PES www.ijpes.com	**International Journal of Psychology and Educational Studies**	

Pre-Service Mathematics Teachers' Concept Definitions and Examples Regarding Sets[1]

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ARTICLE INFO

ABSTRACT

The purpose of this study is to reveal the concept definitions and examples of pre-service elementary mathematics teachers regarding sets. In this context, students' formal definitions, personal definitions and examples in sets were investigated. Concept of equal sets, subsets, union, intersection, compliment, and difference were considered as basic concepts of sets in this study. The participants of the study consisted of 79 freshmen studying in the department of elementary mathematics teacher training at a state university of Turkey. The study adopted qualitative research approach and is a sample of a case study. The data of the study was collected from Definition Knowledge Form for Sets and unstructured interviews. The results of the study indicated that most of the pre-service teachers had difficulty in giving notational definitions and daily-life examples while they gave successfully verbal definitions of the basic concepts in sets. It was revealed that verbal definitions, notational definitions, prototype examples with schema, symbols and operational properties were parts of some pre-service teachers' concept images. Furthermore, some pre-service teachers confused equal sets with equivalent sets and difference with symmetric difference. They also did not consider the universal set in defining the compliment. It was found that most of the pre-service teachers had difficulty in expressing their thoughts using mathematical language throughout the study.

Keywords:[4]
Sets, concept definition, concept image, mathematics education

1. Introduction

The concept is abstract and general design of an object or thought in mind (Turkish Language Society [TDK], 2015). Mathematical concepts are composed of mathematical (mental) objects and relationships between them (Simon, 2017). Some mathematical concepts need to be defined, while others are accepted undefined. Concepts specified by definitions are expressed with the help of concepts accepted without definition and predefined concepts (Baykul, 2014). Baykul (2002) describes the properties of mathematical definitions; "based on previously defined or undefined concepts, being meaningful and meaningful with concept it describes, covering all possible situations related to the concept it describes and excluding other situations measuring definitions must be operational (p.21)."

The level of knowledge and understanding of Bloom Taxonomy comes to the fore in cognitive skills of students for concept definitions. At knowledge level, students have a superficial knowledge of rules, facts and relationships (Baki, 2014). At this level, remembering and recognizing behavior is at forefront (Demirel, 2015). For example, it can be said that a student who can express definition of a mathematical concept in

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textbook or as teacher presents it is at knowledge level. Students at level of understanding require skills such as transporting, interpreting, summarizing, sampling, estimating (Baki, 2014; Demirel, 2015). Accordingly, it can be said that students who can explain, interpret or give examples of a mathematical definition in their own sentences are at level of understanding for relevant definition.

Concept definition-concept image (CD&CI) theoretical framework is also used to comprehend concepts specified by mathematical definitions. The conceptual framework used to guide this research was based on CD & CI framework of Tall and Vinner (1981). Concept images are all cognitive structures in a person's mind regarding a given concept (Tall & Vinner, 1981). Concept images are all cognitive structures in a person's mind regarding a given concept (Tall & Vinner, 1981). The concept image is the mental picture associated with the concept name and all properties that characterize the concept in the student's mind. With mental picture, it means any representation-picture, symbolic form, diagram, graph etc. (Vinner & Dryfus, 1989). Concept definition consists of formal and personal definitions (Tall & Vinner, 1981). Formal concept definition is accepted by mathematical communities and generally included in textbooks (Tall & Vinner, 1981). Personal concept definition is the restructuring of a definition in the mind and is a collection of words that one uses to explain concept image (Tall & Vinner, 1981). This definition can be different from the formal definition (Tall & Vinner, 1981).

Studies conducted on CD&CI in the literature of mathematics education were examined and it was found that studies were carried out with pre-service teachers and teachers regarding concept images in function (Vinner, 1983; Vinner & Dreyfus, 1989), limit, continuity, derivatives and integral (Erdogan, 2017; Kabael, Barak & Ozdas, 2015), geometric concepts (Karakuş, 2018; Yilmaz, 2015), slope (Aydeniz, 2011; Dündar, 2015), complex numbers (Nordlander & Nordlander, 2012), algebraic expression and equation (Tekin Sitrava, 2017), period (Öner & Ertekin, 2015) and rational numbers (Macit & Nacar, 2019). In a small number of studies with pre-service teachers, difficulties of understanding basic concepts of sets were focused (Baki & Sahin, 2002; Fiscbein & Baltsan, 1999; Zehir, Işık & Zehir, 2008). Especially, pre-service teachers' skills of giving definition on sets, universal set, subset, equal sets, finite set, infinite set, empty set concepts were considered in a few studies (Baki & Şahin, 2002; Yazıcı, 2017). There was no study examining concept definitions and examples that are part of the concept image of pre-service teachers in sets. For this reason, it is thought that studies to be done on this subject would contribute to the field.

Concept definitions are seen as the basis of mathematics and mathematics education (Zavlavsky & Shir, 2005). Teachers need to have a deep understanding of the subjects they will teach, to master different mathematical definitions of these concepts, as well as to make instructional assessments about concept definitions (Çakıroğlu, 2013). In this way, individuals who can use mathematical language and understand concepts, express them in different ways and use them in daily life can be raised as aimed in middle school mathematics course curriculum (Ministry of National Education [MEB], 2018a). In summary, mathematics teachers are expected to have a correct understanding of the subjects they undertake their education on. In order to do this, it is necessary to identify deficiencies of pre-service teachers in these subjects and to make remedial applications for elimination of these deficiencies. In this study, concept definitions and examples of pre-service teachers for set concepts would be tried to be revealed. As a result of the study, in addition to contributing literature, pre-service teachers' giving notational and verbal definitions and daily-life examples skills related to set concepts would emerge. It was thought that results obtained from the study would contribute to the teachers and academicians involved in the teaching of the subject to understand the way students' thought. For this purpose, answers to following research questions were sought.

1.   What are the mathematical (formal) definitions of the pre-service teachers for set concepts?

2.   What are the verbal (personal) definitions of the pre-service teachers for set concepts?

3.   What are daily-life examples of the pre-service teachers for set concepts?

2. Method

2.1. Research Design

The study adopted qualitative research approach and is a sample of case study. Case studies are in-depth explanation and examination of limited system (Merriam, 2013).

2.2. Research Group

Research group consisted of 79 freshmen studying in the department of elementary mathematics teacher training at a state university in Turkey. Criterion sampling method was used in selecting of the research group from purposeful sampling methods. In addition, unstructured interviews were conducted with 10 pre-service teachers. Names of interviewed pre-service teachers were coded as P1, P2 ... P10.

Pre-service mathematics teachers in Turkey encountered set concepts at middle school, high school and university level. Middle school students encounter set concepts in sixth grade of middle school. Union and intersection concepts are included (MEB, 2018a). In the course book (Kucukkeleş & Aktas, 2018, p.72), union and intersection definitions are given verbally. Along with the verbal definition, examples are presented using diagram. In the first year of high school, equal set, subset, union, intersection, difference and complement concepts are included (MEB, 2018b). The definition of concepts is presented with mathematical definition, example of concept with diagram, along with the verbal definition (Uçak, Emir, Uçkun, Kutlu & Kahraman, 2018, p.64). In university level, pre-service teachers came across set concepts in Basic Mathematics I and Abstract Mathematics courses. While the Basics Mathematics I course focuses on the operational properties, mathematical definitions and proof activities of set concepts are more in Abstract Mathematics course (Council of Higher Education [YOK], 2018). In most of the course books, presentation types of concept definitions are similar to high school (Balci, 2005; Çallıalp, 2009; Çelik, 2018).

2.3. Data Collection Process

The data of the study was collected from students writing responses in the form Definition Knowledge Form for Sets [DKFS]) developed by the researchers, and interviews were carried out with 10 pre-service teachers selected from the research group. In DKFS, it was aimed to obtain three types of information on concepts of "equal set, subset, union, intersection, difference and complement" labeled as basic set concepts in the study. First, the pre-service teachers were asked to explain these concepts using mathematical symbols. Secondly, pre-service teachers were asked to explain concepts using their own sentences. Pre-service teachers were finally asked to give daily-life example of concepts. DKFS was applied to the pre-service teachers within one hour. There was no time limit for them to fill out DKFS. In interviews, answers given to DKFS were examined together with them. DKFS was presented as an appendix.

2.4. Data Analysis

Content analysis was used to analyze the written answers of the pre-service teachers. After the researchers reached consensus on the categories, opinion of an academician specializing in abstract mathematics was taken about the categories. The expert academician stated that the categories were compatible with each other with the answers. He confirmed that mathematical interpretations were totally correct. The data obtained from interviews are presented descriptively.

3. Results

Mathematical Definitions of the Pre-service Teachers for Set Concepts

Pre-service teachers were asked to define basic set concepts using mathematical symbols. As a result of the analysis, mathematical definitions of the pre-service teachers were collected under eight categories. Figure 1 presents information about these categories.
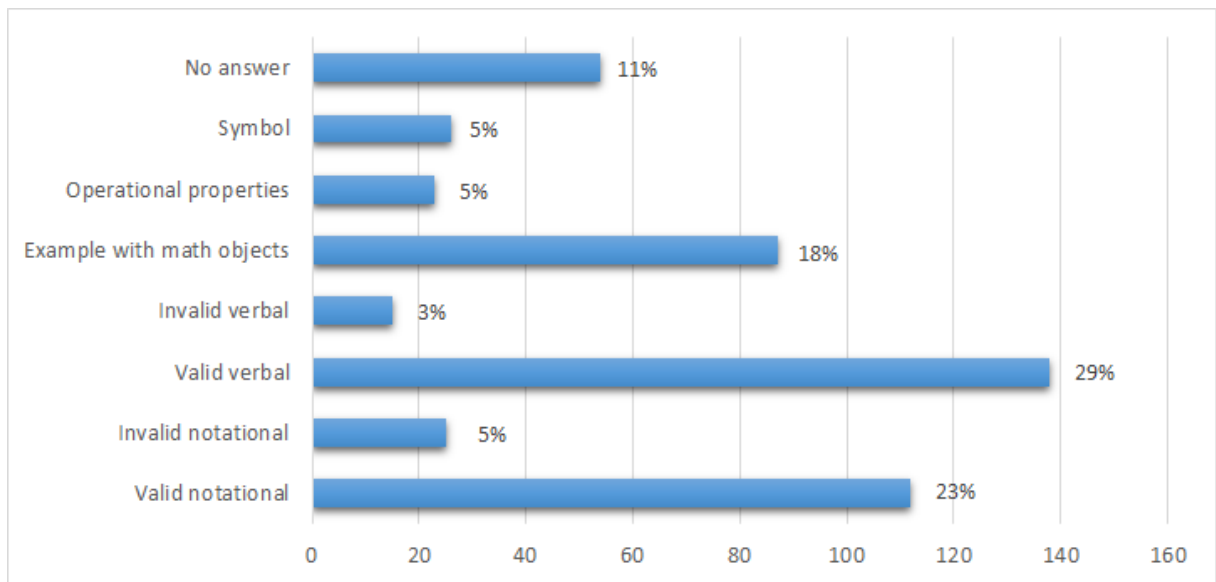
Figure 1. Mathematical definition types of the pre-service teachers

When Figure 1 was examined, it was found that 23% of definitions were valid definitions using mathematical notations. It was determined that approximately one fifth of the definitions are compatible with the formal definition in the textbooks. Accordingly, it can be said that most of the pre-service teachers had difficulty in making notational definition of set concepts. In 5% of the definitions, concepts tried to be defined using mathematical symbols, but failed. These pre-service teachers were evaluated in invalid notational definition category. Figure 2 presents samples of valid notational definition for union and invalid notational definition for intersection.
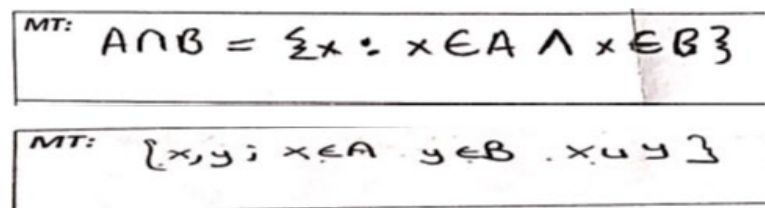

Figure 2. Valid and invalid notational definition

It was found that the pre-service teachers made verbal definitions of concepts at the most rate. Twenty-nine percent of the verbal definitions were mathematically valid. Accordingly, it could be said that the pre-service teachers prioritized verbal definitions rather than notational definitions. Three percent of the definitions were verbally explained, but mathematically was not valid. Figure 3 includes valid verbal definition for subset and invalid verbal definition for equal set. In the invalid verbal definition, it was observed that equivalent was defined instead of equal set.

Figure 3. Valid and invalid verbal definition

It was found that 18% of the definitions were mostly valid examples given by diagram or list method. These definitions were evaluated in examples with mathematical objects category. Accordingly, it could be said that some pre-service teachers considered examples with mathematical objects as mathematical definitions. Figure 4 shows the sample definitions for complement in this category.
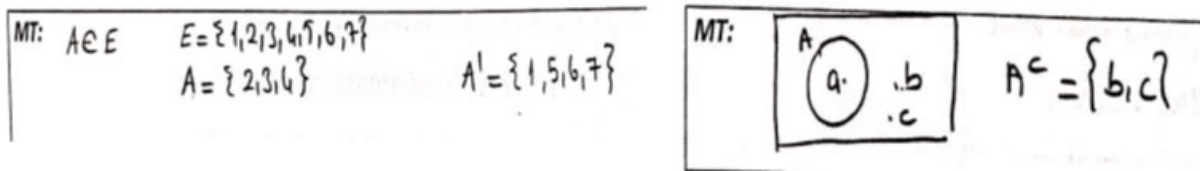
Figure 4. Definitions made using examples

In 10% of the definitions, the pre-service teachers either specified operational properties or symbols of concepts. Samples of these definitions are presented in Figure 5.
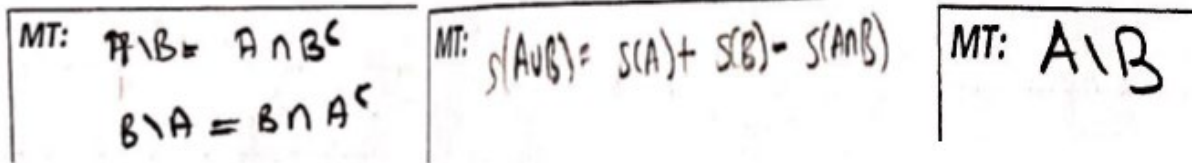

Figure 5. Definitions in operational properties and symbols categories

The mathematical definition categories were evaluated according to the concepts. The information is presented in Figure 6.
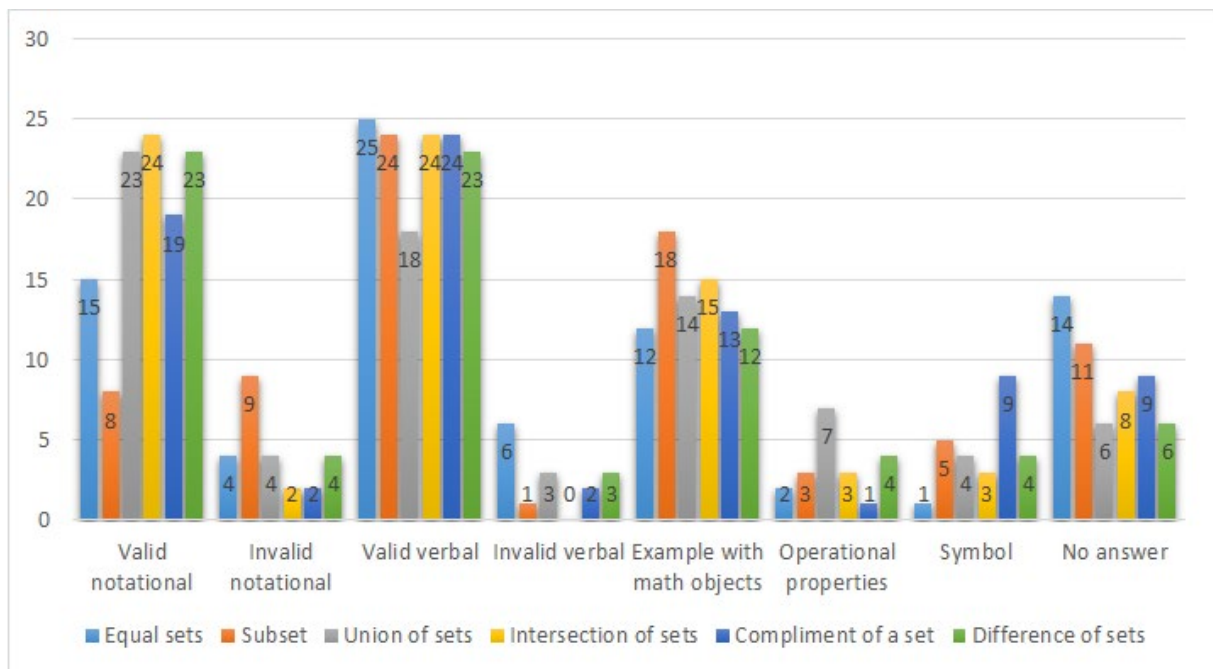

Figure 6. Mathematical definition types of the pre-service teachers according to the concepts

When the mathematical definitions were evaluated according to the concepts, it was found that the pre-service teachers were most successful in making notational definitions of union, intersection and difference. The pre-service teachers made valid and invalid notational definitions in similar numbers for these concepts. The pre-service teachers made the minimum number of valid notational definitions and the most invalid notational definitions in subset. Accordingly, it could be said that the concept that is most difficult to mathematically define was subset. It was emerged that the pre-service teachers had difficulty in using mathematical notations to express their thoughts while making mathematical definition of subset. In these definitions, it was observed that subset was confused with intersection and mathematical notations were misused. Sample subset definitions are presented in Figure 7.
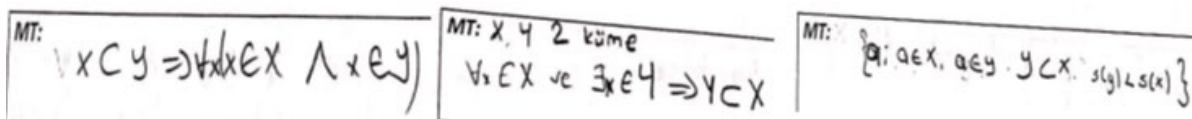
Figure 7. Sample of difficulties in making the notational definition of subset

When the pre-service teachers made mathematical definitions, it was found that the concept that had the most difficulties in the second rank was equal set. They confused equal set with equivalent and had difficulty in expressing their thoughts with notations. Samples of these definitions are shown below.
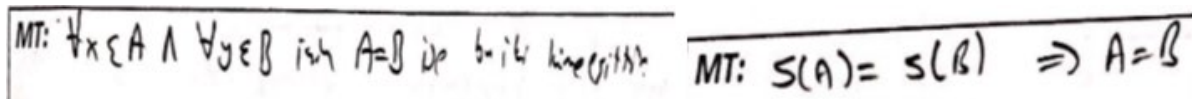

Figure 8. Difficulties in making notational definition of equal set

Although not as much as subset and equal set concepts, pre-service teachers gave few valid mathematical definitions of complement. It was determined that these pre-service teachers did not properly understand complement, made definitions for special cases, and could not use mathematical notations correctly. Samples are presented below in Figure 9.

	Let A be a set. The compliment is $\{x : x \in A \land x \notin A^c\}$
	$A' = x \notin A$ and $x \in B$ in the union of $A \cup B$

Figure 9. Difficulties in making notational definition of complement

Personal Definitions of Pre-service Teachers for Set Concepts

In order to determine how the pre-service teachers interpret the basic set concepts with their own sentences, the pre-service teachers were asked to express the concepts using their own sentences. When the personal definitions of the pre-service teachers were examined, it was found that the definitions were divided into seven categories. The information about the categories is presented below Figure 10 below.
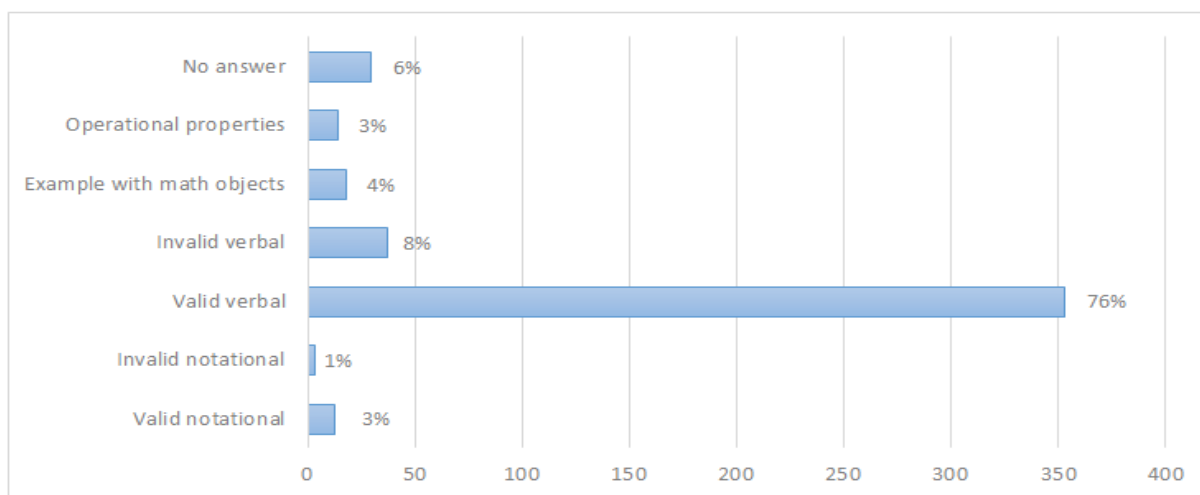
Figure 10. Personal definition types of the pre-service teachers

According to Figure 10, the personal definitions were often valid verbal definitions. It was found that 76% of the definitions reflected the verbal definition. Accordingly, it could be said that most pre-service teachers had no difficulty in defining concepts informally. In 8% of personal definitions, pre-service teachers tried to explain concepts verbally, but were unable to make valid verbal definitions. Below is a sample valid verbal

definition for complement and invalid for difference. In the definition made for difference, the verbal definition defined the concept of symmetrical difference rather than difference.

KT: *Bir kümenin kendi elemanları hariç bulunduğu evrensel kümenin elemanlarından oluşan kümeye o kümenin tümleyeni denir. A' ile gösterilir.*	*The set that has elements of universal set except for its elements is called compliment of this set. It is denoted by A'.*
KT: *İki kümenin farklı olan elemanlarıdır.*	*These are different elements of two sets.*

Figure 11. Valid and invalid verbal definitions

Mathematical notations were used in 4% of the definitions. Three percent of these definitions were valid but 1% invalid. Sample definitions are presented below.

KT: $A \subset B = \{ x : \forall x \in A \land x \in B \}$

KT: $A \cup B = \{ x : x \in A \lor x \in B \}$

Figure 12. Invalid and valid notational definitions

4% of the definitions were valid examples with mathematical objects, while 3% contained operational properties. Some pre-service teachers did not answer or used expressions that could not be analyzed. The following are samples for these categories.
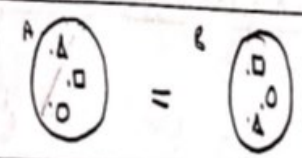
KT:

KT: $A = \{1, 3\} \quad E = \{1, 3, 5, 8, 9\}$
$A' = \{5, 8, 9\}$

KT: $S(A \cup B) = S(A) + S(B) - S(A \cap B)$

Figure 13. Examples with mathematical objects and operational properties

In this study, it was found that the pre-service teachers were mostly successful in explaining the concepts verbally. Analyses were conducted to reveal in which concepts the pre-service teachers were more successful or unsuccessful in making verbal definition. Distribution of verbal definitions according to the concepts is presented in Figure 14.

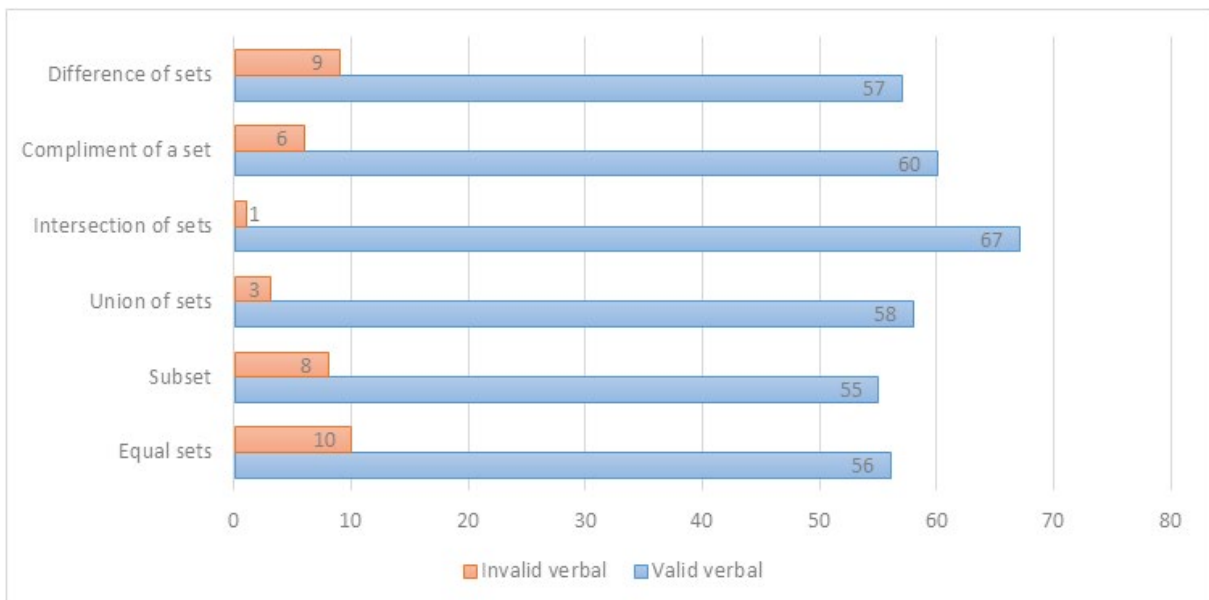| Concept	Invalid verbal	Valid verbal
Difference of sets	9	57
Compliment of a set	6	60
Intersection of sets	1	67
Union of sets	3	58
Subset	8	55
Equal sets	10	56

Figure 14. Verbal definitions of pre-service teachers according to the concepts

When Figure 14 was examined, it was revealed that pre-service teachers were most successful in making valid verbal definitions of intersection. Similar number of pre-service teachers made valid verbal definitions of other concepts. It was emerged that the pre-service teachers mostly had difficulty in making valid verbal definitions of equal set, difference and subset, respectively. Ten pre-service teachers could not explain equal set mathematical appropriately. It was revealed that these pre-service teachers made equivalent definition instead of equal sets. Figure 15 presents a sample definition.

KT: kendi tanımında A nın eleman sayısı ile B nih eleman sayısı birbirine eşit ise Bu kümeler birbirine eşittir.	*In my personal definition, if number of elements of A is equal to B, then these sets are equal.*

Figure 15. Sample verbal definition of equal set

Nine pre-service teachers made invalid verbal definitions of difference. In personal definitions of the pre-service teachers, it was seen that symmetrical difference is more defined than difference. Accordingly, it could be said that these pre-service teachers had difficulty in distinguishing between difference and symmetrical difference. Figure 16 shows sample definition.

KT: İki kümenin ortak elemanları dışında kalan elemanlarda dışa küme	*The set that has elements except common elements of two sets is the difference.*

Figure 16. Sample verbal definition of difference

Eight pre-service teachers tried to explain subset definition verbally, but failed. In invalid verbal definitions, pre-service teachers did not fully reflect the logic of subset or used expressions that were difficult to reconcile with the subset definition. In these statements, it could be said that the pre-service teachers had difficulty in using the quantifiers in particular. Sample definitions are presented in Figure 17.

KT: B'de var olan elemanların en az biri'si A' da da var ise B⊃A veya A⊂B 'yi denilebilir.	*If at least one element of B is also element of A, then it can be said that $A \subset B$ or $B \supset A$.*
KT: A kümenin bazı elemanları, B kümenin elemanlarıyla B, A'nın alt kümesidir denir.	*If some elements of A are also elements of B, then B is subset of A.*

Figure 17. Sample verbal definition of subset

Six pre-service teachers gave invalid definitions of complement and only three pre-service teachers gave invalid definitions of union. Pre-service teachers making invalid verbal definitions of complement often used expressions that did not reflect complement. In some of these statements, it is possible to say that the universal set was not considered. Sample definitions are presented below.

KT: Bir kümenin Evrensel kümeden farkına	*The compliment is called the difference of a set from the universal set.*
KT: Herhangi bir kümenin aynı tüm elemanlarını örtüyorsa tümleyeni olduk gösterilir.	*If a set covers all elements of any set, then it can be demonstrated as its compliment.*

Figure 18. Sample verbal definition of complement

Daily-Life Examples of Pre-service Teachers in Set Concepts

In this study, teachers were asked to give daily-life examples of basic set concepts. It was revealed that examples were collected in seven categories. Figure 19 provides information about categories.
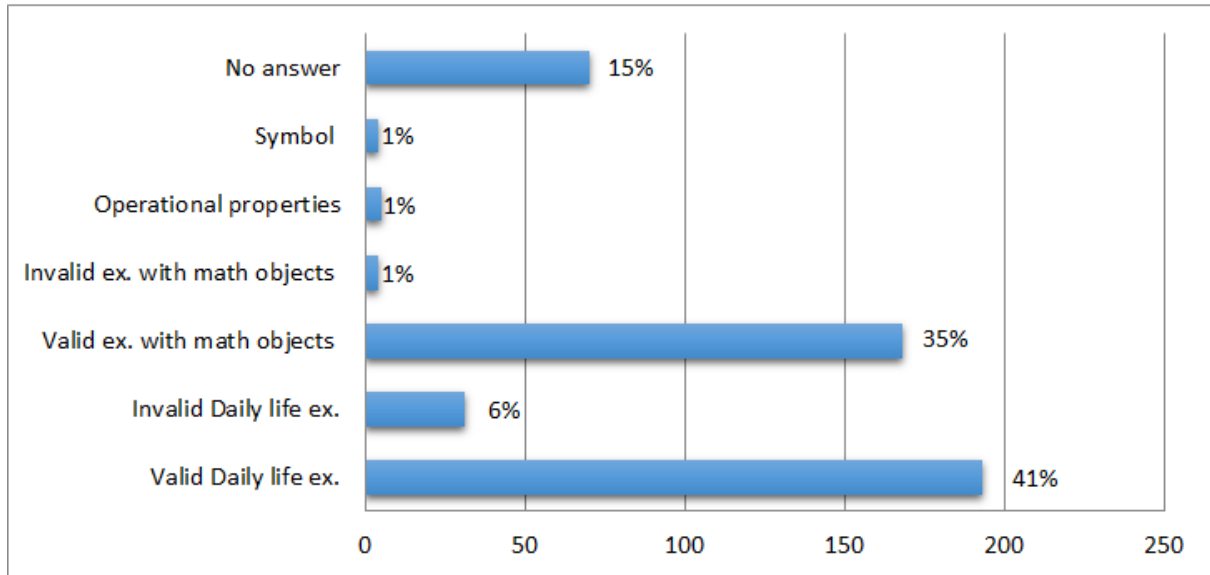
Figure 19. Daily-life example types of pre-service teachers

Figure 19 shows that 76% of pre-service teachers gave valid examples. However, it was found that examples given by 41% of the pre-service teachers were daily-life examples requested from them. In total, 44% of pre-service teachers did not give valid daily-life examples. In 15% of the examples, the pre-service teachers either did not respond or used expressions that could not be analyzed. It was determined that pre-service teachers had difficulty in giving daily-life examples while they gave mathematically valid examples. Although 6% of the examples were real-life examples, they did not fully reflect the concept. Figure 20 presents real-life examples that are valid for difference and invalid for intersection.

Figure 20. Valid and invalid real-life examples

In 36% of the examples, pre-service teachers tried to give examples using mathematical objects such as diagrams and lists instead of daily-life examples. Nearly all of these examples reflected the relevant concepts. 1% of the examples with mathematical objects, did not reflect relevant concepts correctly. Below are samples of these two categories.

Figure 21. Valid and invalid examples with mathematical objects

In 2% of the examples, pre-service teachers specified operational properties and symbols of concepts rather than giving examples. Samples are presented below in Figure 22.
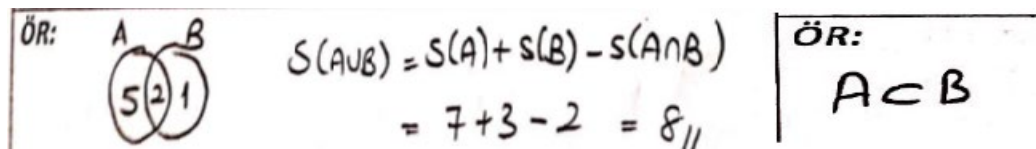
Figure 22. Samples of operational properties and symbol categories

It was intended to examine distribution of example types given frequently by the pre-service teachers according to the concepts. The following figure describes distribution of daily-life examples and examples with mathematical objects according to the concepts.
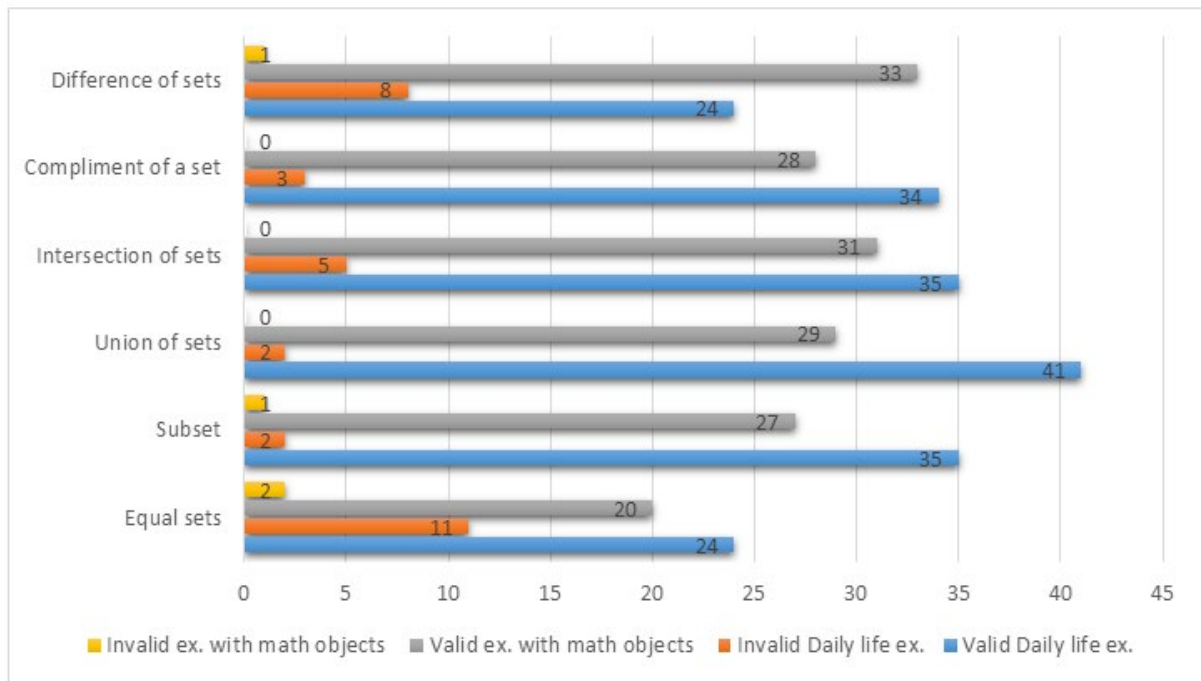

Figure 23. Pre-service teachers' daily life examples and examples with mathematical objects according to the concepts

When Figure 23 was examined, it was found that the pre-service teachers were most successful in giving valid daily-life examples of union. The concepts that the pre-service teachers had most difficulties were equal set and difference. When invalid daily-life examples were examined, it was determined that pre-service teachers mostly gave examples related to equivalent for equal set. When invalid daily life examples for difference were examined, it was found that the examples did not match the mathematical meaning of difference. Samples are presented in Figure 24.

Figure 24. Difficulties in giving daily-life examples of equal set and difference

When examples with mathematical objects were examined, it was found that the pre-service teachers gave more examples in difference. As mentioned above, it was emerged that pre-service teachers having difficulty in giving daily-life examples tend to use mathematical objects. The only one concept that pre-service teachers gave more examples using mathematical objects than daily-life was difference. This showed that the concept that pre-service teachers had most difficulty in associating with daily-life was difference. However, equal set and intersection also drew attention as concepts where difference between valid daily-life examples and

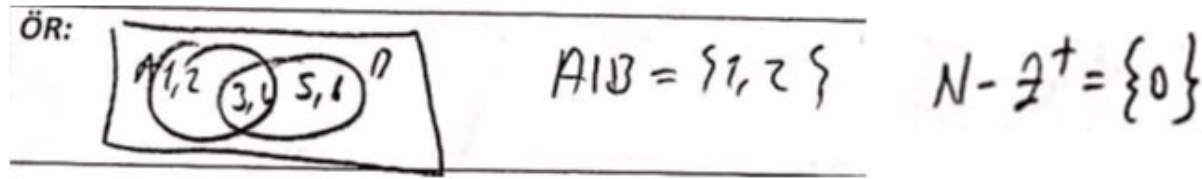examples with mathematical objects. Below are samples of the examples with mathematical objects for difference

Figure 25. Samples of examples with mathematical objects for difference.

Opinions of Pre-service Teachers

Interviews were conducted with ten participants selected from among 79 pre-service teachers responding to DKFS. The first consideration in the interviews was whether written responses reflected their true thoughts on the concepts. For this reason, written responses were evaluated together with them. It was found that all the pre-service teachers' opinions in the interview were parallel with their written responses. For example, interviews were carried out with P9. In DKFS, P9 often left mathematical definitions blank and gave only personal definitions. In this interview, it was revealed that P9 could not make definitions that left unwritten. Below are excerpts from the conversation between the researcher and P9.

> *Researcher: You didn't make a mathematical definition of subset. Why?*
>
> *P9: I didn't specifically define it mathematically. Because I was going to write something missing there for sure. After all, it's a memorization thing. That's why I always wrote my own definitions.*
>
> *Researcher: Do you want to do it now?*
>
> *P9: [She tries to make a mathematical definition, but she can't]. I can't define it mathematically. There's no mathematical definition.*

The pre-service teachers stated that they had difficulty in making mathematical definitions in the study. Most of the pre-service teachers stated that they used their personal definitions instead of mathematical definitions. These pre-service teachers stated that they could not remember their mathematical definitions, that they had difficulty expressing their thoughts mathematically, and that personal definitions were more logical and understandable to them. Below are same opinions of these pre-service teachers.

> *P3. When we make our definitions, we can explain our thoughts with simple things, but when it comes to mathematical definition, it's a little harder.*
>
> *P5: My own definition sounds more logical and understandable. Mathematical definition is more difficult to explain. I used my own definitions instead of mathematical definitions.*
>
> *P6: I've always used my own definition here instead of mathematical definition. I showed some of them in diagram. I didn't remember mathematical definition. I haven't been able to give a mathematical definition in any of the concepts.*

P7 specified operational properties and diagram instead of mathematical definition. She thought that operational properties and diagrams could be given as mathematical definitions. Below is the dialogue between P7 and the researcher.

> *Researcher: In the study, you gave operational property for mathematical definition. Why?*
>
> *P7: That could be a mathematical definition. It's a property in a sense, but it could be a mathematical definition.*
>
> *Researcher: Here are notational definition [showing a valid notational definition] and diagram [showing an valid example with the Venn diagram], which one is more mathematical?*

> *P7: I think, diagram is more mathematical because it's more visual. But notional definition is used in proofs. According to the professors at university, notational definition. But the diagram is easier. I couldn't write the notational definition. I didn't remember.*

P2 also often used examples, especially in personal definitions. Below are P2's views.

> *P2: I use these examples more as my definition. Because when I evaluate my own definition mathematically, it means the same thing. It makes more sense to present definitions by giving examples. When given an example, we think about what the definition might be.*

Some pre-service teachers were asked what method should be used to teach these concepts. They stated that personal definitions and examples should be more prominent in the teaching of set concepts and that it would be useful to apply mathematical definitions to the last. Below are the opinions of P1.

> *P1: First, we provide the information by our definition. Then, we give an example of everyday life. After that, I think it's best to put it into mathematical language. If we do it this way, students will understand better. When we start a concept by giving examples rather than definitions, the issues are better understood. If courses start with definitions, students can't understand anything as there is nothing in their mind. We're having similar problems ourselves. Courses such as linear algebra, for example, are courses about definitions. We don't understand anything when definitions are written by professor. It can be better understood when you give an example and the definition is explained through that example.*

Some difficulties, identified from DKFS before, were revealed in the interviews. For example, in interview with P1, difficulties were detected in understanding of difference and complement. Below is the dialogue between P1 and the researcher.

> *Researcher: You didn't write mathematical definition for complement. Why?*
>
> *P1: I didn't remember.*
>
> *Researcher: Do you want to define it now?*
>
> *P1: Let me give you an example. Let's think of a class. Because the classroom is inside the school, the school is the complement of the class. We can think of it that way.*
>
> *Researcher: All the school? Are we including the class in the complement?*
>
> *P1: We're including the class. The whole school is happening.*
>
> *Researcher: What is the complement of set A in this example? [Researcher wrote an example with Venn diagram such as $A \subset E$].*
>
> *P1: It's a universal set.*
>
> *Researcher: Can you define the difference?*
>
> *P1: It is a set consisting of the removal of common elements from two sets.*

4. Discussion

As a result of the analysis of the data obtained with the help of DKFS and interviews, it was revealed that the pre-service teachers had difficulty in making notational definition of the concepts. Accordingly, it can be said that most pre-service teachers were not even at knowledge level for notational definitions. Pre-service teachers mostly made verbal definitions instead of notational definitions. This means that these pre-service teachers used verbal definitions as mathematical definitions of set concepts. Some pre-service teachers gave examples using mathematical objects instead of notational definitions. Some pre-service teachers also presented operational properties and symbols as mathematical definitions. This conclusion is consistent with the results of studies in which pre-service teachers had difficulties in understanding formal definitions of concepts (Açıkyıldız, 2013; Doruk et al., 2018).

Secondly, it was determined that the majority of pre-service teachers were able to make valid verbal definitions Seventy six percent of the definitions were valid verbal definitions. Therefore, it can be said that

the majority of pre-service teachers were at conceptual level regarding the concepts. In the last part of the study, pre-service teachers were asked to give daily-life examples for set concepts. It was found that 76% of examples were mathematically valid examples. This percentage obtained from this section was found to be equal to the percentage obtained from the previous section. This astonishing result is a meaningful result in terms of validity of the study. Because the skills investigated in these parts are indicators of the same cognitive level. The behavior of explaining a concept, interpreting it, explaining it in different ways, and giving examples are indicators of understanding level (Baki, 2014; Demirel, 2014). This conclusion confirmed that the pre-service teachers were at the conceptual level of set concepts. Similarly, Yazıcı (2017) found that mathematics teachers could often accurately describe equal sets, subsets, and complement.

However, 41% of examples were found to be valid daily life examples. According to these reasons, it can be said that students had difficulty in establishing relationship between set concepts and daily-life. Most of the pre-service teachers gave examples with mathematical objects. Some pre-service teachers presented operational properties and symbols of concept as daily-life examples. Accordingly, in order to associate the concepts of sets with daily life, studies can be carried out in teaching of related courses. Unfortunately, mathematics teachers used more abstract examples in teaching of set concepts (Yazici, 2007). Similarly, Schoenfeld (1991) stated that students in schools could not relate to mathematics and daily-life.

Although this study focused on the concept definition section of CD&CI, it provided important information on the concept images of pre-service teachers for set concepts. Vinner (1981) stated that for some concepts, concept definition was part of concept image. He stated that some of the concepts in this expression are concepts that can be presented by verbal definition. Set concepts are among the some concepts stated by Vinner (1981). In this study, it was found that verbal definitions were considered more than notational definitions. The categories identified in this study can also be said to be part of their concept images. Because the questions asked in every part of the study changed, but the categories always remained same. Accordingly, concept images of the pre-service teachers were: verbal definition, examples with mathematical objects (mostly diagram representation), notational definition, operational properties, and concept symbols. Verbal definitions and examples with mathematical objects were stronger images than others. One of the sources of these images may be the learning backgrounds of the pre-service teachers. The images expressed in the study are included in the course books used from middle school to university level. These images may have been transferred to students through instructions and textbooks for set concepts. The results of this study supported Tall and Vinner's (1981) views that "the concept image may differ from the features in the concept definition".

It was emerged that the pre-service teachers had more difficulty in some concepts. As a result of the examinations, it was determined that the pre-service teachers had various difficulties. These difficulties adversely affected the performance of the pre-service teachers in the study. The difficulties identified in the study are listed below.

1. Pre-service teachers confused notational definition of subset with intersection.
2. Pre-service teachers used equivalent instead of equal set.
3. The pre-service teachers neglected universal set in making definitions and giving examples of complement.
4. The pre-service teachers considered complement for special cases.
5. Pre-service teachers used symmetrical difference instead of difference.
6. Pre-service teachers had difficulty expressing their thoughts using mathematical notations.
7. Pre-service teachers had particular difficulty in using quantifiers and in symbolically expressing concepts.

In previous studies, pre-service teachers confused equal set with equivalent set (Baki & Sahin, 2002), neglected the universal set while working with the concept of complement (Zehir et al., 2008), difficulty in using mathematical notations (Zehir et al., 2008). The first and fifth difficulties identified in this study were never mentioned, and the fourth, sixth and seventh difficulties were partially addressed. For example, Kabael, Barak and Özdas (2015) stated that students could not connect the formal definition of the limit with the concept image, and that the reason for this situation was that they could not use quantifiers in a meaningful way.

5. Recommendations

In order to care about mathematical definitions, mathematical definitions can be used in the teaching and evaluation process of the related courses. In the course teaching process, daily-life examples and problems should be included. Efforts can be made to help students develop an appropriate conceptual image for concepts. Because developing an appropriate conceptual image for concepts paves the way for meaningful learning, while the wrong concept images prevent them from learning correctly (Gülkılık, 2008). The course books used can be reviewed so that students can relate concepts to daily-life and develop appropriate concept images. Teaching plans of the courses can be arranged considered the difficulties of the pre-service teachers.

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Appendix

DEFINITION KNOWLEDGE FORM FOR SETS

- *Make mathematical definition using mathematical symbols of the following concepts (**MD: Mathematical definition**).*
- *Express the following concepts by using your own words (**PD: Personal definition**).*
- *Give an example from everyday life of the following concepts (**EX: Example**).*

EQUAL SETS

MD:	
PD:	
EX:	

SUBSET

MD:	
PD:	
EX:	

UNION OF SETS

MD:	
PD:	
EX:	

INTERSCTION OF SETS

MD:	
PD:	
EX:	

COMPLIMENT OF A SET

MD:	
PD:	
EX:	

DIFFERENCE OF SETS

MD:	
PD:	
EX:	