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Deformed Statistics and Micro Black Holes

Deforme İstatistik ve Mikro Kara Delikler

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Abstract

In this study, we focus on the micro black holes which are formed due to the Hawking radiation of a charged macro black hole. We will analyze the physical behavior of the micro black holes by obtaining the deformed Einstein equations with the aid of statistical approaches of Verlinde and Strominger. Since Strominger proposed that the underlying statistics obeyed by the micro black holes is deformed statistics, we will consider the micro black holes as (q,p)-deformed fermions and obtain the deformed Einstein equations by following the Verlinde's entropic gravity approach. After obtaining the deformed Einstein equations, we will analyze the physical behavior of these black holes by representing and interpreting the solutions of these equations.

Keywords: Deformed statistics, General relativity, Micro black holes, Quantum statistics

Öz

Bu çalışmada, yüklü bir makro kara deliğin Hawking ışımasıyla oluşturduğu mikro kara delikler ele alınacaktır. Stromiger ve Verlinde'nin istatistiksel yaklaşımları vasıtasıyla deforme Einstein denklemleri elde edilerek mikro kara deliklerin fiziksel davranışlarını analiz edeceğiz. Strominger mikro kara deliklerin uyduğu istatistiğin deforme istatistik olduğunu önermesinden dolayı, mikro kara delikleri (q,p)-deforme fermiyonlar olarak ele alacağız ve Verlinde'nin entropik gravite yaklaşımını izleyerek deforme Einstein denklemlerini elde edeceğiz. Deforme Einstein denklemlerini elde ettikten sonra, bu denklemlerin çözümlerini elde ederek ve yorumlayarak bu kara deliklerin fiziksel davranışlarını analiz edeceğiz.

Anahtar Kelimeler: Deforme istatistik, Genel rölativite, Mikro kara delikler, Kuantum İstatistiği

1. Introduction

Statistical mechanics is the field of physics using the statistical methods in order to obtain the results of the classical thermodynamics which uses the empirical relations. For investigating a macroscopic system, thermodynamics laws are led by the experiments; however the microscopic theory requires more fundamental laws more than experimental results. Therefore, it is reasonable to consider the system as composed of micro particles which will be investigated in the framework of statistics, in order to obtain a theoretical aspect for the thermodynamical laws.

Statistical mechanics uses the probability concept and Binomial distribution as the simplest example of the statistical methods, in order to investigate the macroscopic properties of the physical systems, because the statistical mechanics considers the individual states of the particles constituting the whole macroscopic system without a detailed analysis of the states.

As a fundamental assumption, statistical physics investigates the macroscopic systems in the equilibrium state and it tries to understand the distribution of the particles into the states. Basis statistical physics considers the system as composed of identical non-interacting particles. The distribution of the particles over the states differs according to the nature of the particles. These distributions lead to obtain the very important thermostatistical quantity, entropy. After obtaining the entropy of the system differing with respect to the nature of the particles, we can find all the other thermodynamical relations from the statistical quantity entropy (Apaydın 2004).

As the nature of the particles determines the distribution function of the system, this in fact leads to the existing the various types of statistics used in physics. Namely, the number of ways of placing some distinguishable particles

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into some states differs from the case for the same number of indistinguishable particles. For instance, if the particles are identical and distinguishable which means the particles are to be investigated classical mechanically, the underlying statistics used here is the Maxwell-Boltzmann statistics. For an equilibrium system having a total energy E and total particle number N as the fundamental condition of the microcanonical set, the total number of microstates (the way of particles occupying the sub-states) is given as

$$\Omega = \sum_k \Omega_k = \sum_k \left(\prod_i \frac{N! g_i^{N_i}}{N_i!} \right)_k, \tag{1}$$

where i labels the different allowed sub-states in the whole system, N_i represents the number of particles in this state, k is one of the macrostates satisfying the fundamental condition and g_i is the number of degenerate states in the sub-state i.

Whereas, if the particles are identical and indistinguishable meaning the particles should be considered quantum mechanically, and the underlying statistics used here is the quantum statistics. However, the quantum statistics is also grouped into two categories depending on the nature of the indistinguishable particles. Such that: the particles are said to be bosons, if they are allowed to be in the same sub-state with more than one particle. On the other hand, the particles are called as fermions, if they are not allowed to be in the same state with more than one particle. This is the nature of Fermions and first stated by Pauli and this fact is known as the Pauli Exclusion Principle. Bosons obey the Bose-Einstein statistics, while fermions obey the Fermi-Dirac statistics. Therefore, the total number of microstates for bosons is

$$\Omega = \sum_k \Omega_k = \sum_k \left(\prod_i \frac{(N_i + g_i - 1)!}{N_i!(g_i - 1)!} \right)_k, \tag{2}$$

and for fermions

$$\Omega = \sum_k \Omega_k = \sum_k \left(\prod_i \frac{g_i!}{N_i!(g_i - N_i)!} \right)_k, \tag{3}$$

where $N_i \leq g_i$ for the Fermi-Dirac statistics due to the Pauli Exclusion Principle (Apaydın 2004).

The statistical quantity, total number of microstates Ω becomes a bridge between statistics and the thermodynamics, such that the entropy of the system is obtained by Boltzmann as (Pathria 1997)

$$S = k \ln \Omega. \tag{4}$$

Then the other thermodynamical quantities can be obtained from (4), as follows

$$\left(\frac{\partial S}{\partial E} \right)_{N,V} = k \left(\frac{\partial \ln \Omega}{\partial E} \right)_{N,V} = \frac{1}{T}, \tag{5}$$

$$\left(\frac{\partial S}{\partial N} \right)_{E,V} = k \left(\frac{\partial \ln \Omega}{\partial N} \right)_{E,V} = \frac{-\mu}{T}, \tag{6}$$

$$\left(\frac{\partial S}{\partial V} \right)_{E,N} = k \left(\frac{\partial \ln \Omega}{\partial V} \right)_{E,N} = \frac{P}{T}, \tag{7}$$

where k is the Boltzmann constant. These methods and conditions can be generalized to more general situations. For example, we can consider the system is allowed to transfer of energy, while the particle number is fixed. These systems are called as the canonical set and use the partition function instead of the total microstate number. Entropy and all other thermodynamical functions of the system are obtained from the partition function. If both energy and particle transfer is allowed, these are the grand canonical set, and use grand partition function (Huang 1987).

Statistical mechanics interferes with the classical and quantum mechanical structure of the system. To know the statistics of the system leads to know the structure of the system, or vice versa. Therefore to understand the quantum nature of black holes, it is sensible to investigate the statistics of the black holes by considering them as the micro (or extremal) black holes. Firstly the extremal black holes are treated as point particles to investigate the statistics of black holes for scattering phenomena in Strominger's study. The core of his study is to find the answer whether the extremal black holes scatter as bosons, fermions or something else. Finally, it is obtained that the extremal black holes obey neither Bose, nor Fermi statistics, but they obey the deformed statistics. The understanding the quantum statistics of the micro black holes is an important progress on unraveling quantum black hole puzzles (Strominger 1993).

Recently the q-deformed Einstein equations (Dil 2015, Dil et al. 2016) have been studied for describing the gravitational fields of the Strominger's extremal quantum black holes obeying the deformed statistics. Here Verlinde's entropic gravity approach (Verlinde 2011) have been used to obtain the one parameter deformed Eintein equations being the gravitational field equations of quantum black holes which are the charged black holes with possible minimal mass. A charged black hole loses its mass via Hawking radiation and reaches a minimum mass proportional to the charge. The final black hole having a minimal mass is considered to be a micro quantum black hole. Since these black holes can be considered as deformed bosons or deformed fermions, the micro black holes defined by q-deformed Einstein equations are assumed be the Ubriaco's q-deformed bosons (Ubriaco 1997).

In this study we consider another type of micro black hole obeying the (q,p)-deformed Fermi statistics and try to obtain the (q,p)-deformed Einstein equations from the Verlinde's entropic gravity approach and solve these equations for a charged non-rotating micro black hole. Then we compare the masses of the classical and quantum deformed charged black holes with respect to the deformation parameters in order to see the effect of the deformation on quantum masses.

2. Entropy of the (q,p)-Deformed Fermions

To find the (q,p)-deformed Einstein equations for the (q,p)-deformed micro black holes, we first need to obtain the entropy of the (q,p)-deformed fermions and use Verlinde's entropic gravity proposal for this deformed entropy. Here, for the (q,p)-deformed fermions, the grand canonical set is used as the most general case for a non-interacting system. Therefore, we first need to find the grand partition function for the (q,p)-deformed Fermi gas model by using the objects of Fibonacci calculus. Since the total deformed number operator of the (q,p)-deformed fermions is given as (Algin 2014)

$$\sum_{i=1}^{d} c_i^* c_i = [\hat{N}_1 + \hat{N}_2 + ... + \hat{N}_d] = [\hat{N}], \tag{8}$$

its eigenvalue spectrum is known as the Fibonacci basic integer

$$[n] \equiv [n]_{q,p} = \frac{q^{2n} - p^{2n}}{q^2 - p^2}. \tag{9}$$

Here q and p are the real positive independent deformation parameters. From the Fibonacci calculus, the modified Fibonacci difference operator for this system is given as,

$$\hat{D}_z^{(q,p)} = \frac{q^2 - p^2}{ln(q^2/p^2)} \partial^{(q,p)}, \text{ and } \partial^{(q,p)} f(z) = \frac{f(q^2 z) - f(p^2 z)}{(q^2 - p^2) z} \tag{10}$$

In the grand canonical ensemble, the Hamiltonian of the model is given by (Ubriaco 1997),

$$\hat{H}_{q,p} = \sum_i (\varepsilon_i - \mu) \hat{N}_i, \tag{11}$$

where ε_i is the kinetic energy of a particle in state i. Then the grand partition function of the model is defined to be

$$Z = Tr(e^{-\beta \hat{H}_{q,p}}), \tag{12}$$

then

$$\ln Z = \sum_i (1 + z e^{-\beta \varepsilon_i}). \tag{13}$$

where $\beta = 1/kT, k$ is the Boltzmann's constant, and μ is the chemical potential in the range $-\infty < \mu < \infty$ and $0 \le z < \infty$ is the fugacity of the model within the range. This grand partition function is statistical function and used to obtain the thermodynamical relations, such as the number of particle $N = z\hat{D}_z^{(q,p)} \ln Z$, pressure $PV/kT = \ln Z$ and internal energy $U = -\hat{D}_z^{(q,p)} \ln Z$ for the (q,p)-deformed Fermi gas model. The summations can be converted into the integrals for $N, V \to \infty$ limit and by taking $x = \beta\varepsilon$, we obtain N, P and U as

$$\frac{N}{V} = \frac{1}{\lambda^3} \frac{2}{\sqrt{\pi}} \int_0^\infty x^{1/2} dx \frac{1}{|\ln(q^2/p^2)|} \left| \ln\left(\frac{z^{-1} e^x + q^2}{z^{-1} e^x + p^2} \right) \right|, \tag{14}$$

$$\frac{P}{kT} = \frac{1}{\lambda^3} \frac{4}{3\sqrt{\pi}} \int_0^\infty x^{3/2} dx \frac{1}{|\ln(q^2/p^2)|} \left| \ln\left(\frac{z^{-1} e^x + q^2}{z^{-1} e^x + p^2} \right) \right|, \tag{15}$$

$$U = \frac{3}{2} kT \frac{V}{\lambda^3} \frac{4}{3\sqrt{\pi}} \int_0^\infty x^{3/2} dx \frac{1}{|\ln(q^2/p^2)|} \left| \ln\left(\frac{z^{-1} e^x + q^2}{z^{-1} e^x + p^2} \right) \right|, \tag{16}$$

where $\lambda = h/\sqrt{2\pi mkT}$ is the thermal wavelength. We can expand three integrals in (14)-(16) in Taylor series, for $z << 1$ meaning the high temperature limit and

$$\frac{N}{V} = \frac{1}{\lambda^3} f_{3/2}(z,q,p), \tag{17}$$

$$\frac{P}{kT} = \frac{1}{\lambda^3} f_{5/2}(z,q,p), \tag{18}$$

$$U = \frac{3}{2} kT \frac{V}{\lambda^3} f_{5/2}(z,q,p), \tag{19}$$

where

$$f_n(z,q,p) = \frac{1}{|\ln(q^2/p^2)|} \left[\sum_{l=1}^\infty (-1)^{l-1} \frac{(q^2 z)^l}{l^{n+1}} - \sum_{l=1}^\infty (-1)^{l-1} \frac{(p^2 z)^l}{l^{n+1}} \right]. \tag{20}$$

$f_n(z,q,p)$ functions are the deformed Fermi-Dirac functions and for the limit $q,p \to 1$ they turn to be the well-known non-deformed Fermi-Dirac functions $f_n(z)$ (Huang 1987). By using (17) and (18), the Helmholtz free energy of the model can be determined as

$$A = \mu N - PV = \frac{kTV}{\lambda^3} [f_{3/2}(z,q,p) \ln z - f_{5/2}(z,q,p)], \tag{21}$$

and the deformed entropy from (19) and (21) as

$$S = \frac{1}{T}(U - A) = \frac{kV}{\lambda^3} \left[\frac{5}{2} f_{5/2}(z,q,p) - f_{3/2}(z,q,p) \ln z \right]. \tag{22}$$

By using one particle average kinetic energy E instead of kT in λ, the deformed entropy in (25) turns out to be

$$S = \frac{(2\pi\, m)^{3/2} V}{Th^3} F(z,q,p) E^{5/2}, \tag{23}$$

where

$$F(z,q,p)=\frac{5}{2}f_{5/2}(z,q,p)-f_{3/2}(z,q,p)\ln z. \tag{24}$$

In this model, all deformed thermostatistical functions reproduces the standard thermostatistical functions of the ideal Fermi gas model in $q,p\to 1$ limit.

3. (*q,p*)-Deformed Einstein Equations

In Verlinde's approach, the fundamental notion needed to derive the gravity is information. The amount of information associated with the matter and its location is measured in terms of entropy. There exists an entropy change and this change causes a reaction force in order to make the system old equilibrium state, during the matter is moving in space from a reason. This reaction force is the gravity which is accepted as the entropic force acting as an inertial reaction against to the force attempting to increase the entropy (Verlinde 2011).

Verlinde's central assumption is that the information on a space obeys the holographic principle implying the minimum part of the microscopic degrees of freedom is represented holographically on the boundary or horizons of spacetime. Verlinde connects the gravity and inertia and proposes the emergence of spacetime by the holographic scenario. The energy (or matter) is distributed evenly over the bits, namely degrees of freedom. The energy on every bit causes a temperature in the spacetime. The product of entropy change and temperature is equal to the work done by the gravitational force.

Verlinde uses the Bekenstein idea for a test particle near the horizon of a black hole. The increase in the mass and horizon area of the black hole while the particle gets closer to the horizon by one Compton wavelength is identified by one bit of information. Because the information is related to the entropy, this leads Bekenstein to propose area-entropy law for black holes. Verlinde takes into account a piece of holographic screen and a particle with mass *m* approaches to the screen. While the particle moves toward the screen, the information stored on the screen changes. Verlinde assumes the screen to be a closed spherical surface and the boundary as an information storage device. Because of the holographic principle, the maximal storage space, or total number of bits is proportional to the area of the screen.

In order to derive the (*q,p*)-deformed Einstein equations from the entropy of the (*q,p*)-deformed fermions, we assume the matter inside the holographic screen is the (*q,p*)-fermions. Therefore, the change of deformed entropy by an external force leads the (*q,p*)-fermions to produce their gravity which will be assumed as the gravitational field of the micro black holes in accordance with the Strominger's proposal. We use the Verlinde's method to obtain the (*q,p*)-deformed Einstein equation. Firstly, for an equilibrium state, the entropy change is zero, then

$$\frac{d}{dx^a}S(E,x^a)=\frac{\partial S}{\partial E}\frac{\partial E}{\partial x^a}+\frac{\partial S}{\partial x^a}=0. \tag{25}$$

Using the change of entropy at screen for the displacement of particle by one Compton wavelength normal to the screen $\partial S/\partial x^a=-2\pi\, mN_a/\hbar$ obtained by the Bekenstein's argument and using (23) in (25) gives

$$T=\frac{5V(2\pi\, mE)^{3/2}}{8\pi^2h^2}F(z,q,p)e^{\phi}N^a\nabla_a\phi, \tag{26}$$

where $\partial E/\partial x_a=-F_a=me^{\phi}\nabla_a\phi$ is used. By using this temperature value in the equipartition formula of "the distributed (*q,p*)-fermions mass/energy over the bits" and "produced temperature by this mass-energy", such that

$$\begin{aligned} M&=\frac{1}{2}\int_S Tdn=\frac{1}{2}\int_S T\frac{dA}{G\hbar}\\ &=\frac{1}{8\pi G}\frac{5V(2\pi\, mE)^{3/2}}{h^3}F(z,q,p)\int_S e^{\phi}\nabla_a\phi.dA \quad .\\ &=-\frac{1}{8\pi G}\frac{5V(2\pi\, mE)^{3/2}}{h^3}F(z,q,p)\int_\Sigma R_{ab}\eta^a\zeta^b d\tau \end{aligned} \tag{27}$$

In the last step, the redshift factor $e^{\phi}=(-\zeta_a\zeta^a)^{1/2}$, normalized timelike Killing vector field ζ^a of the static asymptotically flat spacetime and the acceleration $\alpha^a=-\nabla^a\phi=e^{-2\phi}\zeta^b\nabla_b\zeta^a$ is used with the identity $\nabla^a\nabla_a\zeta^b=-R^b{}_a\zeta^a$ for the Ricci curvature quantity *R*. We now use the Komar's mass formula for a volume integral with a general stationary metric and equate it to (27), then

$$2\int_\Sigma\left(T_{ab}-\frac{1}{2}g_{ab}T\right)\eta^a\zeta^b d\tau=\frac{1}{8\pi G}\frac{5V(2\pi\, mE)^{3/2}}{h^3}F(z,q,p)\int_\Sigma R_{ab}\eta^a\zeta^b d\tau. \tag{28}$$

This gives the final (*q,p*)-deformed Einstein equations from the arrangement of (28), such that

$$\frac{5V(2\pi\, mE)^{3/2}}{2h^3}F(z,q,p)\left(R_{ab}-\frac{1}{2}g_{ab}R\right)=8\pi GT_{ab}, \tag{29}$$

and by rewriting

$$\Psi^{q,p}\left(R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R\right)=8\pi GT_{\mu\nu}, \tag{30}$$

where $\Psi^{q,p}=5V(2\pi\, mE)^{3/2}F(z,q,p)/2h^3$.

Now we are about the solve the (*q,p*)-deformed Einstein equations in order to interpret the implications for the micro black holes. We therefore consider the charged black holes in order to obtain minimum mass micro (quantum) black holes due to the Hawking radiation.

4. Solution of the (*q,p*)-Deformed Einstein Equations

Deformed Einstein field equations will be used to describe the geometry of the spacetime around a charged and spherically symmetric quantum black hole. The reason why we cannot consider a neutral black hole is that a micro black hole cannot be created from a neutral black hole due to the Hawking radiation. Neutral spherically symmetric black holes refer to the Schwarzschild black holes, and they do not lead to a micro black hole, because mass of the radiating macro black hole is governed by the charge-mass ratio of the micro black hole. Therefore we need to solve the deformed Einstein-Maxwell equations due to the charge of the quantum black holes. Also, because of the spherical symmetry the metric has to be in the form for 4-dimension is (Wald 1984)

$$ds^2 = -e^{2\alpha(r,t)}dt^2 + e^{2\beta(r,t)}dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2. \tag{31}$$

The energy-momentum tensor $T_{\mu\nu}$ in (30) is

$$T_{\mu\nu} = F_{\mu\rho}F_{\nu}^{\rho} - \frac{1}{4}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}, \tag{32}$$

where $F_{\mu\nu}$ is the electromagnetic field strength tensor and the trace of $T_{\mu\nu}$ is zero due to the electromagnetic strength tensor. Taking the trace of (30) leads to

$$\Psi^{q,p}R_{\mu\nu} = 8\pi G T_{\mu\nu}. \tag{33}$$

Spherical symmetry and absence of magnetic charge for our quantum black hole gives the electromagnetic field strength tensor has only radial electric field components $E_r = F_{tr} = -F_{rt} = f(r,t)$. By equating the non-zero components of the Ricci tensor for the metric (31) and the corresponding non-zero components of the energy-momentum tensor obtained by (32), we find $\alpha(r,t) = \alpha(r) = -\beta(r)$. The solution of the Maxwell equations, $g^{\mu\nu}\nabla_{\mu}F_{\nu\sigma} = 0$, and $\nabla_{[\mu}F_{\nu\rho]} = 0$ gives $f(r,t) = f(r) = Q/\sqrt{4\pi}\, r^2$. Using this electromagnetic field strength tensor for one of the components of the Ricci tensor and the corresponding energy-momentum tensor in (33) gives the solutions of the deformed Einstein equations:

$$ds^2 = \Delta dt^2 + \Delta^{-1}dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\phi^2, \tag{34}$$

where

$$\Delta = 1 - \frac{2Gm}{r} + \frac{1}{\Psi^{q,p}}\frac{GQ^2}{r^2}. \tag{35}$$

This result is consistent with the argument that the neutral black holes cannot be considered for micro black hole creation, and also shows that the uncharged black holes do not induce a micro black hole with the factor $\Psi^{q,p}$ in (35). Furthermore, the singularities and the event horizons for these black holes are determined by the function Δ and the radius r. There is a true curvature singularity at $r = 0$, since the metric goes to infinity for this value. The coordinate singularity also occurs at $\Delta = 0$ and the conditions giving this singularity occur from the solution of $\Delta = 0$, such as

$$r_{\pm} = Gm \pm \sqrt{G^2 m^2 - \frac{GQ^2}{\Psi^{q,p}}}. \tag{36}$$

The case $Gm^2 = Q^2/\Psi^{q,p}$ stands for the quantum charged black hole case, since the mass of the black hole decreases to the minimum value from the case $Gm^2 > Q^2/\Psi^{q,p}$. A minimum mass solution for the quantum black holes is reached by the Hawking radiation and remains stationary. $Gm^2 = Q^2/\Psi^{q,p}$ case makes $\Delta = 0$ at a single radius $r_{\pm} = Gm$ and refers to a single event horizon. This deformed case solution $Gm^2 = Q^2/\Psi^{q,p}$ is the analogue of classical Reissner-Nordström solution $m = Q/\sqrt{G}$ which is often examined in the studies of quantum gravity. For $Gm^2 > Q^2/\Psi^{q,p}$ case, the mass of the black hole is allowed to be in very large classical scales due to the ability of getting bigger values than the charge. Whereas the mass of the deformed black hole is allowed to decrease very small values which could fall into the quantum regime, because the decrease of the mass is governed by a very small term $1/\Psi^{q,p}$ being order of $h^{6/7}$.

Mass reduction with respect to the classical Reissner-Nordström case can be compared for (*q,p*)-deformed cases. Using $1/\Psi^{q,p}$ from (30) gives the mass of the extremal quantum black hole in case of $Gm^2 = Q^2/\Psi^{q,p}$, such as

$$m^{(q,p)} = \left(\frac{2h^3}{5V(2\pi E)^{3/2}}\frac{1}{F(z,q,p)}\right)^{\frac{2}{7}}\left(\frac{Q^2}{G}\right)^{\frac{2}{7}}. \tag{37}$$

The comparison of the deformed black hole mass and classical Reissner-Nordström black hole mass $m = Q/\sqrt{G}$ reads as

$$m^{(q,p)} = \left(\frac{2}{5V(2\pi E)^{3/2}Q^{3/2}}\right)^{\frac{2}{7}}\left(\frac{h^3 G^{3/4}}{F(z,q,p)}\right)^{\frac{2}{7}} m. \tag{38}$$

The equation in (38) implies that the mass of the charged extremal black hole in the deformed quantum case can decrease to a smaller value than that of the classical Reissner-Nordström case. We figure out the decrease in the mass by examining the behaviors of the factor $(h^3 G^{3/4}/F(z,q,p))^{2/7}$ in the classical mass of the charged black hole in (38). We illustrate the behavior of $(h^3 G^{3/4}/F(z,q,p))^{2/7}$ with respect

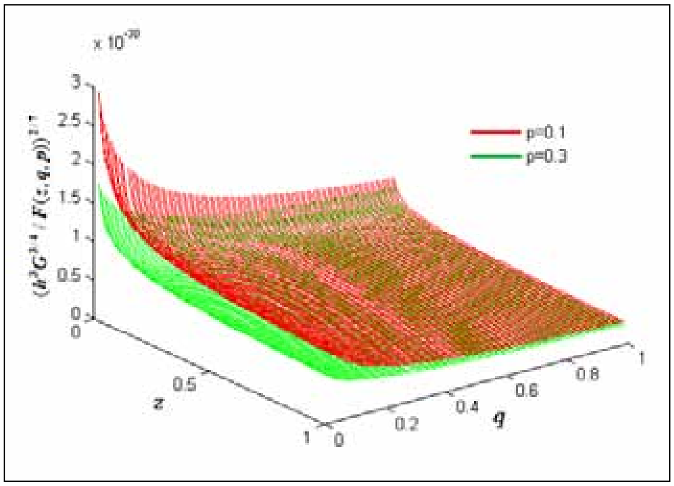

Figure 1. Mass reduction factor $(h^3 G^{3/4}/F(z,q,p))^{2/7}$ for various values of the deformation parameters p and q<1.

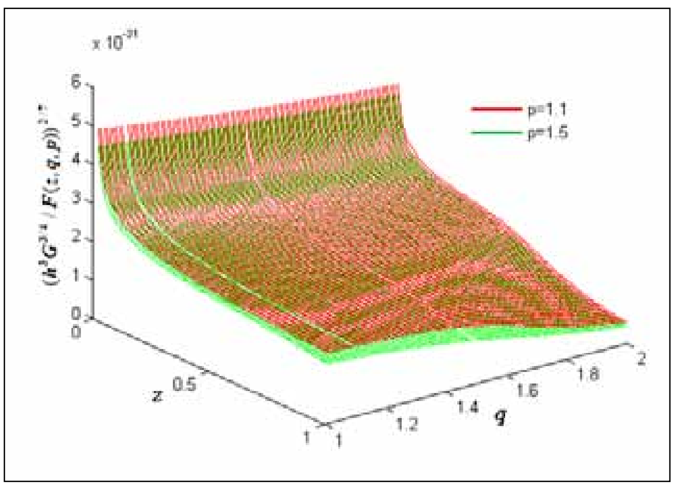

Figure 2. Mass reduction factor $(h^3 G^{3/4}/F(z,q,p))^{2/7}$ for various values of the deformation parameters p and q>1.

to z, q and p in Figure 1 and Figure 2, for q, p<1 and 1< q, p, respectively.

5. Conclusions

In this study, we review the (q,p)-deformed statistics of fermions and obtained the (q,p)-deformed Einstein equations, which is based on the Strominger's idea, such that the micro quantum black holes obey the deformed statistics. We then obtain the solutions of (q,p)-deformed Einstein equations for the charged extremal quantum black holes. We then interpret the solutions for (q,p)-deformed Einstein equations.

There occur two singularities from the solutions of deformed Einstein equations, such that the true and coordinate singularities. The possible decrease in mass via Hawking radiation to the minimum value which is determined by the charge of quantum black hole is investigated and the difference between the classical black holes and quantum black holes is compared from the equations (38). We represent the decrease in quantum mass $m^{q,p}$ in Figure 1 and Figure 2. According to the Figures 1 and 2, the mass of the quantum black hole $m^{q,p}$ in (38) is at least 10^{-30} times smaller than the classical black hole mass m, in the deformed case, with the inverse of the volume, charge and energy factors, mass $m^{q,p}$ gets smaller than 10^{-30} m.

Three independent studies, Strominger's micro black holes obeying the deformed statistics, Verlinde's entropic gravity approach and the thermostatistics of deformed Fermi gas model, seem to be consistent with each other, since the theoretical possibility of concentrating a mass into its reduced Planck mass leads to a quantum black hole whose dynamics described by the deformed Einstein equations and the mass reduction to a smaller value than that of the classical black holes.

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