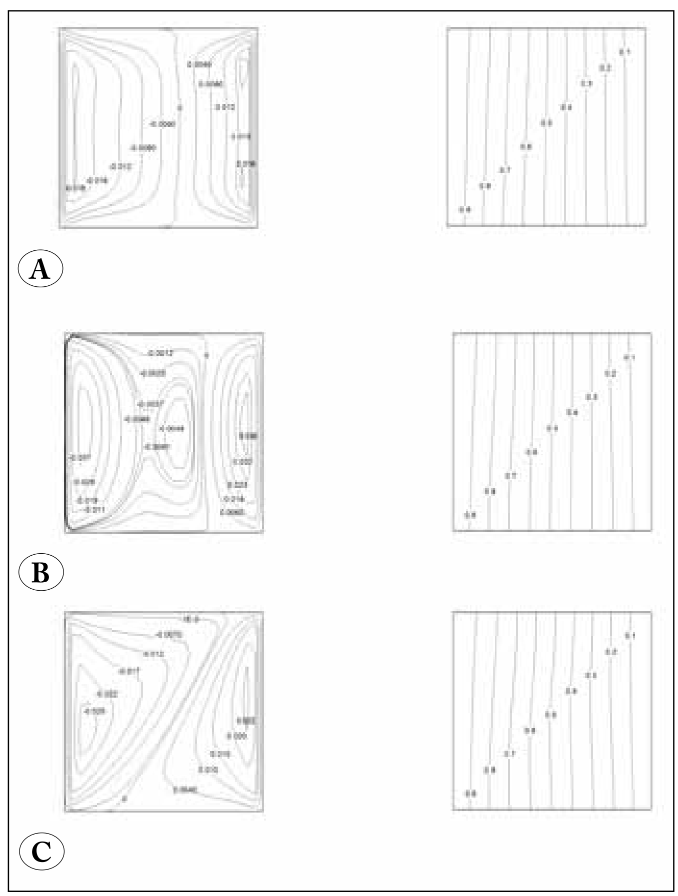

Figure 6. Streamline and isotherm for different Ri numbers at Pr = 0.054, Ha = 50, Ri = 1 and $\gamma = 0°$, **A)** $\phi = 0°$, **B)** $\phi = 90°$, **C)** $\phi = 45°$.

Table 3. Variation of the average Nusselt number.

Pr	Ri	Ha	ϕ	γ	Nu_a
1	0.1	50	0°	0°	1.880
	1.0	50	0°	0°	1.187
	10	50	0°	0°	1.055
	100	50	0°	0°	1.039
0.71	0.1	0	-	0°	4.069
	1.0	0	-	0°	2.192
	10	0	-	0°	1.990
	100	0	-	0°	2.007
0.71	1	10	0°	0°	1.882
	1	50	0°	0°	1.104
	1	100	0°	0°	1.033
0.71	0.1	10	0°	30°	3.372
	0.1	10	0°	60°	3.384
	0.1	10	0°	90°	3.392
	0.1	10	0°	120°	3.387
	0.1	10	0°	180°	3.353
0.1	0.1	50	0°	0°	1.024
	1.0	50	0°	0°	1.008
	10	50	0°	0°	1.002
	100	50	0°	0°	1.001
0.054	1	50	0°	0°	1.008
	1	50	90°	0°	1.006
	1	50	45°	0°	1.007

- Direction of magnetic field an important parameter on temperature distribution and flow field. For reducing heat transfer rate, magnetic field applied parallel to the side walls is more effective
- For different values of Ri, the influence of Prandtl number on streamlines and isotherms are considerable.
- The average Nusselt number at the heated surface increases with increasing the Prandtl number
- With a decrease in the Richardson number there is a remarkable increase in the heat transfer rate.
- While inclination angle of enclosure is γ = 90° hot wall approaches to bottom of enclosure, thus convection rises and average Nusselt number reaches maximum value accordingly.

6. References

Bellman, RE., Kashef, BG., Casti, J. 1972. Differential quadrature: a technique for the rapid solution of nonlinear partial differential equations. *J. Comput. Phys.,* 10: 40-52.

Büyük, E. 2006. Kapalı düzlemsel bölgelerde magnetik alan etkisi altında konvektif ısı transferi, Ph.D. thesis, Trakya University, Edirne, Turkey, (In Turkish).

Chamkha, AJ. 2002. Hydromagnetic combined convection flow in a vertical lid-driven cavity with internal heat generation or absorption. *Num. Heat Transfer, Part A,* 41:529-546.

Chamkha, A.J. 1998. Mixed convection flow along a vertical permeable plate embedded in a porous medium in the presence of a transverse magnetic field. *Num. Heat Transfer, Part A,* 34, pp. 93-103.

De Vahl Davis, G. 1983. Natural convection of air in a square cavity: a benchmark numerical solution. *Int. J. Numer. Methods Fluids,* 3: 249-264.

Ece, M.C., Büyük, E. 2006. Natural convection flow under a magnetic field in an inclined rectangular enclosure heated and cooled on adjacent walls. *Fluid Dynamics Res.,* 38: 564-590.

Ece, M.C., Öğüt**, E.B. 2009.** Mixed convection in a lid-driven and inclined square enclosure under a magnetic field. *Progress in Comput. Fluid Dynamics*, 9:43–51.

Ghia, U., Ghia, K.N., Shin, C.T. 1982. High-Re solutions for incompressible flow using the navier-stokes equations and a multigrid method. *J. Comp. Physics.,* 48: 387-411.

Iwatsu, R., Hyun, J.M., Kuwahara, K. 1993. Mixed convection in a driven cavity with a stable vertical temperature gradient., *Int. J. Heat Mass Transfer*, 36: 1601-1608.

Mohammad, A.A., Viskanta, R. 1993. Flow and thermal structures in a lid-driven cavity heated from below. *Fluid Dynamics Res.,* 12: 173-184.

Nasrin, R., Parvin, S. 2011. Hydromagnetic effect on mixed convection in a lid-driven cavity with sinusoidal corrugated bottom surface. *Int. Comm. Heat and Mass Transfer*, 38: 781–789.

Oztop, H.F., Dagtekin, I. 2004. Mixed convection in two-sided lid-driven differentially heated square cavity. *Int. J. Heat Mass Transfer*, 47:1761-1769.

Shankar, PN., Despande, MD. 2000. Fluid mechanics in the driven cavity. *Annu. Rev. Fluid Mech.,* 136: 93-136.

Sharif, M.A.R. 2007. Laminar mixed convection in shallow inclined driven cavities with hot moving lid on top and cooled from bottom. *Appl. Therm. Eng.,* 27: 1036-1042.

Shu, C. 2000. Differential quadrature and its application in engineering, Springer-Verlag London Limited.

Shu, C., Ding, H., Yeo, K.S. 2003. Local radial basis function-based differential quadrature method and its application to solve two-dimensional in compressible navier-stokes equations. *Computer Methods Appl. Mech. Eng.,* 192: 941-954.

Teamah, M.A., El-Maghlany, W.M. 2010. Numerical simulation of double-diffusive mixed convective flow in rectangular enclosure with insulated moving lid. *Int. J. of Thermal Sciences,* 49: 1625-1638.

Torrance, K., Davis, R., Eike, K., Gill, D., Gutman, D., Hsui, A., Lyons, S., Zien, H. 1972. Cavity flows driven by buoyancy and shear. *J. Fluid Mech.,* 51: 221-231.