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Optimal Capital and Labor Investment in Price Regulated Service Firms

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Abstract

Optimal investment behavior of a price-regulated service firm which faces a fluctuating demand curve, and whose price is determined by governments and municipalities, is studied through the use of the Optimal Control Theory. The aim of our study is to analyze the behavior of the firm when faced with the expected dynamic demand curve, and also to explain the inefficiencies of the firm in terms of machinery and labor. The firm is required to meet the demand at all times. Cobb-Douglas type of demand function is utilized in the analyses where both labor and capital are assumed to be quasi-fixed factors of production. Constant Elasticity of Substitution type of production function is also used in this analysis resulting in similar results. The rates of change of demand, the attrition rate of labor, and the depreciation rate of capital are the primary factors determining the optimal behavior. In other words, as long as the effect of the loss of capacity on capital and labor is greater than the decrease in demand, the investor will continue to invest in factors of production. The switching times between periods when investment in both factors is zero, when investment in capital is zero, and when investment in labor is zero are determined allowing better planning of maintenance periods of machinery and vacation planning.

Keywords: Optimal Control Theory, Labor Investment, Price Regulated Service Firms

Fiyat Düzenlemeli Hizmet Firmalarında Optimum Sermaye ve İşgücü Yatırımı

Öz

Fiyatı hükümetler ve belediyeler tarafından belirlenen ve dalgalı bir talep eğrisiyle karşı karşıya olan fiyat düzenlemeli bir hizmet firmasının optimum yatırım davranışı, Optimal Kontrol Teorisi kullanılarak incelenmiştir. Çalışmamızın amacı, beklenen dinamik talep eğrisi karşısında firmanın davranışını analiz etmek ve ayrıca firmanın makine ve iş gücü açısından verimsizliklerini açıklamaktır. Firmanın talebi her zaman karşılaması gerekmektedir. Cobb-Douglas tipi talep fonksiyonu, hem emeğin hem de sermayenin yarı sabit üretim faktörleri olduğunun varsayıldığı analizlerde kullanılır. Sabit Elastikiyetli İkame tipi üretim foknsiyonu da incelemede kullanılmış ancak sonuçların mahiyeti değişmemiştir. Talebin değişim oranları, emeğin yıpranma oranı ve sermayenin amortisman oranı, optimal davranışı belirleyen temel faktörlerdir. Yani, kapasite kaybının sermaye ve iş gücü üzerindeki etkisi talepteki azalmadan daha büyük olduğu sürece, yatırımcı üretim faktörlerine yatırım yapmaya devam edecektir. Her iki faktöre yapılan yatırımın sıfır olduğu, sermayeye yatırımın sıfır olduğu ve işgücüne yatırımın sıfır olduğu dönemler arasındaki geçiş süreleri, makinelerin bakım dönemlerinin ve tatil planlamasının daha iyi planlanmasına olanak tanıyacak şekilde belirlenir.

Anahtar Kelimeler: Optimal Kontrol Teori, İşgücü Yatırımı, Fiyat Düzenlemeli Hizmet Firmaları

1. Introduction

Capital has long been regarded as a quasi-fixed input by economists. The level of capital can be increased by additional investment in it. It cannot be reduced freely except by its depreciation. The level of optimal capital is determined such that the value of its marginal product is equal to the value of its unit cost plus the value of its depreciation at any time. Arrow (1968:6) was one of the first to introduce the quasi fixity or irreversibility concept of investment. The quasi-fixity of capital is now commonly used concept in economic theory in dynamic settings.

Labor on the other hand has mostly been regarded as completely variable. It can be increased or decreased freely. The optimal level of labor is such that the value of its marginal contribution to output is equal to the wage rate at any time. However, labor is not always freely determined input. Oi (1962:538) was a seminal paper on the subject showing that labor should be considered as a quasi-fixed input to production process. Barceló (2007:46) takes both capital and labor as quasi-fixed factors. A similar model was developed by Tsurumi (1971:4). Abel (1981:381) considered both capital and labor as quasi fixed assets and their utilization rates to develop a dynamic model of investment and capacity utilization. Guyomard (1988:23) is just another example of research where labor was considered as a quasi-fixed factor.

In any case, labor is considered a fixed asset in many strategic sectors operated by the municipalities and the central governments in most countries. Price regulated firms such as railroads, telecommunications, power distribution, and municipal transportation companies are all required to meet the demand at a price determined by the regulators. The results about the investment in labor and capital are important for use in maintenance planning of machinery and vacation planning of labor. In case of a steep decline in demand such as the world is experiencing, planning for optimal use of labor and capital is an important issue. We consider a price-regulated service firm that can invest in both labor and capital but cannot disinvest both resources. It is required to meet the demand with a Cobb-Douglas type production function. Basically, two functions emerge in the production functions: the Walras-Leontief-Harrod-Domar function and the Cobb-Douglas function Arrow (1961:1). The Cobb-Douglas is preferred due to its simplicity and its frequent use in related literature. Constant Elasticity of Substitution (CES) production is a type of Cobb-Douglas production function. Its usage is easy and it is used widely. Arrow *et al.* (1961:230) presented the first usage of the CES production function, i.e., the original specification of the CES function was addressed by Arrow et al. In the literature there are different studies about production function implementation and CES production function usage. Brown (1967:3-13) is an excellent book on production functions. Nerlove's (1967:66) paper in that book is another excellent study on CES type of production functions. However, Gechert *et al.* (2019:4-7) showed that literature rejects the Cobb-Douglas type of production model. Akay and Dogan, (2013:124-128) examined the relationship between labor supply and industry level output in the context of specific factors model using CES production functions. Lagomarsino (2020:2-5) presents a good review on CES production functions and advocation the use of CES functions in empirical analysis. Miller (2008:10-17) concluded that for the purpose of forecasting, there may not be a strong reason to prefer Cobb-Douglas production function over CES. For analysis of policies affecting factor returns, such as taxes on capital and labor income, the Cobb-Douglas specification may be too restrictive. Klump and Preissler (2000:42-50) examined inconsistencies and controversies related to the use of CES production functions in growth models. They showed that not all variants of CES functions commonly used are consistently specified. They also presented the following types of CES production function which we present here also for completeness. They are given in Table 1 where ψ=(σ-1)/σ. Note that in this table *K* indicates capital, whereas *L* indicates the labor. σ, on the other hand, gives the elasticity substitution between capital and labor.

Table 1. Types of CES Production Function

Author(s)	Year	CES production function
Pitchford	1960	$Y = \left(aK^{\psi} + bL^{\psi}\right)^{1/\psi}$
Arrow *et al.*	1961	$Y = C\left[aK^{\psi} + (1-a)L^{\psi}\right]^{1/\psi}$
David and van de Klundert	1965	$Y = \left((BK)^{\psi} + (AL)^{\psi}\right)^{1/\psi}$
Barro and Sala-i-Martin	1995	$Y = C\left[a(BK)^{\psi} + (1-a)[(1-B)L]^{\psi}\right]^{1/\psi}$

The analysis is also performed using the CES production function yielding essentially the same results. This type of firm should expect to have excess capacity to occur as demand changes (fluctuates). The switching times from periods of full capacity to excess capacity and vice-versa are important for planning function. The demand for all companies generally change through time due to seasonal factors, trends, business cycles, and random factors and businesses take measures to respond to these changes as to maximize their profits. In periods of normal economic and stable political periods, companies can deal with these changes without major disruptions in their operations. The Corona virus pandemic was a major random event most companies, or even countries were not prepared to confront economically. It will, in all probability, continue to impact all economies for some time to come at a cyclical fashion.

Outbreak of Corona virus in will continue to have a negative impact on growth. AFP (2020) estimates that the impact could be 0.1-0.2%. Giles et al. (2020) have reported that OECD lowered its growth estimates for the World from 2.9% to 2.4% for 2020. All governments of the World have taken monetary and health (confinement, vaccination) measures to minimize the impact of health crisis on the economies. The interest rates in major economies of the World are already low (zero in Eurozone, negative in Japan) implying that increasing money supply will not have major impact on growth. The pandemic is likely to be on the health agenda of the World for the foreseeable future. The number of cases reached very high levels in Europe, USA, and South American countries around the end of July (2020), remained stable between July and October (2020), and increased until January (2021), decreased until middle of February, and is increasing as of the beginning of March generally and it is still high in countries like the USA and Brazil Brazil (URL-1).

There are many vaccines already being administered in various countries (URL-2). However, the large scale effectives of these vaccines are in doubt to eradicate this virus. It is reported that many variants of the virus have emerged (URL-3). It is not known whether existing vaccines will be effective against these new variants. So, it is possible that the world may face a cyclical demand for goods and services in the future since, as mentioned above, the pandemic seems to exhibit a cyclical behavior.

Under the macro circumstances described above, individual firms have to take measures to survive. They will continue to sell as much as they can even under the cyclical demand conditions. They will take measures and make plans to use their resources (labor and capital) as effectively as possible. Companies operating in competitive markets will reduce price, stop operations or will reduce labor. However, many price regulated companies providing services (electricity, gas, bus, metro, water) to the public cannot act as freely as private sector companies. The principle factor is that they cannot reduce (fire) labor freely. The motivation of this paper is to determine the optimal investment (in labor and capital) behavior of these companies. That is, our purpose here is to analyze the behavior of a cost-minimizing, price-regulated firm facing an expected dynamic demand curve to explain its ”inefficiencies” in terms of excess capacity in machinery and labor. The demand is assumed to be cyclical in this paper. In this study, effect of the parameters on the production function is examined. However, the results are applicable for any differentiable demand function.

The rest of the paper is given as follows. Section 2 gives the material and method that is optimal control theory approach, whereas, section 3 represents the solution of the problem with phase diagram. Section 4 presents the conclusions.

2. Material and Method

In this section, the firm structure is formulated as a mathematical model. For this purpose, the current expenditures in labor and capital are given, and then the model using the Optimal Control Theory approach is presented below.

The current expenditures of the firm can be expressed as:

$$E(t) = wL(t) + cI(t) \qquad (1)$$

where;

$E(t)$	: Current costs/expenditure
$L(t)$	: Labor level
w	: Wage rate, a constant
$I(t)$	: Investment in capital at time *t*
c	: Investment cost per unit of investment, a constant

The firm's current expenditure is formulated to be the sum of the two parts: labor and capital. The first part on the right hand side of equation (1) shows the total expenditure on labors. The second part on the right hand side of the equation (1), on the other hand, shows the expenditures in capital.

In this model, h is used as recruitment cost per unit of labor, a constant with $h = 0$. That is hiring cost is assumed to be zero. The firm's capital is assumed to follow the differential equation:

$$K'(t) = I(t) - \delta K(t) \qquad (2)$$

where $K(t)$ is the level of capital at time *t* and δ is depreciation rate of capital, a constant. Moreover, labor is assumed to follow the differential equation:

$$L'(t) = R(t) - \epsilon L(t) \tag{3}$$

where $R(t)$ is the recruitment level and ϵ is the attrition rate of active labor, a constant. We assume that the firm has a constant return to scale Cobb-Douglas function;

$$F(K, L) = K(t)^{\alpha} L(t)^{\beta} \tag{4}$$

Function in equation (4) is a concave function in $K(t)$ and $L(t)$, since $\alpha + \beta$ is less than one. With this production function, the firm is required to meet the demand, $Q(t)$. $Q(t)$ will be assumed to be a sinusoidal function with a periodicity of one, the length of the planning horizon as an example. We assume that the firm wants to minimize (this maximizes the profits since the price is fixed) its costs over the planning horizon of length one ($T = 1$). Thus, the mathematical model becomes:

$$Min \int_0^{\infty} \big(wL(t) + cI(t)\big)dt$$

St.

$$K' = I(t) - \delta K(t) \qquad K(0) = K_0, K(1) = Free$$

$$L'(t) = R(t) - \varepsilon L(t) \qquad L(0) = L_0, L(1) = Free$$

$$K(t)^{\alpha} L(t)^{\beta} \geq Q(t)$$

$$I(t), R(t) \geq 0$$

And the differential equations (2) and (3). In the mathematical model, K_0 and L_0 indicate the initial value of capital and labor, respectively.

3. The Solution Method and Results

In this section, the solution of the proposed model for the optimal investment behavior of a price-regulated service firm is presented. For given problem, the Lagrangian and the necessary conditions are:

$$H = -(wL(t) + cI(t)) + \lambda_1(I(t) - \delta K(t)) + \lambda_2(R(t) - \varepsilon L(t)) + \eta\left(K(t)^{\alpha}L(t)^{\beta} - Q(t)\right) \quad (5)$$

$$H_I = -c + \lambda_1 \leq 0, H_I I(t) = 0 \quad (6)$$

$$H_R = \lambda_2 \leq 0,\ H_R R(t) = 0 \quad (7)$$

$$\lambda_1' = \lambda_1\delta - \eta\alpha K(t)^{\alpha-1}L(t)^{\beta} \quad (8)$$

$$\lambda_2' = w + \lambda_2\varepsilon - \eta\beta K(t)^{\alpha}L(t)^{\beta-1} \quad (9)$$

$$\eta \geq 0, \eta\left(K(t)^{\alpha}L(t)^{\beta} - Q(t)\right) = 0 \quad (10)$$

To solve a problem that has a non-linear structure, Lagrangian is used. Moreover, to generate the required conditions for the solution of Lagrangian, Hamiltonian is required. For this purpose, the Hamiltonian is given in equation (5) with the Lagrangian multipliers, λ_1 and λ_2, which are any continuously differentiable functions. Equations (6)-(7) represent partial derivatives of Hamiltonian function H, according to the investment and recruitment, respectively. Equations (8)-(10), on the other hand, illustrates the necessary conditions of Lagrangian multipliers. Furthermore, the necessary conditions are also sufficient since the Lagrangian is concave both control variables (I and R) and the state variables (K and L). Note that optimal control contains two type of variables: control variable and state variable. Then by using the equations, to solve problem *phase diagram* is used. Since the solution conditions have non-linear structure and to solve this complex model is hard, phase diagrams are used. The phase diagrams provide the graphical representation of the problem with the different conditions. Here, three phases are examined and we will assume that we start at a phase where both R and I are positive.

3.1. Phase 1: $I > 0, R > 0$

Then, from equations (6)-(8) and equation (10),

$$\lambda_1 = c\,\lambda_1' = 0 \tag{11}$$

$$\eta\alpha K(t)^{\alpha-1}L(t)^{1-\alpha} = c\,\delta \;\Rightarrow \eta > 0 \Rightarrow K(t)^{\alpha}L(t)^{\beta} = Q(t) \tag{12}$$

$$\lambda_2 = 0 \;\Rightarrow \lambda_2' = 0 \Rightarrow \eta\beta(t)K^{\alpha}L(t)^{\beta-1} = w \tag{13}$$

Then,

$$\frac{\beta K(t)}{\alpha L(t)} = \frac{w}{c\delta} \tag{14}$$

defines a relationship between $K(t)$ and $L(t)$. We then can determine $K(t)$ and $L(t)$ using equation (12) and (14). This enables the definition of I and R using equations (2) and (3). From the demand and production equality above, we must have;

$$\frac{(\alpha+\beta)K(t)'}{K(t)} = \frac{Q(t)'}{Q(t)} \Rightarrow \frac{(\alpha+\beta)I(t)}{K(t)} = (\alpha+\beta)\delta + \frac{Q(t)'}{Q(t)} > 0\text{, and}$$
$$\frac{(\alpha+\beta)R(t)}{L(t)} = \frac{Q(t)'}{Q(t)} + (\alpha+\beta)\varepsilon > 0 \tag{15}$$

Then in Phase 1, the firm will be hiring labor and investing in capital if the demand satisfies both equations (14) and (15). Investment in capital will continue even if the demand is declining at a rate less than $(\alpha+\beta)\delta$. Investment in labor will also continue if the demand is declining less than $(\alpha+\beta)$ that implies that the investor will continue to invest in the factors of production as long as the impact of lost capacity in capital, $(\alpha+\beta)\delta$, and labor $(\alpha+\beta)\varepsilon$ is greater than the decline in demand. Then using equations (12) and (14), we have:

$$K(t)^{\alpha+\beta}\left(\frac{\beta\, c\, \delta}{\alpha\, w}\right)^{\beta} = Q(t) \tag{16}$$

This equation determines $K(t)$ and $I(t)$ and using equation (13) determines $L(t)$ and $R(t)$. Notice that an increase in the cost of capital, decreases the level of capital (K) and an increase in cost of labor, increases it. The impact of a change in labor cost is more than the impact of cost of capital since w is in the denominator of equation above. Assuming the impact of depreciation of capital on the output ($\alpha\delta$) is higher than that of labor attrition ($\beta\varepsilon$).

3.2. Phase 2: $I > 0, R = 0$

The investment in labor will stop when $Q(t)'/Q(t) = -(\alpha+\beta)\,\varepsilon$ which will determine the switching time (t_1) from phase 1 to phase 2 where $I > 0$ and $R = 0$. Then, we will have $L(t)' = -\varepsilon L(t)$ or $L(t) = L(t_1)e^{-\varepsilon(t-t_1)}$ and $K(t)^{\alpha}L(t)^{\beta} = Q(t)$ which defines $K(t)$ which, in turn, defines $I(t)$ through equation (2).

Investment in capital will continue until $Q(t)'/Q(t) = -\,(\alpha+\beta)\delta$. The time at which ($t_2$) this equality is satisfied determines the switching time into phase 3.

3.3. Phase 3 where $I = 0$ and $R = 0$

At the switching time, we have $L(t_2) = L(t_1)e^{-\varepsilon(t_2-t_1)}$ and $K(t_2)$ satisfying $Q(t_2) = L(t_2)^{\beta}K(t_2)$. During this phase the differential equations (2) and (3) will satisfy $K(t) = K(t_2)e^{-\delta(t-t_2)}$ and $L(t) = L(t_2)e^{-\varepsilon(t-t_2)}$

which completely determine K and L. For better visual exposition, the following figure is presented. So, the phases with their time interval are given in the Table 2, where the times t_3 and t_4 are determined by the same methodology used to determine t_1 and t_2.

Table 2. Phases and their time intervals

Time Interval	Phase
$0 \leq t \leq t_1$	Phase 1
$t_1 \leq t \leq t_2$	Phase 2
$t_2 \leq t \leq t_3$	Phase 3
$t_3 \leq t \leq t_4$	Phase 2
$t_4 \leq t$	Phase 1

The graphical representation of these phases for a sinusoidal demand curve with no growth is presented in Figure 1.

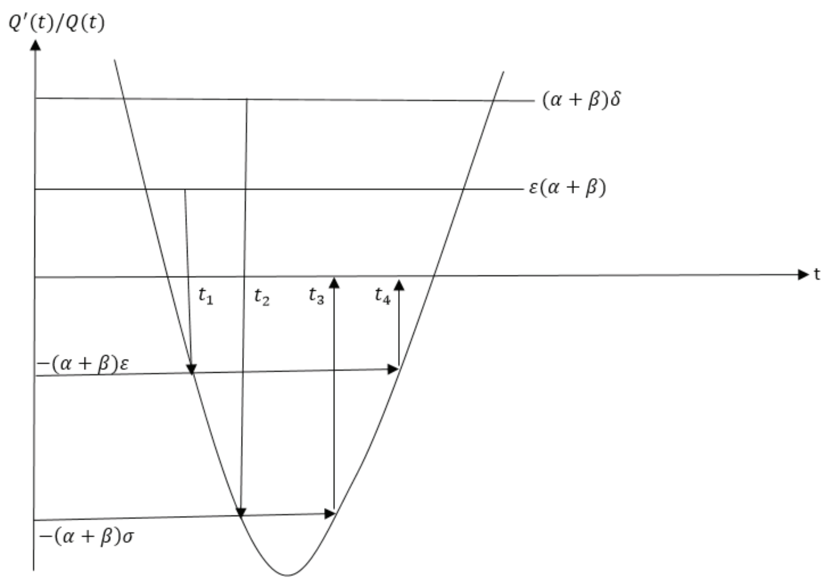

Figure 1. Graphical Representation of Phases

Moreover, given that CES type of production function is also employed in service industries we performed the same analysis above using the Pitchford type of CES production function where $F(K,L) = d(\alpha K^p + (1-\alpha)L^p)^{1/p}$. We have a similar result except we replace the term (α+β) everywhere it appears with the term the term ρ is such that $\sigma = 1/(1-p)$ is the elasticity of substitution.

4. Conclusions

In this study, capital and labor investments are considered as quasi-fixed for a firm-regulated service firm. The problem discussed is modeled using the optimal control theory method, using the Cobb-Douglas type production function, and a solution is obtained with phase diagrams. According to the results, the firm will cease to invest in labor when the demand falls only faster than the incremental value (in units of output) of lost labor (attrition). The firm will cease to invest in capital when the demand falls faster than the incremental value of capital. Investment in capital starts when the demand is still falling but at a lesser rate. Similar analysis can be made if $\alpha < \varepsilon$. The determination of switching points are important for timing of small repairs on capital, annual service of machinery, leaves of personnel, and allowing personnel to take longer vacations. According to the results, it is observed that the effects of parameters on the production function affects the production function like in the literature, e.g., Akay and Dogan, (2013:129). Moreover, the results support the Abel (1981:402) that there is opposite relationship between labor and capital investments.

In this study, the analysis are made without taking into account the discount factor that can be provided by interest rate. Thus, improvement may be the addition of present value factor where the interest rate may be constant or time dependent. In this study, to make analysis, we assume that the demand function is sinusoidal function as an instance. In the future studies, growth factor can be added into the objective function instead of sinusoidal function.

References

Abel, A. B. (1981). Dynamic Model of Investment and Capacity Utilization. *The Quarterly Journal of Economics*, 96(3), 379–403.

Akay, G. H. and Dogan, C. (2013). The Effect of Labor Supply Changes on Output: Empirical Evidence from US Industries. *Journal of Productivity Analysis*, 39(2), 123–130.

Arrow, K. J., Chenery, H. B., Minhas, B. S. and Solow, R. M. (1961). Capital-Labor Substitution and Economic Efficiency. *The Review of Economics And Statistics*, *43*, 225–250.

Arrow, K.J. (1968). Optimal Capital Policy with Irreversible Investment. *Value, Capital and Growth*, 1–20.

AFP. https://www.bangkokpost.com/world/1859319/coronavirus-could-damage-global-growth-in-2020-imf, (February 17, 2020), (Access Date: 07.03.2021).

Barceló, C. (2007). A Q-Model of Labour Demand. *Investigaciones Económicas*, 31(1), 43–78.

Barro, R. J. and Sala-I-Martin, X. (1995). *Economic Growth,* London-England, The MIT Press.

Brown, M. (1967). *The Theory and Empirical Analysis of Production*, NBER, 3-13.

David, P. A. and Van De Klundert, T. (1965). Biased Efficiency Growth and Capital-Labor Substitution in The US, 1899-1960. *American Economic Review*, 55, 357–394.

Gechert, S., Havránek, T., Havránková, Z., and Kolcunova, D. (2019). *Death to The Cobb-Douglas Production Function* (No. 201), IMK Working Paper.

Giles, G., Arnold, M., and Greely, B. OECD Warns That Coronavirus Could Halve Global Growth. France24 TV. https://www.ft.com/content/1356af8c-5c6c-11ea-8033-fa40a0d65a98, (March 3, 2020), (Access Date: 07.03.2021).

Guyomard, H. (1988). Quasi-Fixed Factors and Production Theory: The Case of Self-Employed Labour in French Agriculture. *Irish Journal of Agricultural Economics and Rural Sociology*, 13, 21–33.

Klump, R. and Preissler, H. (2000). CES Production Functions and Economic Growth. *Scandinavian Journal of Economics*, 102(1), 41–56.

Lagomarsino, E. (2020). Estimating Elasticities of Substitution with Nested CES Production Functions: Where Do We Stand?. *Energy Economics*, 88, 104752. https://Doi.Org/10.1016/J.Eneco.2020.104752

Miller, E. (2008). *An Assessment of CES and Cobb-Douglas Production Functions,* Washington, DC: Congressional Budget Office.

Nerlove, M. (1967). Recent Empirical Studies of The CES and Related Production Functions. In *The Theory and Empirical Analysis of Production,* NBER, 55-136.

URL-1: COVID-19 Dashboard (2021). https://coronavirus.jhu.edu/map.html, (Access Date: 07.03.2021)

URL-2: Different COVID-19 Vaccines (2021). https://www.cdc.gov/coronavirus/2019-ncov/vaccines/different-vaccines.html, (Access Date: 07.03.2021).

URL-3: About Variants of the Virus that Cause COVID-19 (2021). https://www.cdc.gov/coronavirus/2019-ncov/transmission/variant.html, (Access Date: 07.03.2021).

Oi, W. Y. (1962). Labor as a Quasi-Fixed Factor. *Journal of Political Economy*, 70(6), 538–555.

Pitchford, J. D. (1960). Growth and The Elasticity of Substitution. *Economic Record*, 36, 491–503.

Tsurumi, H. (1971). S*uboptimization Model of Demand for Investment and Labor: An Optimal Control Theoretic Approach,* Kingston, Ont: Institute for Economic Research, Queen's University.