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On a Different Method For Determining the Primary Numbers

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Abstract: In this study, firstly, ordinal numbers of the odd integers were defined to reach prime numbers, and a new prime number sieve was obtained with the help of these numbers. A general formula was given to examine this sieve. Then, by representing the ordinal numbers with the help of matrices, some properties of these numbers were also examined.

Keywords: Prime Numbers, Prime Number Sieves, Applications of Sieves Methods.

1 Introduction and Preliminaries

While identifying prime numbers is simple, finding a new and large prime number is difficult. To date, the distribution of prime numbers, the next major prime number, prime number calculation algorithms, etc. Mathematicians have developed many theories on topics. The study of the most common and known prime numbers is the study of Euclid in about 300 BC and this also constitutes the basic principles of arithmetic [1]. The Eratosthenes sieve, handled by the Greek mathematicians, is not sufficient in calculating large numbers in terms of cost and time . Then, in the $17th$ century, Fermat and Euler worked on the properties of prime numbers and to make them more understandable. Today, efforts are being made to find the largest prime number within the scope of the Great Internet Mersenne Prime Search (GIMPS) project [2]. As is known, Euclid proved that the prime numbers are infinite. However, no exact formula has yet been found to find these prime numbers. Therefore, studies are still ongoing on prime numbers. Since it is not easy to find prime numbers or to distinguish them from composite numbers, prime numbers are grouped according to some properties [1].
To date, a formula found and proven to generate prime numbers is not yet available in the literature [3]. Through trial-and-error, the search for prime numbers using mathematical calculations and algorithms continues. In this study, we gave a screening algorithm different from previous studies. We supported this method first with the help of tables.
Since we will work on set of the odd integers in our study, and every element of this set is likely to be prime, we named this set of numbers as a possible set of prime numbers and denoted this set with the letter M_s. To further narrow the single set of integers than the probability of being prime, we examined the finale digits of potentially prime numbers. It should be noted that the numbers in the last digit should end with one of the numbers 1, 3, 7 and 9, but the numbers that end in each of the last digits with one of these numbers are not prime. In this method, unlike other prime number tests, we have defined and used the numbers of possible prime numbers. Let's give some definitions that we use and which are not available in the literature.

Definition 1. *The numbers obtained by subtracting 1 from M_s numbers and dividing by 2 are called the number of rows of possible prime numbers and are indicated M_{ss}. For example,*
$M_{ss}(3) = 1, M_{ss}(5) = 2, M_{ss}(7) = 3, \cdots$

It is not possible to find prime numbers directly in M_s array. Therefore, it will be found that the non-prime numbers are found and the remaining numbers are prime numbers. These operations will be done using the row numbers of possible numbers. For $b \in Z^{odd}$ and $s, a \in Z$ the number s shows the number of rows of the number b. Let's define the following numbers with the help of this number b:

$$a = (s, s+1, s+2, s+3, \cdots) = (s+n);\;\; n \in N.$$

Using these numbers, let's call the numbers described below as the number of rows of non-prime numbers.

$$M'_{ss} = s\,(1, 1, 1, \ldots) + b\,(s, s+1, s+2, \cdots)\,.$$

$Number\ of\ rows\ with\ multiples\ of\ 3$										
$M_{ss}^{'}$	4	7	10	13	16	19	22	25	28	31
s	1	1	1	1	1	1	1	1	1	1
a	1	2	3	4	5	6	7	8	9	10
$Number\ of\ rows\ with\ multiples\ of\ 5$										
$M_{ss}^{'}$	12	17	22	27	32	37	42	47	52	57
s	2	2	2	2	2	2	2	2	2	2
a	2	3	4	5	6	7	8	9	10	11
$Number\ of\ rows\ with\ multiples\ of\ 7$										
$M_{ss}^{'}$	24	31	38	45	52	59	66	73	80	87
s	3	3	3	3	3	3	3	3	3	3
a	3	4	5	6	7	8	9	10	11	12
$Number\ of\ rows\ with\ multiples\ of\ 9$										
$M_{ss}^{'}$	40	49	58	67	76	85	94	103	112	121
s	4	4	4	4	4	4	4	4	4	4
a	4	5	6	7	8	9	10	11	12	13
$Number\ of\ rows\ with\ multiples\ of\ 11$										
$M_{ss}^{'}$	60	71	82	93	104	115	126	137	148	159
s	5	5	5	5	5	5	5	5	5	5
a	5	6	7	8	9	10	11	12	13	14
$Number\ of\ rows\ with\ multiples\ of\ 13$										
$M_{ss}^{'}$	84	97	110	123	136	149	162	175	188	201
s	6	6	6	6	6	6	6	6	6	6
a	6	7	8	9	10	11	12	13	14	15
$Number\ of\ rows\ with\ multiples\ of\ 15$										
$M_{ss}^{'}$	112	127	142	157	172	187	202	217	232	247
s	7	7	7	7	7	7	7	7	7	7
a	7	8	9	10	11	12	13	14	15	16
$Number\ of\ rows\ with\ multiples\ of\ 17$										
$M_{ss}^{'}$	144	161	178	195	212	229	246	263	280	297
s	8	8	8	8	8	8	8	8	8	8
a	8	9	10	11	12	13	14	15	16	17

For example,
for $s \in Z^{+}$, $b = 3, 5, 7, 9$ a sequence of numbers as follows is obtained using the formula above, respectively.

$$(1, 1, 1, \cdots) + 3(1, 2, 3, \cdots) = (4, 7, 10, \cdots),$$

$$(2, 2, 2, \cdots) + 5(2, 3, 4, \cdots) = (12, 17, 22, \cdots),$$

$$(3, 3, 3, \cdots) + 7(3, 4, 5, \cdots) = (24, 31, 38, \cdots),$$

$$(4, 4, 4, \cdots) + 9(4, 5, 6, \cdots) = (40, 49, 58, \cdots).$$

The first sequence $(4, 7, 10, \cdots)$ here is the sequence numbers of the odd multiples of the number 3, respectively $(12, 17, 22, \cdots)$, respectively. The sequence numbers of the odd multiples floors of the number 5, $(24, 31, 38, \cdots)$, respectively, gives the number of rows of odd multiples of 7, respectively. Tables for $M_{ss}^{'}$ numbers can also be edited:

According to the table above, the following theorem can be given.

Theorem 1. *The following formula is true for the ordinal numbers of non-prime numbers.*

$$a \times b + s = M_{ss}^{'}$$

By looking at the $M_{ss}^{'}$ numbers in the table above, we can give information about whether a given number is a prime number: If the given number is included in this table, it is not a prime number. For example, let's examine whether the number 299 is prime or not with the above table. First, let's write the number of rows of 299 using.

Definition 2. $M_{ss}^{'}$=149. *Since the number* 149 *is included in above table, this number cannot be a prime number. Non-prime numbers can also be obtained on the matrix by using* $M_{ss}^{'}$ *numbers, that is, the ordinal numbers of non-prime numbers. If the sequence numbers,* $M_{ss}^{'}$ *of non-prime numbers for the integer* $s \leq 8$ *are placed on the matrix, then the following equation can be written using the above theorem.*

$$M_{ss}^{'} = \begin{bmatrix} 1&0&0&0&0&0&0&0\\ 2&2&0&0&0&0&0&0\\ 3&3&3&0&0&0&0&0\\ 4&4&4&4&0&0&0&0\\ 5&5&5&5&5&0&0&0\\ 6&6&6&6&6&6&0&0\\ 7&7&7&7&7&7&7&0\\ 8&8&8&8&8&8&8&8 \end{bmatrix} \times \begin{bmatrix} 3&0&0&0&0&0&0&0\\ 0&5&0&0&0&0&0&0\\ 0&0&7&0&0&0&0&0\\ 0&0&0&9&0&0&0&0\\ 0&0&0&0&11&0&0&0\\ 0&0&0&0&0&13&0&0\\ 0&0&0&0&0&0&15&0\\ 0&0&0&0&0&0&0&17 \end{bmatrix} + \begin{bmatrix} 1&0&0&0&0&0&0&0\\ 1&2&0&0&0&0&0&0\\ 1&2&3&0&0&0&0&0\\ 1&2&3&4&0&0&0&0\\ 1&2&3&4&5&0&0&0\\ 1&2&3&4&5&6&0&0\\ 1&2&3&4&5&6&7&0\\ 1&2&3&4&5&6&7&8 \end{bmatrix}$$

$$\begin{bmatrix} 4&0&0&0&0&0&0&0\\ 7&12&0&0&0&0&0&0\\ 10&17&24&0&0&0&0&0\\ 13&22&31&40&0&0&0&0\\ 16&27&38&49&60&0&0&0\\ 19&32&45&58&71&84&0&0\\ 22&37&52&67&82&97&112&0\\ 25&42&59&76&93&110&127&144 \end{bmatrix}$$

The diagonal matrix b in this last formula consists of odd integers. Matrix a is obtained from the row numbers of the numbers in matrix b. This $M_{ss}^{'}$ matrix gives us the ordinal numbers of non-prime numbers. Also, the formed $M_{ss}^{'}$ matrix is a sub-triangular matrix, and the numbers on the prime diagonal of the $M_{ss}^{'}$ matrix give the sequence numbers of the exact squares of the odd non-prime numbers. For example, the main diagonal of the matrix $M_{ss}^{'}$; 12, 24, 40, 60, 84, 112, 144 non-prime rank numbers correspond to 9, 25, 49, 81, 121, 169, 225, 289 respectively. Here, the $M_{ss}^{'}$ matrix can be further generalized if desired. With the help of a matrix with an infinite number of rows and columns, formulas of the number of rows of non-prime numbers will also be given in the following theorem.

Theorem 2. *$k, n \in Z^{+}$, k is the number of lines, we have*

$$M_{ss}^{'row} = 2n(n+1) + (k-1)(2n+1), \ \ k \geq 1.$$

We will call this formula the matrix row formula for non-prime numbers. Let's examine this formula on an example below. Line 1 formula is as follows, so for $k = 1$ we have $M_{ss}^{'row} = 2n^2 + 2n$. For $k = 2$,

$$M_{ss}^{'row} = 2n^2 + 2n + 1 + 1.(2n+1) = 2n^2 + 4n + 1.$$

Continuing in this way, the desired equations for lines $k = 3, 4, 5, \ldots$ can be written similarly. These operations can also be written in matrix form as follows:

n=1 n=2 n=3 $\cdots$

$$\begin{matrix} 2n^2+2n \\ 2n^2+4n+1 \\ 2n^2+6n+2 \\ \vdots \end{matrix} \begin{bmatrix} 4 & 12 & 24 & \cdots \\ 7 & 17 & 31 & \ldots \\ 10 & 22 & 38 & \ldots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

The formula given here is the generalized form of the formula $a \times b + s$=$M_{ss}^{'}$. The first column in this matrix returns the number of rows of odd multiples of the number 3. The second column gives the number of rows of odd multiples of number 5. The third column gives the number of rows of odd multiples of number 7, and so on. Now, let's give the column formula below for the expanded $M_{ss}^{'}$ matrix. The sequence numbers of non-prime numbers from the matrix row formula can be written in the matrix form, or the same operations can be arranged as a column formula:

Theorem 3. *For $a, b \in Z^{+}$, $b \geq 1$, we have*

$$M_{ss}^{'column} = (3a+1) + (b-1)(2a+1).$$

Where b is the number of columns.

Let's examine this formula for some values: According to the formula, when the 1st column formula, $b = 1$, the following equation is obtained.

$$M_{ss}^{'column} = (3a+1).$$

For $b = 2$, we get

$$M_{ss}^{'column} = (3a+1) + 1(2a+1) = (5a+2).$$

Continuing with the same idea, similar equations can be written for b=3 ,4, 5 $\ldots$ lines. These equations can be written again in matrix form as follows:

$$
\begin{array}{c} \\ 3a+1 \\ 5a+2 \\ 7a+3 \\ \vdots \end{array}
\begin{array}{c} \begin{array}{cccc} a=1 & a=2 & a=3 & \cdots \end{array} \\ \begin{bmatrix} 4 & 7 & 10 & \cdots \\ 7 & 12 & 17 & \dots \\ 10 & 17 & 24 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \end{array}
$$

Note that this matrix has a diagonal and symmetrical matrix feature. Lower and upper triangular matrices can also be created with the help of symmetrical matrix.

$$
\begin{bmatrix} 4 & 0 & 0 & 0 \\ 7 & 12 & 0 & 0 \\ 10 & 17 & 24 & 0 \\ 13 & 22 & 31 & 40 \end{bmatrix} or \begin{bmatrix} 4 & 7 & 10 & 13 \\ 0 & 12 & 17 & 22 \\ 0 & 0 & 24 & 31 \\ 0 & 0 & 0 & 40 \end{bmatrix}
$$

The formula given in the theorem can be considered as the formula that gives the number of rows in the column. Therefore, we will call this formula the matrix column formula for non-prime numbers. Similarly, we will create a matrix of row numbers of prime numbers, and we will show the matrix of row numbers of these prime numbers as M_{ss}. First, some prime numbers are given below.

$$
\{3, 5, 7, 11, 13, 17, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, \dots\} .
$$

The line numbers of these prime numbers are as follows.

$$
M_{ss} = \{1, 2, 3, 5, 6, 8, 9, 11, 14, 15, 18, 20, 21, 23, 26, 28, 30, 33, \dots\} .
$$

Here, the elements of the following x and y matrices are integers, and the columns of the z matrix are the following formula, consisting of the ordinal numbers of the prime numbers:

$$
M_{ss} = xy + z
$$

$$
M_{ss}^{'} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 8 & 8 & 8 & 8 & 8 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 4 & 3 & 10 & 24 \\ 1 & 2 & 3 & 5 & 6 \\ 2 & 6 & 6 & 10 & 12 \\ 1 & 4 & 6 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{bmatrix}
$$

$$
M_{ss}^{'} = \begin{bmatrix} 1 & 2 & 3 & 5 & 6 \\ 2 & 6 & 6 & 15 & 30 \\ 3 & 8 & 9 & 20 & 36 \\ 5 & 14 & 15 & 30 & 48 \\ 6 & 18 & 21 & 35 & 54 \end{bmatrix}
$$

$M_{ss}^{'}$ and M_{ss} matrices are applications on the matrices, except for the table representation of non-prime numbers and the sequence numbers of prime numbers, respectively.

Definition 3. *The formula giving the $M_{ss}^{'}$ numbers on the prime diagonal can be given as follows. K_{ss}=M_{ss} $(1 + M_s)$. Here K_{ss} is the sequence number of the square of non-prime numbers. For example,*

$$
M_s = 7,\ M_{ss} = 3,\ K_{ss} = 3(1 + 7) = 24,\ M_{ss}^{'}(24) = 49 = 7^2
$$

$$
M_s = 11,\ M_{ss} = 5,\ K_{ss} = 5(1 + 11) = 60,\ M_{ss}^{'}(60) = 121 = 11^2
$$

Definition 4. *Let $M_s = x$. Then, we write*

$$
M_{ss} = \frac{(x-1)}{2} \ and\ K_{ss} = (\frac{x-1}{2})(1+x) = \frac{(x^2-1)}{2}.
$$

Here, the following definition can be given if the formula obtained to find non-prime numbers in the row and column in the graph of the formulas K_{ss} and $M_{ss}^{('column)}$ is developed.

Theorem 4. *For $b \in Z^{odd}$ and $b > 2$, we have*

$$
\frac{(b^2-1)}{2},\ \frac{(b^2+2b-1)}{2},\ \frac{(b^2+4b-1)}{2},\ \frac{(b^2+6b-1)}{2},\ \dots \tag{1}
$$

For $b = 3$, the ordinal numbers of the multiples of 3 are obtained. So, $4, 7, 10, 13, 16, 19, 22, 25$, ... Accordingly, since the number of rows is 13, the number itself is 39, this number is not prime. For $b = 5$, that is, the number of rows of multiples of 5.

$$12, 17, 22, 27, 32, 37, 42, \ldots$$

The same operations are repeated for other odd numbers.

b	$\frac{b^2-1}{2}$	$\frac{b^2+2b-1}{2}$	$\frac{b^2+4b-1}{2}$	$\frac{b^2+6b-1}{2}$	$\frac{b^2+8b-1}{2}$	$\frac{b^2+10b-1}{2}$
2	1,5	3,5	5,5	7,5	9,5	11,5
3	4	7	10	13	16	19
4	7,5	11,5	15,5	19,5	23,5	27,5
5	12	17	22	27	32	37
6	17,5	23,5	29,5	35,5	41,5	47,5
7	24	31	38	45	52	59
8	31,5	39,5	47,5	55,5	63,5	71,5
9	40	49	58	67	76	85
10	49,5	59,5	69,5	79,5	89,5	99,5
11	60	71	82	93	104	115
12	71,5	83,5	95,5	107,5	119,5	131,5
13	84	97	110	123	136	149

Observation results

1) The number of rows of no prime numbers is found in the table above.
2) When an even number is given instead of b, it is seen that even numbers correspond to the number of rows.
3) When odd number values are given instead of b, it is seen that the result is odd non-prime numbers.
4) When odd numbers are given instead of b, single multiples of the given number are obtained, respectively.

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