Modeling and Forecasting the Markets Volatility and VaR Dynamics of Commodity

Pınar KAYA*
Bülent GÜLOĞLU**

Abstract

The purpose of this paper is to model and forecast the risk of six commodities namely, crude oil, copper, gold, silver, palladium, and platinum during the period from 02/01/2002 to 29/04/2016 using volatility, value at risk and expected shortfall as risk measures. After showing that squared returns of all six commodities have a significant long memory, the volatility, the value at risk and expected shortfall based on fractional GARCH models are estimated and forecasted. Both forecast performance of volatility models and backtest for value at risk indicate that in many cases FIAPARCH model outperforms the other GARCH models. Then volatility, value at risk and expected shortfall estimates based on FIAPARCH model show that the volatility and market risk of oil is much higher than the other commodities. This casts doubt on the use of oil as a hedging tool.

Keywords: *Volatility Modelling, Commodity Markets, VaR Forecasting, Expected Shortfall*

JEL Classification: *C22, C53, C58, G17, Q02*

Özet - Emtia Piyasalarının Oynaklık ve Riske Maruz Değer Dinamiklerinin Modellenmesi ve Öngörüsü

Bu çalışmanın amacı, ham petrol, bakır, altın, gümüş, paladyum ve platinden oluşan altı temel emtiaya ait zaman serisinin 02/01/2002 - 29/04/2016 arasını kapsayan dönemde, oynaklık, riske maruz değer ve beklenen açık risk ölçümlerini kullanarak, emtia piyasalarının riskini modellemek ve öngörmektir. Bu altı emtianın getiri karelerinin önemli ölçüde uzun hafıza özelliğine sahip olduğu gösterildikten sonra oynaklık, riske maruz değer ve beklenen açık, kesirli bütünleşik GARCH modelleri kullanılarak tahmin edilmiş ve öngörülmüştür. Hem oynaklık modellerinin öngörü performansı hem de riske maruz değer için yapılan geri testler birçok durumda FIAPARCH modelinin diğer GARCH modellerinden bariz biçimde üstün olduğunu göstermektedir. FIAPARCH modelini kullanarak yapılan oynaklık, riske maruz değer ve beklenen açık tahmin sonuçları petrolün diğer emtialardan daha riskli olduğunu göstermekte ve petrolün riskten korunma aracı olarak kullanılmasını sorgulanır hale getirmektedir.

Anahtar Kelimeler: *Oynaklık Öngörüsü, Emtia Piyasaları, VaR Öngörüsü, Beklenen Açık*

JEL Sınıflandırması: *C22, C53, C58, G17, Q02*

* Dr., Marmara University
**Prof. Dr., Istanbul Technical University

1. Introduction

In the light of recent and still growing literature of long memory property, this paper investigates the conditional volatility of six major commodities, namely, crude oil, copper, gold, silver, palladium, and platinum. Various forms of GARCH-class models (linear and non-linear) allowing for asymmetry and long memory characteristic of returns are used for this purpose.

Over recent years, we have witnessed an acceleration of financial integration across stock markets and other assets. This in turn has weakened diversification opportunities across financial markets and brought about the financialization of commodity markets. Especially gold and oil have become important hedging tools for investors seeking to diversify their risk (Kang et al. 2016). Moreover, it is well known that gold is considered as a safe asset worldwide. In risk management, portfolio allocation, and hedging, precious metals have been widely used as assets. The appropriate risk measurement of an asset is therefore crucial both for investors and fund managers. Volatility and value at risk of an asset are important indicators for investors and would play an important role for short and long run trading decisions along with the attitude of investors towards risk. Over recent years in addition to Value at Risk, Expected Shortfall has also been widely used as a risk measurement in the literature. Since expected shortfall (henceforth ESF) is the expected value of the losses conditional on the loss being larger than the VaR (Scaillet, 2004), it is also known as conditional value at risk. As pointed out by Tsay (2010), VaR estimates the potential financial loss. However when an extreme event happens it could underestimate the actual loss. Thus for appropriate measurement of risk, ESF besides the VaR should be used.

There are several ways to measure market risk in the literature. In this study all risk measures (volatility, VaR, ESF) have been based on the same GARCH models to avoid inconsistency between different risk measurements and make them comparable.

In this framework, forecasting of volatility becomes essential for appropriate measurement of risk. This leads us to appropriate modeling of volatility to measure and forecast the volatility. To address this issue we employ various linear and non-linear GARCH models allowing for asymmetry and long memory and compare their forecasting performance.

This study contributes to the literature in several ways. First, we employ three different risk measures for commodities. More precisely we use dynamic volatility, VaR and ESF. All of them are coherent as they are based on the same GARCH models. Second, forecast performance of GARCH models and backtests for VaR are carried out. Third unlike the previous studies, which use fractionally integrated GARCH models without testing for long memory for commodities, in this study we have employed various tests for long memory in returns and squared returns. Fourth the results have also practical implications for investors in the sense that using appropriate GARCH models they could accurately measure and forecast the future volatility. Then the decision to invest on the commodity market could be taken. In addition, comparing with the risk of other assets (such as equity) commodities could be used as hedging instruments.

This paper is structured as follows. After the introduction, the second section is devoted to the literature. The third section presents data and descriptive statistics. The methodology used in the paper is illustrated in section four. The fifth section presents empirical findings. The last section concludes the paper.

2. Literature

Recent empirical studies on commodity markets volatilities focus on the long memory behavior of data (Aloui and Mabrouk, 2010). In this respect, several studies have been carried out allowing for the long memory to model both price variations and price volatilities.

Firstly, Mandelbrot (1971) analyzes long memory in financial markets and suggests using Hurst's 'rescaled range' statistic to test for long memory process in financial return series. He argues that arbitrage opportunities might exist since in the presence of long memory shocks cannot be absorbed quickly. As shown by Yajima (1985), if security prices expose long memory, then the standard testing procedures of asset pricing and the martingale models will be inappropriate. In addition, Lo (1991) states that both the capital asset pricing model and the arbitrage pricing theory based on standard testing procedures may not be valid if the security returns display long memory behavior.

Long memory models are known with their autocovariances falling into decay slowly. According to Ding et al. (1993) fully decaying of a shock can last for long. Long memory property can be clearly explained as a significant correlation between distant observations of time series. This means that a shock cannot be eliminated immediately by the market. Thus, the distinction between I(0) stationary and I(1) nonstationary processes is prominently too restrictive (Baillie et al., 1996). While the propagation of shocks in a stationary process arises at an exponential rate of decay, for a nonstationary process shocks exhibit infinite persistence.

In order to bridge the gap between short and complete persistence while modeling the conditional mean, fractionally integrated autoregressive moving average (ARFIMA) specifications have been suggested. In this class of the model, the short-run conduct of time-series is represented by the ARMA parameters, while the fractional differencing parameter allows for the long-run dependence (Conrad and Haag, 2006).

According to fractional integration theory, the order of integration namely the fractional difference parameter is a fractional value (Baillie, 1996). Fractionally integrated processes are different from both stationary and unit root processes with their persistence and mean reverting features. Especially, the long memory parameter is given by $d \in (0, 0.5)$. When $d > 0.5$ the time series is considered as nonstationary and when $d \in (-0.5, 0)$ the series is considered as antipersistent (Kumar, 2014).

Long-memory property can occur in volatility of financial returns as well, which points out the wide-distant correlation of time-varying volatility elements. Robinson (1991) comes up with the linear autoregressive conditional heteroskedastic model (LARCH) which permits long memory in the conditional variance. Later extensions of generalized ARCH (GARCH)-type models taking into account long-memory behavior have been proposed by many researchers. Baillie, Bollerslev, and Mikkelsen (1996) develop the fractionally integrated GARCH (FIGARCH) model which are more convenient for this type of data in various empirical analyses (Bollerslev and Mikkelsen, 1996; Beine and Laurent 2003; Conrad and Karanasos, 2005a, b).

Following the studies by Granger (1980), Granger and Joyeux (1980) and Hosking (1981), extended use of long memory model is observed in empirical studies. Two strands of literature in the field of long memory have received a great amount of attention. One strand investigates modeling the volatility of commodities. Based on standard volatility models, previous studies on the volatility of commodity focus on single commodity’s dynamic volatility characteristics or volatility spillovers across various commodities. More recent studies, however, extend the existing literature by incorporating a variety of volatility behavior of several commodities (Arouri et al., 2012a, b; Wei et al., 2010).

The other strand of literature focuses on the relationship between commodities and stock markets. Kang et al.(2017) investigate spillover effects among six commodity futures markets (gold, silver, crude oil, corn, wheat, and rice) for the period from January 4, 2002 to July 28, 2016. They find out bidirectional return and volatility spillovers between commodity markets. The findings of the study show that both gold and silver are information transmitters to other commodity futures markets. With respect to crude oil, corn, wheat, and rice, the results indicate that those commodities are receivers of spillovers during the financial crisis period in 2017.

Jain and Biswal(2016) examine the relationships between gold, oil USD-INR(Indian Rupee) exchange rate and SENSEX. Using the daily data spanning the period 2006-2015 and employing the DCC GARCH model and linear and non-linear causality tests, they show that a decrease in gold and oil prices causes the exchange rate to depreciate. The results of the study also show that gold becomes an investment asset class among the investors. Besides, the authors argue that gold and oil can be used as hedging tools against to volatility in SENSEX and fluctuations in the exchange rate.

Creti et al. (2013) analyze the links between returns for 25 commodities and stocks over the period from January 2001 to November 2011. Using the dynamic conditional correlation (DCC) GARCH model, they show that the correlations between commodity and stock markets are highly volatile especially since the 2007–2008 financial crisis and vary through time.

In a recent study by Dahl and Iglesias (2009), an alternative functional relationships (from GARCH (1, 1) to GARCH (1, 1)-AR (m)) are used to model the spot price risk and spot prices. Based on the classical rational expectations and ARCH-M model proposed by Engle et al. (1987), they investigate empirically the rational expectations model of Muth's (1961). In their paper, they conclude that lagged conditional variance should enter into the mean equation.

Kang and Yoon (2013) explore the effectiveness of a volatility model for three crude oil markets, namely, Brent, Dubai, and West Texas Intermediate (WTI) regarding persistence and long memory. Relying on the conditional volatility models they found that the CGARCH and FIGARCH models are better equipped to capture persistence and provide superior performance in terms of out-of-sample volatility forecasts in comparison with the GARCH and IGARCH models.

In their paper, Thuraisamy et al. (2013) investigate spillover effects between the volatility of Asian equity market and that of the crude oil and gold futures. Their results show that volatility shocks in fully-fledged equity markets overflow into the crude oil and gold futures markets, while fledgling markets are in a tendency to spill over from commodity futures to equity markets. They also provide strong evidence of increased bi-directional volatility transmission during the 2008 financial crisis period. As for equity market volatility, not only volatility shocks from the crude oil futures market are important but also the volatility shocks from the gold futures market.

In a recent study, Vivian and Wohar (2012), investigate whether there are structural breaks in commodity spot return volatility using an iterative cumulative sum of squares procedure and the GARCH (1,1) model for each regime. Their findings provide very limited evidence of commodity volatility breaks during the recent financial crisis compared to the 1985–2010 sample period as a whole.

In their paper, Aloui and Mabrouk (2010) appraise the value-at-risk (VaR) for some major crude oil and gas commodities for both short and long trading positions. They calculate the VaR for various ARCH/GARCH-type models, namely FIGARCH, FIAPARCH and HYGARCH. Their findings show that considering long-range memory, fat-tails, and asymmetry performs better in predicting a one-day-ahead VaR for both short and long trading positions.

Wang et al. (2010) use two non-parametric methods, detrended fluctuation analysis (DFA) and rescaled range analysis (R/S). They check the long memory properties of conditional volatility series obtained from GARCH-class models against actual volatility series for WTI crude oil returns. As they are interested in the long memory of volatility, GARCH models are appropriate for the time scale larger than a year.

Choi and Hammoudeh (2009) conclude that forecasting the commodity volatility relying on long memory univariate GARCH models is more accurate than the standard GARCH models for oil and refined products markets.

Other papers allowing for structural breaks and long memory properties in return and volatility of commodity markets show that the volatility of precious metals is better represented by long memory than by structural breaks (Arouri et al., 2012a,b).

The studies, which concentrate on commodity volatility, have been becoming crucial because of the increasing importance of commodities in the financial sector (Regnier, 2007; Chkili et al. 2014). Not only financial sector but also real sector, and economic growth are significantly affected by prices of commodities. Another reason for their popularities are their empirical features such as their pronounced excess kurtosis or fat tailedness, asymmetry, and structural breaks that influence the model fitting, these series need to be estimated with varied volatility models (Aloui and Mabrouk, 2010; Cheng and Hung, 2011; Cheong, 2009; Hung et al., 2008). Existing researches on volatility forecasting for commodity markets are limited since they have concentrated more on forecasting conditional return than conditional volatility.

3. Data

In this study, daily data of six major commodity spot prices; gold, silver, copper, crude oil, palladium, and platinum are used. Prices of two main metals gold and silver are determined by Handy & Harman (H & H) which is operated as dealers in silver and gold and these base prices are taken for transactions worldwide. These precious metals are measured in US dollars per troy ounce. The other precious metals copper and platinum are traded on the London Metal Exchange (LME) as the second largest traded contract. West Texas Intermediate (WTI) crude oil produced in the US serves as a reference price in the oil market. The data set is extracted from the Thomson and Reuters DataStream database, and the whole sample period spans from 02/01/2002 to 29/04/2016. All estimations are based on the daily period from January 2, 2002 to February 26, 2016. The out-of-sample forecast performances of the competing long memory-based GARCH models are based on the period from February 29, 2016 to April 29, 2016. Daily return series are calculated as log differences in price levels as follows:

$$r_t = 100 \times \left[\ln P_{i,t} - \ln P_{i,t-1} \right] \tag{1}$$

The return series of gold, copper, silver, crude oil, palladium, and platinum are plotted in Figure 1. As illustrated in Table A1, all returns series are stationary according to ADF and KPSS test statistics.

Descriptive statistics for the commodity return series are shown in Table 1. As seen, all series have high excess kurtosis and negative skewness. In addition, the distribution of the series seems to be leptokurtic. Furthermore, Ljung-Box Q (20) statistic for returns are significant except for silver and it is significant for squared returns of all commodities. As seen, regardless of the lag chosen, all ARCH test statistics are highly significant. As a result, while modeling the returns, conditional heteroscedasticity and serial correlations are taken into account.

Figure 1: Time series plots of spot price series

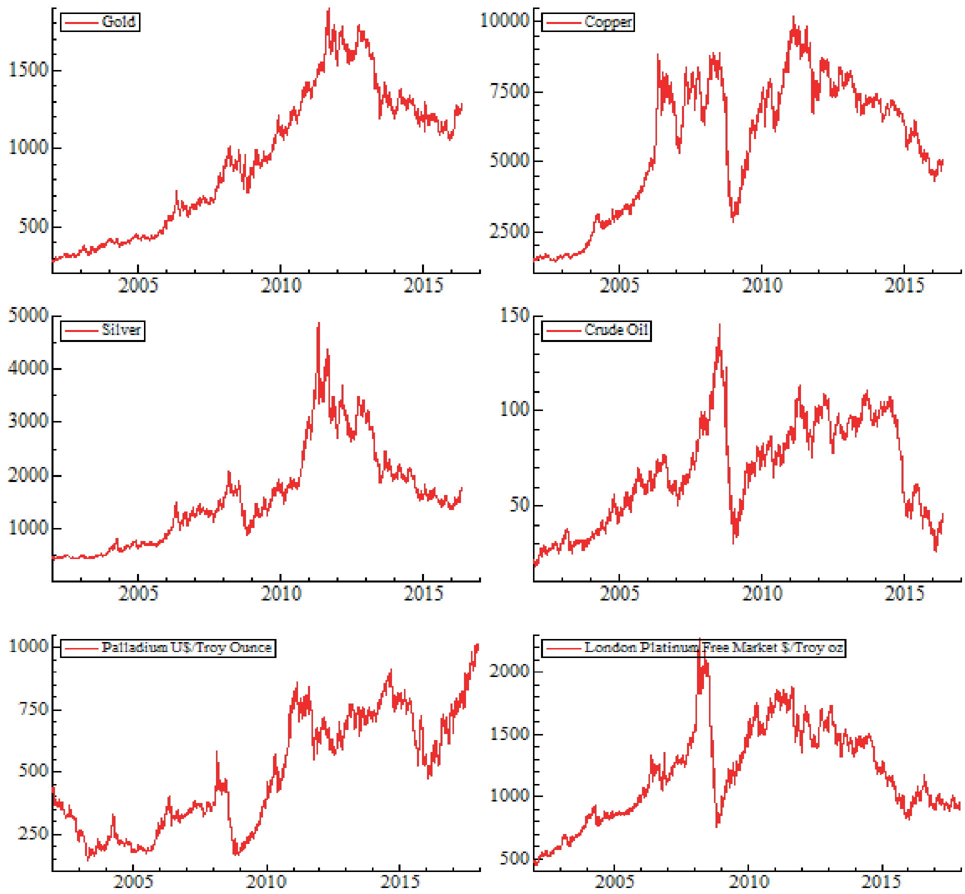


Figure 2: Time series plots of spot return series

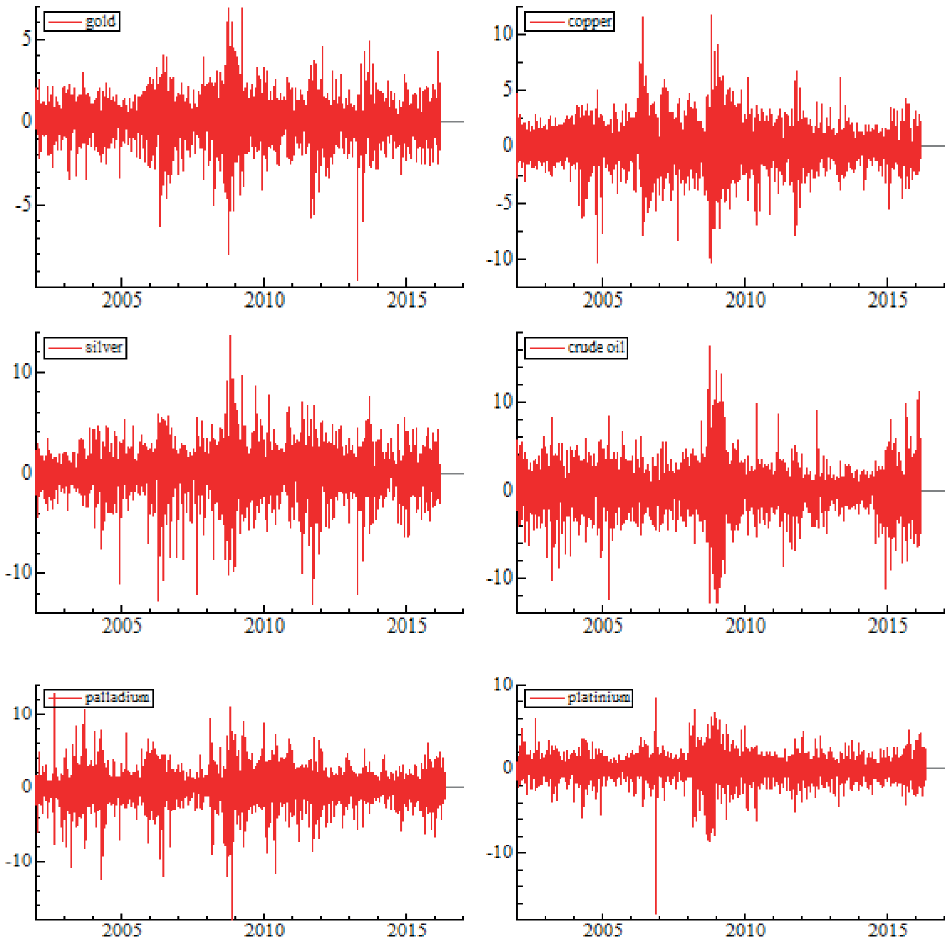
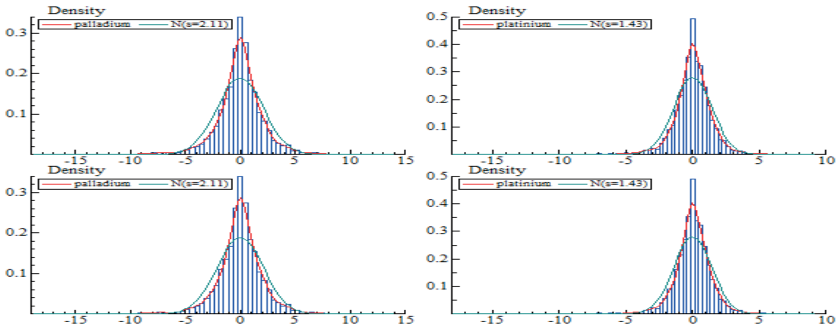


Table 1: Descriptive Statistics

	GOLD	COPPER	SILVER	CRUDE_OIL	PALLADIUM	PLATINIUM
Mean	0.040513	0.031673	0.031497	0.013397	0.0031338	0.017
Maximum	6.855528	11.72590	13.66480	16.41370	12.846	8.427
Minimum	-9.596	-10.358	-12.982	-12.827	-17.859	-17.2
Std. Dev.	1.170235	1.786288	2.023908	2.380344	2.1148	1.42
Skewness	-0.407524	-0.134399	-0.565387	-0.020894	-0.42313	-0.82
Kurtosis	8.248861	7.025565	7.966751	7.369698	5.5092	10.21
Jarque-Bera	4342.735	2505.367	3993.717	2939.193	4781.9	16481
ARCH 1-2	31.544[0.000]	211.86[0.000]	76.64[0.000]	145.9[0.000]	61.7[0.000]	126.7[0.000]
ARCH 1-5	41.230[0.000]	123.61[0.000]	39.59[0.000]	128.06[0.000]	38.6[0.000]	61.7[0.000]
ARCH 1-10	33.808[0.000]	74.510[0.000]	24.39[0.000]	69.27[0.000]	23.9[0.000]	34.3[0.000]
Q(20)	36.46[0.013]	52.95[0.000]	22.09[0.335]	57.13[0.000]	36.03[0.015]	24.7[0.209]
$Q^2(20)$	1150.08[0.000]	3197.8[0.000]	635.34[0.000]	2803.92[0.000]	646.8[0.000]	964.5[0.000]
Observations	3693	3693	3693	3693	3693	3693

Figure 3: Density plots of return series



4. Methodology

In this section, the tests of long memory as well as various forms of GARCH-type models allowing for asymmetry and long memory characteristic of commodity price are used to model volatility. Then based on the GARCH models value at risk and expected shortfall is calculated.

4.1. Long Memory Tests

The presence of long memory in the data implies the persistence of observed autocorrelations. Long memory in volatility is an important phenomenon since it is characterized by a slowly decaying autocovariance function. When long memory in time series exists then various forms of models can be employed taking into account intermediate degrees of volatility persistence.

In this paper, the Hurst-Mandelbrot Rescaled Range (R/S) statistics, Lo (1991) Rescaled Range R/S, Geweke and Porter-Hudak (1983) (GPH), and the Robinson and Hendry (1999) Gaussian Semiparametric (GSP) test statistics are used in order to test for long memory components in the returns. In addition, considering long memory in the volatility process, these tests are applied to commodities' squared returns, which are widely regarded as a proxy of conditional volatility (Choi & Hammoudeh, 2009; Lobato & Savin, 1998). These tests have been extensively used in the related literature.

4.1.1. Rescaled Range (R/S) statistics

Rescaled range statistic R/S is one of the oldest and famous tests which was introduced by Mandelbrot and Wallis (1969) and Hurst (1951) to examine the presence of long-term memory in time series. Mandelbrot (1971) suggests that R/S analysis can be used in economic and financial investigations. Basically, Rescaled range statistic R/S is the range of partial sums of deviations of a time series from its mean scaled with its standard deviation. Hence, Hurst exponent H stands for the scaling behavior of the range of cumulative departures of a time series from its mean. This statistic is robust to data non-normality, but in the presence of autocorrelation, the coefficients may be biased. Therefore, Mandelbrot's null hypothesis is "there is no long-term dependence" under the assumption of no autocorrelation.

Consequently, Lo (1991) developed a modified rescaled range, which adjusts for possible short term dependence by applying the Newey West heteroscedasticity and autocorrelation consistent estimator instead of the sample standard deviation. Hence, Lo's null hypothesis is "there is no long-term dependence".

Geweke and Porter-Hudak (1983) propose a semi-nonparametric approach to test for long memory with regard to a fractionally integrated process. Furthermore, Geweke and Porter-Hudak (1983) use Fourier transformation and spectral density into the test equation. GPH test for the null hypothesis is "there is no long memory (d=0)".

Gaussian Semiparametric estimation model developed by Robinson and Henry (1999) depends on low-frequency periodogram estimates and the specification of the shape of the spectral density of the time series. Robinson and Henry (1999) demonstrate that the Gaussian semiparametric estimator is asymptotically normally distributed and it is robust to conditional heteroskedasticity including the long-range dependence.

4.2. Results of Long Memory Tests

As can be seen from the Tables 2-7, the long memory property in return and squared returns are analyzed using test statistics of Hurst-Mandelbrot R/S, Lo R/S, GPH, and GSP. The results show that these test statistics do not reject the null hypothesis of no long-range dependence. So, it seems that the commodity return series does not exhibit a long memory effect.

On the other hand, for the squared returns, tests lend a support to long memory effect at the 1% level. Since d parameter is significantly lies into interval (0, 0.5) one may conclude that the squared return series follow the long memory process.

Other studies dealing with commodity markets also attain almost identical results (Aloui and Mabrouk, 2010; Cheong, 2009; Choi and Hammoudeh, 2009; Mohammadi and Su, 2010; Wei et al., 2010).

To sum up, according to the long memory test results, the most appropriate method to model and forecast of commodity volatility is the GARCH-class models allowing for long memory property.

Table 2: Long Memory Test for Gold return and gold squared return

RETURN	Hurst-mandelbrot R/S	Lo R/S	GPH	GSP
d parameter	-	-	0.000377 (0.0163172)	-0.0118 (0.0116342)
Test Statistics	1.46717	1.47824		
Critical values			Probability	Probability
90%	[0.861, 1.747]		[0.9815]	[0.3088]
95%	[0.809, 1.862]			
99%	[0.721, 2.098]			
SQUARED RETURN	Hurst-mandelbrot R/S	Lo R/S	GPH	GSP
d parameter	-	-	0.0702662 (0.0163172)	0.125672 (0.0116342)
Test Statistics	4.51543	4.29048		
Critical values			Probability	Probability
90%	[0.861, 1.747]		[0.0000]	[0.0000]
95%	[0.809, 1.862]			
99%	[0.721, 2.098]			

Table 3: Long Memory Test for Copper return and Copper squared return

RETURN	Hurst-mandelbrot R/S	Lo R/S	GPH	GSP
d parameter	-	-	-0.0205351 (0.0163172)	-0.022516 (0.0116342)
Test Statistics	1.39618	1.44774		
Critical values			Probability	Probability
90%	[0.861, 1.747]		[0.2082]	[0.0529]
95%	[0.809, 1.862]			
99%	[0.721, 2.098]			
SQUARED RETURN	Hurst-mandelbrot R/S	Lo R/S	GPH	GSP
d parameter	-	-	0.18836 (0.0163172)	0.193619 (0.0116342)
Test Statistics	6.95016	6.1849		
Critical values			Probability	Probability
90%	[0.861, 1.747]		[0.0000]	[0.0000]
95%	[0.809, 1.862]			
99%	[0.721, 2.098]			

Table 4 : Long Memory Test for Silver return and Silver squared return

	Hurst-mandelbrot R/S	Lo R/S	GPH	GSP
d parameter	-	-	0.0092(0.0163172)	0.002(0.0116342)
Test Statistics	1.4115	1.4015		
Critical values			Probability	Probability
90%	[0.861, 1.747]		[0.5696]	[0.859]
95%	[0.809, 1.862]			
99%	[0.721, 2.098]			
	Hurst-mandelbrot R/S	Lo R/S	GPH	GSP
d parameter	-	-	0.152654 (0.0163172)	0.1494 (0.0116342)
Test Statistics	4.83328	4.4280		
Critical values			Probability	Probability
90%	[0.861, 1.747]		[0.0000]	[0.0000]
95%	[0.809, 1.862]			
99%	[0.721, 2.098]			

Table 5: Long Memory Test for Crude oil return and Crude oil squared return

	Hurst-mandelbrot R/S	Lo R/S	GPH	GSP
d parameter	-	-	-0.0085 (0.0163172)	-0.0220 (0.0116342)
Test Statistics	1.3682	1.402		
Critical values			Probability	Probability
99%	[0.861, 1.747]		[0.6002]	[0.0585]
95%	[0.809, 1.862]			
99%	[0.721, 2.098]			
	Hurst-mandelbrot R/S	Lo R/S	GPH	GSP
d parameter	-	-	0.16218 (0.0163172)	0.181869 (0.0116342)
Test Statistics	4.8752	4.392		
Critical values			Probability	Probability
90%	[0.861, 1.747]		[0.0000]	[0.0000]
95%	[0.809, 1.862]			
99%	[0.721, 2.098]			

Table 6: Long Memory Test for Palladium return and Palladium squared return

	Hurst-mandelbrot R/S	Lo R/S	GPH	GSP
d parameter	-	-	0.01508 (0.0163172)	-0.0048 (0.0116342)
Test Statistics	1.352	1.348		
Critical values			Probability	Probability
90%	[0.861, 1.747]		[0.3551]	[0.6790]
95%	[0.809, 1.862]			
99%	[0.721, 2.098]			
	Hurst-mandelbrot R/S	Lo R/S	GPH	GSP
d parameter	-	-	0.141 (0.0163172)	0.139 (0.0116342)
Test Statistics	3.654	3.391		
Critical values			Probability	Probability
90%	[0.861, 1.747]		[0.0000]	[0.0000]
95%	[0.809, 1.862]			
99%	[0.721, 2.098]			

Table 7: Long Memory Test for Platinium return and Platinium squared return

	Hurst-mandelbrot R/S	Lo R/S	GPH	GSP
d parameter	-	-	0.0114 (0.0163172)	0.00483 (0.0116342)
Test Statistics	1.605	1.598		
Critical values			Probability	Probability
90%	[0.861, 1.747]		[0.4832]	[0.6780]
95%	[0.809, 1.862]			
99%	[0.721, 2.098]			
	Hurst-mandelbrot R/S	Lo R/S	GPH	GSP
d parameter	-	-	0.204 (0.0163172)	0.171 (0.0116342)
Test Statistics	4.586	4.190		
Critical values			Probability	Probability
90%	[0.861, 1.747]		[0.0000]	[0.0000]
95%	[0.809, 1.862]			
99%	[0.721, 2.098]			

4.3. GARCH models

The GARCH (Bollerslev, 1986) models assume that the market variance is based on both past conditional market variance and past market shocks. Generalized Auto Regressive Conditional Heteroskedasticity GARCH (p, q) process is given by:

$$R_t = \alpha_0 + \sum_{i=1}^{k} \beta_i X_i + \sum_{j=1}^{h} \psi_j R_{t-j} + \varepsilon_t \tag{8}$$

where $\varepsilon_t \left| \Omega_{t-1} \sim N(0, h_t) \right.$

$$h_t = \omega + \sum_{i=1}^{p} \beta_i h_{t-i} + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2 \tag{9}$$

where $h_t = \sigma_t^2 \left| \Omega_{t-1} \right.$

The parameters in this model should satisfy $\omega > 0, \alpha > 0, \beta \geq 0, \left(\sum_{i=1}^{p} \alpha_i + \sum_{i=1}^{q} \beta_i < 1 \right)$. ε_t represents disturbance term for the mean equation, R_t denotes the return of the asset at time t, and X's are explanatory variables.

Equation (8) is the mean equation while Equation (9) is the conditional variance equation. Note that in the GARCH model the parameters are restricted to be strictly non-negative in order to satisfy the positive variance condition. Therefore GARCH models give information about the magnitude of the shock but not about its sign.

4.3.1.Fractionally Integrated Asymmetric Power ARCH (FIAPARCH) Model

Tse (1998) has proposed the fractionally integrated asymmetric power (FIAPARCH) model, which allows for long memory and asymmetry features in the conditional variance. The FIAPARCH model can be written as follows:

$$h_t^{\delta/2} = \omega\left(1-\beta L\right)^{-1} + \left[1-\left(1-\beta L\right)^{-1}\left(1-\lambda L\right)\left(1-L\right)^d\right]\left(\left|\varepsilon_t\right|-\gamma\varepsilon_t\right)^{\delta} \qquad (11)$$

where $\omega \succ 0, \delta > 0, \beta \prec 1, and\, \lambda < 1$.

In the above equation (11) if the fractional integration parameter d is between zero and 1 ($0 \leq d \leq 1$), then volatility displays the long memory property. The asymmetric parameter γ satisfies the condition $-1 < \gamma < 1$. If $\gamma > 0$ it means that negative shocks are more effective on volatility than positive shocks of equal dimension. When $\gamma = 0$ and $\delta = 2$, the FIAPARCH model reduces to the FIGARCH model, and when d = 0 it degrades to the APARCH model.

Conrad and Haag (2006) have created necessary and sufficient conditions for the positivity of the conditional variance in the FIGARCH model. According to Conrad et al. (2011), non-negativity condition for the conditional variance h_t is sufficient for all t when $\gamma > -1$ and the parameter combination (λ, d, β) satisfies the inequality constraints, which are:

(i) even if all parameters are nonnegative, the conditional variance can become negative and

(ii) even if all parameters are negative (apart from d), the conditional variance can be nonnegative almost surely.

4.3.2. Hyperbolic GARCH (HYGARCH) Model

Hyperbolic GARCH is derived by Davidson (2004). The hyperbolic GARCH (HYGARCH) model extends the conditional variance of the FIGARCH model by introducing weights into the difference operator. The HYGARCH model allows for modeling long memory property in conditional volatility with hyperbolic convergence rates. The HYGARCH (1,d,1) model can be written as follows:

$$h_t = \omega + \left[1-\left(1-\beta L\right)^{-1}\lambda L\left\{1+\alpha\left(\left(1-L\right)^d-1\right)\right\}\right]\varepsilon_t^2 \qquad (12)$$

where $\omega > 0$, $\alpha \geq 0$, $\beta < 1$, $\lambda < 1$, and $0 \leq d \leq 1$.

Davidson (2004) argues that the HYGARCH model allows for the existence of the second moment and greater extremes of amplitudes and it could be considered as a more general version of FIGARCH.

In fact, the hyperbolic GARCH model can be considered as a more general version of the FIGARCH model with hyperbolic convergence rates, and permits even more extreme amplitudes than the simple IGARCH and FIGARCH models.

4.4.Choosing between models

4.4.1. Forecast evaluation

Measuring comparative performance of out of sample forecasts obtained from various GARCH type models is an essential part of an empirical study. In this paper among the various forecast evaluation criteria, Mean Absolute Error (MAE), and Theil Inequality Coefficient (TIC) are used to measure the out-of-sample forecasting performance of the various GARCH-class models.

We use the rolling forecasting methodology in order to constitute the one- and twenty-day out-of-sample forecasts of the various volatility models like previous studies (Chkili et al., Arouri et al., 2012b; Kang et al., 2009). The best forecasting GARCH-based model for forecasting the volatility of commodity returns is the one that creates the lowest prediction error.

4.4.2. The VaR and backtesting

Value at Risk is a measure that determines the potential loss in value of a risky asset or portfolio over a certain period of time for a given confidence level. The significance level α (confidence level 1-α) and the risk horizon (h), which is the period of time in terms of trading days, constitute two main parameters of VaR. According to the Basel II Accord, banks should measure VaR at the 99 % confidence level and use internal VaR models in order to determine their market risk capital requirement.

Like other financial assets, commodities are sensitive to market-oriented fluctuations. Because of this, investors who hold a portfolio of commodities are interested in measuring market risk of their portfolio (Chikili et al, 2014). VaR is an attractive risk metric because it measures not only the risk factors but also the sensitivity to risk factor. One of the most important characteristics of VaR is its universality as applicable to all activities and to all type of risk (Carol, 2008).

While an attractive risk metric, the Value-at-Risk has however some drawbacks. One of these is that it is not a coherent measure of risk from the viewpoint of Artzner et al. (1999). A coherent risk measure should satisfy the four axioms of translation invariance, subadditivity, positive homogeneity, and monotonicity (Artzner et al. 1999). VaR can identify extreme events, however it need not be sub-additive which means the total risk on a portfolio should never exceed the sum of individual risks. In order to deal with this shortcoming "Expected Shortfall" is used as a measure of risk (Scaillet, 2000).

Expected shortfall is a coherent measure for such risk and it is used to predict the expected value of the losses conditional on the loss larger than the VaR (Scaillet, 2004). ESF can be defined as follows:

$ESF_t = E(|L_t| > |VaR_t|)$, where L_t is the expected value of loss if a VaR_t violation occurs.

In this study, both long and short trading positions are taken into account and the VaRs are estimated for the GARCH, IGARCH, FIAPARCH, and FIAPARCH models under skewed student distribution. The daily VaR for long and short trading positions at time t can be calculated as

$VaR_{L,t} = \hat{\mu}_t + z_\alpha \hat{\sigma}_t$ and $VaR_{S,t} = \hat{\mu}_t + z_{1-\alpha} \hat{\sigma}_t$

Where z_α represents the left quantile at α percent of the normal distribution and $z_{1-\alpha}$ is the right quantile at α percent. $\hat{\mu}_t$ denotes to the estimated daily conditional mean whereas $\hat{\sigma}_t$ represents the estimated standard deviation of the commodity returns obtained from a GARCH-class model.

The daily VaR's for the skewed-Student distribution for long and short positions is given by

$VaR_{L,t} = \hat{\mu}_t + skst_\alpha \hat{\sigma}_t$ and $VaR_{S,t} = \hat{\mu}_t + skst_\alpha \hat{\sigma}_t$

VaR estimations should be backtested for their reliability and consistency. This testing method enables us to compare actual profits and losses with projected VaR. The most widely known test is Kupiec's (1995) POF-test that examines the frequency of losses in excess of VaR. This test defined as a likelihood ratio test (LR) which examines whether the failure rate of the model is statistically equal to the expected one.

$N = \sum_{t=1}^{T} I_t$ is the number of exceptions in the sample size T. Then

$$I_{t+1} = \begin{cases} 1 \text{ if } r_{t+1} < \mathrm{VaR}_{t+1|t(\boldsymbol{\alpha})} \\ 0 \text{ if } r_{t+1} \geq \mathrm{VaR}_{t+1|t(\boldsymbol{\alpha})} \end{cases}$$

follows a binomial distribution, $N \sim B(T,\alpha)$. If $p = E(N/T)$ is the expected exception frequency, then the hypothesis for testing whether the failure rate of the model is equal to the expected one is expressed as follows: $H_0 : \boldsymbol{\alpha} = \boldsymbol{\alpha}_0$ and is $\boldsymbol{\alpha}_0$ the prescribed VaR level.

Under the null hypothesis, the Kupiec's (1995) likelihood ratio test is given by

$$LR_{uc} = -2\log\left\{\boldsymbol{\alpha}_0^N (1-\boldsymbol{\alpha}_0)^{T-N}\right\} + 2\log\left\{\left(\frac{N}{T}\right)^N \left(1-\left(\frac{N}{T}\right)\right)^{T-N}\right\}$$

and is asymptotically $\chi^2(1)$ chi squared distributed with one degree of freedom. Therefore, the model will be favored for VaR prediction which exhibits the property that the unconditional coverage measured by p=E(N/T) equals the desired coverage level p_0.

5.Empirical Findings

5.1. Estimates of GARCH-type models

In this section, the findings obtained from different GARCH models are presented. Since normal distribution hypothesis is rejected for all returns, student t distribution is used to model volatility.

GARCH estimation results for gold are illustrated in Table 8. As seen, for all models, d-FIGARCH coefficients are statistically significant at 1% level, which implies the existence of long memory in gold return.

The conditional volatility of gold returns reacts asymmetrically to shocks since the APARCH (γ) parameter is negative and significant. This implies that positive and negative shocks have an asymmetric impact on conditional volatility. Positive shocks have an impact on the conditional volatility more than negative shocks. Among all models with different estimation technique and distributions, FIAPARCH (1, 0.894, 1) appears to be the best model according to AIC.

Table 8: Estimation results of GARCH methods for gold

Estimation Method	GARCH	IGARCH	FIGARCH	FIAPARCH	FIAPARCH skewed	HYGARCH
Cst(M)	0.058***	0.058	0.055***	0.061***	0.052***	0.056***
Cst(V)	0.011***	0.007***	0.012***	0.017***	0.016***	0.018***
d-Figarch			0.903***	0.894***	0.89***	0.923***
ARCH(Alpha1)	0.037***	0.041***	0.028***	0.015	0.014	0.017
GARCH(Beta1)	0.955***	0.958***	0.929***	0.925***	0.92***	0.928***
APARCH(Gamma1)				-0.284***	-0.28	
APARCH(Delta)				1.556***	1.54***	
Student(DF)	4.769***	4.372***	4.359***	4.874***		4.745***
Asymmetry					-.028	
Tail					4.94***	
Log Alpha (HY)						-0.011*
No. Observations	3693	3693	3693	3693	3693	3693
No. Parameters	5	4	6	8	9	7
Log Likelihood	-5348.61	-5350.9	-5346.43	-5335.9	-5335	-5344.2
AIC	2.899	2.900	2.898	2.894	2.894	2.898
SW	2.907	2.906	2.908	2.907	2.909	2.909
SB	2.899	2.900	2.898	2.894	2.894	2.898
H-quinn	2.902	2.902	2.902	2.898	2.899	2.902
JB	4520***	5287***	8672.9	12759	12986	7010.9
Nyblom stability test	2.032	1.894	2.157	2.64	2.86	2.3
Pearson (50)	387.9***	359.1***	387.8***	370.78***	409.42***	378.87***

Note: *, ** and *** indicate that statistics are significance at the 10%, 5% and 1% level of significant respectively.

In Table 9, the estimated GARCH models for copper returns are compared. Similarly for all models, d-FIGARCH coefficients are statistically significant at 1% level implying the existence of long memory in copper return.

ARCH and GARCH coefficients are also highly significant for all models except IGARCH model. GARCH coefficient is generally greater than 0.9 which implies highly persistence nature of volatility. GARCH coefficients of the FIGARCH, FIAPARCH, and HYGARCH models decrease gradually because these models take into consideration of long memory property in return series.

In addition, the conditional volatility of copper return reacts asymmetrically to shocks since the APARCH (γ) parameter is positive and significant. Therefore, negative shocks have impacts on volatility more than positive shocks. The APARCH (δ) power coefficient is also positive and significant.

Student t distribution reveals better performance than the normal distribution and the skewed student-t distribution since the t-statistics of the Student DF parameter is significant at 1% level for all estimation methods.

According to AIC, FIAPARCH (1, 0.419, 1) can be chosen as to be the most appropriate model.

Table 9: Estimation results of GARCH methods for copper

	GARCH	IGARCH	FIGARCH	FIAPARCH	FIAPARCH	HYGARCH
Estimation Method					Skewed	
Cst(M)	0.036***	0.035*	0.0358*	0.029	0.02	0.036*
AR(1)	-0.069***	-0.069***	-0.067***	-0.067***		-0.06***
Cst(V)	0.014***	0.01***			0.05**	
d-Figarch			0.492***	0.419***	0.49***	0.439***
ARCH(Alpha1)	0.05***	0.052***	0.262***	0.251***	0.22***	0.268***
GARCH(Beta1)	0.945***	0.947	0.717***	0.635***	0.68***	0.686***
APARCH(Gamma1)				0.124**	0.13**	
APARCH(Delta)				2.137***	1.93***	
Student(DF)	6.863***	6.528***	7.648***	7.008***		6.78***
Asymmetry					-.015	
Tail					7.44***	
Log Alpha (HY)						0.04*
No. Observations	3693	3693	3693	3693	3693	3693
No. Parameters	6	5	6	8	9	7
Log Likelihood	-6781.97	-6782.71	-6786.99	-6780.13	-6788.6	-6783.2
AIC	3.676	3.675	3.678	3.676	3.681	3.677
SW	3.686	3.684	3.688	3.689	3.690	3.689
SB	3.676	3.675	3.678	3.676	3.681	3.677
H-quinn	3.679	3.678	3.682	3.681	3.686	3.681
JB	677.9	693.4	709.8	680.7	625.8	690.61
Nyblom stability test	2.518	2.29	2.54	4.946	4.5	2.855
Pearson (50)	100.29***	97.29***	104.71***	104.92***	260.8***	97.45***

Note: *, ** and *** indicate that statistics are significance at the 10%, 5% and 1% level of significant respectively.

The results for silver and crude oil are illustrated in Table 10 and 11. The best model for silver is FIAPARCH (1, 0.414, 1) while it is FIAPARCH (1, 0.46, 1) with skewed student distribution for crude oil. The conditional volatility of silver returns reacts asymmetrically to shocks since the APARCH (γ) parameter is negative and significant implying that positive shocks have more impacts on conditional volatility than negative shocks. The power coefficient APARCH (δ) is also positive and significant. On the other hand, asymmetric volatility parameter (γ) is positive and significant for crude oil volatility estimation in Table 11. This result reveals that past negative shocks are more effective than past positive shocks on current conditional volatility.

Table 10: Estimation results of GARCH methods for silver

	GARCH	IGARCH	FIGARCH	FIAPARCH	FIAPARCH	HYGARCH
Estimation Method					skewed	
Cst(M)	0.086***	0.086***	0.081***	0.092***	0.057**	0.085***
Cst(V)	0.024**	0.022***			0.05	
d-Figarch			0.452***	0.414***	0.47***	0.345***
ARCH(Alpha1)	0.04***	0.041***	0.416***	0.408***	0.37***	0.478***
GARCH(Beta1)	0.958***	0.958	0.777***	0.742***	0.77***	0.761***
APARCH(Gamma1)				-0.21***	-0.2***	
APARCH(Delta)				2.341***	2.17***	
Student(DF)	3.774***	3.696***	4.672***	3.934***		3.818***
Asymmetry					-.05***	
Tail					3.98***	
Log Alpha (HY)						0.11**
No. Observations	3693	3693	3693	3693	3693	3693
No. Parameters	5	4	5	7	9	6
Log Likelihood	-7321	-7321.1	-7327.9	-7312.9	-7309.04	-7318.72
AIC	3.967	3.967	3.971	3.964	3.96	3.966
SW	3.975	3.973	3.979	3.975	3.97	3.976
SB	3.967	3.967	3.971	3.964	3.963	3.966
H-quinn	3.970	3.969	3.974	3.968	3.96	3.970
JB	3855.1	3898.6	3118.4	3555.7	3452.7	3009.3
Nyblom stability test	2.483	2.370	2.184	3.26	4.36	2.67
Pearson (50)	386.05***	384.59***	358.7***	360.98***	558.48***	348.33***

Note: *, ** and *** indicate that statistics are significance at the 10%, 5% and 1% level of significant respectively.

Table 11: Estimation results of GARCH methods for crude oil

	GARCH	IGARCH	FIGARCH	FIAPARCH	FIAPARCH	HYGARCH
Estimation Method					skewed	
Cst(M)	0.06**	0.06**	0.066**	0.058*	0.02	0.066***
AR(1)	-0.046***	-0.046***	-0.04***	-0.048***		-0.047***
Cst(V)	0.024**	0.018***			0.076*	
d-Figarch			0.517***	0.416***	0.46***	0.459***
ARCH(Alpha1)	0.053***	0.055***	0.359***	0.406***	0.37***	0.384***
GARCH(Beta1)	0.943***	0.944	0.774***	0.718***	0.72***	0.752***
APARCH(Gamma1)				0.296***	0.34***	
APARCH(Delta)				1.98***	1.76***	
Student(DF)	7.064***	6.834***	7.995***	7.433***		7.06***
Asymmetry					-0.05***	
Tail					7.62***	
Log Alpha (HY)						0.04**
No. Observations	3693	3693	3693	3693	3693	3693
No. Parameters	6	5	6	8	9	7
Log Likelihood	-7863.4	-7863.85	-7867.6	-7852.6	-7854.6	-7862.8
AIC	4.261	4.261	4.264	4.257	4.25	4.262
SW	4.271	4.269	4.274	4.270	4.27	4.273
SB	4.261	4.261	4.264	4.257	4.25	4.262
H-quinn	4.265	4.264	4.267	4.261	4.26	4.266
JB	933.05	940.73	1000.3	942.27	901.5	999.83
Nyblom stability test	2.457	1.712	2.18	2.94	3.55	2.68
Pearson (50)	128.27***	132.68***	123.9***	124.1***	235.1***	129.29***

Note: *, ** and *** indicate that statistics are significance at the 10%, 5% and 1% level of significant respectively.

GARCH estimation results for palladium is illustrated in Table 12. As seen for all models, d-FIGARCH coefficients are statistically significant at 1% level implying the existence of long memory in palladium return. On the other hand, the APARCH (γ) parameter is positive and insignificant. Here, neither positive nor negative shocks have the asymmetric effect on conditional volatility. The APARCH (δ) power coefficient is also positive and significant. According to AIC, FIAPARCH model could be chosen to be the most appropriate model. Regarding the log likelihood, FIAPARCH model with skewed student-t distributed innovations presents the most accurate model.

Table 12: Estimation results of GARCH methods for Palladium

	GARCH	IGARCH	FIGARCH	FIAPARCH	FIAPARCH	HYGARCH
Estimation Method					skewed	
Cst(M)	0.022	0.022	0.025	0.023	0.004	0.026
Cst(V)	0.059***	0.058***	0.109***	0.109***	0.109***	0.078
d-Figarch			0.578***	0.571***	0.571***	0.53***
ARCH(Alpha1)	0.10***	0.10***	0.27***	0.289***	0.288***	0.297***
GARCH(Beta1)	0.899***	0.899***	0.709***	0.708***	0.708***	0.694***
APARCH(Gamma1)				0.043	0.044	
APARCH(Delta)				1.98***	1.98***	
Student(DF)	3.985***	3.97***	4.19***	4.19***		4.036***
Asymmetry					-.031*	
Tail					4.19***	
Log Alpha (HY)						0.035
No. Observations	3693	3693	3693	3693	3693	3693
No. Parameters	5	4	6	8	9	7
Log Likelihood	-7383.2	-7383.2	-7376.6	-7376.6	-7375	-7376.1
AIC	4	4	4	3.998	3.998	3.998
SW	4	4	4.008	4.012	4.014	4.010
SB	4	4	3.998	3.998	3.998	3.998
H-quinn	4	4	4	4.003	4	4.002
JB	16344	16576	15382	14914	14909	18114
Nyblom stability test	2.241	2	3	4.479	4.94	3.44
Pearson (50)	1704.25***	1703.8***	1713.29***	1717.04***	1709.55***	1706.95***

Note: *, ** and *** indicate that statistics are significance at the 10%, 5% and 1% level of significant respectively.

In Table 13, various GARCH estimation results for platinum return are displayed. The results show evidence of long memory in platinum return. The APARCH (δ) power coefficient is also positive and significant. However, the APARCH (γ) parameter is negative and insignificant. Here it couldn't be mentioned the asymmetric effects of positive and negative shocks. According to information criteria and log likelihood ratio, FIAPARCH (1,0.74,1) model with skewed student-t distributed innovations could be interpreted as the most appropriate model.

Table 13: Estimation results of GARCH methods for Platinum

	GARCH	IGARCH	FIGARCH	FIAPARCH	FIAPARCH	HYGARCH
Estimation Method					skewed	
Cst(M)	0.046***	0.046***	0.045***	0.052***	0.038**	0.03
Cst(V)	0.029***	0.019***	0.049**	0.049	0.051	0.06
d-Figarch			0.592***	0.767***	0.746***	0.764***
ARCH(Alpha1)	0.07***	0.07***	0.274***	0.177	0.185	0.177
GARCH(Beta1)	0.916***	0.922***	0.75***	0.841***	0.832***	0.816***
APARCH(Gamma1)				-0.039	-0.042	
APARCH(Delta)				1.432***	1.469***	
Student(DF)	5.16***	4.72***	4.9***	5.27***		
Asymmetry					-.037	-0.043**
Tail					5.3***	5.22***
Log Alpha (HY)						-0.032
No. Observations	3693	3693	3693	3693	3693	3693
No. Parameters	5	4	6	8	9	8
Log Likelihood	-5938.4	-5941.06	-5938.57	-5931.6	-5930.14	-5.935
AIC	3.218	3.219	3.219	3.216	3.216	3.218
SW	3.227	3.219	3.229	3.230	3.231	3.231
SB	3.218	3.226	3.219	3.216	3.216	3.218
H-quinn	3.221	3.222	3.222	3.221	3.221	3.223
JB	867.96	898.6	857.75	855.49	859.55	858.97
Nyblom stability test	3.728	3.368	4.505	5	5.31	5.09
Pearson (50)	200.54***	211.1***	206.6***	252.85***	443.59***	327.21***

Note: *, ** and *** indicate that statistics are significance at the 10%, 5% and 1% level of significant respectively.

We estimate FIAPARCH model both assuming Student-t and skewed Student-t distributed innovations. Asymmetric parameters are negative and statistically significant for silver, crude oil and palladium respectively at the %1, %5 and %10 level. Therefore, one may observe from these results that silver, crude oil and palladium innovations are skewed to the left. In addition, the tail parameters in all the FIAPARCH models are statistically significant and positive. This reveals that the commodity return series are fat-tailed. To sum up, FIAPARCH model with skewed student-t distributed innovations is the most accurate model for most of these commodity return series. It takes into consideration both asymmetry and long memory properties of series.

5.2. Forecast evaluation

To evaluate the forecast performance of different GARCH models, we constitute the one and twenty-day-ahead volatility forecasts over the out of sample period from February 29, 2016 through April 29, 2016 to assess the forecasting performance of the eight competing GARCH models. Theil Inequality Coefficient (TIC) and Mean Absolute Error (MAE) are reported in tables 14, 15 for the mean equation of these commodities. As can be seen from Table 14, with regard to the one-day forecasting horizon, FIAPARCH model with student distribution outperforms the other models for metals but HYGARCH model performs better for crude oil, palladium and platinum.

Table 14: Forecast Comparison of Volatility models

Model	Criteria	GOLD	COPPER	SILVER	CRUDE OIL	PALLADIUM	PLATINIUM
GARCH	MAE	0.623	0.077	0.655	3.36	0.383	1.021
	TIC	0.84	0.247	0.79	0.984	0.896	1
IGARCH	MAE	0.624	0.076	0.655	3.36	0.383	1.021
	TIC	0.842	0.242	0.791	0.984	0.897	1
FIGARCH	MAE	0.626	0.081	0.659	**3.354**	**0.379**	1.02
	TIC	0.849	0.26	0.801	**0.981**	**0.879**	1
FIAPARCH	MAE	**0.62**	**0.075**	**0.649**	3.363	0.382	1.027
	TIC	**0.833**	**0.236**	**0.779**	0.986	0.891	1
FIAPARCH	MAE	0.63	0.21	0.68	3.35	0.401	1.014
skewed	TIC	0.85	1	0.85	0.98	0.978	1
HYGARCH	MAE	0.626	0.079	0.656	**3.354**	**0.379**	**1.005**
	TIC	0.847	0.255	0.792	**0.981**	**0.879**	**1**

Notes: This table reports the results of the one-day out-of-sample prediction errors of return series for the different competing volatility models.

When looking at the results of twenty-day forecast horizon in Table 15, it can be seen that the FIAPARCH model has superiority in terms of volatility forecast accuracy for all commodity returns except that of copper and palladium. With respect to Theil Inequality Coefficient (TIC), while IGARCH model produces the lowest mean loss for copper, FIGARCH and HYGARCH model produce that of for Palladium and the latter produces the lowest mean absolute error for Platinum.

Table 15: Forecast Comparison of Volatility models

Model	Criteria	GOLD	COPPER	SILVER	CRUDE OIL	PALLADIUM	PLATINIUM
GARCH	MAE	0.989	1.043	1.189	2.849	1.63	1.396
	TIC	0.959	0.965	0.948	0.979	0.986	0.972
IGARCH	MAE	0.989	1.043	1.189	2.849	1.63	1.396
	TIC	0.959	**0.965**	0.948	0.979	0.986	0.972
FIGARCH	MAE	0.988	1.043	1.188	2.847	**1.629**	1.396
	TIC	0.961	0.966	0.95	0.977	**0.983**	0.973
FIAPARCH	MAE	**0.989**	1.043	1.19	**2.84**	1.63	1.397
	TIC	**0.957**	0.969	**0.945**	**0.98**	0.985	**0.969**
FIAPARCH	MAE	0.98	1.05	1.18	2.85	1.637	1.396
skewed	TIC	0.96	0.98	0.96	0.99	0.997	0.977
HYGARCH	MAE	0.988	1.043	1.189	2.847	**1.629**	**1.395**
	TIC	0.96	0.966	0.948	0.977	**0.983**	0.981

Notes: This table reports the results of the Twenty-day out-of-sample prediction errors of return series for the different competing volatility models.

Once again, out of sample forecast analysis implies that allowing for asymmetry and long memory properties leads to enhance the quality of volatility forecasts of commodity returns. In most cases, volatility estimates show that the FIAPARCH model is favored to the other five GARCH-class models.

5.3 VaR estimations

In this part of our analysis, VaR of commodities are estimated based on three GARCH models for level α from 5% to 0.25%. Besides we compute the failure rate for the long trading position as a percentage of negative returns smaller than one-step-ahead VaR for long positions. For the short trading position, the failure rate is defined as the percentage of positive returns larger than the one-step-ahead VaR for short positions.

Table 16 shows the in sample VaR estimations based on GARCH, FIGARCH, and FIAPARCH models. In this respect, the Kupiec LR tests are carried out for each of these GARCH models, which investigate whether the empirical failure rate is equal to the pre-specified VaR level α . The other risk measures Expected Shortfalls (ESF) are presented for long and short positions and labelled ESF1 and ESF2 respectively in Table 17-18. Hendricks (1996) defines the ESF1 as the excess value of the losses over the VaR, the ESF2 as the expected value of loss exceeding the VaR level, divided by the associated VaR values.

According to our results, expected shortfalls are the highest for crude oil and the lowest for gold for all risk levels. The Kupiec test statistics are highly significant at all levels (1%, 5%, and 10%) in most cases. This result suggests that the standard GARCH model performs inadequately at all events regardless of the commodity analyzed. For the short trading position, the use of the FIGARCH model for Copper, Silver, and Crude oil returns performs well. On the other hand, for these three commodity return series, the Kupiec test statistics are significant at three conventional levels for the long trading positions with the FIGARCH model. However, the Kupiec test statistics are insignificant for all commodities and for long trading position at % 1 level of significance with skewed t distribution FIAPARCH model. More precisely the findings show that the FIAPARCH model with skewed student distributed innovations outperforms the other methods for long position and it has more or less similar performance with FIGARCH model for short trading positions. It can be concluded that in the models which take into account volatility clustering, asymmetry and long memory in the commodity returns, VaR and ESF are more accurate measure of risk for both long and short trading positions.

Table 16: Kupiec LR test statistics based on in sample VaR

	Short positions					Long positions			
α quantile	GARCH	FIGARCH	FIAPARCH	FIAPARCH skewed	α quantile	GARCH	FIGARCH	FIAPARCH	FIAPARCH skewed
GOLD					GOLD				
0.95	0.2274	0.0103	0.0024	0.39	0.05	7.105***	2.991*	7.885***	4.682**
0.975	0.9993	3.998**	1.766	0.61	0.025	6.244***	1.984	6.74***	3.27*
0.99	15.214***	15.214***	10.694***	10.69***	0.01	0.9572	0.0238	3.613**	1.66
0.995	6.018***	11.483***	7.576***	6.01***	0.005	1.184	2.598	0.6984	1.81
0.9975	5.7308***	12.038***	8.36***	3.78**	0.0025	0.1728	0.5908	0.59088	1.29
COPPER					COPPER				
0.95	0.1242	1.483	0.391	0.39	0.05	0.993	4.078**	0.6001	0.00069
0.975	0.612	1.006	0.005	0.14	0.025	1.455	4.437**	0.637	0.078
0.99	2.97*	0.023	1.856	1.85	0.01	0.957	8.588***	2.087	0
0.995	7.576***	2.598	6.018***	1.18	0.005	0.011	2.091	0.011	0.011
0.9975	3.78**	3.78**	3.78**	3.78*	0.0025	1.366	3.033*	0.319	0.31
SILVER					SILVER				
0.95	0.78963	1.8621	2.2794	0.015	0.05	5.662***	15.471***	8.7031***	3.24*
0.975	6.614***	0.2389	4.9591**	0.019	0.025	4.862**	27.275***	6.7408***	2.27
0.99	10.694***	0.24122	7.1297***	2.97*	0.01	0.4374	8.5883***	2.5534	0.0238
0.995	7.5768***	1.1845	9.3862***	3.54**	0.005	1.038	9.3092***	1.038	0.015
0.9975	.NaN	3.7809**	12.038***	8.36***	0.0025	0.0622	6.521***	0.75932	0.319
CRUDE OIL					CRUDE OIL				
0.95	2.521	0.107	0.789	0.72	0.05	4.078**	11.38***	1.309	.015
0.975	2.762*	0.148	2.762*	0.124	0.025	1.221	8.334***	2.277	.078
0.99	8.22***	0.696	7.129***	2.37	0.01	0.437	4.206**	0.673	0.000133
0.995	6.018***	4.683**	7.576***	3.54*	0.005	0.64	4.269***	2.74*	0.334
0.9975	0.59	0.172	1.296	1.29	0.0025	1.366	5.237**	3.033*	.759
PALLADIUM					PALLADIUM				
0.95	0.00069	0.227	0.49	2.067	0.05	0.063	1.146	0.993	0.227
0.975	0.211	0.321	0.612	0.005	0.025	0.483	0.637	0.637	0.03
0.99	2.974*	2.379	2.974*	1.017	0.01	1288	5.513***	4.206**	1.665
0.995	0.698	0.12	0.0118	0.011	0.005	0.011	0.64	1.038	0.015
0.9975	2.336	0.5908	0.5908	0.172	0.0025	0.062	0.319	0.319	0.319
PLATINIUM					PLATINIUM				
0.95	1.832	2.777*	1.627	.015	0.05	4.078	2.991*	4.998**	2.508
0.975	2.404	5.481***	1.485	.321	0.025	2.277	.238	2.277	.350
0.99	1.017	2.974*	.696	.103	0.01	2.087	.673	2.553	.673
0.995	1.184	3.549***	1.814	.346	0.005	.334	.015	1.038	.124
0.9975	.590	1.296	.590	.172	0.0025	.172	1.296	0	5.730***

Notes: The table reports the Kupiec test statistics. *, ** and *** indicate that statistics are significance at the 10%, 5% and 1% level of significant respectively. The best model is the one with the least rejections.

Table 17: Expected Shortfalls based on in-sample Value-at-Risk

Short positions								
α quantile	GARCH	GARCH	FIGARCH	FIGARCH	FIAPARCH	FIAPARCH	FIAPARCH skewed	FIAPARCH skewed
	ESF1	ESF2	ESF1	ESF2	ESF1	ESF2	ESF1	ESF2
GOLD								
0.95	2.42	1.3	2.38	1.28	2.41	1.3	2.39	1.31
0.975	2.82	1.2	2.82	1.19	2.88	1.22	2.84	1.23
0.99	3.95	1.22	3.9	1.17	3.9	1.19	3.9	1.23
0.995	4.55	1.1	4.49	1.11	4.64	1.14	4.43	1.15
0.9975	5.11	1.02	3.66	1	5.84	1.07	5.32	1.06
COPPER								
0.95	3.53	1.3	3.4	1.31	3.43	1.3	3.48	1.31
0.975	4.1	1.21	3.98	1.23	3.96	1.22	4.07	1.21
0.99	5.07	1.13	4.69	1.15	4.83	1.16	5.01	1.17
0.995	5.71	1.11	4.83	1.12	5.16	1.14	5.02	1.1
0.9975	7.14	1.04	7.14	1.12	7.15	1.08	7.14	1.1
SILVER								
0.95	4.17	1.34	3.91	1.37	4.17	1.36	4.07	1.37
0.975	5.05	1.28	4.55	1.29	5	1.27	4.76	1.26
0.99	6.07	1.18	5.37	1.22	5.67	1.18	5.49	1.19
0.995	8	1.06	6.61	1.16	8.05	1.1	6.91	1.11
0.9975	.NaN	.NaN	5.48	1.08	4.7	1.03	9.18	1.07
CRUDE OIL								
0.95	4.9	1.31	4.78	1.31	4.86	1.29	4.76	1.29
0.975	5.91	1.24	5.56	1.24	5.68	1.23	5.61	1.24
0.99	7.31	1.29	6.63	1.22	7.2	1.24	6.93	1.23
0.995	8.86	1.38	8.82	1.37	9.14	1.34	8.16	1.29
0.9975	9.66	1.28	9.14	1.27	9.62	1.26	9.62	1.31
PALLADIUM								
0.95	4.23	1.44	4.21	1.43	4.18	1.43	4.12	1.44
0.975	5.15	1.37	5.24	1.39	5.27	1.4	5.09	1.4
0.99	6.56	1.42	6.54	1.43	6.55	1.45	6.34	1.43
0.995	7.21	1.37	7.1	1.35	6.99	1.35	6.99	1.39
0.9975	8.24	1.58	8.67	1.45	8.67	1.47	8.06	1.45
PLATINIUM								
0.95	2.87	1.38	2.81	1.38	2.9	1.39	2.82	1.39
0.975	3.47	1.33	3.48	1.34	3.44	1.33	3.37	1.34
0.99	4.16	1.28	4.17	1.29	4.26	1.28	4.12	1.29
0.995	4.98	1.29	5.27	1.31	5.14	1.31	4.79	1.29
0.9975	5.4	1.29	5.52	1.27	5.4	1.28	5.78	1.29

Table 18: Expected Shortfalls based on in-sample Value-at-Risk

Long positions								
α quantile	GARCH	GARCH	FIGARCH	FIGARCH	FIAPARCH	FIAPARCH	FIAPARCH skewed	FIAPARCH skewed
	ESF1	ESF2	ESF1	ESF2	ESF1	ESF2	ESF1	ESF2
GOLD								
0.05	-2.46	1.45	-2.5	1.46	-2.46	1.45	-2.49	1.45
0.025	-3.01	1.35	-3.02	1.36	-2.95	1.36	-3.01	1.36
0.01	-3.83	1.29	-3.85	1.31	-3.77	1.26	-3.83	1.26
0.005	-4.72	1.41	-4.81	1.44	-4.85	1.4	-4.99	1.44
0.0025	-5.55	1.39	-5.45	1.42	-5.59	1.49	-5.48	1.54
COPPER								
0.05	-3.6	1.41	-3.5	1.43	-3.54	1.4	-3.6	1.39
0.025	-4.06	1.34	-4.01	1.37	-4.1	1.34	-4.28	1.33
0.01	-4.95	1.31	-4.58	1.29	-4.64	1.28	-4.97	1.31
0.005	-5.67	1.4	-5.35	1.36	-5.5	1.39	-5.66	1.35
0.0025	-6.17	1.32	-5.87	1.37	-6.23	1.38	-6.23	1.35
SILVER								
0.05	-4.54	1.55	-4.31	1.57	-4.45	1.53	-4.61	1.51
0.025	-5.63	1.44	-5.03	1.43	-5.51	1.43	-5.73	1.42
0.01	-7.34	1.43	-6.58	1.43	-6.9	1.39	-7.45	1.43
0.005	-8.25	1.35	-7.54	1.38	-8.15	1.38	-8.74	1.38
0.0025	-9.97	1.39	-8.24	1.35	-9.51	1.33	-9.9	1.29
CRUDE OIL								
0.05	-4.6	1.38	-4.54	1.41	-4.71	1.41	-4.86	1.4
0.025	-5.39	1.34	-5.3	1.34	-5.67	1.34	-5.7	1.34
0.01	-6.01	1.31	-5.53	1.34	-6.05	1.34	-6.52	1.33
0.005	-6.93	1.3	-6.31	1.34	-6.56	1.3	-7.08	1.31
0.0025	-8.15	1.28	-7.24	1.33	-7.64	1.3	-8.27	1.31
PALLADIUM								
0.05	-4.64	1.51	-4.56	1.49	-4.57	1.49	-4.64	1.48
0.025	-5.54	1.43	-5.74	1.44	-5.74	1.43	-5.9	1.43
0.01	-6.95	1.31	-6.56	1.29	-6.69	1.29	-6.87	1.29
0.005	-8.95	1.3	-7.95	1.26	-7.89	1.25	-8.22	1.26
0.0025	-9.67	1.21	-9.88	1.22	-9.88	1.21	-9.88	1.18
PLATINIUM								
0.05	-3	1.4	-3	1.39	-2.99	1.41	-3.05	1.39
0.025	-3.73	1.33	-3.7	1.33	-3.7	1.33	-3.88	1.33
0.01	-4.57	1.23	-4.61	1.21	-4.64	1.24	-4.67	1.23
0.005	-5.39	1.18	-5.48	1.15	-5.36	1.17	-5.6	1.16
0.0025	-6.42	1.14	-6.37	1.14	-6.42	1.15	-9.05	1.35

5.4 Out of Sample Forecast Results

As mentioned above, the FIAPARCH model with skewed t seems to outperforms the others in terms of forecasting performance. Moreover, 95 % the value at risk obtained from that model passes the backtesting (Kupiec test) and confirms that value at risk captures well the losses and gains for short and long positions respectively. Based on these considerations we perform out of sample forecasts for conditional mean, conditional variance and value at risk. The findings are illustrated in figures 4-9. As seen there are major differences in volatility and VaR among markets. It seems that gold and platinum have similar volatility and VaR patterns. In both markets, volatility diminishes over the forecast period and risk of losses decreases. In contrast, the volatility of silver copper and palladium tends to increase rapidly over time. As a result, in those markets value at risk for short positions tend to increase. With respect to oil we have a somewhat different picture. As shown in figure 7 oil volatility is much higher than the others. Moreover, the dynamic value at risk plot also confirms that oil market risk is considerably large compared with other commodities.

Figure 4: Skewed-student FIAPARCH forecasting for Gold

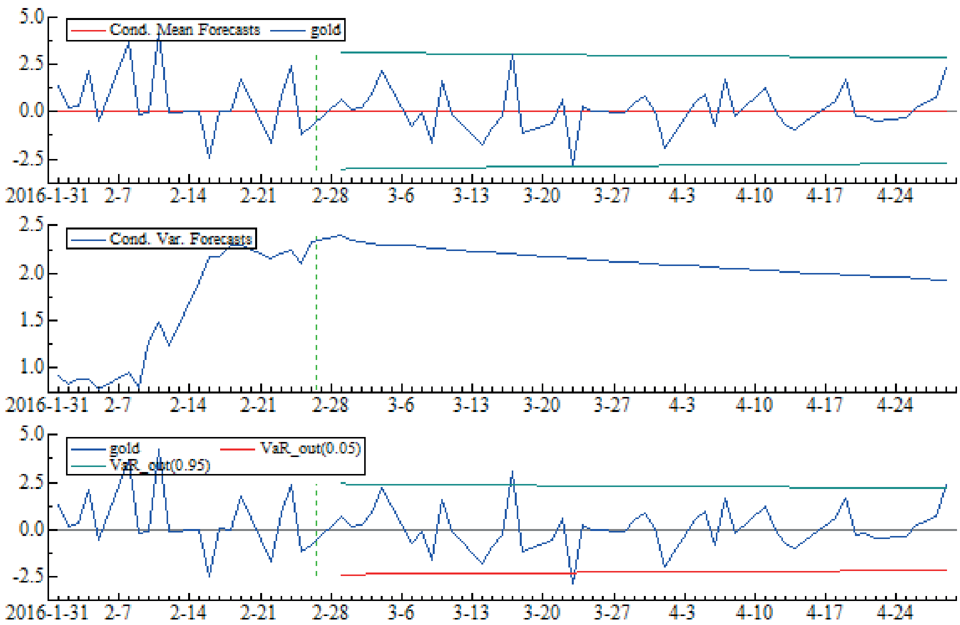


Figure 5: Skewed-student FIAPARCH forecasting for Copper

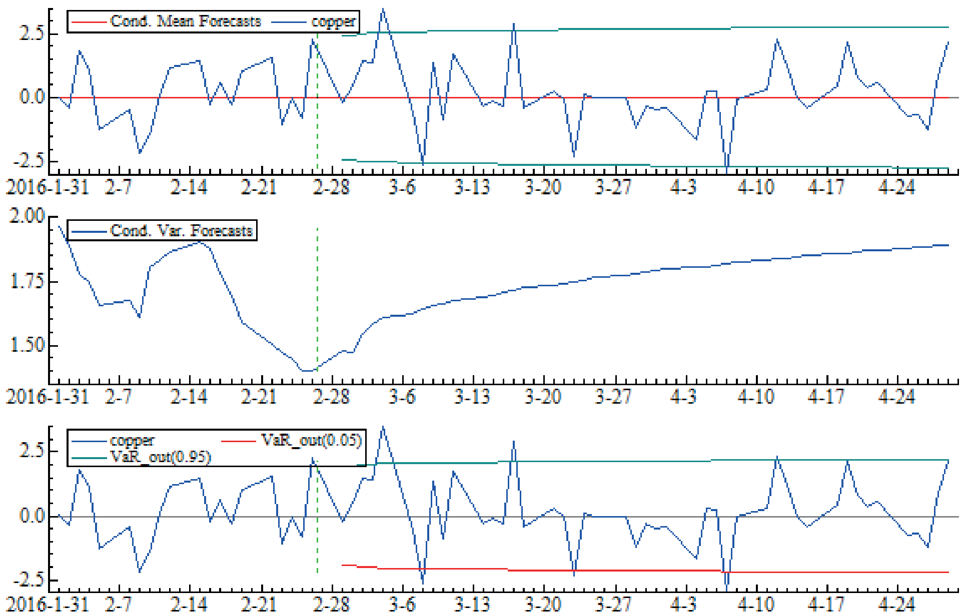


Figure 6: Skewed-student FIAPARCH forecasting for Silver

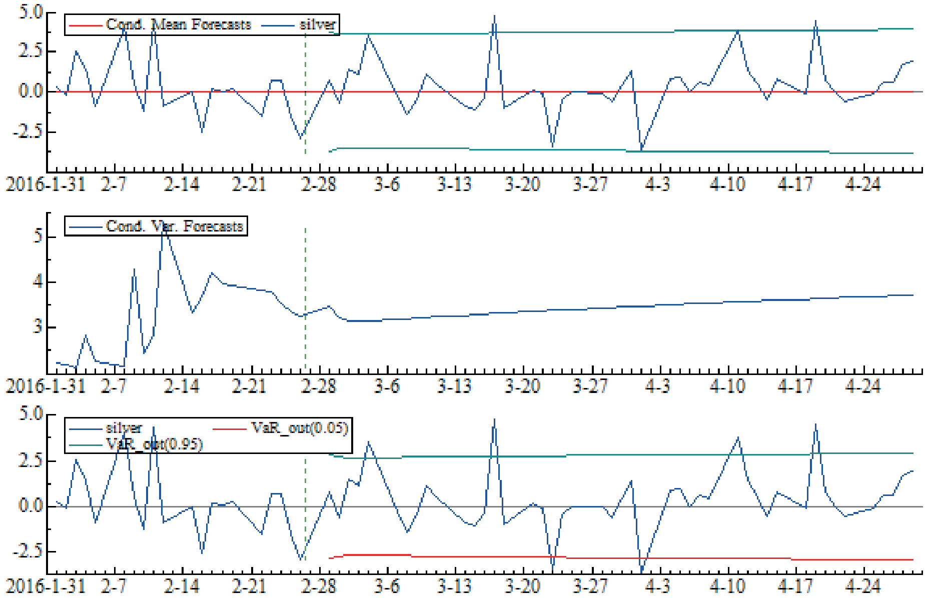


Figure 7: Skewed-student FIAPARCH forecasting for Crude oil

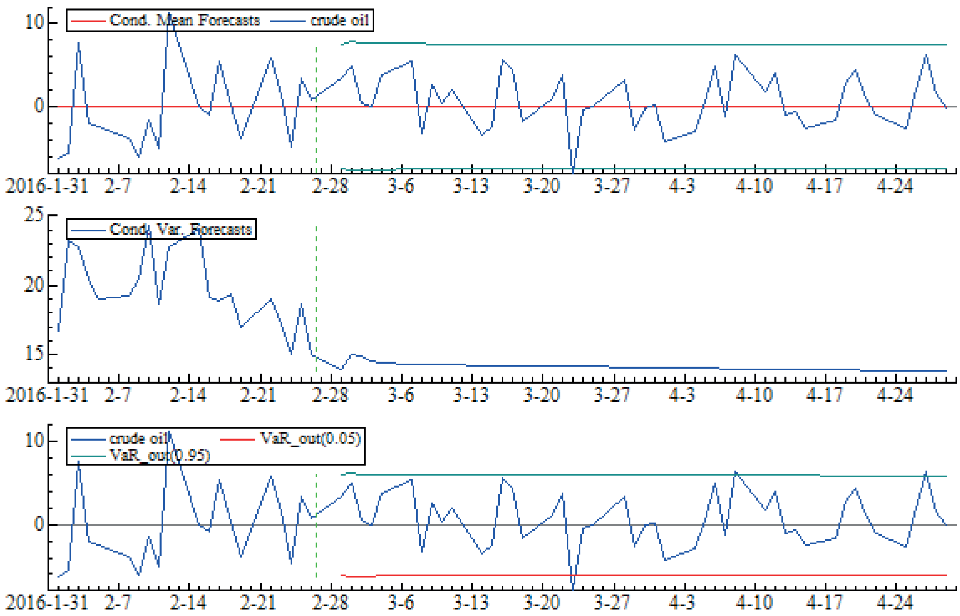


Figure 8: Skewed-student FIAPARCH forecasting for Palladium

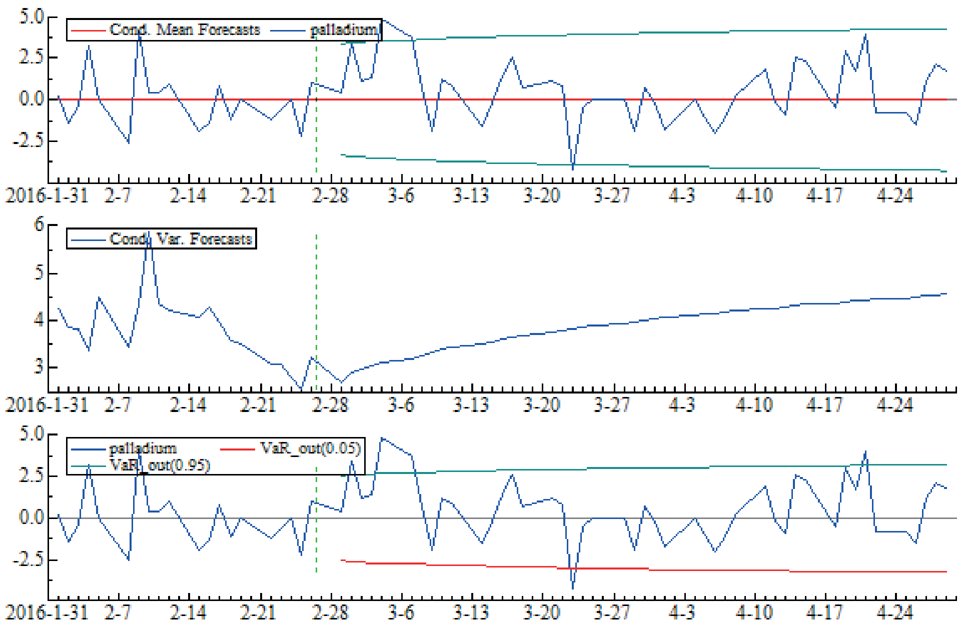
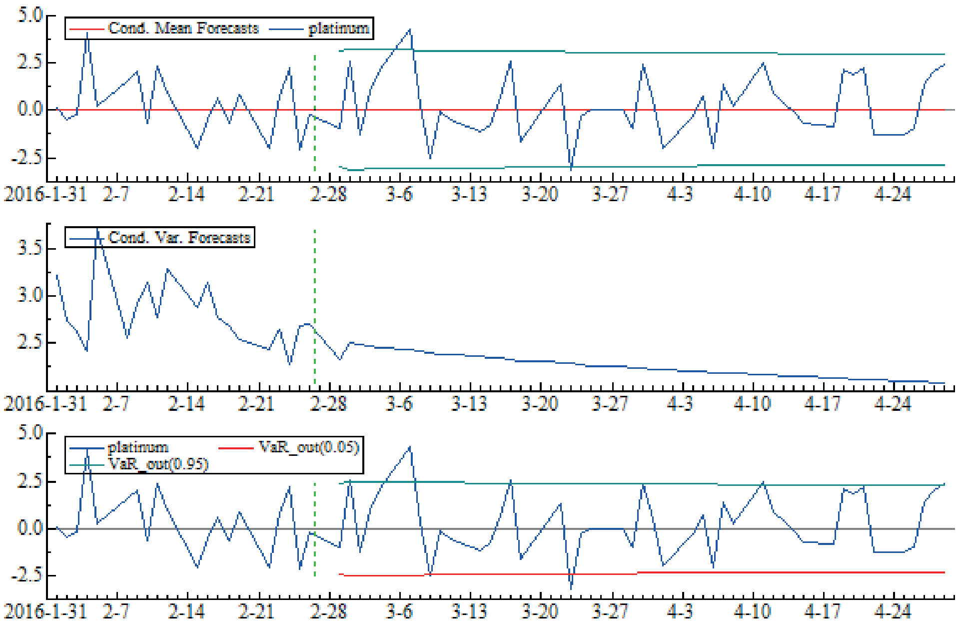


Figure 9: Skewed-student FIAPARCH forecasting for Platinum



6. Conclusion

Risk measurement of an asset is essential for investors in portfolio allocation, hedging and risk management. In this paper using various GARCH models, the riskiness of gold, silver, copper, crude oil, palladium, and platinum are modelled and forecasted. Volatility, value at risk, and expected shortfall are employed as risk measures. All risk measures are comparable as they are all based on the same GARCH model.

It turns out that among existing GARCH models those allowing for long memory and asymmetry outperform the others in terms of out of sample forecast. Moreover, it seems that those models perform better in backtesting. In other words, the value at risk and expected shortfall risk measures are more credible. More specifically for most of these commodities, FIAPARCH model appears to be the best model not only in terms of goodness of fit measures such as AIC, SW, SB and H-Quinn criteria, and Log likelihood but also in terms of predicting volatility and value at risk. It seems that taking into account asymmetry and long memory also improves the performance of out of sample forecasts. In this respect, the FIAPARCH model outperforms in most of the cases except for palladium and platinum. Other studies dealing with commodity markets also attain almost identical results. For instance Aloui and Mabrouk (2010) found that considering for long-range memory, fat-tails and asymmetry performs better in predicting a one-day-ahead VaR for both short and long trading positions. They also argue that the FIAPARCH model outperforms the other models in the VaR's prediction. Chkili et al. (2014) suggest also that the FIAPARCH model is the best suited for estimating the VaR forecasts for both short and long trading positions of commodities' returns.

The results based on FIAPARCH model(with skewed t) suggest that oil volatility is high and the risk of oil measured by value at risk and expected shortfall is significantly higher than the other commodities considered in the study. The finding is in contradiction to the findings obtained in previous studies. However, it is consistent with the reality. It is well known that oil price is very sensitive to political issues. Over recent years many political events such as the invasion of Iraq, sanctions on oil exporting countries like Iran, Russia, Venezuela and the slowdown of Chinese economic growth cause oil price to fluctuate more. Moreover, after the improvement in shell gas technology, the USA becomes an oil exporting country affecting supply and the price of oil.

The above considerations suggest that oil is no longer safe heaven and cast doubt on the use of oil as a hedging tool. The results on other commodities are more and less in accord with those obtained in earlier studies.

References

1. Aloui, C. and Mabrouk S.. (2010). Value-at-risk estimations of energy commodities via long- memory, asymmetry and fat-tailed GARCH models, Energy Policy, 38 (5). 2326–2339.

2. Arouri, M. Hammoudeh, S. Lahiani, A. Nguyen, D. K.. (2012a). Long memory and structural breaks in modeling the return and volatility dynamics of precious metals, The Quarterly Review of Economics and Finance, 52: 207–218.

3. Arouri, M. Lahiani, A,. Lévy, A. Nguyen, D. K.. (2012b). Forecasting the conditional volatility of oil spot and futures prices with structural breaks and long memory models, Energy Economics, 34 (1):283–293.

4. Baillie, R. T.. (1996). Long memory processes and fractional integration in econometrics, Journal of Econometrics, 73: 5–59.

5. Baillie, R. T. Bollerslev, T. and Mikkelsen, H. O.. (1996). „Fractionally integrated generalized autoregressive conditional heteroskedasticity, J. Econometrics, 74:3-30.

6. Bollerslev, T.. (1986). Generalized autoregressive conditional heteroscedasticity, Journal of econometrics, 31(3), 307-327.

7. Bollerslev, T. and Mikkelsen, H. O.. (1996). Modelling and pricing long memory in stock market volatility, Journal of Econometrics, 73:151–184.

8. Beine, M. and Laurent, S.. (2003). Central bank interventions and jumps in double long memory models of daily exchange rates, Journal of Empirical Finance, 10:641–660.

9. Cheng, W. H. and Hung, J. C.. (2011). „Skewness and leptokurtosis in GARCH-typed VaR estimation of petroleum and metal asset returns, Journal of Empirical Finance, 18:160–173.

10. Cheong, C. W.. (2009). Modeling and forecasting crude oil markets using ARCH-type models, Energy Policy, 37:2346–2355.

11. Chkili, W. Hammoudeh, S. D. and Nguyen, K.. (2014). Volatility forecasting and risk management for commodity markets in the presence of asymmetry and long memory, Energy Economics, 41:1-18.

12. Choi, K. and Hammoudeh, S.. (2009). Long memory in oil and refined products markets, Energy Journal, 30: 97–116.

13. Conrad, C.. (2010). Non-negativity conditions for the hyperbolic GARCH model, Journal of Econometrics, 157(2):441-457.

14. Conrad, C. and Haag, B. R.. (2006). „Inequality constraints in the fractionally integrated GARCH model, Journal of Financial Econometrics, 4:413–449.

15. Conrad, C. and Karanasos, M.. (2005a). On the inflation-uncertainty hypothesis in the USA, Japan and the UK: a dual long memory approach, Japan and the World Economy, 17:327–343.

16. Conrad, C. and Karanasos, M.. (2005b). „Dual long memory in inflation dynamics across countries of the Euro area and the link between inflation uncertainty and macroeconomic performance, Studies in Nonlinear Dynamics & Econometrics, 4 (5).

17. Conrad, C. Karanasos, M. and Zeng, N.. (2011). Multivariate fractionally integrated APARCH modelling of stock market volatility: multi-country study, Journal of Empirical Finance, 18:147–159.

18. Creti, A. Joëts, M. and Mignon, V.. (2013). On the links between stock and commodity markets' volatility, Energy Econonomics, 37:16–28.

19. Dahl, C. M. and Iglesias, E. M.. (2009). „Volatility spillovers in commodity spot prices: new empirical results, Economic Modelling, 26:601–607.

20. Davidson, J.. (2004). Moment and memory properties of linear conditional heteroscedasticity models, and a new model, Journal of Business and Economic Statistics, 22:16–29.

21. Ding, Z. Granger, C. W. J. and Engle, R. F.. (1993). A Long Memory Property of Stock Market Returns and a New Model, Journal of Empirical Finance, 1:83-106

22. Engle, R. F.. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of Variance of United Kingdom Inflation, Econometrica, 50 (4):987–1008.

23. Engle, R. F. Lilien, D. M. and Robins, R. P.. (1987). Estimating time varying risk premia in the term structure: The ARCH-M model, Econometrica: Journal

of the Econometric Society, 391-407.

24. Geweke, J. and Porter-Hudak, S.. (1983). The estimation and application of long memory time series models, Journal of Time Series Analysis, 4:221-238.

25. Granger, C.. (1980). Long Memory Relationships and the Aggregation of Dynamic Models, Journal of Econometrics, 14:227-238.

26. Granger, C. and Joyeux, R.. (1980). An Introduction to Long-Memory Time Series Models and Fractional Differencing, Journal of Time Series Analysis, 1:15-29.

27. Hosking, J.. (1981). Fractional Differencing, Biometrika, 68:165-176.

28. Hung, J. C. Lee, M. C. and Liu, H. C.. (2008). Estimation of value-at-risk for energy commodities via fat-tailed GARCH models, Energy Ecoonomics. 30:1173–1191.

29. Hurst, H. E.. (1951). Long-term storage capacity of reservoirs, Transactions of the American Society of Civil Engineers, 116:770-799.

30. Jain A,.Biswal P.C(2016) Dynamic linkages among oil price, gold price, exchange rate, and stock market in India, Resources Policy, 49,179-185

31. Jarque, C. M. and Bera, A. K.. (1987). A Test for Normality of Observations and Regression Residuals, International Statistical Review / Revue Internationale de Statistique, 55(2):163-172.

32. Kang, S. H. Kang, S. M. and Yoon, S. M.. (2009). Forecasting volatility of crude oil markets, Energy Economics, 31:119–125.

33. Kang, S. H. McIver R. and Yoon, S. M. (2016). Modeling Time-Varying Correlations in Volatility Between BRICS and Commodity Markets Emerging Markets Finance & Trade, 52(7), 1698-1723.

34. Kang, S. H. McIver R. and Yoon, S. M. (2017). Dynamic spillover effects among crude oil, precious metal, and agricultural commodity futures markets, Energy Economics, 62,19-32.

35. Kang, S. H. and Yoon, S. M.. (2013). Modeling and forecasting the volatility of petroleum futures prices, Energy Economics, 36:354–362.

36. Kumar, D.. (2014). Long Memory in the Volatility of Indian Financial Market:

An Empirical Analysis Based on Indian Data. Anchor Academic Publishing, Hamburg.

37. Lo, A. W.. (1991). Long-term memory in stock market prices, Econometrica, 59:1279-1313.

38. Lobato, I. N. and Savin, N. E.. (1998). Real and spurious long memory properties of stock market data, Journal of Business and Economic Statistics, 16:261–268.

39. Mandelbrot, B. B.. (1971). When Can Price be Arbitraged Efficiently? A Limit to the Validity of the Random Walk and Martingale Models, Review of Economics and Statistics, 53(3):225-236.

40. Mandelbrot, B. B, and Wallis, J. R.. (1969). „Some long-run properties of geophysical records, Water Resouces. Research, 5(2):321-340.

41. Mohammadi, H. and Su, L.. (2010). International evidence on crude oil price dynamics: applications of ARIMA-GARCH models, Energy Economics, 32:1001–1008.

42. Muth, J. F.. (1961). Rational expectations and the theory of price movements, Econometrica: Journal of the Econometric Society, 315-335.

43. Regnier, E.. (2007). Oil and energy price volatility, Energy Economics, 29:405–427.

44. Robinson, P. M.. (1991). Testing for Strong Serial Correlation and Dynamic Conditional Heteroscedasticity in Multiple Regression, Journal of Econometrics, 47:67–84.

45. Robinson, P. M. and Henry, M.. (1999). Long and short memory conditional heteroskedasticity in estimating the memory parameter of levels, Econometric Theory, 15(3):299-336.

46. Thuraisamy, K. S. Sharma, S. S. and Ahmed, H. J. A.. (2013). The relationship between Asian equity and commodity futures markets, Journal of Asian Economics, 28: 67–75.

47. Tsay R S(2010). Analysis of Financial Time Series 3rd Edition, Wiley

48. Tse, Y.. (1998). The conditional heteroscedasticity of the yen–dollar ex-

change rate, Journal of Applied Economics, 13:49–55.

49. Vivian, A. and Wohar, M. E.. (2012). Commodity volatility breaks, Journal of International Financial Markets Institutions and Money, 22:395–422.

50. Wang, Y. Wei, Y. and Wu, C.. (2010). Auto-correlated behavior of WTI crude oil volatilities: A multiscale perspective, Physica A: Statistical Mechanics and its Applications, 389(24):5759-5768.

51. Wei, Y. Wang, Y. and Huang, D.. (2010). „Forecasting crude oil market volatility: further evidence using GARCH-class models, Energy Economics 32:1477–1484.

52. Yajima, Y.. (1985). On estimation of long memory time series models, Australian Journal of Statistics, 27:303-20.