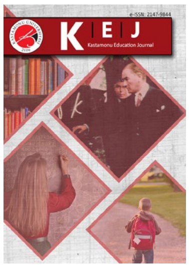
Kastamonu Education Journal, 2022, Vol. 30, No:1, 205-216
doi: 10.24106/kefdergi. 833879



| Research Article / Araştırma Makalesi |

An Investigation of Prospective Mathematics Teachers' Ability to Subtract Integers with a Number Line and Counters

Matematik Öğretmeni Adaylarının Tam Sayılarda Çıkarma İşlemini Sayı Doğrusu ve Sayma Pulları İle Yapabilme Durumlarının İncelenmesi

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Keywords
1.Integers
2.Number line
3.Counters
4.Prospective mathematics teachers

Anahtar Kelimeler
1. Tam sayılar
2.Sayı doğrusu
3.Sayma pulu
4.Matematik öğretmeni adayları

Received/Başvuru Tarihi
30.11.2020

Accepted / Kabul Tarihi
24.04.2021

Abstract

Purpose: In the present study, the ability of middle school prospective mathematics teachers to subtract integers with counters and a number line was investigated.

Design/Methodology/Approach: A case study method was used for the investigation. The research was conducted with 37 prospective teachers who were training in a state university's undergraduate program on primary mathematics in northern Turkey. Four questions developed by the researcher for subtraction in integers were used as a data collection tool. The data of the study were analyzed in two stages. In the first stage, the answers of the prospective teachers were determined as right or wrong; subsequently, in the second stage, the mistakes that led to the wrong answers were determined.

Findings: The prospective teachers were relatively successful with respect to both modeling types. The prospective teachers had the highest rates of success in modeling with counters, with a rate of approximately 97% in subtracting a negative integer from a positive integer ((+3) − (−5)), and the lowest rates of success in subtracting a negative integer from a negative integer, with a rate of around 89% for the operation ((−4) − (−7)). In modeling with the number line, the highest success rate was approximately 91% for subtracting a positive integer ((+2) − (+5)) from another positive integer, whereas the lowest success rate was around 86% for subtracting a negative integer from a negative integer in the operation of ((−4) − (−7)).

Highlights: According to the results of the research, the prospective teachers were relatively successful in modeling the subtraction of integers using counters and a number line.

Öz

Çalışmanın amacı: Bu çalışmada ortaokul matematik öğretmeni adaylarının tam sayılarda çıkarma işlemini sayma pulları ve sayı doğrusu ile yapabilme durumları araştırılmıştır.

Materyal ve Yöntem: Durum çalışması yöntemi kullanılmıştır. Araştırma Türkiye'nin kuzeyinde yer alan bir devlet üniversitesinin ilköğretim matematik öğretmenliği lisans programında eğitim gören 37 öğretmen adayı ile yürütülmüştür. Veri toplama aracı olarak araştırmacı tarafından geliştirilen ve tam sayılarda çıkarma işlemine yönelik 4 soru kullanılmıştır. Çalışmanın verileri iki aşamada analiz edilmiştir. Birinci aşamada öğretmen adaylarının doğru veya yanlış cevaplara ulaşma durumları belirlenmiş, ikinci aşamada ise yanlış cevaplara sebep olan hatalar tespit edilmiştir.

Bulgular: Çalışmada öğretmen adayları tam sayılarda çıkarma işlemlerini her iki modelleme türünde de oldukça başarılı olmuşlardır. Öğretmen adayları sayma pulları ile modellemede en yüksek başarıyı yaklaşık %97'lik oranla pozitif bir tam sayıdan negatif bir tam sayının çıkarılması ((+3)-(-5)), en düşük başarıyı ise yaklaşık %89'luk oranla negatif bir tam sayıdan negatif bir tam sayının çıkarılması ((-4)-(-7)) işleminde göstermişlerdir. Sayı doğrusu ile modellemede en yüksek başarıyı yaklaşık %91'lik oranla pozitif bir tam sayıdan pozitif bir tam sayının çıkarılması ((+2)-(+5)), en düşük başarıyı ise yaklaşık %86'lık oranla negatif bir tam sayıdan negatif bir tam sayının çıkarılması ((-4)-(-7)) işleminde göstermişlerdir.

Önemli Vurgular: Araştırma sonuçlarına göre, öğretmen adayları tam sayılarda çıkarma işlemini sayma pulları ve sayı doğrusu ile modellemede oldukça başarılı olmuşlardır.

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Citation/Alıntı: Özdemir, E. (2022). An investigation of prospective mathematics teachers' ability to subtract integers with a number line and counters, Kastamonu Education Journal, 30(1), 205-216. doi: 10.24106/kefdergi. 833879

INTRODUCTION

National and international mathematics teaching programs aim to develop students' skills, such as problem-solving, communication, association, representation, reasoning, and providing evidence to learn and use mathematics effectively (Ministry of National Education [MEB], 2013; National Council of Teachers of Mathematics [NCTM], 2000). The skill of representation includes the effective use of manipulatives, diagrams, graphs, tables, and symbols, which are important indicators of expressing mathematical ideas and relationships (NCTM, 2000). There are various classifications for representations. One is a classification in which mathematical ideas are divided into external representation and internal representation (Hiebert & Carpenter, 1992). In this classification, external representations are divided into five categories by Lesh, Post, and Behr. (1987): concrete models, pictures, symbolic expression, spoken language, and real-life situations. The ability of students to switch between these different styles of representation improves their relational understanding (Lesh, Post, & Behr, 1987). Lesh et al. (1987) stated that multiple representations should be used in mathematics teaching. Some examples include moving from counters, fraction bars, and decimal base blocks to concrete models; from a number line model to a picture or a diagram; written symbols or symbolic representations, 5!, and a-1 <6 examples (Özdemir & İpek, 2020).

The arithmetic of whole numbers is relatively intuitive for children because they can think of it in ways that are based on real-world contexts (Carpenter et al., 1999). Children can learn by representing fractions, decimal notations, percentages, and non-negative rational numbers in different ways and by associating them with aspects of daily life (Whitacre et al., 2017). Natural numbers are used to denote multiplicities. However, the set of natural numbers is insufficient in expressing quantities, such as temperature, height, average, and credit–debt; solving equations, such as $x + 3 = 0$; and doing subtraction operations, such as $3 - 7$ (wherein the minuend is less than the subtractive). To eliminate such deficiencies, a set of integers has been defined (Argün, Arıkan, Bulut, & Halıcıoğlu, 2014, p. 507; Baykul, 2009, p. 240; Van de Walle, Karp, & Williams, 2012, pp. 473–482). Integers have two components: direction and quantity. In expressions such as −3 and +7, the signs − and + indicate the direction, and 3 and 7 indicate the quantity (multiplicity or size).

Integers comprise an important and challenging issue in the transition from arithmetic to algebra (Peled & Carraher, 2007). Students experience various difficulties while processing integers (Avcu & Durmaz, 2011; Stephan & Akyüz, 2012). Hativa and Cohen (1995) identified five types of mistakes related to the addition and subtraction of integers. The first is an mistake in subtracting a positive number from zero. For example, in the process of $0 - 6$, answers such as 4, 0, or 6 can be given by considering it to mean $10 - 6$. The second is an mistake in subtracting a larger positive integer from a positive integer. For example, answers such as 5, 11, −11, 3, or 8 might be given for the operation $(+3) - (+8)$. The third type of mistake is an mistake made with the sum of two negative integers. For example, in the operation $(-3) + (-8)$, the answers of −5, 3, 5, or 11 can be given. The fourth is the mistake of adding an integer and its reverse according to the addition operation. For example, answers such as 6 or 3 might be given for the $(-3) + (+3)$ operation. Finally, the fifth type of mistake is an mistake in adding a positive integer and the number with the reverse sign. For example, answers such as 5, 11, −11, 3, or 8 can be given in the operation $(+3) + (-8)$. (In the study by Hativa and Cohen (1995), mistakes that could be made with negative numbers in addition and subtraction, such as $(-4) - (-7)$, were not mentioned because only a + b, a-b, -a + b, -a-b (with $a > 0$ and $b > 0$) operations were included.) The reasons for mistakes and difficulties when dealing with integers include the meanings of the arithmetic operations in integers (Vlassis, 2004), the different meanings of the minus sign (Gallardo & Rojano, 1994; Janvier, 1985; Vlassis 2004; Vlassis, 2008), an inability to physically model negative numbers (Stephan & Akyüz, 2012), transferring the operations and generalizations in natural numbers to integers (Hativa & Cohen, 1995; Kilhamn, 2011), the opposite relationship between negative numbers and size notation (Fischbein, 1987), and all the algebraic properties of negative numbers.

Contexts and models are used when teaching integers. The contexts can be divided into two categories: those that contain quantity and those that contain linearity. Debt-receivable and profit–loss contexts include quantity, whereas contexts related to temperatures, altitudes, elevators, and timelines include linearity. Further, models can also be divided into two as either quantitative or linear. Counters can be given as an example of a quantitative model and a number line as that of the linear model (Van de Walle, Karp, & Williams, 2012). Janvier (1983) defined two models for teaching integers: the equilibrium model and the number line. The equilibrium model can be said to correspond to the quantitative model in the classification of Van de Walle et al. (2012). In the equilibrium model, numbers are represented by elements of two opposite types, such as black and white marbles and negative-positive electric charges. In this model, the addition process is defined as merging and the extraction process as removing or vice versa. In the number line model, numbers are represented either by their position on the number line or by their displacement. Here, addition is a "combination of two movements" or a "displacement from one location to another"; subtraction is defined as "moving in the opposite direction" or a "difference between two positions" (Hativa & Cohen, 1995; Janvier, 1983).

When examining the literature, it is evident that no consensus exists as to which of the two models of the number line and counters are more useful in the teaching of integer and integer operations. Some studies recommended the use of the number line model (Cemen, 1993; Cunningham, 2009; Fischbein, 1977; Peled, Mukhopadahyay, & Resnick, 1989; Thompson & Dreyfus, 1988), while others recommended the use of the counting scale model (Battista, 1983; Hart, 1981; Liebeck, 1990).

Cemen (1993) argued that the most effective method among the monetary model, the two-color tile model, and the number line model in teaching the addition and subtraction of integers is that of the number line. The monetary model includes receiving and giving the money that is owed, which does not make a clear distinction between negative numbers and subtraction. In the two-color tile model, one color is used for positive numbers and a different color for negative numbers, which helps explain the distinction between negative numbers and subtraction by using various colors. In this approach, operation −2 − (−5) involves removing five negative counters, and the −2 + 5 process requires the addition of five positive counters. However, describing that the removal of five negative counters and the addition of five positive counters give both processes the same result, and these processes remain abstract. In the number line model, positive numbers are considered to represent a forward movement while and negative numbers represent a backward movement. Here, addition is defined as "preserving its direction," and subtraction is defined as "turning in the opposite direction." Thus, it is argued that these properties make the number line model the most useful one.

Liebeck's (1990) "score and forfeits" model and Battista's (1983) "positive and negative charge" model can be regarded to be the same as that of counters because the implementation of both models is done with concrete objects. These researchers argued that the counting scale model was more effective than the number line model because counters are concrete, whereas the number line is pictorial; because counters are appropriate for four integer operations, whereas there are different restrictions with the number line; and so on.

Almeida and Bruno (2014) investigated the strategies used by prospective middle schoolteachers to solve verbal problems requiring addition with negative numbers. From their results, it was determined that the prospective teachers used six different strategies: using operations with positive numbers, using operations with negative numbers, using number lines, counting in order/over, verbal explanations, and drawings. It was observed that the prospective teachers preferred the strategy of "making operations with negative numbers" for the problems they found easy to solve and other strategies for more complex problems; the rate of reaching the correct answers ranged from 76% to 97%.

Durmaz (2017) investigated the situation for teachers and prospective teachers in modeling four processes for integers with counters. Teachers and prospective teachers were more successful in modeling the addition process than in modeling other operations. Moreover, it was determined that teachers were more successful in all types of modeling than prospective teachers.

Kubar and Çakıroğlu (2017) investigated the knowledge of middle-school mathematics prospective teachers about the possible mistakes and mistakes of middle-school students in defining integers. It was observed that prospective teachers had a general knowledge of students' mistakes but lacked detailed information in terms of content and pedagogical aspects.

Research Significance and Problems

Students, prospective teachers, and even teachers are known to experience some difficulties in performing the addition and subtraction operations for integers with number lines and counters (Battista, 1983; Durmaz, 2017; Liebeck, 1990). The lack of studies investigating both the number line and the counting counter model together reveals a gap in the field. Furthermore, prospective teachers might develop misunderstandings or a limited understanding of different basic mathematical concepts (Ball, 1988; Ma, 1999). Such understandings are often passed on by prospective teachers to their future students (Reeder & Bateiha, 2016). This situation reveals the importance of detecting and eliminating the incorrect or limited understandings, if such understandings exist, of prospective teachers. Accordingly, this study investigated the ability of prospective teachers to subtract integers using the counters and number line models. It is thought that the study will contribute to filling the gap in this area and to enhance the knowledge of prospective teachers, teachers, and, consequently, students. For this purpose, the following research problems were identified.

1. How are the prospective teachers' success in modeling subtraction with integers with number lines and counters?

2. What are the mistakes that prospective teachers make in modeling the subtraction of integers with number lines and counters?

Subtracting Integers with the Number Line and Counters

According to the model considered in this analysis, processing begins by looking at the zero point on the number line in the positive direction (to the right). Positive numbers have been defined as a "move forward," whereas negative numbers as a "backward movement." The addition process is considered a process where you "keep your current direction," and the subtraction process is considered one where you "reverse your current direction" (Billstein et al., 2016, p. 230; Cemen, 1993; Teppo & Heuvel-Panhuizen, 2014).

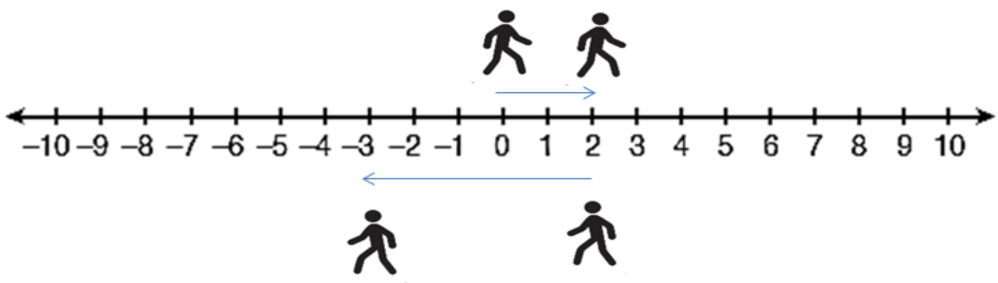


Figure 1. Operation of (+2) − (+5) with the number line

According to this model, the operation of (+2) − (+5) is as follows. On the number line, the zero point is looked at in the positive direction. Because the value +2 is given, we move two units forward. Owing to the subtraction, it is turned from the current point (from the +2 point) to the opposite direction (i.e., the negative axis). Because of the value of +5, we move five units forward from the current point. The point of arrival is −3. Therefore, the result of (+2) − (+5) operation is −3.

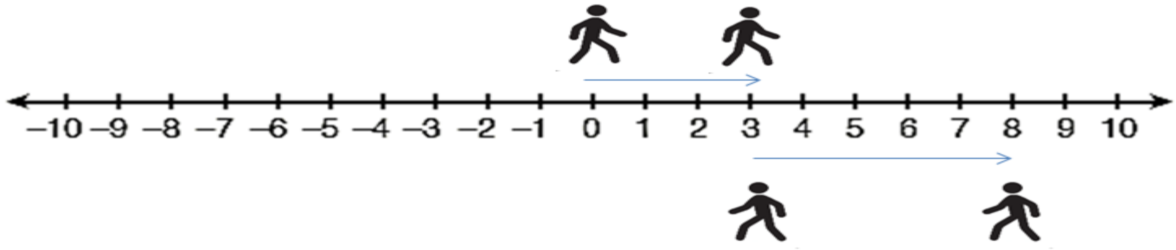


Figure 2. Operation of (+3) − (−5) with the number line

According to this model, the operation of (+3) − (−5) is as follows. The positive direction is looked at from the zero point on the number line. Because of the value of +3, we move three units forward. Owing to the subtraction process, we return to the opposite direction (i.e., the negative axis) from the current point (point +3). Because of the −5 value, we move back five units from the current point. The reached point is +8. Therefore, the result of (+3) − (−5) is +8.

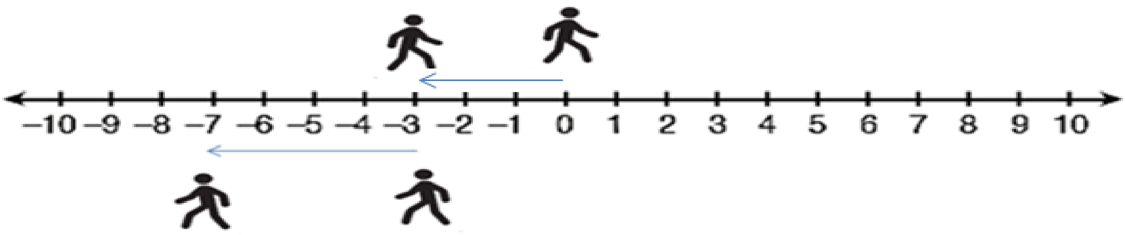


Figure 3. Operation of (−3) − (+4) with the number line

According to this model, the operation of (−3) − (+4) is as follows. The positive direction is looked at from the point of zero on the number line. Because of the −3 value, we move back three units. Owing to the subtraction operation, we turn from the current point (from −3 point) to the opposite direction (i.e., the negative axis). Because of the value of +4, we move four units forward from the current point. The reached point is −7. Therefore, the result of (−3) − (+4) is −7.

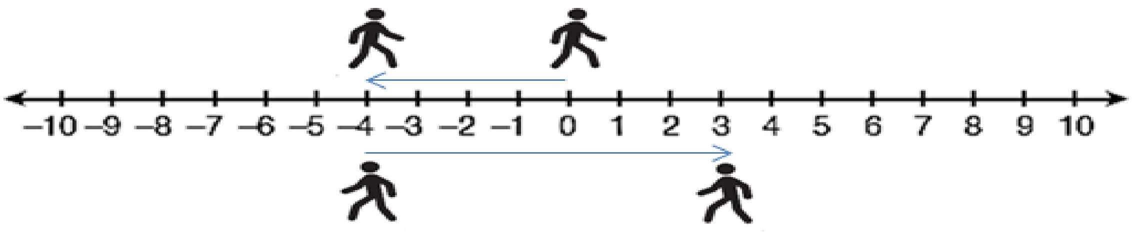


Figure 4. Operation of (−4) − (−7) with the number line

According to this model, the operation of (−4) − (−7) is as follows. On the number line, the positive direction is looked at from the point of zero. Owing to the value of −4, we move four units back. Because of the subtraction operation, we turn from the current point (point −4) to the opposite direction (i.e., the negative axis). Because of the −7 value, we move seven units back from the current point. The reached point is +3. Therefore, the result of (−4) − (−7) is +3.

In this study, in modeling the subtraction of whole numbers with counters, the model used by Billstein et al. (2016, p. 229) has been considered. White scales are used to represent negative integers; blue scales are used to represent positive integers. The addition process is considered to involve adding counters to the box, while the subtraction process involves removing counters from the box. These operations can also be done with electrical charges instead of counters (Battista, 1983; Billstein et al., 2016, p. 229).

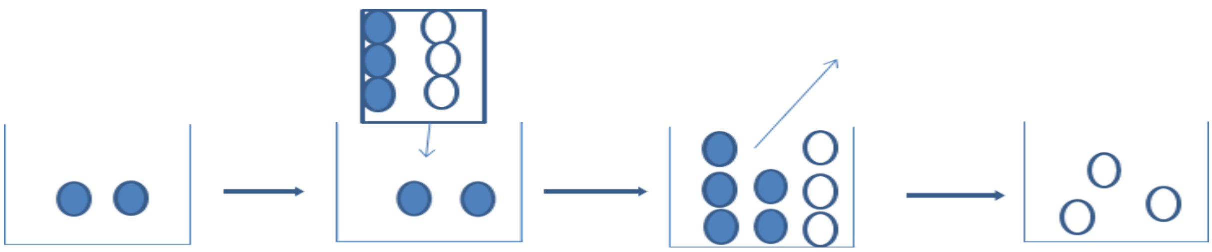


Figure 5. Operation of (+2) − (+5) with counters

According to this model, the operation of (+2) − (+5) is as follows. There are two blue counters inside the box. For subtraction, the counter must be removed from the box. Because of the value of +5, five blue counters must be removed from the box. Since there are five blue counters in the box, three blue and three white scales are added. (This can be expressed as "3 zero pairs or 3 neutral pairs are added"). There are five blue and three white scales in the box. Subsequently, five blue counters are removed from the box, and three white counters remain in the box. Therefore, the result of (+2) − (+5) operation is −3.

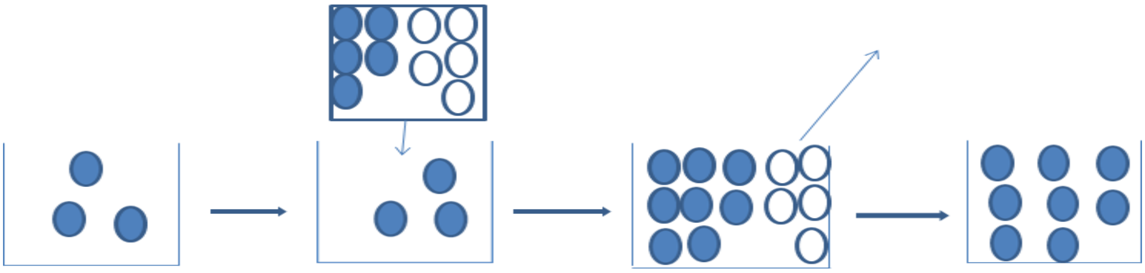


Figure 6. Operation of (+3) − (−5) with counters

According to this model, the operation of (+3) − (−5) is as follows. There are three blue counters inside the box. For subtraction, the counter must be removed from the box. For the value of −5, five white counters should be removed from the box. Since there are no white counters in the box, five white and five blue counters are added. (This can be expressed as "5 zero pairs or 5 neutral pairs are added"). In this case, there are eight blue and five white counters in the box. When five white counters are removed, eight blue counters remain in the box. Therefore, the result of (+3) − (−5) is +8.

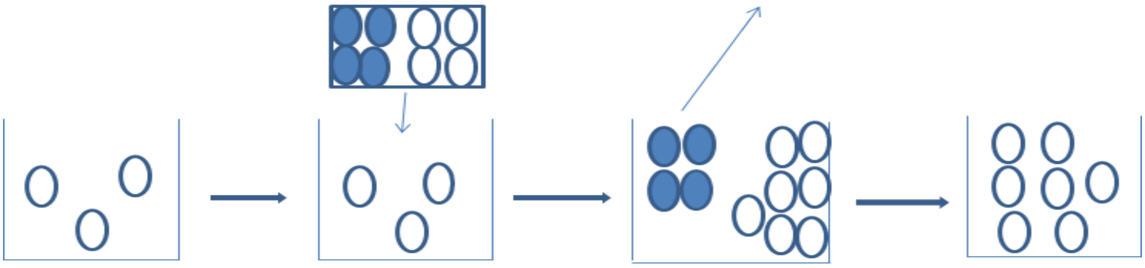


Figure 7. Operation of (−3) − (+4) with counters

According to this model, the operation of (−3) − (+4) is as follows. There are three white counters inside the box. For subtraction, the counter must be removed from the box. Because of the +4 value, four blue counters should be removed from the box. Since there is no blue counter in the box, four white and four blue scales are added. (This can be expressed as "4 zero pairs or 4 neutral pairs are added"). In this case, there are four blue and seven white counters in the box. When four blue counters are removed, seven white counters remain in the box. Therefore, the result of (−3) − (+4) is −7.

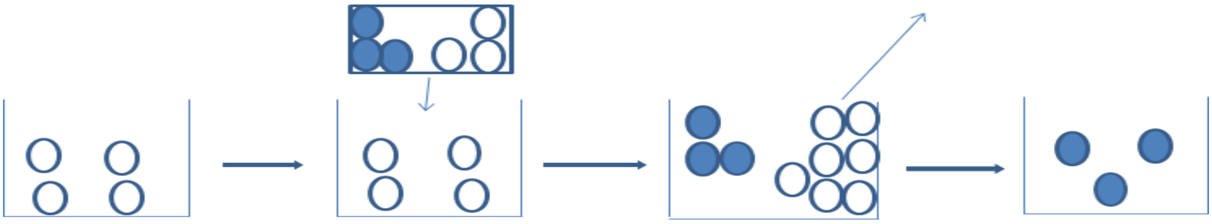


Figure 8. Operation of (−4) − (−7) with counters

According to this model, the operation of (−4) − (−7) is as follows. There are four white counters inside the box. For subtraction, the counter must be removed from the box. Because of the value of −7, seven white counters should be removed from the box. Since there are seven white counters in the box, three white and three blue counters are added. (This can be expressed as "3 zero pairs or 3 neutral pairs are added"). There are three blue and seven white counters in the box. Next, seven white counters are removed from the box, and three blue counters remain in the box. Therefore, the result of (−4) − (−7) is +3.

METHOD/MATERIALS

A case study design was used in the study. Case studies aim to disclose findings regarding a specific situation. The main feature of a case study is an in-depth investigation of one or a few cases (Yıldırım & Şimşek, 2008). This method was used to reveal results concerning prospective teachers' ability to subtract integers with counters and number lines.

Study Group

The research was conducted with 37 prospective teachers who were enrolled in an undergraduate program of a state university on mathematics education in middle school and were in the third grade. In the process of determining the study group, criteria sampling and easily accessible situation sampling methods were used. Easily accessible situation sampling is preferred because it brings speed and practicality to the research (Yıldırım & Şimşek, 2008). Regarding the criteria sampling, the criteria were “to take the Special Teaching Methods 1 course and to be successful.” In the scope of this course, prospective teachers learn about the “basic concepts specific to the field, the relationship of these concepts with field teaching, and the tools and materials used in field teaching.” Moreover, an “examination of the relevant aspects of the curriculum in terms of acquisition, unit, activity, and so on” was also carried out. In the course, the concepts of counters, the number line, and zero pairs were introduced to prospective teachers. The information that the prospective teachers received in this course on modeling with integers was limited, and no special training was given in relation to modeling the subtraction process in integers. Prospective teachers who completed the Special Teaching Methods 1 course were informed about the contents of the present study. The study was conducted with 37 prospective teachers who voluntarily agreed to participate after being informed about the research.

Data Collection

A subtraction of Integers Test (SoIT) was developed by the researcher and was used as a data collection tool. In the SoIT, four questions regarding subtraction in integers were included, and the prospective teachers were asked to solve the given questions using counters and number lines. The questions in SIT involved subtracting a positive integer from another positive integer ((+2) − (+5)), subtracting a negative integer from a positive integer ((+3) − (−5)), subtracting a positive integer from a negative integer ((−3) − (+4)), and subtracting a negative integer from another negative integer ((−4) − (−7)). During the process of creating the questions, existing studies on integers were reviewed (Cemen, 1993; Gallardo & Romero, 1999; Stephan & Akyüz, 2012; Teppo & Heuvel-Panhuizen, 2014). The prospective teachers were given 40 minutes to complete the SoIT. The necessary arrangements were made in the classroom where the test was taken so that the prospective teachers would not be affected by each other’s solutions.

Data Analysis

The data were analyzed in two stages. In the first stage, the answers given by the prospective teachers for the questions in SoIT were coded as correct, incorrect, or blank. Operations with counters were carried out with the model of Billstein et al. (2016, p. 229). The execution of operations according to this model is given in Figures 5, 6, 7, and 8 with accompanying explanations. Models that were made as indicated in the figures were accepted as correct; the ones that were different were accepted as incorrect. Incorrect modeling examples are presented in Figures 15 and 16 in the Results section. The operations on the number line were evaluated according to the model used by Billstein et al. (2016), Cemen (1993), and Teppo and Heuvel-Panhuizen (2014). The operations according to this model are given in Figures 1, 2, 3, and 4 with accompanying explanations. Models that were made as indicated in the figures were accepted as correct; the ones that were different were accepted as incorrect. Incorrect modeling examples are presented in Figures 11, 12, 13, and 14 in the Results section. In the second stage, the mistakes that caused the wrong answers were determined. The next section of the present study provides some examples of the mistakes and explains the possible reasons for the mistakes. The two stages of the analysis were performed by two math educators. Differences in the analysis were discussed until a consensus was reached between the two experts (Miles & Huberman, 1994). To ensure the validity of the research, how the results obtained herein were acquired has been shown clearly (Yıldırım & Şimşek, 2008). Further, to increase the reliability of the case studies, researchers should clearly define the processes followed and support them with the relevant documents. Therefore, the prospective teachers, who were the sources of the research data, have been clearly defined herein, and the study group and process for determining the group have also been explained for other researchers who may conduct similar studies. The research method, stages, data collection methods, research analysis, and details regarding what was done in terms of obtaining and interpreting the research results have been clearly stated. The data were described in detail by providing direct quotations concerning the operations made by the prospective teachers in the SoIT (Yıldırım & Şimşek, 2008).

FINDINGS

In this section, the data obtained from the solutions of the prospective teachers in the SoIT are presented in accordance with the problems of the study.

Findings Regarding the First Problem of the Study

Table 1 shows the percentage frequency distribution of correct, blank, and incorrect answers given by the prospective teachers to the questions in the SoIT.

Table 1. Percentage frequency distribution of the answers given by the prospective teachers to the questions in the SoIT

Question	Number Line				Counters			
	C	I	B	T	C	I	B	T
	f (%)	f (%)	f (%)	f (%)	f (%)	f (%)	f (%)	f (%)
(+2) − (+5)	34 (91.8)	3 (8.1)	0 (0)	37 (100)	34 (91.8)	2 (5.4)	1 (2.7)	37 (100)
(+3) − (−5)	33 (89.1)	4 (10.8)	0 (0)	37 (100)	36 (97.2)	1 (2.7)	0 (0)	37 (100)
(−3) − (+4)	33 (89.1)	3 (8.1)	1 (2.7)	37 (100)	34 (91.8)	1 (2.7)	2 (5.4)	37 (100)
(−4) − (−7)	32 (86.4)	3 (8.1)	2 (5.4)	37 (100)	33 (89.1)	1 (2.7)	3 (8.1)	37 (100)

(C: Correct, I:Incorrect, B: Blank, T: Total)

Please It can be seen from Table 1 that the prospective teachers were relatively successful in both types of modeling. The prospective teachers showed the highest rate of success for the (+3) − (−5) operation with counters and the lowest rate of success for the operation of (−4) − (−7) with the number line.

In relation to subtraction with the number line, the prospective teachers showed the highest success rate for the operation of (+2) − (+5) and the lowest for (−4) − (−7). Thus, it is understood that the problem that the prospective teachers had the lowest success in subtracting with counters and the number line concerned subtracting a negative integer from another negative integer. Moreover, it can be seen that the questions with which the prospective teachers showed the highest success in differ in terms of the models used. The highest successes were related to subtracting a negative integer from a positive integer with counters and subtracting a positive integer from a positive integer with the number line.

When the solutions in the SoIT were examined, 35 prospective teachers supported the procedures that they expressed in writing with figures. An example of a solution is given in Figure 9.

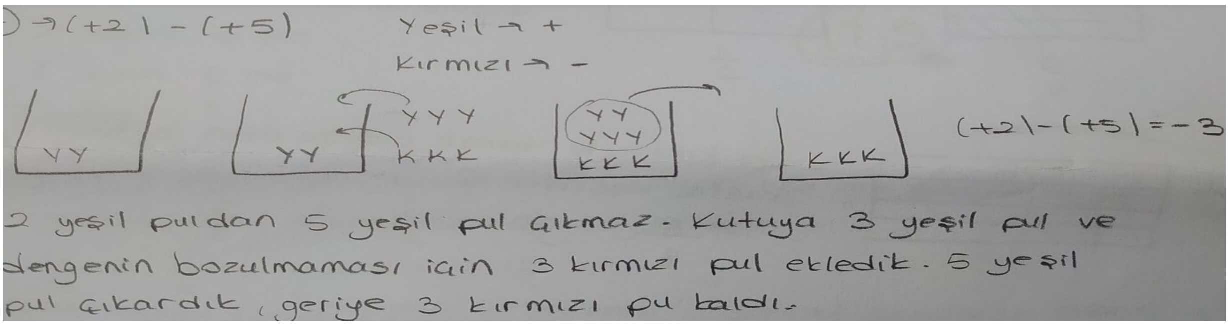
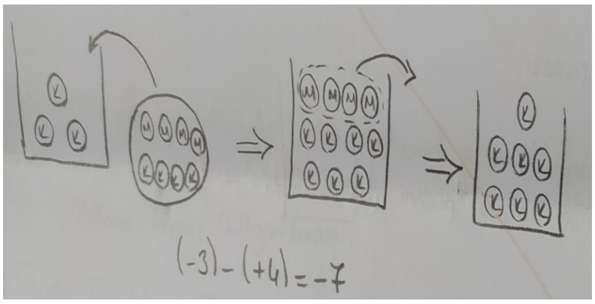


Figure 9. Modeling example for the operation of (+2) − (+5)

(*English translation of the text in the figure*: *Yeşil = Green, Kırmızı = Red..As 5 green counters cannot be subtracted from 2 green counters, I added 3 green counters, and then, I added 3 red counters so as not to break the balance. I removed 5 green counters. There remained 3 red counters.*)

Furthermore, it was found that two prospective teachers solved the questions using only figures. Two examples of solutions are given in Figure 10.

a) 

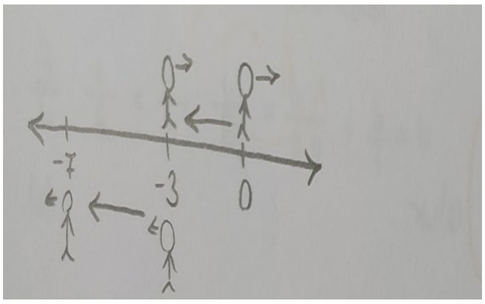
b) 

Figure 10. Modeling examples for the operation of (−3) − (+4)

Prospective teachers made more mistakes in the subtraction of integers with a number line than with counters. The types of mistakes made using the number line and their frequencies are given in Table 2.

Table 2. Mistakes in modeling with the number line while subtracting integers

Type of Mistake	Frequency
Not starting the modeling from the starting point (point 0)	7
Ignoring the sign of the number	3
Giving the same meaning to the operation sign and the number sign	1
Ignoring the sign of the operation	5
Arithmetic	1

Table 1 shows that the prospective teachers gave 13 wrong answers when modeling with the number line; however, the number of mistakes in Table 2 is 17. This is because some of the prospective teachers made more than one mistake in solving a problem.

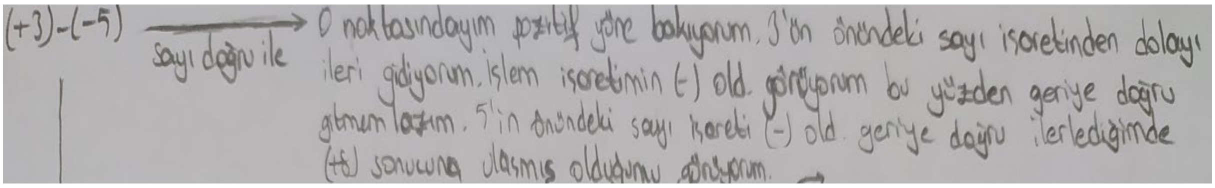


Figure 11. Example of the "giving the same meaning to the operation sign and the number sign" mistake

(*English translation of the text in the figure*: *(+3) − (-5) the number with line >>> I am at point 0 and facing the positive direction. I am moving forward because of the sign in front of the number 3. I see that the sign is (−), because of which, I should go back. When I do so, because of the sign (−) in front of the number 5, I arrive at point (+8).*)

When we look at Figure 11, it is evident that the sign of the operation and the number after the subtraction process are given the same meaning. In addition, if we move back five units while looking to the positive direction from +3, −2 instead of +8 would be reached. Thus, it can be understood that the prospective teacher did not check the action that they performed or took action according to the result.

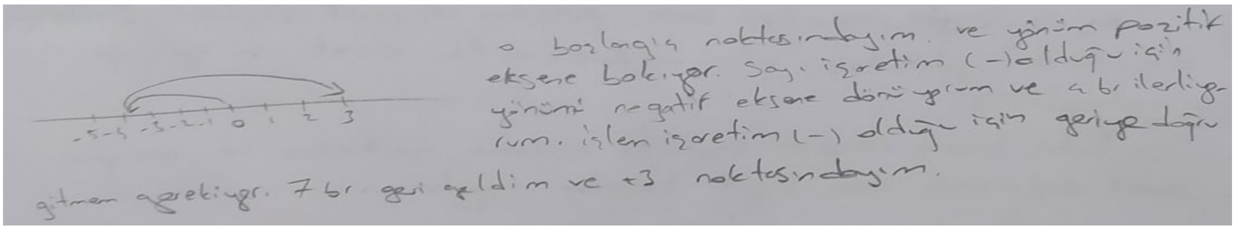


Figure 12. Example of the "ignoring the sign of the number" mistake

(*English translation of the text in the figure*: *I am at point 0 and facing the positive direction. Because the sign of the number is (−), I turn to face the negative direction and move forward 4 units. Because the sign of the process is (−), I should go back. I came back 7 units and now I am at point (+3).)*

In Figure 12, while modeling with the number line, the sign of the number seven is not taken into account. Moreover, the minus sign in front of the number has been given the meaning of turning to the negative axis and the subtraction to go backward. Giving such a meaning to the signs of the number and operation is the opposite of the model adopted in this study. The prospective teachers did not consider the number sign (or did not include it in the modeling process) regardless of whether the modeling was accurate or incorrect.

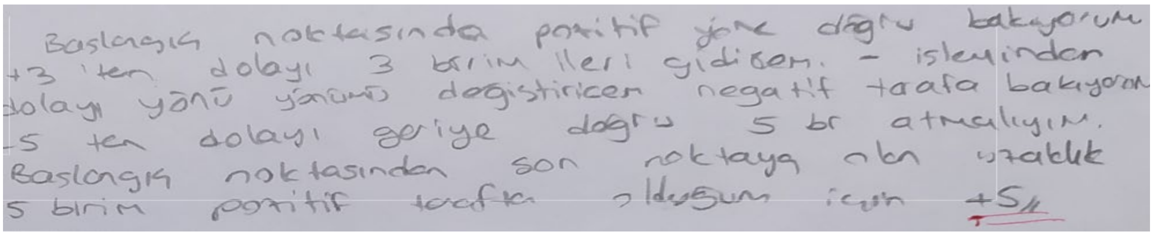
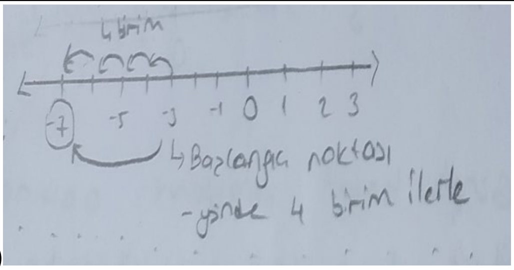


Figure 13. Example of the "arithmetic" mistake

(*English translation of the text in the figure*: *I am facing the positive direction from the starting point. I go forward 3 units because of the number (+3). I change my direction to the negative because of the sign (−). I am facing the negative direction. I should go 5 units back because of the number (−5). This is because the distance from the starting point to the finish point is 5 units in the positive direction (+5).)*

Figure 13 shows that the prospective teacher reached the point of +5 when looking at the negative axis at the +3 point while moving five units back. The prospective teacher correctly applied all stages of modeling with the number line and found the answer to be +5 instead of +8. Thus, it is thought that the wrong answer in this case was reached owing to an arithmetic mistake.

a) 

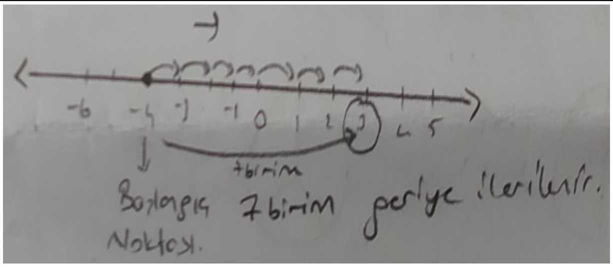
b) 

Figure 14. Examples of the "not starting the modeling from the starting point" and "ignoring the sign of the operation" mistakes

In Figures 14a and 14b, the modeling was not started at the starting point (i.e., point 0). It was stated that the sign of the second number will move forward or backward by ignoring the operation signs. Depending on the direction, the concepts of "forward and backward" differ. Although the conclusions reached here were correct, the process was wrong. Clearly, these mistakes arise from not starting the model by looking in the positive direction at the zero point and not including the sign of the subtraction process in the modeling.

Table 3 shows the types of mistakes made by the prospective teachers in modeling with counters and their frequencies.

Table 3. Mistakes in modeling subtraction of integers with counters

Type of Mistake	Frequency
Editing according to the result	3
Arithmetic	2

The prospective teachers used counters less frequently than the number line (see Table 1). Three of the mistakes made in modeling with counters were due to "editing according to the result," while two of the mistakes were related to "arithmetic."

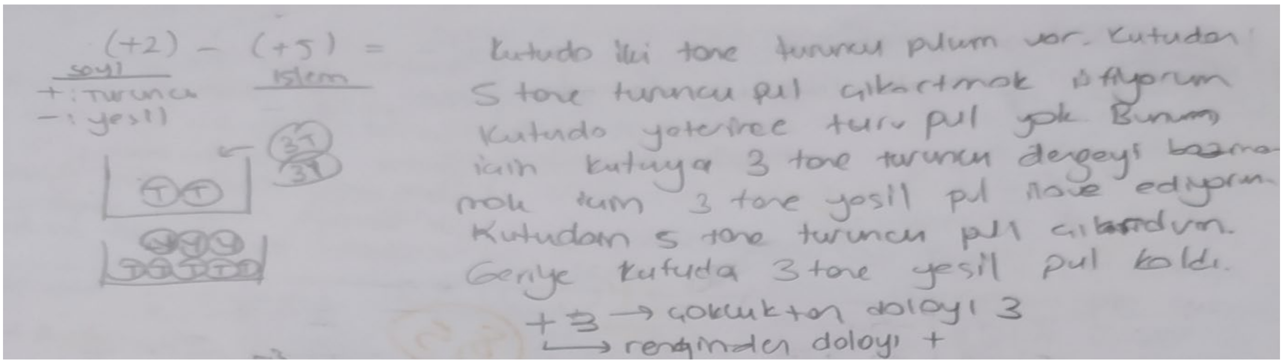
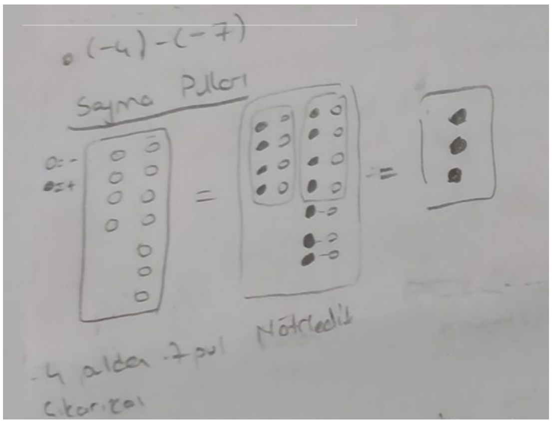


Figure 15. Example of an "arithmetic" mistake in modeling with counters

(*English translation of the text in the figure*: *I have 2 orange counters in my box. I want to remove 5 orange counters from the box. There are not enough orange counters in the box. So as not to break the balance, I add 3 orange counters and 3 green counters to the box. I remove 5 orange counters from the box. There remain 3 green counters in the box… + is because of its color …3 is because of the multiplicity.)*

Figure 15 shows that the prospective teacher represented positive integers with an orange counter and negative integers with a green counter, as indicated in the upper left of the figure. Although the prospective teacher provided the correct answer by stating that there were three green counters in the box using the counters model, they stated the answer was +3. At the last stage, it was understood that the green counters, which represented negative integers, had escaped the attention of the prospective teacher.

a) 

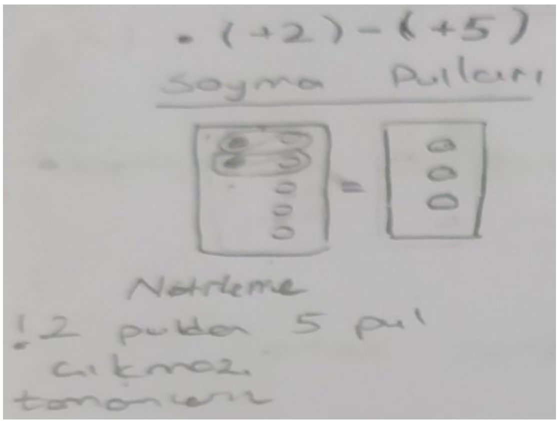
b) 

Figure 16. Examples of the “editing according to the result” mistake in modeling with counters

In Figure 16a, it is evident that the prospective teacher started modeling the counters to be used in the subtracting process by initially accepting them in the box. In the second stage, they added a positive counter for each negative counter, neutralized (or added a zero pair) the process, and then found the result to be +3. It can be seen that the prospective teacher performed operations to obtain +3, which is the result of (−4) − (−7). Figure 16b depicts a similar situation.

DISCUSSION AND CONCLUSION

Prospective teachers may develop misunderstandings or a limited understanding of different basic mathematical concepts (Ball, 1988; Ma, 1999). Moreover, prospective teachers often transfer their understandings to their future students (Reeder & Bateiha, 2016). Thus, identifying and amending their misunderstandings, if any exist, is essential. Thus, the present study investigated and analyzed the modeling abilities of prospective teachers of middle school mathematics for subtraction operations in integers with counters and number lines.

In the study, the prospective teachers were relatively successful with respect to both modeling types. The prospective teachers had the highest rates of success in modeling with counters, with a rate of approximately 97% in subtracting a negative integer from a positive integer ((+3) − (−5)), and the lowest rates of success in subtracting a negative integer from a negative integer, with a rate of around 89% for the operation ((−4) − (−7)) (see Table 1). In modeling with the number line, the highest success rate was approximately 91% for subtracting a positive integer ((+2) − (+5)) from another positive integer, whereas the lowest success rate was around 86% for subtracting a negative integer from a negative integer in the operation of ((−4) − (−7)) (see Table 1). Accordingly, it was understood that the question type with which the prospective teachers showed the highest success differed across modeling types, while the question type with the lowest success was the same. One reason for the high success rate of the prospective teachers might be their success in the “special teaching methods 1” course, as efforts in courses on mathematical teaching methods might positively affect prospective teachers’ understanding of mathematical concepts and planned pedagogical practices (Reeder & Bateiha, 2016; Sowder, Phillip, Armstrong, & Schappelle, 1998).

It was observed that the mistakes made by the prospective teachers in modeling the subtraction process with counters were caused by a lack of attention and resulted in an editing of the modeling according to the answer because they did not fully know or understand how the model worked. Specifically, prospective teachers who made the mistake of “editing according to the result” may have first found the answer of the process without using a model and then tried to model it with counters according to that answer. Taking Figures 16a and 16b into consideration, the steps of starting the modeling, adding a zero pair, and reaching the answer were not applied correctly; however, the answer itself was correct. This shows that the prospective teacher did not know how to model the procedure with counters and made arrangements according to the answer of the procedure. Furthermore, the prospective teachers made five different types of mistakes in modeling with the number line. These were (1) not starting the modeling at the zero point, (2) ignoring the sign of the resulting number, (3) giving the same meaning to the operation sign and the number sign, (4) ignoring the sign of the operation, and (5) arithmetic mistakes. The literature indicates that difficulties exist in relation to subtracting integers and working with negative numbers. The mistakes revealed in this study are also thought to stem from a lack of understanding of the different meanings of the minus sign (Gallardo & Rojano 1994; Janvier, 1985; Vlassis, 2004) and the limitations of the models used (Battista, 1983; Cemen, 1993; Cunningham, 2009; Liebeck, 1990).

RECOMMENDATIONS

In this study, it was observed that prospective teachers made more mistakes in modeling with a number line than modeling with counters. More activities and studies should be conducted with prospective teachers to improve their ability to operate using the number line because this model is widely used in subjects related to fractions, natural numbers, integers, and real numbers. Thus, it should be ensured that prospective teachers use the number line effectively.

Clinical interviews were not conducted with the prospective teachers in the present research, thereby limiting the information available with respect to their thought processes and sources of mistake in modeling the extraction process. Future studies can overcome this limitation by interviewing the participants regarding these aspects.

The prospective teachers herein were considerably successful in modeling procedures with the number line and counters. Future research can explore teachers’ knowledge of the mistakes of middle school students while using these models for integers as well as the teachers’ suggestions for potential solutions for such mistakes.

Declaration of Conflicting Interests

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, author-ship, and/or publication of this article.

Statements of publication ethics

I hereby declare that the study has not unethical issues and that research and publication ethics have been observed carefully.

Ethics Committee Approval Information

The study was approved by the Ethics Committee of Recep Tayyip Erdoğan University, and it was determined to be compliant with ethical standards with the letter dated November 10, 2020 and numbered 2020/129.

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