NEAR APPROXIMATIONS IN VECTOR SPACES

H. TASBOZAN

ABSTRACT. Near set theory is a concept that emerged from the comparison of similarities between digital images. Another set theory, soft set, was obtained by Molodstov[3]. The space where we obtain the near sets is the nearness approximation space. Tasbozan [25] introduce the soft sets based on a near approximation space. The group and ring structure of algebraic structures on near sets are defined. In this article, the structure of vector spaces will be discussed with close approximations. First I defined a pair of near approximation operators with respect to congruence based on a subspace, investigated near subsets in the vector space and its basic properties. Then considered a near soft set over a vector space has a properties of vector space defined lower and upper near soft approximation operators in vector spaces.

1. INTRODUCTION

The rough set was created by pawlak with an equivalence relation on the universal set.[14, 15]. The remaining part of the equivalence classes in this equivalence relation is defined as the upper approximation of the rough set and the part that does not intersect with the lower approximation equivalence class is empty. The rough set is expressed by a pair of these approximations. The concept of near set, which is a more general form of the rough set, was built by pawlak[18, 17, 20, 21, 22, 23, 24, 19]. Accordingly, each rough set is a near set. In near sets, equivalence classes are given directly using real-valued functions instead of tables from which equivalence classes in rough sets are obtained. That's less complicated than that. By trying to make transitions between these theories, different sets have been obtained by studying the theories together. Many studies have been done in the field of algebraic structure as well as the studies in engineering field with these theories.

The concept of soft set, which is another different set theory, was found by Molodstov[3] and studied by many authors[2, 4, 5]. This set theory has been highly recognized in both theoretical and engineering. Feng and li[1] have obtained the rough soft set by associating the concepts of rough set and soft set, and similarly, Tasbozan[25] has obtained the near soft set builts up a connection between soft set and near set. The algebraic side of the rough set has been studied by many

scientists[7, 6, 26, 27, 10, 11, 13, 12, 8, 9] . Biswas and Nanda made the definition of subgroup with the approximation and Kuroki[12] made the definition of sub-ring with the approximation. Wang [8] studied some properties of the approximations for normal subgroups. Among the algebraic studies in near sets, firstly, Bağırmaz[11] gave near approximations on groups and davvas[6] gave near approximations on rings which is another one. My goal in this article is to create some properties of vector space on near approximation spaces. The notions of near subsets in the vector space and near soft approximation operators in vector spaces are defined and their basic properties are investigated.

2. Preliminary

2.1. Congruences and vector subspaces.
A congruence relation δ defined on the vector space V and the properties provided for a subspace W of this vector space are known by [26, 27].

Proposition 1. Let W, W_1, W_2 be subspaces of V. Then for all $\alpha, \beta \in V$,

(1) $\delta_W(\alpha) + \delta_W(\beta) = (\alpha + W) + (\beta + W) = \delta_W(\alpha + \beta)$
(2) $\delta_W(k\alpha) = (k\alpha + W) = k\delta_W, 0 \neq k \in K.$
(3) $\delta_{W1+W_2}(\alpha + \beta) = \delta_{W_1}(\alpha) + \delta_{W2}(\beta).$
(4) $\delta_{W1 \cap W_2}(\alpha) = \delta_{W_1}(\alpha) \cap \delta_{W2}(\alpha).$[27]

2.2. Near Sets.

Definition 2.1. Let $(\mathcal{O}, F, \sim_{B_r}, N_r, v_{N_r}) = NAS$ be an nearness approximation space where $\mathcal{O}$ is a nonempty set of perceptual objects, F is a countable set of probe functions and $B \subseteq F$. The sets

(1): $N_r(B)_* X = \bigcup\limits_{x : [x]_{B_r} \subseteq X} [x]_{B_r} = \{x \in \mathcal{O} : [x]_{B_r} \subseteq X\},$

(2): $N_r(B)^* X = \bigcup\limits_{x : [x]_{B_r} \cap X \neq \emptyset} [x]_{B_r.} = \{x \in \mathcal{O} : [x]_{B_r} \cap X \neq \emptyset\}$

(3): $Bnd_{N_r(B)}(X) = N_r(B)^* X \setminus N_r(B)_* X = \{x \in N_r(B)^* X : x \notin N_r(B)_* X\},$

are called $N_r(B) - lower approximation N_r(B)_* X$, $N_r(B) - upper approximation$ $N_r(B)^* X$ and boundary region of X in $(\mathcal{O}, F, \sim_{B_r}, N_r, v_{N_r})$, respectively. A set X with an approximation boundary $\left| Bnd_{N_r(B)}(X) \right| \geq 0$ is a near set[25].

2.3. Soft Sets.

Definition 2.2. Let O be an objects set and F be a properties of objects in O and $\mathcal{P}(O)$ is the set of all subsets of O . A pair (K, B) is called soft set over O if $B \subseteq F$ and $K : B \to \mathcal{P}(O)$ [4].

2.4. Near Soft Sets.

Definition 2.3. Let NAS be a nearness approximation space and $\sigma = (F, B)$ be a soft set over $\mathcal{O}$ where $\mathcal{O}$ is a nonempty set of perceptual objects, F is a countable set of probe functions and $B \subseteq F$. The sets $N_r^*(\sigma) = (F^*, B)$, $N_r*(\sigma) = (F_*, B)$ are called the upper and lower near approximation of $\sigma = (F, B)$ in NAS.

(1): $F_*(\phi) = N_r*(F(\phi)) = \cup\{x \in \mathcal{O} : [x]_{Br} \subseteq F(\phi)\}$,

(2): $F^*(\phi) = N_r^*(F(\phi)) = \cup\{x \in \mathcal{O} : [x]_{Br} \cap F(\phi) \neq \emptyset\}$ where all $\phi \in B$.

: Then the soft set $\sigma = (F, B)$ is called a near soft set and denotes NSS [25].

2.5. **Near subsets of vector spaces.** Let V be a vector space over K, Z be a subspace of V, and A a nonempty subset of V. δ is a congruence on V, $\delta = \delta_Z$ where $Z = [0]_\delta$. δ_Z is denote a congruence determined by a subspace Z.

The sets $N_{r\delta *}(A) = \{\alpha \in V : \delta(\alpha) \subseteq A\}$ and $N_r\delta^*(A) = \{\alpha \in V : \delta(\alpha) \cap A \neq \emptyset\}$ are called $\delta - lower$ and $\delta - upper$ near approximations of a set A with respect to congruence relation δ, respectively. If $BndN_r\delta(A) = N_r^*\delta(A) \backslash N_{r*}\delta(A) \geq 0$, A is called a near set in vector space.

Definition 2.4. Let Z be a subspace of vector space V, and $A \subseteq V$. $\delta = \delta_Z$ where $Z = [0]_\delta$ and $\delta(\alpha) = \delta_Z(\alpha) = \alpha + Z$ for any $\alpha \in V$. Then $\delta_r = \delta_{Zr} = \alpha + Z_r = [\alpha]_{Z_r}$ The sets

$$N_{r*}(Z)(A) = \{\alpha \in V : \alpha + Z = [\alpha]_{Z_r} \subseteq A\}$$
$$N_r^*(Z)(A) = \{\alpha \in V : [\alpha]_{Z_r} \cap A \neq \emptyset\}$$

are called the lower and upper near approximations of a set A with respect to subspace Z respectively. (V, Z) is called a near approximation space.

Example 2.5. Let $V = Z_{12}$ be a vector space on K. Where $K = \{0, 1\}$ is two elements field. $Z_1 = \{0, 6\}, Z_2 = \{0, 4, 8\}$ is a subspace of V. $A = \{0, 2, 3, 6, 8, 9, 10\}$. Then

For $r = 1$, $\quad \delta_r = \delta_{Zr} = \alpha + Z_r = [\alpha]_{Z_r}, \quad \delta_1 = \{[\alpha]_{Z_1}, [\alpha]_{Z_2}\},$

$$\delta_{Z_1} = \alpha + Z_1 = [\alpha]_{Z_1}, [0]_{Z_1} = \{0, 6\}, [1]_{Z_1} = \{1, 7\},$$
$$[2]_{Z_1} = \{2, 8\}, [3]_{Z_1} = \{3, 9\}, [4]_{Z_1} = \{4, 10\}, [5]_{Z_1} = \{5, 11\}$$
$$\delta_{Z_2} = \alpha + Z_2 = [\alpha]_{Z_2}, [0]_{Z_2} = \{0, 4, 8\}, [1]_{Z_2} = \{1, 5, 9\},$$
$$[2]_{Z_2} = \{2, 6, 10\}, [3]_{Z_2} = \{3, 7, 11\}$$

$N_{1*}(Z)(A) = \{\alpha \in V : \alpha + Z = [\alpha]_{Z_r} \subseteq A\} = \{0, 3, 6, 8, 9\},$ $\quad N_1^*(Z)(A) = \{\alpha \in V : [\alpha]_{Z_r} \cap A \neq \emptyset\} = Z_{12},$ $\quad BndN_1(Z)(A) = Z_{12} \backslash \{0, 3, 6, 9\} \geq 0.$

for $r = 2$, $\quad \delta_r = \delta_{Zr} = \alpha + Z_r = [\alpha]_{Z_r}, \quad \delta_2 = \{[\alpha]_{\{Z_1, Z_2\}}\}, \{Z_1, Z_2\} = \{0, 4, 6, 8\}$

$$\delta_{\{Z_1, Z_2\}} = \alpha + (\{Z_1, Z_2\}) = [\alpha]_{\{Z_1, Z_2\}}, [0]_{Z_1} = \{0, 4, 6, 8\},$$
$$[1]_{\{Z_1, Z_2\}} = \{1, 5, 7, 9\}, [2]_{\{Z_1, Z_2\}} = \{2, 6, 8, 10\}, [3]_{\{Z_1, Z_2\}} = \{3, 7, 9, 11\},$$

$N_{2*}(Z)(A) = \{\alpha \in V : \alpha + Z = [\alpha]_{Z_r} \subseteq A\} = \{2, 6, 8, 10\},$ $\quad N_1^*(Z)(A) = \{\alpha \in V : [\alpha]_{Z_r} \cap A \neq \emptyset\} = Z_{12},$ $\quad BndN_1(Z)(A) = Z_{12} \backslash \{2, 6, 8, 10\} \geq 0.$ Then A becomes a near set in vector space.

Theorem 2.6. *Let Z be a subspace of V, A and B nonempty subsets of V. Then*

(1) $N_{r*}(Z)(A) \subseteq A \subseteq N_r^*(Z)(A)$
(2) $N_{r*}(Z)(A \cap B) = N_{r*}(Z)(A) \cap N_{r*}(Z)(B)$
(3) $N_r^*(Z)(A \cup B) = N_r^*(Z)(A) \cup N_r^*(Z)(B)$
(4) $N_{r*}(Z)(A \cup B) \supseteq N_{r*}(Z)(A) \cup N_{r*}(Z)(B)$
(5) $N_r^*(Z)(A \cap B) = N_r^*(Z)(A) \cap N_r^*(Z)(B)$
(6) $A \subseteq B$ then $N_{r*}(Z)(A) \subseteq N_{r*}(Z)(B), N_r^*(Z)(A) \subseteq N_r^*(Z)(B)$.

Example 2.7. Let $V = Z_{12}$ be a vector space on K. Where $K = \{0,1\}$ is two elements field. $Z_1 = \{0,6\}, Z_2 = \{0,4,8\}$ is a subspace of V. $A = \{0,2,3,6,8,9,10\}, .B = \{1,5,7,9\}$ Then

$$\text{For } r = 1 , \quad \delta_r = \delta_{Zr} = \alpha + Z_r = [\alpha]_{Z_r}, \quad \delta_1 = \{[\alpha]_{Z_1}, [\alpha]_{Z_2}\},$$
$$\delta_{Z_1} = \alpha + Z_1 = [\alpha]_{Z_1}, [0]_{Z_1} = \{0,6\}, [1]_{Z_1} = \{1,7\},$$
$$[2]_{Z_1} = \{2,8\}, [3]_{Z_1} = \{3,9\}, [4]_{Z_1} = \{4,10\}, [5]_{Z_1} = \{5,11\}$$
$$\delta_{Z_2} = \alpha + Z_2 = [\alpha]_{Z_2}, [0]_{Z_2} = \{0,4,8\}, [1]_{Z_2} = \{1,5,9\},$$
$$[2]_{Z_2} = \{2,6,10\}, [3]_{Z_2} = \{3,7,11\}$$

$N_{1*}(Z)(A) = \{\alpha \in V : \alpha + Z = [\alpha]_{Z_r} \subseteq A\} = \{0,3,6,8,9\}, \quad N_{1*}(Z)(B) = \{1,7\}$
$N_1{}^*(Z)(A) = \{\alpha \in V : [\alpha]_{Z_r} \cap B \neq \emptyset\} = Z_{12}, N_1{}^*(Z)(B) = \{1,3,5,7,9,11\}$
$BndN_1(Z)(A) = Z_{12} \backslash \{0,3,6,9\} \geq 0.$ $BndN_1(Z)(B) = \{1,3,5,7,9,11\} \backslash \{1,7\} \geq 0.$

for $r = 2$, $\delta_r = \delta_{Zr} = \alpha + Z_r = [\alpha]_{Z_r}, \quad \delta_2 = \{[\alpha]_{\{Z_1,Z_2\}}\}, \{Z_1,Z_2\} = \{0,4,6,8\}$

$\delta_{\{Z_1,Z_2\}} = \alpha + (\{Z_1,Z_2\}) = [\alpha]_{\{Z_1,Z_2\}}, [0]_{Z_1} = \{0,4,6,8\}, [1]_{\{Z_1,Z_2\}} = \{1,5,7,9\},$

$[2]_{\{Z_1,Z_2\}} = \{2,6,8,10\}, [3]_{Z_1} = \{3,7,9,11\},$

$N_{2*}(Z)(A) = \{\alpha \in V : \alpha + Z = [\alpha]_{Z_r} \subseteq A\} = \{2,6,8,10\}, \quad N_1{}^*(Z)(A) = \{\alpha \in V : [\alpha]_{Z_r} \cap A \neq \emptyset\} = Z_{12}, \quad BndN_1(Z)(A) = Z_{12} \backslash \{2,6,8,10\} \geq 0.$ $N_{2*}(Z)(B) = \{\alpha \in V : \alpha + Z = [\alpha]_{Z_r} \subseteq B\} = \{1,5,7,9\}, \quad N_1{}^*(Z)(B) = \{\alpha \in V : [\alpha]_{Z_r} \cap B \neq \emptyset\} = \{1,5,7,9\}, \quad BndN_1(Z)(B) = \emptyset \geq 0.$ Then A, B is a NSS in vector space. Then the following statements are true;

(1) $N_{r*}(Z)(A) \subseteq A \subseteq N_r{}^*(Z)(A)$, $\{0,3,6,8,9\} \subseteq \{0,2,3,6,8,9,10\} \subseteq Z_{12}$
(2) $N_{r*}(Z)(A \cap B) = N_{r*}(Z)(A) \cap N_{r*}(Z)(B) = \emptyset$
(3) $N_r^*(Z)(A \cup B) = N_r{}^*(Z)(A) \cup N_r{}^*(Z)(B) = Z_{12}$
(4) $N_{r*}(Z)(A \cup B) \supseteq N_{r*}(Z)(A) \cup N_{r*}(Z)(B) = \{0,1,3,6,7,8,9\}$
(5) $N_r^*(Z)(A \cap B) = N_r^*(Z)(A) \cap N_r^*(Z)(B) = \{1,3,5,7,9,11\}$

Theorem 2.8. *Let Z be a subspace of V, A and B nonempty subspaces of V. Then*

(1) $N_r^*(Z)(A + B) = N_r{}^*(Z)(A) \cup N_r{}^*(Z)(B)$
(2) $N_{r*}(Z)(A + B) \supseteq N_{r*}(Z)(A) + N_{r*}(Z)(B)$
(3) $N_r^*(Z)(kA) = kN_r{}^*(Z)(A)$ for any $0 \neq k \in K$
(4) $N_{r*}(Z)(kA) = kN_{r*}(Z)(A)$ for any $0 \neq k \in K$.

Theorem 2.9. *Let Z and W be a subspace of V, A and B nonempty subspaces of V. Then*

(1) $N_{r*}(Z \cap W)(A) \supseteq N_{r*}(Z)(A) \cup N_{r*}(W)(A)$
(2) $N_r^*(Z \cap W)(A) \subseteq N_r^*(Z)(A) \cap N_r^*(W)(A)$
(3) $N_r^*(Z + W)(A + B) = N_r{}^*(Z)(A) + N_r{}^*(W)(B)$
(4) $N_{r*}(Z + W)(A + B) \supseteq N_{r*}(Z)(A) + N_{r*}(W)(B)$
(5) $Z \subseteq W \Rightarrow N_{r*}(Z)(A) \subseteq N_{r*}(Z)(A)$ and $N_{r*}(Z)(A) \subseteq N_{r*}(W)(A)$.

2.6. **Near soft approximations in vector spaces.** Let $\sigma = (F, B)$ be a NSS, $\mathcal{O}$ be a vector space over a field K and B be the parameter set. Let $\sigma_1 = (F_1, B), \sigma_2 = (F_2, B), ..., \sigma_n = (F_n, B)$ be n NSS in $(\mathcal{O}, B)$. Then $(F, B) = \sigma_1 + \sigma_2 + ... + \sigma_n$ is a NSS over $(\mathcal{O}, B)$ and is defined as $F(\phi) = \{x_1 + x_2 + ... + x_n : x_i \in F_i(\phi), i = 1, 2, ..., n\}$ for each $\phi \in B$. Let $\alpha \in K$ be any scalar and $\sigma = (F, B)$ be a NSS over

$(\mathcal{O}, B)$, then αF is a NSS over $(\mathcal{O}, B)$ and is defined as follows:$\alpha(F, B) = (G, B)$, $G(\phi) = \{\alpha x : x \in F(\phi)\}$for each $\phi \in B$.

Definition 2.10. Let $\sigma = (F, B)$ be a NSS over $\mathcal{O}$ in NAS and $\mathcal{O}$ be a vector space over a field K and B be the parameter set. If $(F, B) = \sigma_1 + \sigma_2 + ... + \sigma_n$ is a NSS over $(\mathcal{O}, B)$ and is defined as $F(\phi) = \{x_1 + x_2 + ... + x_n : x_i \in F_i(\phi), i = 1, 2, ..., n\}$ for each $\phi \in B$ and $\alpha \in K$ be any scalar,αF is a NSS over $(\mathcal{O}, B)$ and is defined as follows:$\alpha(F, B) = (G, B)$, $G(\phi) = \{\alpha x : x \in F(\phi)\}$for each $\phi \in B$. Then $N_r^*(\sigma)$, $N_{r*}(\sigma)$ are called the upper and lower near soft approximation operators in vector spaces, respectively and (F, B) called as near soft vector space.

Example 2.11. Let $\mathcal{O} = Z_{12}$ be a vector space over a field K and $B = \{\phi_1, \phi_2\} \subseteq \mathcal{F} = \{\phi_1, \phi_2\}$ be a set of functions. $\phi_1 = \{0, 4, 8\}, \phi_2 = \{0, 6\}$ Let $\sigma_1 = (F_1, B), \sigma_2 = (F_2, B)$ be near soft sets in $(\mathcal{O}, B)$. $(F, B) = \{\sigma_1, \sigma_2\} = \{(F_1, B), (F_2, B)\}$

Let $\sigma_1 = (F_1, B)$, $B = \{\phi_1, \phi_2\}$ be a soft set defined by

$$(2.1) \qquad\qquad F_1(\phi_1) = \{0, 4\}, F_1(\phi_2) = \{0, 6, 8\}$$

. for $r = 1, (F_1, B)$ is a NSS;

$[0]_{\phi_1} = \{0, 4, 8\}, [1]_{\phi_1} = \{1, 5, 9\}, [2]_{\phi_1} = \{2, 6, 10\}, [3]_{\phi_1} = \{3, 7, 11\}$
$[0]_{\phi_2} = \{0, 6\}, [1]_{\phi_2} = \{1, 7\}, [2]_{\phi_2} = \{2, 8\}, [3]_{\phi_2} = \{3, 9\}, [4]_{\phi_2} = \{4, 10\}, [5]_{\phi_2} = \{5, 11\}$

$$(2.2) \qquad N_{1*}(\sigma_1) = N_{1*}(F_1(\phi), B) = (N_{1*}F_1(\phi), B) = (F_{1*}(\phi), B)$$

for $\phi_2 \in B$, $N_{1*}(\sigma) = (F_{1*}(\phi_2), B) = \{(\phi_2, \{0, 6\})\}$

$$(2.3) \qquad\qquad N_1^*(\sigma_1) = (F_1^*(\phi), B)$$

for $\phi_1, \phi_2 \in B$, $N_1^*(\sigma) = \{(\phi_1, \{0, 4, 8\}), (\phi_2, \{0, 2, 6, 8\})\}$
$Bnd_N(\sigma) \geq 0$,then (F_1, B) is a NSS.
$r = 2, (F_1, B)$ is a NSS;

$$[0]_{\phi_1, \phi_2} = \{0\},$$
$$[1]_{\phi_1, \phi_2} = \{1\}, ...$$
$$[11]_{\phi_1, \phi_2} = \{11\}$$

$$N_{2*}(\sigma_1) = \{(\phi_1, \{0, 4\}), (\phi_2, \{0, 6, 8\})\}$$
$$N_2^*(\sigma_1) = \{(\phi_1, \{0, 4\}), (\phi_2, \{0, 6, 8\})\}$$

$Bnd_N(\sigma) \geq 0$, then $\sigma_1 = (F_1, B)$ is a NSS.
Let $\sigma_2 = (F_2, B)$, $B = \{\phi_1, \phi_2\}$ be a soft set defined by

$$(2.4) \qquad\qquad F_2(\phi_1) = \{0, 8\}, F_2(\phi_2) = \{1, 2, 7\}$$

. for $r = 1, (F_2, B)$ is a NSS ;

$$(2.5) \qquad N_{1*}(\sigma_2) = N_{1*}(F_2(\phi), B) = (N_{1*}F_2(\phi), B) = (F_{2*}(\phi), B)$$

for $\phi_2 \in B$, $N_{1*}(\sigma_2) = (F_{2*}(\phi_2), B) = \{(\phi_2, \{1, 7\})\}$

$$(2.6) \qquad\qquad N_1^*(\sigma_2) = (F_2^*(\phi), B)$$

for $\phi_1, \phi_2 \in B$, $N_1^*(\sigma_2) = \{(\phi_1, \{0, 4, 8\}), (\phi_2, \{1, 2, 7, 8\})\}$

$Bnd_N(\sigma) \geq 0$, therefore (F_2, B) is a NSS.

Lets see if (F, B) is a near soft vector space.

*: $(F, B) = \sigma_1 + \sigma_2 = (F_1, B) + (F_2, B) = \{(\phi_1, \{0, 4\}), (\phi_2, \{0, 6, 8\})\} \cup \{(\phi_1, \{0, 8\}), (\phi_2, \{1, 2, 7\})\} = \{(\phi_1, \{0, 4, 8\}), (\phi_2, \{1, 2, 6, 7, 8\})\}$ is a NSS.

$N_{1*}(F, B) = \{(\phi_1, \{0, 4, 8\}), (\phi_2, \{1, 2, 6, 7, 8\})\} = N_1^*(F, B), Bnd_N(F, B) \geq 0$

*: $\alpha = 2 \in Z$ be any scalar and $\sigma_1 = (F_1, B)$ is a NSS. Then

$\alpha \sigma_1 = \alpha(F_1, B) = (G, B) = 2.\{(\phi_1, \{0, 4\}), (\phi_2, \{0, 6, 8\})\} = \{(\phi_1, \{0, 8\}), (\phi_2, \{0, 4\})\}$

$N_{1*}(G, B) = \emptyset$ therefore (G, B) is not a NSS. Because of this $\sigma = (F, B)$ is not a near soft vector space .

Definition 2.12. Let $\mathcal{O}$ be a vector space over a field K and B be a parameter set. Let $\sigma = (F, B)$ be a NSS over $\mathcal{O}$. Then G is said to be a near soft vector space(denotes $NSVS$) or a near soft linear space of $\mathcal{O}$ over K, if $G(\phi)$ is a vector subspace of $\mathcal{O}$, for each $\phi \in B$.

Definition 2.13. Let $\sigma = (F, B)$ be a $NSVS$ of $\mathcal{O}$ over K and $Bnd_N(F, B) \geq 0$.

i) (F, B) is said to be null $NSVS$ if $F(\phi) = \{\varphi\}$,for each $\phi \in B$ where φ is null element of $\mathcal{O}$.

ii) (F, B) is said to be absolute $NSVS$ if $F(\phi) = \mathcal{O}$,for each $\phi \in B$.

Definition 2.14. Let $\sigma = (F, B)$ be a $NSVS$ of $\mathcal{O}$ over K.Let $\sigma = (G, B)$ be a NSS over $(\mathcal{O}, B)$. Then (G, B) is said to be a $NSVS$ of (F, B) if it satistified the following conditions;

i) for each $\phi \in B$, $G(\phi)$ is a vector subspace of $\mathcal{O}$ over K.

ii) $F(\phi) \supseteq G(\phi)$ for each $\phi \in B$.

3. Conclusions

In this study, we introduce near subsets in the vector space and near soft approximation operators in vector spaces. Based on the relationships between the congruences and vector subspaces of a vector space, I have given the definition of the lower approximations in the near approximation spaces based on a vector subspace of a subset of the vector space. I think the near soft vector spaces given here will be useful in theory as well as applications of near sets and soft sets. I will try to obtain different results and findings regarding these concepts in my future studies.

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Hatay Mustafa Kemal University, Mathematics of Department, 31060, Hatay, Turkey
Email address: htasbozan@mku.edu.tr