Hacettepe Journal of Mathematics & Statistics

Hacet. J. Math. Stat.
Volume 50 (5) (2021), 1491 – 1499
DOI : 10.15672/hujms.823694

Research Article

Study of generalized networks using graph theory

Tomáš Vetrík

Department of Mathematics and Applied Mathematics, University of the Free State, Bloemfontein, South Africa

Abstract

We study eccentricity-based indices such as Zagreb eccentricity indices, general eccentricity index and total eccentricity index. We present exact values of eccentricity-based indices of generalized networks related to binary and m-ary trees, where $m \geq 2$.

Mathematics Subject Classification (2020). 05C90, 05C12

Keywords. graph theory, eccentricity-based index, network

1. Introduction

Let G be a network with vertex set $V(G)$ and edge set $E(G)$. The distance between two vertices is the number of edges in a shortest path connecting them. The eccentricity $ecc_G(v)$ of a vertex v is the distance between v and a vertex furthest from v in G.

Gao et al. [3] and Imran et al. [5] studied networks $HT(k)$ (see Figure 1) and ST_k^1 (see Figure 2). We generalize those networks.

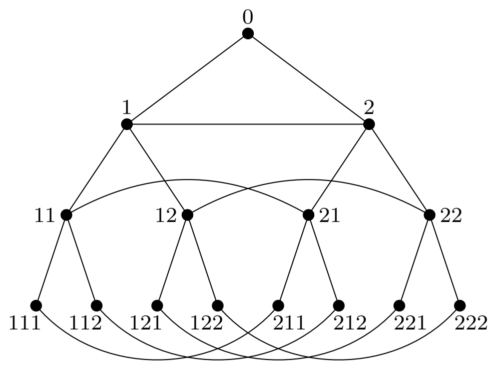

Figure 1. Network $HT(k)$ for $k = 3$.

We introduce the sets $HT_{m,k}$ and $ST_{m,k}$ for $m \geq 2$. The special cases for $m = 2$ contain the networks $HT(k)$ and ST_k^1 studied in [3] and [5].

We define general networks using complete m-ary trees $T_{m,k}$ of height $k \geq 1$. The tree $T_{m,k}$ contains $1 + m + m^2 + \cdots + m^k$ vertices which are divided into $k + 1$ sets V_i for

Email address: vetrikt@ufs.ac.za
Received: 09.11.2020; Accepted: 01.06.2021

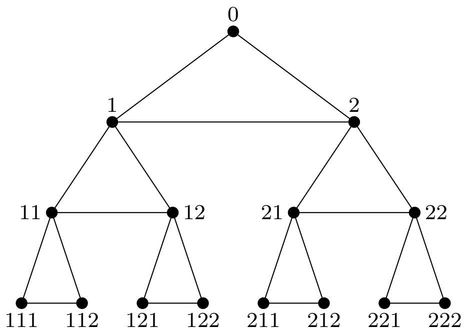

Figure 2. Network ST_k^1 for $k = 3$.

$i = 0, 1, 2, \dots, k$, where $|V_i| = m^i$. The vertex in V_0 is called the root and we denote it by 0. For $i = 1, 2, \dots, k$, we denote the vertices in V_i by $(x_1, x_2, \dots, x_i)$ or simply $x_1 x_2 \dots x_i$, where $x_1, x_2, \dots, x_i \in \{1, 2, \dots, m\}$. So

$$V_i = \{(x_1, x_2, \dots, x_i) \mid 1 \leq x_j \leq m;\ j = 1, 2, \dots, i\}.$$

The root is adjacent to all the m vertices $(1), (2), \dots, (m)$ in V_1. For $i = 1, 2, \dots, k-1$, any vertex $(x_1, x_2, \dots, x_i)$ of V_i is adjacent to m vertices

$$(x_1, x_2, \dots, x_i, 1), (x_1, x_2, \dots, x_i, 2), \dots, (x_1, x_2, \dots, x_i, m)$$

of V_{i+1}. If $m = 2$, then $T_{2,k}$ is called the complete binary tree.

The set $ST_{m,k}$ contains all the networks obtained from $T_{m,k}$ by connecting each vertex v in V_i for $i = 2, 3, \dots, k$, to any $m - 1$ vertices in V_i whose first coordinate is equal to the first coordinate of v, and by connecting every two different vertices in V_1. One of the networks of $ST_{m,k}$ for $m = 3$ and $k = 3$ is presented in Figure 3. Note that if $k = 1$ or 2, there is only one network in $ST_{m,k}$ for every $m \geq 2$.

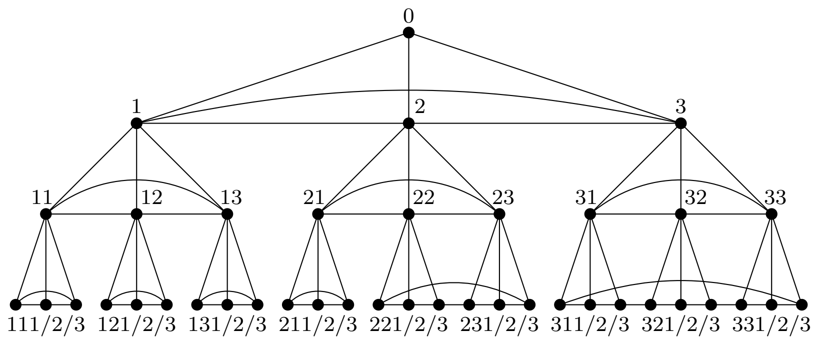

Figure 3. One of the networks in $ST_{m,k}$ for $m = 3$ and $k = 3$.

For $G \in ST_{m,k}$, no vertex of $V(G) \setminus (V_0 \cup V_1)$ with the first coordinate j is adjacent to a vertex with the first coordinate l, where $j \neq l$. Therefore, the eccentricity of $(x_1, x_2, \dots, x_i) \in V_i$ for $i \geq 2$ is $k + i - 1$. A vertex furthest from $(x_1, x_2, \dots, x_i)$ is any vertex $(y_1, y_2, \dots, y_k) \in V_k$, where $x_1 \neq y_1$ and the shortest path between $(x_1, x_2, \dots, x_i)$ and $(y_1, y_2, \dots, y_k)$ is

$$(x_1, x_2, \dots, x_i), (x_1, x_2, \dots, x_{i-1}), \dots, (x_1), (y_1), (y_1, y_2), \dots, (y_1, y_2, \dots, y_k).$$

The eccentricity of any vertex in $V_0 \cup V_1$ is k.

The network ST_k^1 is one of the networks in $ST_{2,k}$. The network ST_k^1 is obtained from the complete binary tree $T_{2,k}$ by joining the vertex $(x_1, x_2, \dots, x_{i-1}, 1)$ and the vertex $(x_1, x_2, \dots, x_{i-1}, 2)$ for each $i = 1, 2, \dots, k$ and $x_1, x_2, \dots, x_{i-1} \in \{1, 2\}$.

The set $HT_{m,k}$ contains all the networks obtained from $T_{m,k}$ by connecting each vertex v in V_i for $i = 2, 3, \ldots, k$, to any $m - 1$ vertices in V_i whose second coordinate is equal to the second coordinate of v, and by connecting any two vertices in V_1. If $k = 1$ or 2, there is only one network in $HT_{m,k}$ for every $m \geq 2$. The network which is in the set $HT_{m,k}$ if $m = 3$ and $k = 2$ is presented in Figure 4.

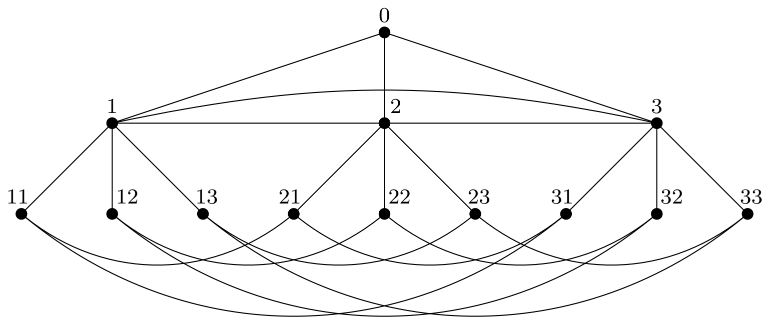

Figure 4. The network which is in the set $HT_{m,k}$ if $m = 3$ and $k = 2$.

For $G \in HT_{m,k}$, no vertex of $V(G)\setminus(V_0 \cup V_1)$ with the second coordinate j is adjacent to a vertex with the second coordinate l, where $j \neq l$. The eccentricity of $(x_1, x_2, \ldots, x_i) \in V_i$ for $i \geq 2$ is $k+i-1$ A vertex furthest from $(x_1, x_2, \ldots, x_i)$ is any vertex $(y_1, y_2, \ldots, y_k) \in V_k$, where $x_1 \neq y_1$ and $x_2 \neq y_2$. A shortest path between $(x_1, x_2, \ldots, x_i)$ and $(y_1, y_2, \ldots, y_k)$ is for example the path

$$(x_1, x_2, \ldots, x_i), (x_1, x_2, \ldots, x_{i-1}), \ldots, (x_1), (y_1), (y_1, y_2), \ldots, (y_1, y_2, \ldots, y_k),$$

where $x_1 \neq y_1$ and $x_2 \neq y_2$. The eccentricity of any vertex in $V_0 \cup V_1$ is k. This implies that any network in $ST_{m,k}$ and any network in $HT_{m,k}$ have the same number of vertices of given eccentricity.

The network $HT(k)$ studied in [3] and [5] is one of the networks in $HT_{2,k}$. The network $HT(k)$ is obtained from the complete binary tree $T_{2,k}$ by joining the vertex $(1, x_2, \ldots, x_i)$ and the vertex $(2, x_2, \ldots, x_i)$ for each $i = 1, 2, \ldots, k$ and $x_2, \ldots, x_i \in \{1, 2\}$.

Zagreb indices belong to the most well-known topological indices. We mention them in order to present the similarity between the definitions of classical Zagreb indices and Zagreb eccentricity indices. The first Zagreb index M_1 and the second Zagreb index M_2 of a network G are defined as

$$M_1(G) = \sum_{v \in V(G)} (deg_G(v))^2 = \sum_{uv \in E(G)} (deg_G(u) + deg_G(v))$$

and

$$M_2(G) = \sum_{uv \in E(G)} deg_G(u) deg_G(v),$$

where $deg_G(u)$ and $deg_G(v)$ are the degrees of vertices u and v in G, respectively. The Zagreb eccentricity indices are defined as

$$\begin{aligned}
\xi_1(G) &= \sum_{v \in V(G)} (ecc_G(v))^2, \\
\xi_1'(G) &= \sum_{uv \in E(G)} (ecc_G(u) + ecc_G(v)), \\
\xi_2(G) &= \sum_{uv \in E(G)} ecc_G(u) ecc_G(v).
\end{aligned}$$

The first Zagreb eccentricity index ξ_1 and the second Zagreb eccentricity index ξ_2 were introduced in [7]. The ξ_1' index was introduced in [4] and we call it the modified Zagreb eccentricity index.

We generalize $\xi_1(G)$ and introduce the general eccentricity index

$$EI_a(G) = \sum_{v\in V(G)} (ecc_G(v))^a$$

for any real number a. If $a = 1$, we obtain the total eccentricity index

$$EI_1(G) = \sum_{v\in V(G)} ecc_G(v)$$

and EI_2 is the first Zagreb eccentricity index ξ_1.

Eccentricity based indices have been studied because of their extensive applications. Eccentricity based indices of nanostar dendrimers were investigated in [2] and other related indices for example in [1] and [6].

2. Results

First we compute $\sum_{i=1}^k m^i$, $\sum_{i=1}^k im^i$ and $\sum_{i=1}^k i^2m^i$ for $m \ge 2$ and $k \ge 1$. Since

$$\sum_{i=1}^k m^i = \frac{m(m^k-1)}{m-1},$$

we obtain

$$(1-m)\sum_{i=1}^k im^i = m + m^2 + m^3 + \cdots + m^k - km^{k+1} = \frac{m(1-m^k)}{1-m} - km^{k+1}$$

and

$$\sum_{i=1}^k im^i = \frac{m(1-m^k)}{(1-m)^2} + \frac{km^{k+1}}{m-1} = \frac{km^{k+2}-(k+1)m^{k+1}+m}{(m-1)^2}.$$

We have $(m-1)^2\sum_{i=1}^k i^2m^i$

$$\begin{aligned}
&= m + 2m^2 + 2m^3 + 2m^4 + \cdots + 2m^k - (k^2+2k-1)m^{k+1} + k^2m^{k+2} \\
&= m + 2m^2(1 + m + m^2 + \cdots + m^{k-2}) - (k^2+2k-1)m^{k+1} + k^2m^{k+2} \\
&= \frac{2m^2(m^{k-1}-1)}{m-1} + m - (k^2+2k-1)m^{k+1} + k^2m^{k+2},
\end{aligned}$$

thus

$$\begin{aligned}
\sum_{i=1}^k i^2m^i &= \frac{2m^{k+1}-2m^2}{(m-1)^3} + \frac{k^2m^{k+2}-(k^2+2k-1)m^{k+1}+m}{(m-1)^2} \\
&= \frac{k^2m^{k+3}-(2k^2+2k-1)m^{k+2}+(k+1)^2m^{k+1}-m^2-m}{(m-1)^3}.
\end{aligned}$$

A vertex v of any network in $ST_{m,k}$ and the same vertex v of any network in $HT_{m,k}$ have the same eccentricities, therefore for every network in $ST_{m,k}$ and every network in $HT_{m,k}$, we obtain the same value of the EI_a index (note that $a \in \mathbb{R}$).

Theorem 2.1. *For any network $G \in ST_{m,k} \cup HT_{m,k}$, we have*

$$EI_a(G) = k^a + \sum_{i=1}^k m^i(k+i-1)^a.$$

Proof. The network G contains $1 + m + m^2 + \cdots + m^k$ vertices. For $i = 0, 1, 2, \ldots, k$, we have $|V_i| = m^i$, where V_i is the set of vertices at distance i from the root 0. For $i = 1, 2, \ldots, k$, any vertex in V_i has eccentricity $k + i - 1$. The vertex in $0 \in V_0$ has eccentricity k. Thus

$$\begin{aligned} EI_a(G) &= \sum_{v \in V_0} k^a + \sum_{v \in V_1} k^a + \sum_{v \in V_2} (k+1)^a + \cdots + \sum_{v \in V_k} (2k-1)^a \\ &= m^0 k^a + m^1 k^a + m^2 (k+1)^a + \cdots + m^k (2k-1)^a \\ &= k^a + \sum_{i=1}^{k} m^i (k+i-1)^a \end{aligned}$$

□

In Theorems 2.2 and 2.3, we investigate the total eccentricity index EI_1 and the first Zagreb eccentricity index EI_2 $(= \xi_1)$ of biological networks, respectively.

Theorem 2.2. *For any network $G \in ST_{m,k} \cup HT_{m,k}$, we have*

$$EI_1(G) = \frac{(2k-1)m^{k+2} - 2km^{k+1} + m^2 - km + k}{(m-1)^2}.$$

Proof. By Theorem 2.1, we have

$$EI_1(G) = k + \sum_{i=1}^{k} m^i (k+i-1).$$

So

$$\begin{aligned} EI_1(G) &= k + (k-1)\sum_{i=1}^{k} m^i + \sum_{i=1}^{k} i m^i \\ &= k + (k-1)\frac{m(m^k - 1)}{m-1} + \frac{km^{k+2} - (k+1)m^{k+1} + m}{(m-1)^2} \\ &= \frac{(2k-1)m^{k+2} - 2km^{k+1} + m^2 - km + k}{(m-1)^2}. \end{aligned}$$

□

Theorem 2.3. *For any network $G \in ST_{m,k} \cup HT_{m,k}$, we have*

$$\begin{aligned} \xi_1(G) &= \frac{(2k-1)^2 m^{k+3} - (8k^2 - 4k - 1)m^{k+2} + 4k^2 m^{k+1}}{(m-1)^3} \\ &\quad + \frac{(2k-1)m^3 - (k+1)^2 m^2 + 2k^2 m - k^2}{(m-1)^3}. \end{aligned}$$

Proof. By Theorem 2.1, we have

$$EI_2 = \xi_1(G) = k^2 + \sum_{i=1}^{k} m^i (k+i-1)^2.$$

So

$$\begin{aligned} \xi_1(G) &= k^2 + \sum_{i=1}^{k} m^i [(k-1)^2 + 2(k-1)i + i^2)] \\ &= k^2 + (k^2 - 2k + 1)\sum_{i=1}^{k} m^i + (2k-2)\sum_{i=1}^{k} i m^i + \sum_{i=1}^{k} i^2 m^i \\ &= k^2 + (k^2 - 2k + 1)\frac{m(m^k - 1)}{m-1} + (2k-2)\frac{km^{k+2} - (k+1)m^{k+1} + m}{(m-1)^2} \end{aligned}$$

$$+\frac{k^2m^{k+3}-(2k^2+2k-1)m^{k+2}+(k+1)^2m^{k+1}-m^2-m}{(m-1)^3}$$
$$=\frac{(2k-1)^2m^{k+3}-(8k^2-4k-1)m^{k+2}+4k^2m^{k+1}}{(m-1)^3}$$
$$+\frac{(2k-1)m^3-(k+1)^2m^2+2k^2m-k^2}{(m-1)^3}.$$

□

In Theorem 2.4, we investigate the invariant

$$I(G)=\sum_{uv\in E(G)} f(ecc_G(u),ecc_G(v)),$$

where $f(ecc_G(u),ecc_G(v))$ is a function of $ecc_G(u)$ and $ecc_G(v)$ such that $f(ecc_G(u),ecc_G(v))=f(ecc_G(v),ecc_G(u))$.

If $f(ecc_G(u),ecc_G(v))=ecc_G(u)+ecc_G(v)$, we obtain the modified Zagreb eccentricity index If $f(ecc_G(u),ecc_G(v))=ecc_G(u)ecc_G(v)$, we obtain the second Zagreb eccentricity index.

Theorem 2.4. *For any network $G\in ST_{m,k}\cup HT_{m,k}$, we have*

$$I(G)=m\cdot f(k,k)+\sum_{i=2}^{k} m^i f(k+i-2,k+i-1)+\sum_{i=1}^{k}\frac{m^{i+1}-m^i}{2}f(k+i-1,k+i-1).$$

Proof. The network G contains

$$\left(m+m^2+\cdots+m^k\right)+\left(\frac{(m-1)m}{2}+\frac{(m-1)m^2}{2}+\cdots+\frac{(m-1)m^k}{2}\right)$$

edges. Let

$$E_{i,j}=\{uv\in E(G)\mid u\in V_i,\ v\in V_j\}.$$

So the set $E_{i,j}$ contains the edges incident with one vertex in V_i and the other vertex in V_j. We obtain

$$|E_{0,1}|=m,\ |E_{1,2}|=m^2,\ldots,\ |E_{k-1,k}|=m^k$$

and

$$|E_{1,1}|=\frac{(m-1)m}{2},\ |E_{2,2}|=\frac{(m-1)m^2}{2},\ldots,\ |E_{k,k}|=\frac{(m-1)m^k}{2}.$$

Note that

$$E(G)=(E_{0,1}\cup E_{1,2}\cup\cdots\cup E_{k-1,k})\cup(E_{1,1}\cup E_{2,2}\cup\cdots\cup E_{k,k}).$$

For $i=1,2,\ldots,k$, any vertex in V_i has eccentricity $k+i-1$ and the vertex $0\in V_0$ has eccentricity k. Thus we obtain

$$\begin{aligned}
I(G) &= \sum_{uv\in E(G)} f(ecc_G(u),ecc_G(v))\\
&= \sum_{uv\in E_{0,1}} f(k,k)+\sum_{uv\in E_{1,2}} f(k,k+1)\\
&\quad+\sum_{uv\in E_{2,3}} f(k+1,k+2)+\cdots+\sum_{uv\in E_{k-1,k}} f(2k-2,2k-1)\\
&\quad+\sum_{uv\in E_{1,1}} f(k,k)+\sum_{uv\in E_{2,2}} f(k+1,k+1)+\cdots+\sum_{uv\in E_{k,k}} f(2k-1,2k-1)\\
&= m\cdot f(k,k)+m^2 f(k,k+1)\\
&\quad+m^3 f(k+1,k+2)+\cdots+m^k f(2k-2,2k-1)+\frac{(m-1)m}{2}f(k,k)
\end{aligned}$$

$$+ \frac{(m-1)m^2}{2} f(k+1,k+1) + \cdots + \frac{(m-1)m^k}{2} f(2k-1,2k-1)$$

$$= m \cdot f(k,k) + \sum_{i=2}^{k} m^i f(k+i-2,k+i-1)$$

$$+ \sum_{i=1}^{k} \frac{m^{i+1}-m^i}{2} f(k+i-1,k+i-1).$$

□

In Theorem 2.5, we obtain the values of the modified Zagreb eccentricity index ξ_1' and the second Zagreb eccentricity index ξ_2 of the generalized networks.

Theorem 2.5. *For any network $G \in ST_{m,k} \cup HT_{m,k}$, we have*

$$\xi_1'(G) = \frac{(2k-1)m^{k+3} - 2m^{k+2} - (2k-1)m^{k+1} - (k-2)m^3 + km}{(m-1)^2}$$

and

$$\begin{aligned} \xi_2(G) &= km + \left[\frac{m+1}{2}(k^2-2k+1) - k + 1\right] \frac{m^{k+1}-m}{m-1} \\ &+ [(m+1)(k-1)-1]\frac{km^{k+2}-(k+1)m^{k+1}+m}{(m-1)^2} \\ &+ \left(\frac{m+1}{2}\right) \frac{k^2m^{k+3}-(2k^2+2k-1)m^{k+2}+(k+1)^2m^{k+1}-m^2-m}{(m-1)^3}. \end{aligned}$$

Proof. For the ξ_1' index, we get $f(ecc_G(u), ecc_G(v)) = ecc_G(u) + ecc_G(v)$, so $f(k+i-2, k+i-1) = 2k+2i-3$ and $f(k+i-1,k+i-1) = 2k+2i-2$. Then by Theorem 2.4,

$$\begin{aligned} \xi_1'(G) &= 2km + \sum_{i=2}^{k} m^i(2k+2i-3) + \sum_{i=1}^{k} \frac{m^{i+1}-m^i}{2}(2k+2i-2) \\ &= 2km - (2k-1)m + \sum_{i=1}^{k} m^i(2k+2i-3) + \sum_{i=1}^{k}(m-1)m^i(k+i-1) \\ &= m + [(k-1)(m-1)+2k-3]\sum_{i=1}^{k} m^i + (m+1)\sum_{i=1}^{k} im^i \\ &= m + [(k-1)m+k-2]\frac{m(m^k-1)}{m-1} + (m+1)\frac{km^{k+2}-(k+1)m^{k+1}+m}{(m-1)^2} \\ &= \frac{(2k-1)m^{k+3}-2m^{k+2}-(2k-1)m^{k+1}-(k-2)m^3+km}{(m-1)^2}. \end{aligned}$$

For the ξ_2 index, we get $f(ecc_G(u), ecc_G(v)) = ecc_G(u)ecc_G(v)$, so

$$f(k+i-2,k+i-1) = (k+i-2)(k+i-1) = (k^2-3k+2) + (2ki-3i) + i^2$$

and

$$f(k+i-1,k+i-1) = (k+i-1)(k+i-1) = (k^2-2k+1) + (2ki-2i) + i^2$$

Then by Thworem 2.4,

$$\begin{aligned} \xi_2(G) &= k^2m + \sum_{i=2}^{k} m^i[(k^2-3k+2) + (2ki-3i) + i^2] \\ &+ \sum_{i=1}^{k} \frac{m^{i+1}-m^i}{2}[(k^2-2k+1) + (2ki-2i) + i^2] \end{aligned}$$

$$\begin{aligned}
&= k^2m - k(k-1)m + \sum_{i=1}^{k} m^i[(k^2-3k+2)+(2ki-3i)+i^2] \\
&\quad + \sum_{i=1}^{k} \frac{(m-1)m^i}{2}[(k^2-2k+1)+(2ki-2i)+i^2] \\
&= km + \left[\frac{m-1}{2}(k^2-2k+1)+(k^2-3k+2)\right]\sum_{i=1}^{k} m^i \\
&\quad + [(m-1)(k-1)+(2k-3)]\sum_{i=1}^{k} im^i + \left(\frac{m-1}{2}+1\right)\sum_{i=1}^{k} i^2m^i \\
&= km + \left[\frac{m+1}{2}(k^2-2k+1)-k+1\right]\frac{m^{k+1}-m}{m-1} \\
&\quad + [(m+1)(k-1)-1]\frac{km^{k+2}-(k+1)m^{k+1}+m}{(m-1)^2} \\
&\quad + \left(\frac{m+1}{2}\right)\frac{k^2m^{k+3}-(2k^2+2k-1)m^{k+2}+(k+1)^2m^{k+1}-m^2-m}{(m-1)^3}.
\end{aligned}$$

□

The following corollary holds for any network in biological networks in which one person transmits a virus or bacteria to two other people, so it holds also for the networks ST_k^1 and $HT(k)$.

Corollary 2.6. *For any network $G \in ST_{2,k} \cup HT_{2,k}$, we have*

$$\xi_1'(G) = (k-1)2^{k+4} - k2^{k+2} + 2^{k+1} - 6k + 16$$

and

$$\xi_2(G) = (1-k)2^{k+4} + k(k-1)2^{k+3} + (k^2-k+1)2^{k+2} + 2^{k+1} - 3k^2 + 16k - 22.$$

Proof. By Theorem 2.5, the modified Zagreb eccentricity index

$$\begin{aligned}
\xi_1'(G) &= (2k-1)2^{k+3} - 2\cdot 2^{k+2} - (2k-1)2^{k+1} - (k-2)2^3 + 3k \\
&= (k-1)2^{k+4} - k2^{k+2} + 2^{k+1} - 6k + 16
\end{aligned}$$

and the second Zagreb eccentricity index

$$\begin{aligned}
\xi_2(G) &= 2k + \left[\frac{3}{2}(k^2-2k+1)-k+1\right](2^{k+1}-2) \\
&\quad + [3(k-1)-1][k2^{k+2}-(k+1)2^{k+1}+2] \\
&\quad + \frac{3}{2}[k^2 2^{k+3}-(2k^2+2k-1)2^{k+2}+(k+1)^2 2^{k+1}-2^2-2] \\
&= (1-k)2^{k+4} + k(k-1)2^{k+3} + (k^2-k+1)2^{k+2} + 2^{k+1} - 3k^2 + 16k - 22.
\end{aligned}$$

□

It is easy to check that for $G \in ST_{2,k} \cup HT_{2,k}$, we obtain $\xi_1'(G) = 6$ and $\xi_2(G) = 3$ if $k = 1$ and $\xi_1'(G) = 44$ and $\xi_2(G) = 54$ if $k = 2$. These values can be computed after drawing the networks of the set $ST_{2,k} \cup HT_{2,k}$. The only network in $ST_{2,k} \cup HT_{2,k}$ for $k = 1$ is the triangle. The only networks in the set $ST_{2,k} \cup HT_{2,k}$ for $k = 2$ are the networks ST_2^1 and $HT(2)$.

Acknowledgment. This work is based on the research supported by the National Research Foundation of South Africa (Grant Number 129252).

References

[1] F. Asif, Z. Zahid, S. Zafar, M.R. Farahani and W. Gao, *On topological properties of some convex polytopes by using line operator on their subdivisions*, Hacet. J. Math. Stat. **49** (1), 136–146, 2020.

[2] R. Farooq and M.A. Malik, *On some eccentricity based topological indices of nanostar dendrimers*, Optoelectronics and Advanced Materials, Rapid Communications **9** (5-6), 842–849, 2015.

[3] W. Gao, H. Wu, M.K. Siddiqui and A.Q. Baig, *Study of biological networks using graph theory*, Saudi J. Biol. Sci. **25** (6), 1212–1219, 2018.

[4] M. Ghorbani and M.A. Hosseinzade, *A new version of Zagreb indices*, Filomat **26** (1), 93–100, 2012.

[5] M. Imran, M.A. Iqbal, Y. Liu, A.Q. Baig, W. Khalid and M.A. Zaighum, *Computing eccentricity-based topological indices of 2-power interconnection networks*, J. Chem. **2020**, 3794592, 2020.

[6] M.K. Siddiqui, M. Imran and M. Saeed, *Topological properties of face-centred cubic lattice*, Hacet. J. Math. Stat. **49** (1), 195–207, 2020.

[7] D. Vukičević and A. Graovac, *Note on the comparison of the first and second normalized Zagreb eccentricity indices*, Acta Chim. Slov. **57** (3), 524–528, 2010.