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Research Article

Robust Image Watermarking using DWT, SVD, Chirp-z and LU Decomposition

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Abstract

The exponential increase of multimedia technologies usage has made connectivity simpler, more convenient and quicker, but has also contributed to a number of infringements of digital content. Digital watermarking has been known to offer a solution to compliance issues related to copyright infringements in multimedia technology. This paper proposes a robust approach to image watermarking by integrating the benefits of all four algorithms. The proposed method is based on discrete wavelet transform (DWT), chirp z-transform (CZT), lower and upper (LU) decomposition and singular decomposition value (SVD). In this method, using 1-level DWT, the image is broken down into its frequency sub-bands. The LL sub-band is then converted into a z domain using CZT. LU decomposition was then used to further decompose the image into two matrixes, each of which was used to insert the watermark. SVD has been applied to each of the matrixes and the unique value of the watermark is inserted to the matrixes of the decomposed image. The robustness and imperceptibility of the proposed solution was tested by checking the watermarked image for various image processing operations. Experimental findings show that the proposed technique has high imperceptibility capability and it shows a reasonable level of robustness against signal processing operations such as filtering, scaling, JPEG, rotating, gamma correction, blurring, cropping, gaussian noise, contrast enhancement, histogram equalization, and salt & pepper noise.

Keywords: Watermarking, LU Decomposition, DWT, CZT, SVD

ADD, TDA, Cıvıltı-z ve AÜ Ayrıştırma kullanarak Sağlam Görüntü Filigranı Oluşturma

Öz

Multimedya teknolojileri kullanımının katlanarak artması, bağlanabilirliği daha basit, daha rahat ve daha hızlı hale getirdi, ancak aynı zamanda bir dizi dijital içerik ihlaline de katkıda bulundu. Dijital filigranlamanın, multimedya teknolojisindeki telif hakkı ihlalleri ile ilgili uyumluluk sorunlarına bir çözüm sunduğu bilinmektedir. Bu makale, dört algoritmanın tümünün faydalarını entegre ederek, görüntü filigranlamaya sağlam bir yaklaşım önermektedir. Önerilen yöntem, ayrık dalgacık dönüşümü (ADD), cıvıltı z-dönüşümü (CZT), alt ve üst (AU) ayrıştırma ve tekil ayrıştırma değerine (TDA) dayanmaktadır. Bu yöntemde, 1 seviyeli ADD kullanılarak, görüntü, frekans alt bantlarına bölünür. LL alt bandı daha sonra CZT kullanılarak bir z alanına dönüştürülür. Daha sonra görüntüyü, her biri filigranı eklemek için kullanılan iki matrise ayrıştırmak için AÜ ayrıştırması kullanıldı. TDA, matrislerin her birine uygulanmış ve filigranın benzersiz değeri, ayrıştırılmış görüntünün matrislerine eklenmiştir. Önerilen çözümün sağlamlığı ve algılanamazlığı, çeşitli görüntü işleme işlemleri için filigranlı görüntünün kontrol edilmesiyle test edildi. Deneysel bulgular, önerilen tekniğin yüksek algılanamazlık yeteneğine sahip olduğunu ve filtreleme, ölçekleme, JPEG, döndürme, gama düzeltme, bulanıklaştırma, kırpma, gauss gürültüsü, kontrast geliştirme, histogram eşitleme ve tuz- biber gürültüsü gibi sinyal işleme işlemlerine karşı makul düzeyde sağlamlık göstermektedir.

Anahtar Kelimeler: Filigranlama, AÜ Ayrıştırma, ADD, CZT, TDA

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1. Introduction

Watermarking has been identified in recent years as the primary approach to copyright enforcement and content authentication. watermarking is the technique of inserting digital content into a multimedia content. Digital watermarking is a validated technique that enables the protection of digital content from improper duplication, modifying and dispersion issues. A effective watermarking scheme has critical characteristics, such as imperceptibility, robustness, data capability and security. (Kamble et al.,2019; Subhashini & Bagan, 2017).

Watermarking schemes are sub grouped into three systems: non blind, semi blind and blind. In the non-blind method, in the embedding process and the extraction process, the original and watermark data are needed. The original data is not needed for extraction in the semi blind method. During the extraction process, both the cover data and the watermark are not needed under the blind scheme (Rasti et al.,2016; Laur, et al., 2015a). Algorithms for embedding watermarking are typically classified in spatial spectrum or in the frequency spectrum. watermarking performed in the spatial spectrum is generally by pixel modification, the data is inserted into the digital content. When watermarking is inserted in the frequency domain, the frequency values of the original data are extracted and the modified data is added by adjusting the coefficient of magnitude of the visual material according to the embedding algorithm. (Preet & Aggarwal, 2017; Boreiry & Keyvanpour, 2017).

Watermarking in frequency spectrum has greater computing costs, it has been found to possess more robust capability and the quality of the perceived data are better than that of the watermarking in spatial spectrum. At present, the discrete wavelet transform (DWT) is a frequently utilized frequency spectrum watermarking algorithms due to its frequency distribution, multi-resolution functionality and the localized and spatialized nature of its wavelet (Abodena et al.,2017; Prabha et al., 2019).

Singular Value Decomposition (SVD) is another widely used frequency domain algorithm. SVD is used in watermarking to gain greater transparency and robustness, as minor differences in its specific values are not affected by the visual perception of the multimedia data (Thakkar & Srivastava, 2017). The chirp z transform (CZT) is another transforming algorithm. The chirp z-transform is an algorithm that gives a better reliable representation of the zeros and poles of the system, it helps to estimate the system's propagation function, resulting in a sharper function at a lower bandwidth. The sharp nature of the spectrum and the substantially improved frequency resolution will help produce a highly imperceptible and durable watermarked data (Agoyi et al., 2015; Laur, et al., 2015b).

Another method of decomposition used by some watermarking researchers is LU (lower and upper) decomposition. Due to its large energy distribution, LU decomposition was used in watermarking, which offered a more explicit guide for the selection of watermark embedding pixels (Su, 2018; Wang et al.,2016).

This paper therefore proposes a new methodology of watermarking by combining DWT, CZT, LU and SVD. The four algorithms make up the proposed algorithm. By substantially providing a quality and good visual representation of the watermarked data and ensuring robustness against traditional signal processing operations and attacks, the proposed solution would help to achieve the robust and imperceptible characteristics of a successful watermarking algorithm.

2. Overview

2.1. DWT

The DWT divides the input signal into four data bands, where each band is usually separated into another four sub-bands. The decomposition of the band includes a low-frequency band (LL) and three high-frequency dictionary bands known as sub-bands of high-high (HH), high-low (HL) and low-high (LH). The LL sub-band contains the bulk of the information contained in the data. The vertical feature information is contained in the LH sub-band. There is horizontal edge information in the HL sub-band, and there is vertical edge information in the HH sub-band. The main benefits of using DWT is due to its multi-resolution and good localized property. Its multi-resolution features allow watermark to be embedded on all sub bands. Although inserting watermark in the high-frequency band usually produce a high visual perception quality, but may not be able to withstand several attacks. However, for Watermark inserted in the LL sub-band, the quality of visual perception of the watermarked image may degrade but it is generally immune to several attacks. (Abodena & Agoyi, 2018; Thakkar & Srivastava, 2017).

2.2. LU Decomposition

LU decomposition is a method by which, as given in Eq, (1,) a matrix M of N x N is decomposed into two matrixes, A and B. The lower triangular matrix is composed of A and its diagonal has 1. B includes the triangular upper matrix

$$M = A \times B = \begin{bmatrix} 1 & 0 & 0 \\ a_{21} & 1 & 0 \\ a_{31} & \mathrm{a}_{32} & 1 \end{bmatrix} \times \begin{bmatrix} d_1 & \mathrm{b}_{12} & \mathrm{b}_{13} \\ 0 & \mathrm{d}_2 & \mathrm{b}_{23} \\ 0 & 0 & \mathrm{d}_3 \end{bmatrix} \tag{1}$$

By then decomposing the B matrix into two, D and B^1 matrixes. As defined in Eq, (2) Matrix M can then be broken down into three matrices (Jane & Elbaşi, 2014)

$$M = A \times D \times B' = \begin{bmatrix} 1 & 0 & 0 \\ a_{21} & 1 & 0 \\ a_{31} & a_{32} & 1 \end{bmatrix} \times \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & \frac{b_{12}}{d_1} & \frac{b_{13}}{d_1} \\ 0 & 1 & \frac{b_{23}}{d_2} \\ 0 & 0 & 1 \end{bmatrix} \quad (2)$$

2.3. Singular Value Decomposition (SVD)

An N x N matrix can be broken down into three matrices in the SVD transform. Let M be a matrix of N x N, then I may be given as in Eq. (3)

$$\begin{aligned} [U\ S\ V] &= SVD(I) \\ I &= USV^T \end{aligned} \quad (3)$$

The main features of SVD are: When a small disturbance is inserted into an multimedia object, due to the stabilization nature of the singular value, which do not change dramatically. As it determines the image layer luminance and the singular vector pairs describe the object geometry. (Zhang & Qin, 2017; Nikbakht & Mahdavi, 2015);

2.4. Chirp-Z transform (CZT)

The Chirp-z transform is used to calculate the z transform of the sample sequence. Transformation facilitates the effective evaluation of Z transfer at points in the Z-plane lying on a circle or spiral contour beginning at some arbitrary point in the Z-plane. In Chirp-z transform, an easy means of quantifying the system's behavior is to characterize the system in terms of poles and zeros; poles are the roots of the transfer function's input function, and zeros are the roots of the forward feed function. By accurately positioning the poles and zeros of their conversion function, the z-transform chirp attempts to analyses the constructs. The greatest advantage of using the z-transform is that the sharpness of the peak point position is improved by calculating the z-transform along a contour near to the pole(s). Since the Chirp-z transform has the ability to calculate the z transformation both in and out the unit circle, by measuring the unit circle transformation, the contour are changed to shift nearer the signal poles, rendering the spectrum sharper. Chirp-z will allow the zooming in and out of the studied frequency range with a very high resolution, which will significantly enhance its frequency resolution. This would help to create a watermarked image that is extremely imperceptible and robust since there has been a sharpening in of frequency spectrum, thereby making it resolution to significantly improved. (Agoyi et al., 2015).

3. Proposed Technique

The proposed technique is laid out as embedding and extraction process. Flowchart for the proposed embedding and extracting process is depicted in Figure 1.

3.1. Watermarking Embedding Process

1. DWT on image I produces sub bands as given Eq. (4).

$$[LL\ LH\ HL\ HH] = DWT(I) \quad (4)$$

2. CZT of the LL band is computed *as* given in Eq. (5)

$$I_2 = CZT(LL) \quad (5)$$

3. Apply LU decomposition to I2 to decompose it. B is further decomposed as given in Eq. (6)

$$\begin{aligned} AB &= LU(I_2) \\ B &= D\ \mathrm{B}_1 \\ I_2 &= A\ D\ \mathrm{B}_1 \end{aligned} \quad (6)$$

4. SVD is applied to D from Eq. (6) to give 3 matrixes as given in Eq. (7)

$$U\ SV = SVD(D) \quad (7)$$

5. SVD is applied to W to give 3 matrixes as given in Eq. (8)

$$U_1\, S_1 V_1 = SVD(W) \tag{8}$$

6. singular values of original image and watermark is modified as given in Eq. (9)

$$S_2 = S + \alpha S_1 \tag{9}$$

7. the matrixes are combined as given in Eq. (10)

$$D_1 = U\, S_2 V^T \tag{10}$$

8. Combine the matrixes A and B_1 with the modified D_1 matrix as given in Eq. (11)

$$II_2 = AD_1 B_{\,1} \tag{11}$$

9. Inverse CZT is computed as given in Eq. (12)

$$LL_2 = ICZT(II_2) \tag{12}$$

10. inverse DWT is computed as given Eq. (13).

$$I_1 = iDWT(LL_2\ \ LH\ HL\ HH\ ) \tag{13}$$

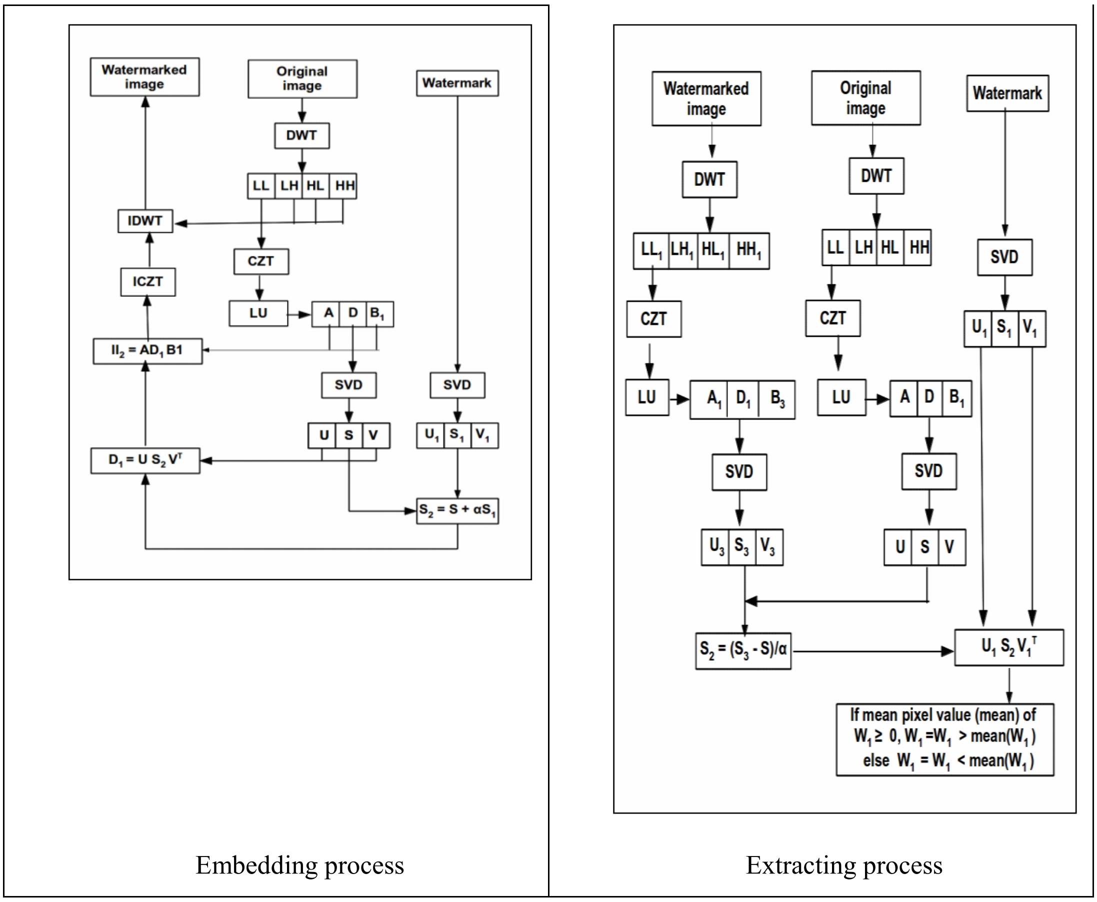


Figure 1: Flowchart for the Embedding and Extracting process

3.2. Watermarking Extraction process

1. DWT of original image produces sub bands as given in Eq. (14).

$$[LL\ LH\ HL\ HH]\ \ = DWT(I) \tag{14}$$

2. DWT of watermarked image produces sub bands as given in Eq. (15).

$$[LL_1\ \ LH_1\ HL_1\ HH_1]\ = DWT(I_1) \tag{15}$$

3. Compute CZT of original image as given in Eq. (16)

$$I_2 = CZT(LL) \tag{16}$$

4. Compute CZT of watermarked image as given in Eq. (17)

$$I_3 = CZT(LL_1) \tag{17}$$

5. LU decomposition is applied to I_2 to decompose it as given in Eq. (18)

$$AB = LU(I_2)$$
$$B = D\ \mathrm{B}_1 \tag{18}$$
$$I_2 = A\ D\ \mathrm{B}_1$$

6. LU decomposition is applied to I_3 to decompose as given in Eq. (19)

$$A_1 B_2 = LU(I_3)$$
$$B_2 = D_1\ \mathrm{B}_3 \tag{19}$$
$$I_3 = A_1\ D_1\ \mathrm{B}_3$$

7. SVD is applied to *D* to further decompose it as follows in Eq. (20)

$$[U\ SV] = SVD(D) \tag{20}$$

8. SVD is applied to D_1 to further decompose it as follows in Eq. (21)

$$[U_3\ S_3 V_3] = SVD(D_1) \tag{21}$$

9. Apply SVD to watermark image *W* to decompose it as follows in Eq. (22)

$$[U_1\ \mathbf{S_1} V_1] = SVD(W) \tag{22}$$

10. Calculate the singular value S_2 as given in Eq. (23)

$$S_2 = (S_3 - S)/\alpha \tag{23}$$

11. Combine the U and V of W with the acquired S3 as given in Eq. (24).

$$W_1 = \mathbf{U_1}\ S_2 {V_1}^T \tag{24}$$

12. Find the mean value of pixels (MP) of extracted watermark. If MP is negative perform the operation in Eq (25) to get the watermark image.

$$W_1(:)) \tag{25}$$

13. If *MP* is positive perform the operation in Eq 26 to get the watermark image

$$W_1(:)) \tag{26}$$

4. Performance Evaluation of the proposed technique

This paper used MATLABr2017a as simulation software. Experiments were conducted using three 512 x 512 gray scale as cover images and a 256 X 256 binary images the watermark. The cover images and watermark image are given in Figure 2.

Cameraman	Lena	Baboon	A-alphabet

Figure 2: Cover Images and watermark image

As given in Figure 2, “Cameraman”, “Lena” and “Baboon” images are used as the cover images and the A-alphabet is used as the watermark image.

The watermarked images are given in Figure 3(a -c) and the extracted watermark image is given as Figure 3d.

(3a) (3b) (3c) (3d)

Figure 3: Watermarked Images and extracted watermark image

It can be seen that the original cover images in Figure 2 and the watermarked images in Figure 3(a -c) had no obvious difference. This indicate that the proposed technique has good imperceptibility. The extracted watermark given in Figure 3d also looks just like the original watermark, indicating the proposed technique is very robust.

4.1. Imperceptibility Test

To measure the imperceptible capability, the peak signal to noise ratio (PSNR) can be used. The PSNR can be used to calculate the degradation caused by the watermarked effect, The PSNR of the original image and the watermarked image of the proposed algorithm is compared with that of Jane & Elbaşi and the results are given in Table 1.

Table 1: PSNR Values of watermarked Images

	Jane & Elbaşi	**Proposed Technique**
Cameraman	39.13	64.06
Lena	34.33	58.18
Baboon	38.82	59.09

As seen in Table 1, the PSNR values of the proposed technique is higher than that of Jane & Elbaşi, indicating that the proposed technique provides a better impeccability capacity.

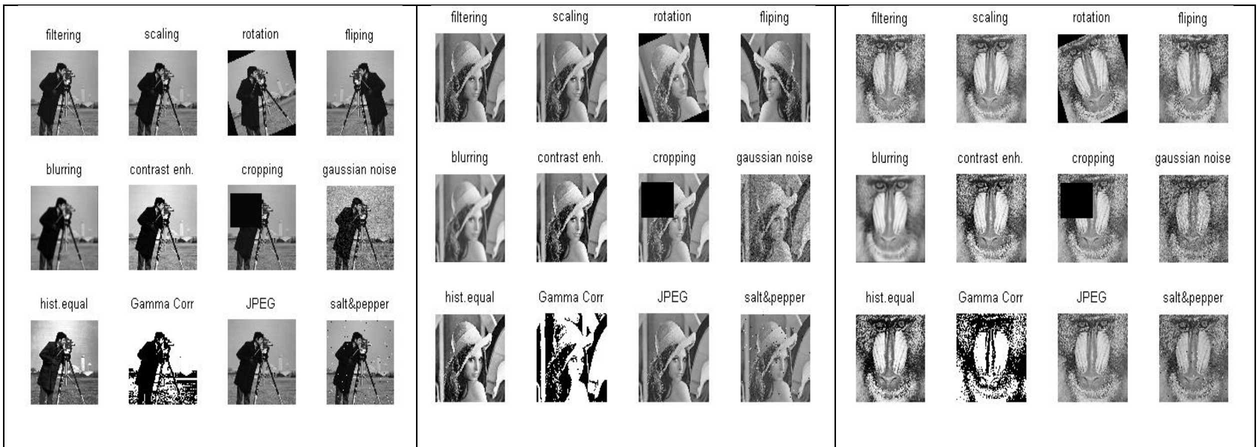
The PSNR values in dB of the watermarked image for the proposed technique are compared with those of Jane & Elbaşi to further discuss the accuracy of the watermarked images quantitatively, and the results are presented in Table 2.

Table 2: PSNR Values of attacked watermarked Images

		Jane & Elbaşi	**Proposed Technique**
Cameraman	Filtering	31.90	32.01
	Scaling	30.80	38.77
	Rotation	10.44	10.54
Lena	Filtering	30.51	31.56
	Scaling	30.20	33.87
	Rotation	11.19	11.15
Baboon	Filtering	23.20	21.75
	Scaling	22.48	22.47
	Rotation	11.54	10.63

PSNR values in Table 2, also shows that proposed technique has a good impeccability capability.

To show the visual properties of the proposed technique, the pictures of the attacked watermarked images Cameraman, Lena and Baboon are given in Figure 4a, Figure 4a and Figure 4c respectively.



(4a) (4b) (4c)

Figiure 4: Attacked watermarked images Cameraman, Lena and Baboon

To further test the imperceptibility of the proposed technique, other kinds of attacks such as flipping, blurring, contrast enhancement, cropping, Gaussian noise, histogram equalization, Gamma correction, JPEG and salt & pepper noise were applied to the watermarked images for Cameraman, Lena and Baboon images. The PSNR for attacked watermarked images are tabulated in Table 3.

Table 3: PSNR Values of attacked watermarked Images

	Cameraman	Lena	Baboon
Flipping	9.8412	11.7777	13.6928
Blurring	23.0955	24.4307	18.8381
Contrast enhancement	17.9085	20.1452	20.1840
Cropping	13.3313	12.4599	11.9062
Gaussian_noise	20.5319	20.0352	20.0271
Histogram_equalization	19.1964	19.1063	17.9479
Gamma_correction	9.6098	8.5694	8.5865
JPEG	42.3791	37.2956	30.4919
Salt & Pepper noise	25.2176	25.6843	25.5361

Pictures and data in Figure 4 and Table 3 shows that the proposed technique provides a good imperceptibility capability.

4.2. Robustness Test

To measure the robustness capability, similarity Ratio (SR) can be used. The SR is utilized to calculate the robustness capability of the proposed technique. To test the robustness of the proposed technique, attacks such as filtering, scaling and rotation were applied to the watermarked images. These attacks include filtering, scaling and rotation. The SR values of the attacked watermarked images for the proposed technique were compared with that of that of Jane & Elbaşı and the results are given in Table 5.

Table 5: SR for extracted watermark

		Jane & Elbaşi	Proposed Technique
Cameraman	Filtering	0.8988	0.9345
	Scaling	0.8901	0.9276
	Rotation	0.7436	0.5970
Lena	Filtering	0.8828	0.9243
	Scaling	0.8854	0.9213
	Rotation	0.8815	0.7903
Baboon	Filtering	0.8865	0.9085
	Scaling	0.8747	0.9069
	Rotation	0.8971	0.8863

As seen in Table 5, the SR values of the proposed technique is higher than that of Jane & Elbaşi, indicating that the proposed technique provides a better robustness capability.

To show the visual properties of proposed technique, the pictures of the attacked watermarked images and the extracted watermark are given. The extracted watermark images from Cameraman, Lena and Baboon are given in Figure 5a, Figure 5b and Figure 5c respectively.

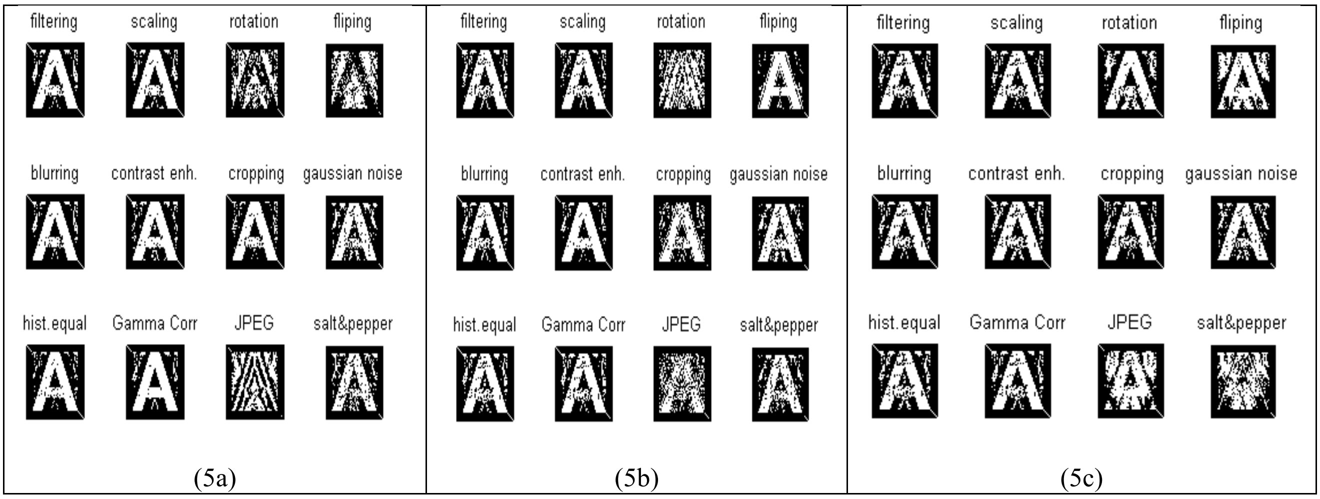


Figure 5: Extracted watermarks for Cameraman, Lena and Baboon attacked images

To further test the robustness capability of the proposed technique, other kinds of attacks such as flipping, blurring, contrast enhancement, cropping, Gaussian noise, histogram equalization, Gamma correction, JPEG and salt & pepper noise were applied to the watermarked images for Cameraman, Lena and Baboon images. The SR for extracted watermark images are tabulated in Table 5.

Table 5: SR for extracted watermark

	Cameraman	Lena	Baboon
Flipping	0.5500	0.8966	0.8379
Blurring	0.9402	0.9130	0.9041
Contrast_enhancement	0.9360	0.9488	0.8879
Cropping	0.9514	0.8379	0.9069
Gaussian_noise	0.8878	0.8934	0.8923
Histogram_equalization	0.9520	0.9035	0.8909
Gamma_correction	0.9502	0.9058	0.9048
JPEG	0.7143	0.7824	0.7624
Salt & Pepper noise	0.8737	0.8918	0.7091

The extracted watermarks for all the tested signal processing operations on Cameraman, Lena and Baboon as show in figure 5 and Table 5 indicates that the proposed techniques is robust against several signal processing operation thus, the proposed technique has high robustness capability.

5. Conclusion

A hybrid watermarking approach using Chirp Z transform, DWT and SVD via Lu decomposition was proposed in this paper. The cover image has been decomposed using DWT into its four frequency sub-bands. Using CZT, the low frequency sub band is then translated into the z domain. Using LU decomposition, the decomposed image derived from the z transform is then decomposed. SVD is added to the diagonal matrix of the upper triangular matrix that was obtained. The singular value obtained by applying SVD is then applied to the watermark's singular value. Using well known images, the proposed technique is tested on multiple signal processing operations. The experimental results show that the proposed watermarking technique has a very imperceptibility capability and it is very robust in diverse signal processing operations.

References

1. Abodena, O., & Agoyi, M. (2018). Colour Image Blind Watermarking Scheme Based on Fast Walsh Hadamard Transform and Hessenberg Decomposition. *Studies in Informatics and Control*, *27*(3), 339-348.
2. Abodena, O., Agoyi, M., & Celebi, E. (2017, May). Hybrid technique for robust image watermarking using discrete time fourier transform. In *2017 25th Signal Processing and Communications Applications Conference (SIU)* (pp. 1-4). IEEE.
3. Agoyi, M., Çelebi, E., & Anbarjafari, G. (2015). A watermarking algorithm based on chirp z-transform, discrete wavelet transform, and singular value decomposition. *Signal, Image and Video Processing*, *9*(3), 735-745.
4. Boreiry, M., & Keyvanpour, M. R. (2017, April). Classification of watermarking methods based on watermarking approaches. In *2017 Artificial Intelligence and Robotics (IRANOPEN)* (pp. 73-76). IEEE.
5. Jane, O., İlk, H. G., & Elbaşı, E. (2013, November). A secure and robust watermarking algorithm based on the combination of DWT, SVD, and LU decomposition with Arnold's Cat Map approach. In *2013 8th International Conference on Electrical and Electronics Engineering (ELECO)* (pp. 306-310). IEEE.
6. Kamble, P., Raut, N., Raut, A., & Naik, S. (2019, April). An Innovative Approach for Data Recovery Using Robust Reversible Watermarking. In 2019 3rd International Conference on Trends in Electronics and Informatics (ICOEI) (pp. 1401-1404). IEEE.
7. Laur, L., Daneshmand, M., Agoyi, M., & Anbarjafari, G. (2015b, May). Robust grayscale watermarking technique based on face detection. In *2015 23nd Signal Processing and Communications Applications Conference (SIU)* (pp. 471-475). IEEE.
8. Laur, L., Rasti, P., Agoyi, M., & Anbarjafari, G. (2015a). A Robust Color Image Watermarking Scheme Using Entropy and QR Decomposition. *Radioengineering*, *24*(4).
9. Nikbakht, P., & Mahdavi, M. (2015, May). Targeted watermark removal of a SVD-based image watermarking scheme. In *2015 7th Conference on Information and Knowledge Technology (IKT)* (pp. 1-6). IEEE.
10. Prabha, K., Vaishnavi, M. J., & Sam, I. S. (2019, April). Quaternion Hadamard transform and QR decomposition based robust color image watermarking. In *2019 3rd International Conference on Trends in Electronics and Informatics (ICOEI)* (pp. 101-106). IEEE.
11. Preet, C., & Aggarwal, R. K. (2017, May). Multiple image watermarking using LWT, DCT and arnold transformation. In *2017 International Conference on Trends in Electronics and Informatics (ICEI)* (pp. 158-162). IEEE.
12. Rasti, P., Samiei, S., Agoyi, M., Escalera, S., & Anbarjafari, G. (2016). Robust non-blind color video watermarking using QR decomposition and entropy analysis. *Journal of Visual Communication and Image Representation*, *38*, 838-847.
13. Su, Q., Wang, G., Zhang, X., Lv, G., & Chen, B. (2018). A new algorithm of blind color image watermarking based on LU decomposition. *Multidimensional Systems and Signal Processing*, *29*(3), 1055-1074.
14. Subhashini, R., & Bagan, K. B. (2017, March). Robust audio watermarking for monitoring and information embedding. In *2017 Fourth International Conference on Signal Processing, Communication and Networking (ICSCN)* (pp. 1-4). IEEE.
15. Thakkar, F. N., & Srivastava, V. K. (2017). A blind medical image watermarking: DWT-SVD based robust and secure approach for telemedicine applications. *Multimedia Tools and Applications*, *76*(3), 3669-3697.
16. Wang, D., Yang, F., & Zhang, H. (2016). Blind Color Image Watermarking Based on DWT and LU Decomposition. *Journal of Information Processing Systems*, *12*(4).
17. Zhang, Q., & Qin, Y. (2017, August). Adaptive Singular Value Decomposition and its Application to the Feature Extraction of Planetary Gearboxes. In *2017 International Conference on Sensing, Diagnostics, Prognostics, and Control (SDPC)* (pp. 488-492). IEEE.