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# CURRENT-VOLTAGE CHARACTERISTIC OF JOSEPHSON JUNCTION WITH FRACTIONAL TERM IN CURRENT-PHASE RELATION

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## ABSTRACT

In this study, we carried out the analysis of the influence of unconventional current-phase relation of Josephson junction with additional fractional term $\sin(\phi/2)$ to first harmonic $\sin\phi$ on current-voltage characteristics. The current-voltage characteristic of the junction was calculated for the different amplitudes of fractional term *m* and McCumber parameter $\beta$. It was shown increasing the amplitude of fractional terms leads to increasing in the hysretesis on the current-voltage characteristics.

**Keywords:** Josephson junction, Current-phase relation, Current-voltage characteristic

## 1. INTRODUCTION

It is well known that the dynamics of Josephson junction for the case of harmonic current-phase relation $I = I_c \sin\phi$ is given by the equation of resistively shunted junction model [1,2]

$$\beta\ddot{\phi} + \dot{\phi} + \sin\phi = i_e \quad , \tag{1}$$

where $i_e$ *is* normalized external current via Josephson junction in units of critical current $I_c$, dots over $\phi$ corresponds to the derivative with respect to dimensionless time $\frac{\Phi_0}{2\pi I_c R_N}$, $\Phi_0$ is the magnetic flux quantum, $R_N$ is the normal resistance of Josephson junction. In Eq. (1) notation $\beta$ is the McCumber parameter of Josephson junction $\beta = \frac{2e}{\hbar} I_c R_N^2 C$, which determines the size of hysteresis in the current-voltage characteristic of Josephson junction. It is well known, that the case of $\beta >> 1$ corresponds to tunnel junction [1, 2]. In the case of $\beta << 1$ hysteresis on current-voltage characteristic is absent and in Eq. (1) we can be neglected by the first term.

Relationship $I = I_{c0}\sin\phi$ is fulfilled with a high accuracy for Josephson junctions on low-temperature superconductors [3]. In the limit of tunnel Josephson junction (high capacitance limit) $\beta >> 1$, the numerical solution of Eq. (1) shows that the current-voltage characteristic has two separate branches: S (superconducting) and R (resistive) branches. In the case of junctions on new topological superconductors, the current-phase relation include additional fractional term [4-5],

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$$I = I_c f_m(\phi) = I_{c0}(\sin\phi + m\sin(\phi/2)) \quad . \tag{2}$$

The second term in Equation (2) related to Majorana quasi-particles and dynamical detection of these particles seems very challenging in solid-state physics. The discovery of Majorana fermions seems interesting from the point of fault-tolerant quantum computing [6]. Some dynamical properties of Josephson junction with unconventional current-phase relation of the type $I = I_{c0}(\sin\phi + \alpha\sin(2\phi))$ were investigated in Ref. [7-8]. In this study, we carried out the analysis of the current-voltage characteristics of the single junction with unconventional relation (2).

## 2. BASIC EQUATIONS AND RESULTS

The dynamics of Josephson junction for the case of current-phase relation (2) is given by the equation of resistive model [4] similar Equation (1)

$$\beta\ddot{\phi} + \dot{\phi} + \sin\phi + m\sin(\phi/2) = i_e \quad . \tag{3}$$

For the overdamped Josephson junctions ( $\beta << 1$) we can be neglected the first term in Eq. (3) and use analytical calculations of current-voltage characteristics similar to Ref. [1]. Result coincide with standard expression from textbooks (see [1] and [2] (Figure 1a)

$$v = \sqrt{i_e^2 - f\ (m)} \quad . \tag{4}$$

However, function $f(m)$ varied with *m* and result presented in Fig. 1 b. It is clear that in the case of *m=0* will be *f=1*. It means that in the case of $\beta << 1$ hysteresis on the current-voltage characteristics of junction is absent and critical current linearly changes with the amplitude of fractional term *m* in current-phase relation (3). This results in agreement with very recent calculations in Ref. [9].

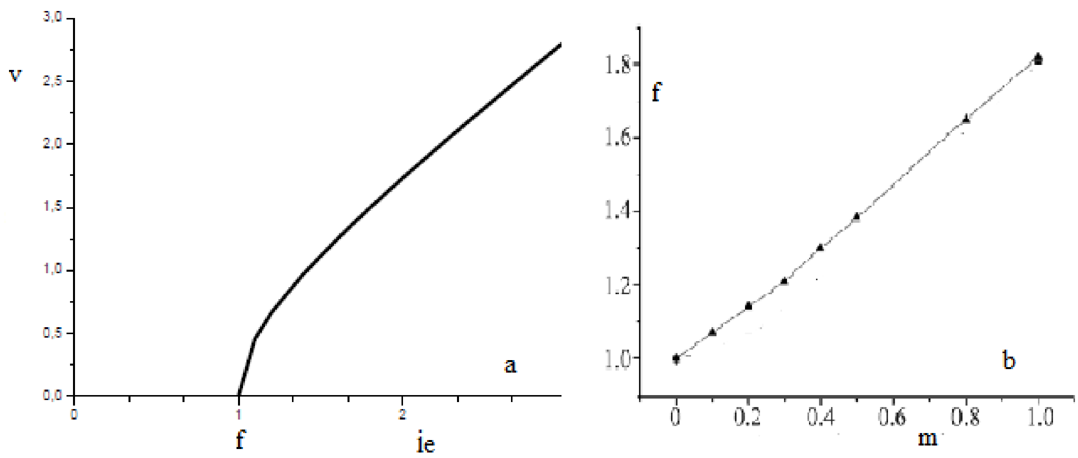


**Figure 1.** Current-voltage characteristic of overdamped Josephson junction with current-phase relation (2) with McCumber parameter $\beta << 1$

For the tunnel Josephson junctions ($\beta = 30$) numerical solution of Equation (3) obtained using the Runge-Kutta method of four order [9]. For average voltage *v*, we use the time averaging procedure of numerical solution of Equation (3). Result of calculation of current-voltage characteristic for the different amplitudes of fractional term $m$ presented in Figure 2. As followed from this Figure 2, the size of hysteresis on current-voltage characteristics grows with increasing amplitude *m* of the fractional term in current-phase relation. It means that the critical current of the junction is increased with amplitude $m$ of fractional term *m*, accordingly [9] (Figure 1b)

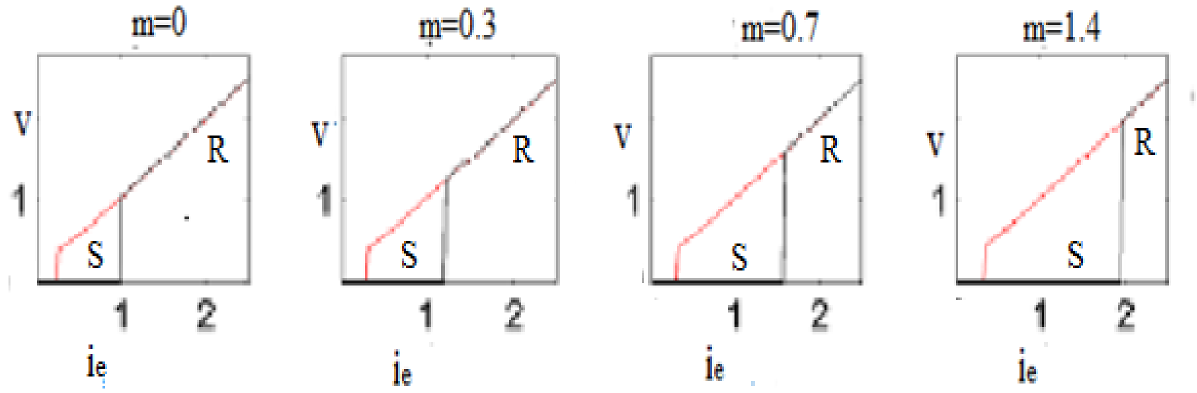


**Figure 2.** Current-voltage characteristic of tunnel Josephson junction with current-phase relation (2) with McCumber parameter $\beta = 30$

Thus, the influence of unconventional current-phase relation with a fractional term of Josephson junction on current-voltage characteristics was investigated. Renormalization of critical current in single Josephson junction with fractional terms leads to increasing of the hysretesis on the current-voltage characteristics for the case $\beta >> 1$. In the case of $\beta << 1$, the current-voltage characteristic reveal classical square root character with linearly growing critical current with an amplitude of fractional term *m*.

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