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Application of Differential Transformation Method for Nonlinear Cutting Tool Vibration

İbrahim DEMİR[1,2*], Mustafa Mehmet Fatih KARAHAN[2], Nizami AKTÜRK[1]

[1]*Gazi University, Faculty of Engineering, Department of Mechanical Engineering, 06570, Ankara, Turkey*
[2]*Manisa Celal Bayar University, Faculty of Engineering, Department of Mechanical Engineering, 45140, Manisa, Turkey*

Keywords

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Abstract

This article aims to solve the mathematical model developed for machine tool vibrations by employing Differential Transformation Method (DTM). Multi-step differential transformation method is used as it has been shown to have good accuracy in physical applications. Nondimensionalization (scaling) technique is used in order to fully understand the physical effects of each varying parameter. Transformed function of delayed nonlinear velocity terms are explained. MatLab® software is used for DTM solutions. The equation of motion is solved with the DDE23 function in MatLab® software as well as with MatLab®/Simulink® software to compare the results. The solution of the fundamental DTM which is obtained by using the constant transformed function values which differ from other methods after a certain time period. However, the results obtained with the transformed functions for different values at each sampling time (Multi-Step DTM) give very close results to Simulink® and DDE23.

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Author ID (ORCID Number)

İ. Demir, 0000-0002-2128-3949
M. M. F. Karahan, 0000-0001-9040-5041
N. Aktürk, 0000-0003-1398-8145

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1. INTRODUCTION

Vibration is a phenomenon that is rarely desired and generally untoward in engineering applications. In addition to the obtaining the mathematical model of vibrating systems, the solution of the established model also requires an effort. Analytical solutions of mathematical models of engineering problems are generally very challenging as models are established to be more realistic. Hence, various semi-analytical and numerical methods are employed to obtain the solution. The differential transformation method (DTM), a semi-analytical method, is employed in this paper for investigating machine tool vibrations as it is quite complicated in nature. Differential Transformation Method, based on Taylor Series, was first used by Zhou (1986) for electrical circuit (Hatami et al., 2017). Jang et al. (2000) promoted an adaptive grid size mechanism. Abdel & Hassan (2002) suggested DTM to obtain eigenvalues, eigenfunctions and to find out a solution the partial differential equations. Ayaz (2004) offered three-dimensional differential transformation method and defined the basic theorems. Varying grid size is employed by Kurnaz & Oturanç (2005) to boost the accuracy of the DTM solutions.

Arıkoğlu & Özkol (2005) introduced DTM to solve integro-differential equations. Difference equations are carried out by utilizing DTM by Arıkoğlu & Özkol (2006a). Arıkoğlu & Özkol (2006b) presented theorems related to DTM for the solution of differential-difference equations (DDEs). Application of DTM on integro-differential and integral equations systems was also discussed by Arıkoğlu & Özkol (2008).

*Corresponding Author, e-mail: ibrahim.demir@cbu.edu.tr

Momani & Ertürk (2008) designed a modified DTM for the good accuracy of non-linear oscillatory system solutions. A different DTM technique worked by El-Shaded (2008) for non-linear oscillatory systems. Karakoç & Bereketoğlu (2009) used DTM in the solution of delay differential equations. Kuo & Lo (2009) utilized DTM to explore the response of a physical system. Chen & Chen (2009) adopted DTM to the non-linear physical system solution. Thongmoon & Pusjuso (2010) compared DTM with the Laplace transform method for a system of differential equations.

Mirzaee (2011) applied DTM to the solution of linear and nonlinear DDEs. Süngü & Demir (2012) compared DTM with Adomian Decomposition Method for the solution of linear and nonlinear differential equations. Villafuerte & Chen-Charpentier (2012) developed the random DTM to solve random differential equations.

Gokdogan et al. (2012) compared numerical results obtained by DTM with analytical solution for the DDE. Šmarda et al. (2013) presented theorems related to DTM for the solution of nonlinear differential and integro-differential equations with proportional delays. Bozdoğan & Ozturk (2014) apply differential transform method to investigate the free vibration analysis of an orthotropic beam model. Tabatabaei & Gunerhan (2014) solved Duffing equations by means of DTM. DTM is also used by Bozdogan & Aydin (2016) in stability analysis for multistory structure that is modeled as a continuous system. Benhammouda & Leal (2016) proposed an analytical solution for the nonlinear delay differential equations by using DTM. Rebenda & Šmarda (2017) presented semi-analytical DTM approach for the solution of functional differential equations with multiple delays. Hatemi et al. (2017) explained DTM on some physical problems.

In this current study, the solution of a mathematical model in cutting tool vibrations is investigated. DTM is applied to the mathematical model presented for the first time. The mathematical model is comprised of delayed, nonlinear differential equations. In this paper, a system of transformed equations is expressed in matrix form. The difficulty caused by the transformed nonlinear terms is overcomed by using transformed functions which are found at previous cycle and sampling time. The results obtained by DTM which is a semi analytical method are in good agreement with the numerical methods, Simulink and DDE23 MatLab® function.

2. DIFFERENTIAL TRANSFORMATION METHOD

In this method, which is based on Taylor Series, the transformed function, $F(k)$ and inverse transform, $f(t)$ is defined as follows.

$$F(k)=\frac{1}{k!}\left[\frac{d^k f(t)}{dt^k}\right]_{t=t_o} \qquad f(t)=\sum_{k=0}^{\infty}F(k)(t-t_0)^k \tag{1}$$

The basic DTM theorems used in this article is below (Zhou, 1986; Arıkoğlu & Özkol, 2006).

$$x(t)=\frac{d^m f(t)}{dt^m} \qquad X(k)=\frac{(k+m)!F(k+m)}{k!} \tag{2}$$

$$x(t)=f(t)g(t) \qquad X(k)=\sum_{l=0}^{k}F(l)G(k-l) \tag{3}$$

$$f(t)=g(t+a) \qquad F(k)=\sum_{h_1=k}^{N}\binom{h_1}{k}a^{h_1-k}G(h_1) \tag{4}$$

3. SOLUTION TECHNIQUE

Multi-step differential transformation method is used in physical applications due to its high accuracy (Hatami et al., 2017). In this study, instead of using a single t_o value and the associated transformed function to obtain the function value at a specific time, t, different t_o values and different transformed functions are used in each step by keeping the step interval, $t - t_0 = T_o$, constant.

$$F_{t_o}(k) = \frac{1}{k!}\left[\frac{d^k f(t)}{dt^k}\right]_{t=t_o} \qquad f(t_{t_o+T_o}) = \sum_{k=0}^{\infty} F_{t_o}(k) T_o^{\,k} \tag{5}$$

4. PROBLEM FORMULATION

In cutting tool vibrations, the equation of motion defining the single degree of freedom and self-excited vibration, including flank wear caused by vibrations, is shown as follows:

$$\ddot{y}(t) + 2\xi\omega_n \dot{y}(t) + \omega_n^2 y(t) = -b\frac{\omega_n^2}{k}\left[K_1\left(y(t) - y(t-T)\right) + K_2 VB_d(t)\right] \tag{6}$$

The relationship between the flank wear and the cutting velocity was expressed first by Taylor (1907) (Altintas, 2012). There is a connection between the relative velocity of the two materials and the wear rate (Maekawa et al., 1989; Chowdhury et al., 2011; Zanger & Schulze, 2013). The mathematical model proposed for the flank wear length which includes the lost material length, $G_m(t)$, depending on the vibration velocity, whose basic approach is based on the above studies is given in Eq 7.

$$VB_d(t+dt) = VB_d(t) + \frac{G_m(t) z_1}{z_2} = VB_d(t) + \frac{z_1}{z_2} p\dot{y}(t)^q \tag{7}$$

Eq. 7 is described with the idea that it will be easier to use previous values.

$$VB_d(t) = VB_d(t-dt) + \frac{z_1}{z_2} p\dot{y}(t-dt)^q \tag{8}$$

The terms in Eqs. 6 and 8 $y(t)$, cutting tool displacement in vibration motion, $\dot{y}(t)$, first derivative of displacement (velocity), $\ddot{y}(t)$, second derivative of displacement (acceleration), $y(t-T)$, displacement in the previous cycle of the workpiece, $VB_d(t)$, flank wear length caused by vibration, K_2, hardness of work material, K_1, specific cutting constant, b, deep of cut, k, cutting tool stiffness, ω_n, natural frequency, ξ, dimensionless damping ratio and the other terms are some process parameters. Initial conditions are $y(0)=0$, $\dot{y}(0)=V$, $VB_d(0)=0$. The term $bK_2VB_d(t)$ in Eq. 6 is the force on the flank face. It is assumed as $b(HB/3)VB_d(t)$ in this mathematical model based on the investigation by Astakhov (2004). HB is the Brinell hardness of the workpiece material. This force will only exist when the vibration velocity is positive due to physical acceptance. Similarly, the term related to velocity in Eq. 8 prevail when the vibration velocity is positive, otherwise it will be $VB_d(t) = VB_d(t-dt)$.

For the convenience and the understanding of the Eqs. 6 and 8, some dimensionless parameters are used as below.

$$u = y / VB_{do} \qquad \tau = \omega_n t \qquad \delta = \omega_n T \qquad w = VB_d / VB_{do} \qquad \delta_o = \omega_n T_o$$

Taking first and second derivative of y:

$$\frac{dy}{dt}=\frac{dy}{d\tau}\frac{d\tau}{dt}=VB_{do}\omega_n\frac{du}{d\tau} \qquad \frac{d^2y}{dt^2}=VB_{do}\omega_n^2\frac{d^2u}{d\tau^2}$$

The above derivatives and dimensionless parameters are substituted in Eq. 6.

$$VB_{do}\omega_n^2\frac{d^2u}{d\tau^2}+2\xi VB_{do}\omega_n^2\frac{du}{d\tau}+VB_{do}\omega_n^2u=-b\frac{\omega_n^2}{k}VB_{do}\left[K_1\left(u-u\left(\tau-\delta\right)\right)+\frac{HB}{3}w\right] \tag{9}$$

Eq. 10 is obtained by dividing Eq. 9 by the coefficient of the second dimensionless derivative.

$$\frac{d^2u}{d\tau^2}+2\xi\frac{du}{d\tau}+u=-\frac{b}{k}\left[K_1\left(u-u\left(\tau-\delta\right)\right)+\frac{HB}{3}w\right] \tag{10}$$

Dimensionless parameters are also substituted in Eq. 8.

$$VB_{do}w(\tau)=VB_{do}w(\tau-\delta_o)+\frac{z_1}{z_2}p\left(VB_{do}\omega_n\right)^q\left(\frac{du(\tau-\delta_o)}{d\tau}\right)^q \tag{11}$$

Eq. 12 is found by dividing Eq. 11 by VB_{do}.

$$w(\tau)=w(\tau-\delta_o)+\frac{z_1}{z_2}p\left(VB_{do}\right)^{q-1}\left(\omega_n\right)^q\left(\frac{du(\tau-\delta_o)}{d\tau}\right)^q \tag{12}$$

The following abbreviations are used to make the Eq. 10 and 12 look simpler.

$$a_1=2\xi \qquad a_2=b\frac{K_1}{k} \qquad a_3=b\frac{HB}{3k} \qquad a_4=\frac{z_1}{z_2}p\omega_n{}^qVB_{do}{}^{q-1}$$

After these operations, Eq. 13 and 14 and dimensionless initial conditions appear as follows.

$$u''(\tau)+a_1u'(\tau)+(1+a_2)u(\tau)-a_2u\left(\tau-\delta\right)+a_3w(\tau)=0 \tag{13}$$

$$w(\tau)=w(\tau-\delta_o)+a_4u'(\tau-\delta_o)^q \tag{14}$$

$$u\left(0\right)=0 \qquad u'\left(0\right)=\frac{V}{VB_{do}\omega_n}=1 \qquad w\left(0\right)=0 \tag{15}$$

5. APPLICATION OF DIFFERENTIAL TRANSFORMATION METHOD

In this section, DTM is applied to solve the presented equation of motion. Eq. 13 and 14 will be solved in three cases where q is equal to 1, 2 and 3.

Case 1

In this case, Eq. 13 and 14 is handled for $q=1$.

$$u''(\tau)+a_1u'(\tau)+(1+a_2)u(\tau)-a_2u\left(\tau-\delta\right)+a_3w(\tau)=0 \tag{13}$$

$$w(\tau)=w(\tau-\delta_o)+a_4u'(\tau-\delta_o) \tag{16}$$

Eq. 13 is transformed by means of the above fundamental transformations as follow. These transformed functions are renewed at each step.

$$\frac{(k+2)!U(k+2)}{k!}+a_1\frac{(k+1)!U(k+1)}{k!}+(1+a_2)U(k)-a_2U(k)_{\tau_o-\delta}+a_3W(k)=0 \tag{17}$$

The $\tau_o-\delta$ value in the lower right index of the term $U(k)_{\tau_o-\delta}$ in this transformed Eq. 17 represents the previous transformed function as δ. The final equation is organized as follows.

$$(k+2)(k+1)U(k+2)+a_1(k+1)U(k+1)+(1+a_2)U(k)-a_2U(k)_{\tau_o-\delta}+a_3W(k)=0 \tag{18}$$

By adapting the above fundamental transformations to Eq. 16:

$$W(k)=\sum_{h_1=k}^{N}\binom{h_1}{k}(-\delta_o)^{h_1-k}W(h_1)+a_4(k+1)\sum_{h_1=k+1}^{N+2}\binom{h_1}{k+1}(-\delta_o)^{h_1-k-1}U(h_1) \tag{19}$$

is obtained. This equation is also expressed as:

$$0=\sum_{h_1=k+1}^{N}\binom{h_1}{k}(-\delta_o)^{h_1-k}W(h_1)+a_4(k+1)\sum_{h_1=k+1}^{N+2}\binom{h_1}{k+1}(-\delta_o)^{h_1-k-1}U(h_1) \tag{20}$$

The equation system obtained by changing k from 0 to N to find the transformed functions, $U(k)$, in the Eq. 18 is shown as follows.

$$\begin{pmatrix} (2)(1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_1(2) & (3)(2) & 0 & 0 & 0 & 0 & 0 & 0 \\ (1+a_2) & a_1(3) & (4)(3) & 0 & 0 & 0 & 0 & 0 \\ 0 & (1+a_2) & a_1(4) & (5)(4) & 0 & 0 & 0 & 0 \\ 0 & 0 & (1+a_2) & a_1(5) & (6)(5) & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddots & \ddots & \ddots & 0 & 0 \\ 0 & 0 & 0 & 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & (1+a_2) & a_1(N+1) & (N+2)(N+1) \end{pmatrix}\begin{pmatrix} U(2) \\ U(3) \\ U(4) \\ U(5) \\ U(6) \\ \vdots \\ \vdots \\ U(N+2) \end{pmatrix}+$$

$$\begin{pmatrix} a_1(1)U(1)+(1+a_2)U(0)-a_2U(0)_{\tau_o-\delta}+a_3W(0) \\ (1+a_2)U(1)-a_2U(1)_{\tau_o-\delta} \\ -a_2U(2)_{\tau_o-\delta} \\ -a_2U(3)_{\tau_o-\delta} \\ -a_2U(4)_{\tau_o-\delta} \\ \vdots \\ \vdots \\ -a_2U(N)_{\tau_o-\delta} \end{pmatrix}+\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_3 \end{pmatrix}\begin{pmatrix} W(1) \\ W(2) \\ W(3) \\ W(4) \\ \vdots \\ \vdots \\ W(N) \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ \vdots \\ 0 \end{pmatrix} \tag{21}$$

The equation system obtained by changing k from 0 to N-1 to find the transformed functions in the Eq. 20 is also shown as follows.

$$\begin{pmatrix} \binom{1}{0}(-\delta_o)^1 & \binom{2}{0}(-\delta_o)^2 & \cdots & \cdots & \binom{N}{0}(-\delta_o)^N \\ 0 & \binom{2}{1}(-\delta_o)^1 & \cdots & \cdots & \binom{N}{1}(-\delta_o)^{N-1} \\ 0 & 0 & \ddots & \cdots & \vdots \\ 0 & 0 & 0 & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \binom{N}{N-1}(-\delta_o)^1 \end{pmatrix} \begin{pmatrix} W(1) \\ W(2) \\ \vdots \\ W(N-1) \\ W(N) \end{pmatrix} +$$

$$\begin{pmatrix} a_4(1)\binom{2}{1}(-\delta_o)^1 & a_4(1)\binom{3}{1}(-\delta_o)^2 & \cdots & \cdots & \cdots & a_4(1)\binom{N+2}{1}(-\delta_o)^{N+1} \\ a_4(2)\binom{2}{2}(-\delta_o)^0 & a_4(2)\binom{3}{2}(-\delta_o)^1 & \cdots & \cdots & \cdots & a_4(2)\binom{N+2}{2}(-\delta_o)^N \\ 0 & a_4(3)\binom{3}{3}(-\delta_o)^0 & \ddots & \cdots & \cdots & \vdots \\ 0 & 0 & \ddots & \ddots & \cdots & \vdots \\ 0 & 0 & 0 & a_4(N)\binom{N}{N}(-\delta_o)^0 & \cdots & a_4(N)\binom{N+2}{N}(-\delta_o)^2 \end{pmatrix} \begin{pmatrix} U(2) \\ U(3) \\ \vdots \\ \vdots \\ U(N+1) \\ U(N+2) \end{pmatrix} +$$

$$\begin{pmatrix} a_4(1)\binom{1}{1}(-\delta_o)^0 U(1) \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix} \tag{22}$$

The equations in these last two matrices form are given in Eqs. 23 and 24.

$$\mathbf{A_1U + A_2 + A_3W = 0} \tag{23}$$

$$\mathbf{A_4W + A_5U + A_6 = 0} \tag{24}$$

After mathematical operations the transformed equations are found as below:

$$\mathbf{U = (A_1 - A_3(A_4^{-1}A_5))^{-1}(-A_2 + A_3(A_4^{-1}A_6))} \tag{25}$$

$$\mathbf{W = -A_4^{-1}(A_5U + A_6)} \tag{26}$$

It should be noted here that there are no transformed function values $\mathbf{U(0)}$, $\mathbf{U(1)}$, $\mathbf{W(0)}$ in the $\mathbf{U}$ and $\mathbf{W}$ vectors. Because these values indicate the initial values in each step.

Case 2

In this case, Eqs. 13 and 14 is handled for $q = 2$. It is needed to make some mathematical operations to determine the transformed function of $u'(\tau - \delta_o)^q$ in Eq. 14. Let the function is $f(\tau) = u'(\tau - \delta_o)^2$ to be applied DTM. The mathematical operations to be done step by step to find the transformed function are as follows.

- $U_1(k)=\sum_{h_1=k}^{N}\binom{h_1}{k}(-\delta_o)^{h_1-k}U(h_1)$ transformed function of $u(\tau-\delta_o)$
- $U_2(k)=(k+1)U_1(k+1)$ transformed function of $u'(\tau-\delta_o)$
- $U_2(l)$ is found as $U_2(l)=(l+1)\sum_{h_1=l+1}^{N}\binom{h_1}{l+1}(-\delta_o)^{h_1-l-1}U(h_1)$ by using $U_1(l)$
- $U_2(k-l)$ is $U_2(k-l)=(k-l+1)\sum_{h_2=k-l+1}^{N}\binom{h_2}{k-l+1}(-\delta_o)^{h_2-k+l-1}U(h_2)$

The transformed function of $f(\tau)$ is $F(k)=\sum_{l=0}^{k}U_2(l)U_2(k-l)$ due to the theorem in Eq. 3. After these operations

$$F(k)=\sum_{l=0}^{k}\sum_{h_1=l+1}^{N}\sum_{h_2=k-l+1}^{N}(l+1)(k-l+1)\binom{h_1}{l+1}\binom{h_2}{k-l+1}(-\delta_o)^{h_1+h_2-k-2}U(h_1)U(h_2) \tag{27}$$

is obtained. If DTM is applied to Eq. 14, the following transformed equation is found.

$$\begin{aligned}0=&\sum_{h_1=k+1}^{N}\binom{h_1}{k}(-\delta_o)^{h_1-k}W(h_1)+\\ &a_4\sum_{l=0}^{k}\sum_{h_1=l+1}^{N}\sum_{h_2=k-l+1}^{N}(l+1)(k-l+1)\binom{h_1}{l+1}\binom{h_2}{k-l+1}(-\delta_o)^{h_1+h_2-k-2}U(h_1)U(h_2)\end{aligned} \tag{28}$$

Since q is equal to 2, this last equation is quite different from Eq. 20. It is quite difficult to obtain equation system in $q=1$ solution for $q=2$. Instead, the transformed function at the previous sampling time is used for $f(\tau)=u'(\tau-\delta_o)^2$.

The transformed function of $f(\tau)=u'(\tau-\delta_o)u'(\tau-\delta_o)$

$$F(k)=\sum_{l=0}^{k}(l+1)(k-l+1)U_{\tau-\delta_o}(l+1)U_{\tau-\delta_o}(k-l+1) \tag{29}$$

Thus, Eq. 14 is transformed as follows.

$$0=\sum_{h_1=k+1}^{N}\binom{h_1}{k}(-\delta_o)^{h_1-k}W(h_1)+a_4\sum_{l=0}^{k}(l+1)(k-l+1)U_{\tau-\delta_o}(l+1)U_{\tau-\delta_o}(k-l+1) \tag{30}$$

Since functions $U_{\tau-\delta_o}$ are found in the previous step, there are no nonlinear terms in Eq. 30. Thus, a system of linear equations for Eq. 30 can be obtained as Eq. 31.

$$\begin{pmatrix} \binom{1}{0}(-\delta_o)^1 & \binom{2}{0}(-\delta_o)^2 & \cdots & \cdots & \binom{N}{0}(-\delta_o)^N \\ 0 & \binom{2}{1}(-\delta_o)^1 & \cdots & \cdots & \binom{N}{1}(-\delta_o)^{N-1} \\ 0 & 0 & \ddots & \cdots & \vdots \\ 0 & 0 & 0 & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \binom{N}{N-1}(-\delta_o)^1 \end{pmatrix} \begin{pmatrix} W(1) \\ W(2) \\ \vdots \\ W(N-1) \\ W(N) \end{pmatrix} + a_4 \begin{pmatrix} F(0) \\ F(1) \\ \vdots \\ F(N-2) \\ F(N-1) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix} \tag{31}$$

Eq. 20 in the first solution is replaced by Eq. 30. Eq. 31 is below as more compact form.

$$\mathbf{A_4 W} + \alpha_4 \mathbf{F} = \mathbf{0} \tag{32}$$

If $\mathbf{W}$ in Eq. 32 is substituted in Eq. 23 equations below are found.

$$\mathbf{U} = -\mathbf{A_1^{-1} A_2} + \alpha_4 \mathbf{A_1^{-1}}\left(\mathbf{A_3}\left(\mathbf{A_4^{-1} F}\right)\right) \tag{33}$$

$$\mathbf{W} = -\alpha_4 \mathbf{A_4^{-1} F} \tag{34}$$

Case 3

In this case, Eq. 13 and 14 is handled for $q=3$. In this solution, the approach in solution $q=2$ is repeated. Let the function is $f(\tau)=u'(\tau-\delta_o)^3$ to be applied DTM. Mathematical operations are included in the appendix and the transformed function is as follows.

$$\begin{aligned} F(k) = &\sum_{l=0}^{k}\sum_{m=0}^{l}\sum_{h_1=m+1}^{N}\sum_{h_2=l-m+1}^{N}\sum_{h_3=k-l+1}^{N}(m+1)(l-m+1)(k-l+1) \\ &\times\binom{h_1}{m+1}\binom{h_2}{l-m+1}\binom{h_3}{k-l+1}(-\delta_o)^{h_1+h_2+h_3-k-3}U(h_1)U(h_2)U(h_3) \end{aligned} \tag{35}$$

Because of the same problem encountered in the second solution, the transformed functions at the previous sampling time (δ_o) are utilized. Mathematical transaction is the following order.

$f(\tau)=u'(\tau-\delta_o)u'(\tau-\delta_o)u'(\tau-\delta_o)$ is transformed to

$$F(k)=\sum_{l=0}^{k}\sum_{m=0}^{l}(m+1)(l-m+1)(k-l+1)U_{\tau-\delta_o}(m+1)U_{\tau-\delta_o}(l-m+1)U_{\tau-\delta_o}(k-l+1) \tag{36}$$

Expression of Eq. 14 for $q=3$ is below.

$$\begin{aligned} 0 = &\sum_{h_1=k+1}^{N}\binom{h_1}{k}(-\delta_o)^{h_1-k}W(h_1) + \\ &a_4\sum_{l=0}^{k}\sum_{m=0}^{l}(m+1)(l-m+1)(k-l+1)U_{\tau-\delta_o}(m+1)U_{\tau-\delta_o}(l-m+1)U_{\tau-\delta_o}(k-l+1) \end{aligned} \tag{37}$$

The linear equation system of Eq. 37 is the same way as the previous solution as follows.

$$\mathbf{A_4 W} + \alpha_4 \mathbf{F} = \mathbf{0} \tag{38}$$

6. RESULTS AND DISCUSSION

Eqs. 13 and 14 are solved with the DDE23 function in MatLab® software and with MatLab®/Simulink software. The solutions of Eqs. 13 and 14 obtained with the Differential Transformation Method (Eqs. 25, 26, 33, 34 and 38) are also solved using MatLab® software. The parameter values used in the solutions are $a_1 = 0.05$, $a_2 = 0.6$, $a_3 = 0.001$, $a_4 = 0.001$, $N = 5$, $\delta = 6.4$ and $\delta = 6$.

The results obtained with four different methods, DTM, Multi-Step DTM, Simulink and DDE23, for $q = 3$ and $\delta = 6$ values are compared in Fig.1. The solution of the fundamental DTM which found by using the constant transformed functions values differs from other methods after a certain time. But the accuracy can be increased by selecting the higher N value. However, the result obtained with the transformed functions that take different values at each sampling time (Multi-Step DTM), gives very similar output to Simulink® and DDE23 solutions.

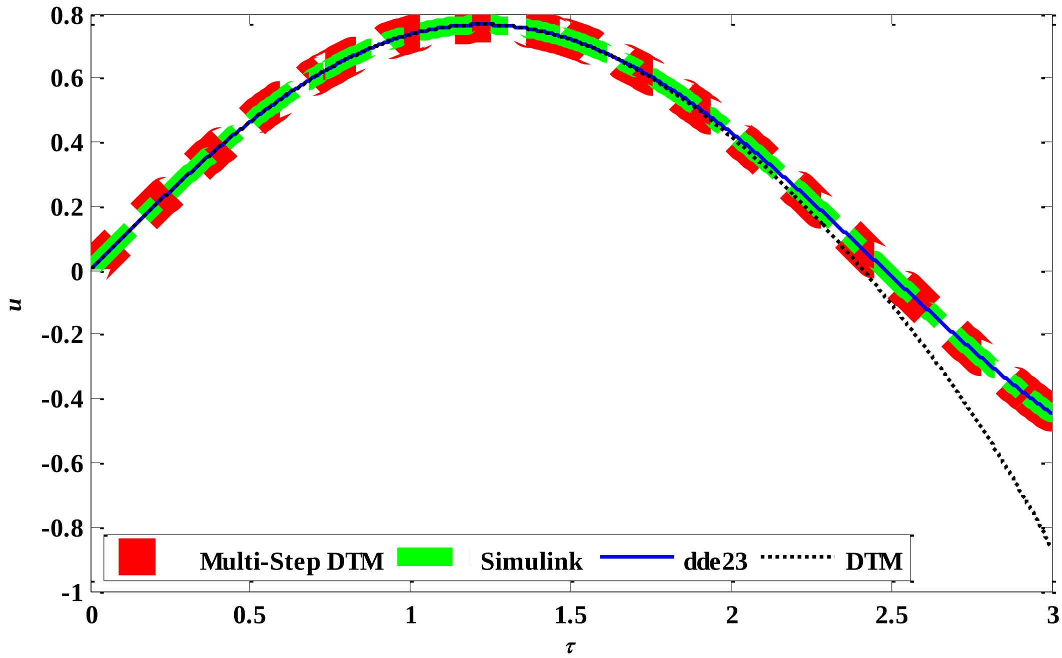

Figure 1. *Comparision of Solutions for $q = 3$ and $\delta = 6$*

A simulation for $q = 1$ and $\delta = 6.4$ values is executed by using Multi-Step DTM, Simulink® and DDE23 is shown in Fig. 2. As seen the system is damped depending on the selected parameters and carries on cutting without vibration. In this graph, the result obtained with Runge-Kutta based Simulink® is very close to the result obtained with Multi-Step DTM. DDE23 also gives close result to these two methods. Multi-Step DTM has higher values than that of the others when the direction of the motion changes.

u versus τ is demonstrated in Fig. 3. $q = 2$ and $\delta = 6.4$ values are taken in this solution. A similar interpretation in Fig. 2 can be made for Fig. 3. There can also be observed a damped oscillation. The difference from Fig. 2 is that damping occurs slowly. The reason for this is the q value in the nonlinear damping effect. Magnitude of derivative of u changes between 0 and 1. Consequently, taking the power of the damping term values with a larger q gives smaller values. This causes less damping effect.

The results of the simulation for $q = 3$ and $\delta = 6.4$ is plotted in Fig. 4. The basic difference of Fig. 4 from Figs. 2 and 3 is that the system is damped slowly which reason as mentioned above. The higher q value makes the magnitude of u' small. Therefore, the amplitude of u is higher than Figs. 2 and 3.

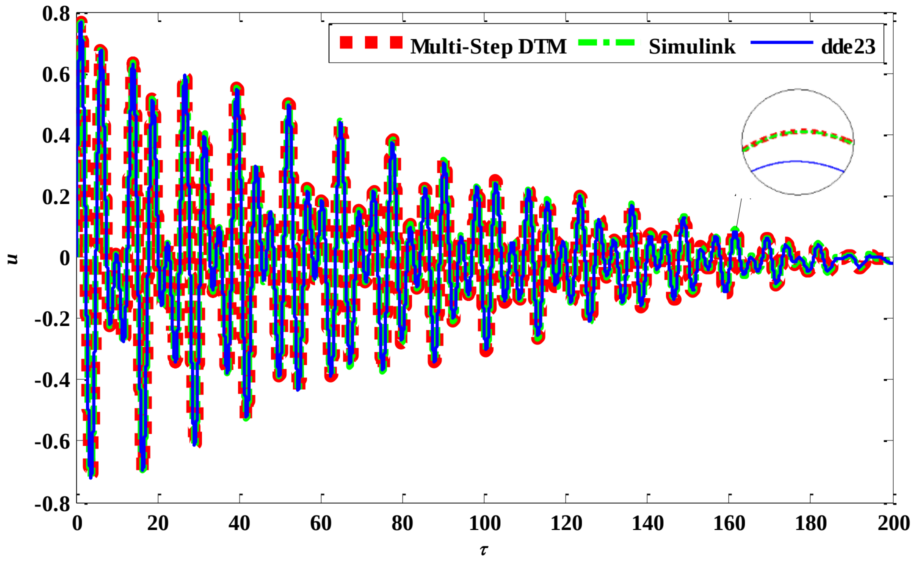

Figure 2. *Time Histories for* $q = 1$ *and* $\delta = 6.4$

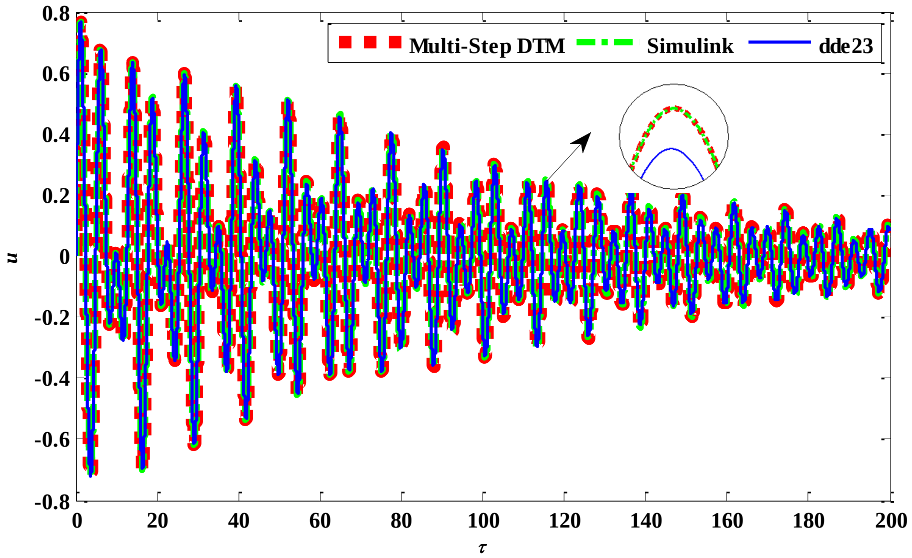

Figure 3. *Time Histories for* $q = 2$ *and* $\delta = 6.4$

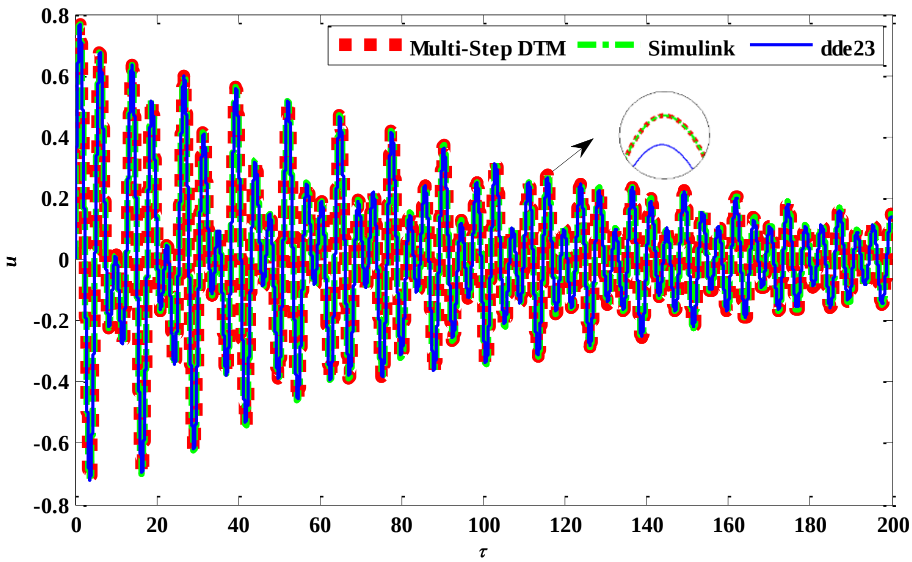

Figure 4. *Time Histories for* $q = 3$ *and* $\delta = 6.4$

The results for the value, $\delta = 6$, the dimesionless time delay, and three different q values are depicted in Fig. 5. Increasing of amplitude for all q values occurs because of the changing of dimensionless time delay. As seen in Fig. 5. (c), incremet of amplitude is slower for larger q values. When taking the power of damping terms whose magnitude exceeds 1 with larger q values, larger damping values come out. This proves that the damping effect is higher. In this way, the amplitude rising is slower. In this solution, Multi-Step DTM and Simulink® methods give closer results than DDE23.

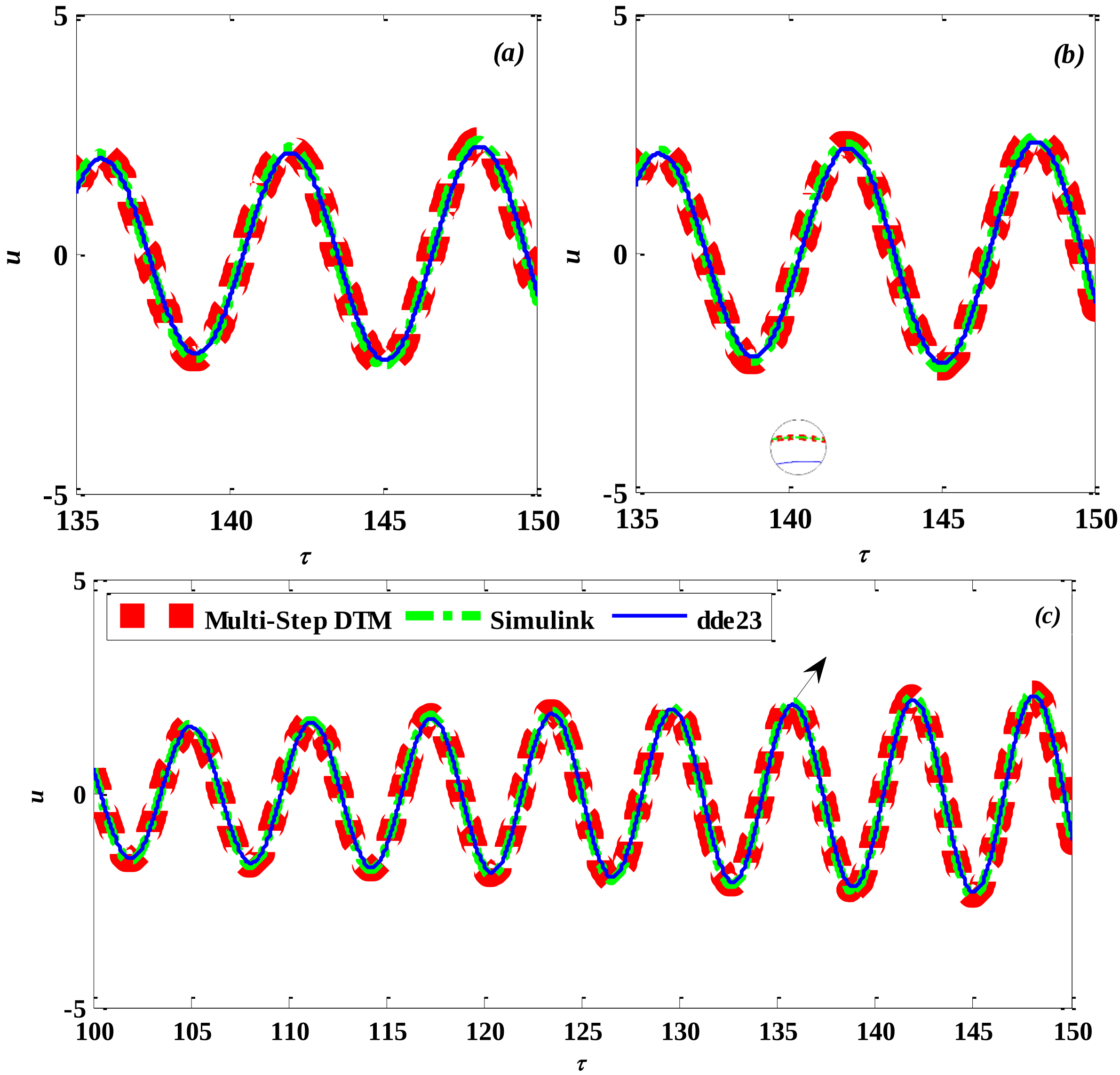

Figure 5. *Time Histories for* $\delta = 6$. *a)* $q = 1$, *b)* $q = 2$, *c)* $q = 3$

7. CONCLUSIONS

In this paper, differential transformation method is used for solving the mathematical model that can be considered in cutting tool vibrations. The results are compared with that of Simulink® and DDE23. This study shows that the fundamental DTM is not giving valid results always. The study also proves that Multi-Step DTM gives competitive results with other methods. It is also a suitable method for the analysis of physical systems. This article offers a method to overcome the difficulty caused by nonlinear delay terms, by using the values of nonlinear delay terms that found before the delay time.

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CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

REFERENCES

Abdel, I. H., & Hassan, H. (2002). Different applications for the differential transformation in the differential equations. *Applied Mathematics and Computation*, *129*(2-3), 183-201. doi:10.1016/S0096-3003(01)00037-6

Altintas, Y. (2012). Manufacturing Automation. New York: Cambridge University Press.

Arıkoğlu, A., & Özkol, İ. (2005). Solution of boundary value problems for integro-differential equations by using differential transform method. *Applied Mathematics and Computation*, *168*(2), 1145-1158. doi:10.1016/j.amc.2004.10.009

Arıkoğlu, A., & Özkol, İ. (2006a). Solution of difference equations by using differential transform method. *Applied Mathematics and Computation*, *174*(2), 1216-1228. https://doi.org/10.1016/j.amc.2005.06.013

Arıkoğlu, A., & Özkol, İ. (2006b). Solution of differential-difference equations by using differential transformation method. *Applied Mathematics and Computation*, *181*(1), 153-162. doi:10.1016/j.amc.2006.01.022

Arıkoğlu, A., & Özkol, İ. (2008). Solutions of integral and integro-differential equation systems by using differential transform method. *Computers and Mathematics with Applications*, *56*(9), 2411-2417. doi:10.1016/j.camwa.2008.05.017

Astakhov, V. P. (2004). The assessment of cutting tool wear. *International Journal of Machine Tools and Manufacture*, *44*(6), 637-647. doi:10.1016/j.ijmachtools.2003.11.006

Ayaz, F. (2004). Solutions of the system of differential equations by differential transform method. *Applied Mathematics and Computation*, *147*(2), 547-567. doi:10.1016/S0096-3003(02)00794-4

Benhammouda, B., & Leal, H. V. (2016). A new multi-step technique with differential transform method for analytical solution of some nonlinear variable delay differential equations. SpringerPlus, 5, 1723. doi:10.1186/s40064-016-3386-8

Bozdogan, K. B., & Ozturk, D. (2014). Free vibration analysis of the tube-in-tube tall buildings with the differential transform method. *Advances in Structural Engineering*, *17*(9), 1271-1279. doi:10.1260/1369-4332.17.9.1271

Bozdogan, K. B., & Aydin, S. (2016). Lateral stability analysis of multistory buildings using the differential transform method. *Structural Engineering and Mechanics*, *57*(5), 861-876. doi:10.12989/sem.2016.57.5.861

Chen, S-S., & Chen, C-K. (2009). Application of the differential transformation method to the free vibrations of strongly non-linear oscillators. *Nonlinear Analysis: Real World Applications*, *10*(2), 881-888. doi:10.1016/j.nonrwa.2005.06.010

Chowdhury, M. A., Khalil, M. K., Nuruzzaman, D. M., & Rahaman, M. L. (2011). The effect of sliding speed and normal load on friction and wear property of aluminium. *International Journal of Mechanical & Mechatronics Engineering*, *11*(1), 53–57.

El-Shahed, M. (2008). Application of differential transformation method to non-linear oscillatory systems. *Communications in Nonlinear Science and Numerical Simulation*, *13*(8), 1714-1720. doi:10.1016/j.cnsns.2007.03.005

Gokdogan, A., Merdan, M., & Yıldırım, A. (2012). Differential transformation method for solving a neutral functional-differential equation with proportional delays. *Caspian Journal of Mathematical Sciences*, *1*(1), 31-37.

Hatami, M., Ganji, D. D., & Sheikholeslami, M. (2017). Differential Transformation Method For Mechanical Engineering Problems. Elsevier Academic Press. WOS:000409365500010

Jang, M-J., Chen, C-L., & Liy, Y-C. (2000). On solving the initial-value problems using the differential transformation method. *Applied Mathematics and Computation*, *115*(2-3), 145-160. doi:10.1016/S0096-3003(99)00137-X

Karakoç, F., & Bereketoğlu, H. (2009). Solutions of delay differential equations by using differential transform method. *International Journal of Computer Mathematics*, *86*(5), 914-923. doi:10.1080/00207160701750575

Kuo, B-L., & Lo, C-Y. (2009). Application of the differential transformation method to the solution of a damped system with high nonlinearity. *Nonlinear Analysis: Theory, Methods & Applications*, *70*(4), 1732-1737. doi:10.1016/j.na.2008.02.056

Kurnaz, A., & Oturanç, G. (2005). The differential transform approximation for the system of ordinary differential equations. International *Journal of Computer Mathematics*, *82*(6), 709-719. doi:10.1080/00207160512331329050

Maekawa, K., Kitagawa, T., Shirakashi, T., & Usui, E. (1989). Analytical prediction of flank wear of carbide tools in turning plain carbon steels II: prediction of flank wear. *Bull. Japan. Soc. Prec. Eng*, *23*(2), 126-133. WOS:A1989AN12800007

Mirzaee, F. (2011). Differential transform method for solving linear and nonlinear systems of ordinary differential equations. *Applied Mathematical Sciences*, *5*(70), 3465-3472.

Momani, S., & Ertürk, V. S. (2008). Solutions of non-linear oscillators by the modified differential transform method. *Computers and Mathematics with Applications*, *55*(4), 833-842. doi:10.1016/j.camwa.2007.05.009

Rebenda, J., & Šmarda, Z. (2017). A differential transformation approach for solving functional differential equations with multiple delays. *Commun Nonlinear Science and Numerical Simulation*, *48*, 246-257. doi:10.1016/j.cnsns.2016.12.027

Süngü, İ. Ç., & Demir, H. (2012). Diferansiyel Dönüşüm/Sonlu Fark Yöntemi İle Denklem Sistemlerinin Çözümleri. *e-Journal of New World Sciences Academy*, *7*(2), 66-73.

Šmarda, Z., Diblík, J., & Khan, Y. (2013). Extension of the differential transformation method to nonlinear differential and integro-differential equations with proportional delays. *Advances in Difference Equations*, 69. doi:10.1186/1687-1847-2013-69

Tabatabaei, K., & Gunerhan, E. (2014). Numerical solution of duffing equation by the differential transform method. *Applied Mathematics & Information Sciences Letters*, *2*(1), 1-6. doi:10.12785/amisl/020101

Taylor, F. W. (1907). On the art of cutting metals. Transactions of the ASME, 28, 31-350.

Thongmoon, M., & Pusjuso, S. (2010). The numerical solutions of differential transform method and the Laplace transform method for a system of differential equations. *Nonlinear Analysis: Hybrid Systems*, *4*(3), 425-431. doi:10.1016/j.nahs.2009.10.006

Villafuerte, L., & Chen-Charpentier, B. M. (2012). A random differential transform method: Theory and applications. *Applied Mathematics Letters*, *25*(10), 1490-1494. doi:10.1016/j.aml.2011.12.033

Zanger, F., & Schulze, V. (2013). Investigations on mechanisms of tool wear in machining of Ti-6Al-4V using FEM simulation. *Procedia CIRP*, *8*, 158-163. doi:10.1016/j.procir.2013.06.082

Zhou, J. K. (1986) Differential Transformation and its Application for Electrical Circuit. Wuhan: Huazhong University Press.

APPENDIX

Transformed function of $u(\tau-\delta_o)$

$$U_1(k)=\sum_{h_1=k}^{N}\binom{h_1}{k}\left(-\delta_o\right)^{h_1-k}U(h_1) \tag{A1}$$

Transformed function of $u'(\tau-\delta_o)$

$$U_2(k)=\left(k+1\right)U_1(k+1) \tag{A2}$$

Transformed function of $h(\tau)=u'(\tau-\delta_o)^2$

$$H(k)=\sum_{l=0}^{k}U_2(l)U_2(k-l) \tag{A3}$$

$f(\tau)=u'(\tau-\delta_o)^3$ is expressed as $f(\tau)=h(\tau)u'(\tau-\delta_o)$.

Transformed function of $f(\tau)=h(\tau)u'(\tau-\delta_o)$

$$F(k)=\sum_{l=0}^{k}H(l)U_2(k-l) \tag{A4}$$

$H(k)$ in A3 is written in A4.

$$F(k)=\sum_{l=0}^{k}\sum_{m=0}^{l}U_2(m)U_2(l-m)U_2(k-l) \tag{A5}$$

Using A1 and A2 equations, $U_2(k)$ in A2 is arranged as follows to be used in A5.

$$U_2(l)=(l+1)\sum_{h_1=l+1}^{N}\binom{h_1}{l+1}\left(-\delta_o\right)^{h_1-l-1}U(h_1) \tag{A6}$$

$$U_2(k-l)=(k-l+1)\sum_{h_2=k-l+1}^{N}\binom{h_2}{k-l+1}\left(-\delta_o\right)^{h_2-k+l-1}U(h_2) \tag{A7}$$

Transformed function of $f(\tau)=u'(\tau-\delta_o)^3$ is found as below.

$$\begin{aligned}F(k)=&\sum_{l=0}^{k}\sum_{m=0}^{l}\sum_{h_1=m+1}^{N}\sum_{h_2=l-m+1}^{N}\sum_{h_3=k-l+1}^{N}(m+1)(l-m+1)(k-l+1)\\&\times\binom{h_1}{m+1}\binom{h_2}{l-m+1}\binom{h_3}{k-l+1}\left(-\delta_o\right)^{h_1+h_2+h_3-k-3}U(h_1)U(h_2)U(h_3)\end{aligned} \tag{A8}$$