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Research article

Analysis and optimization of agro waste composite beam structures

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Abstract

The present work, is focused on the study of beam type structures made of agro waste composite materials, namely of sawdust and palm kernel shell, and comprises two parts. A first one where the static and dynamic behavior characterization of the structure is carried out using a first order shear deformation theory approach, considering the influence of different fiber constituent's percentages. In the second part, a few structural optimization case studies are considered concerning different objective functions and constraints, using sequential quadratic programming technique. Both, analysis and optimization studies were developed on a symbolic computation platform. According to the results obtained in the case studies carried out, it can be concluded that they respect the trends that would be expected, taking into account the nature of the parameters studied: the length to thickness ratio, the boundary conditions and the percentages of the reinforcement agents' mixtures. A final aspect that is worth mentioning is related to the advantages of using such type of platform in research work, which enables for an integrated development and simulation environment, where the libraries access and the functionalities available, enable a more expedite and perceptive way of achieving results.

Keywords: *Natural fiber composites, agro waste composites, static and free vibration analysis, and structural optimization*

1. Introduction

The increasing trend on the use of green composite materials imposes the need for characterizing the corresponding mechanical behavior, as well as to obtain optimum designs for structures made of these materials. This is mainly evident in application fields where the mechanical requisites do not collide to some of these materials shortcomings, namely in terms of durability and strength limits. The trend of using green composite materials can be due to its effective advantages in ecological terms. They have more renewable characteristics and friendly recycling. Moreover, the necessary raw materials

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have lower cost and weight. Following this interest, we can find several published works [1] on the so-called green composites, a significant number of which are related to their manufacturing. There are also some other studies comparing them with more traditional composites.

According to this, Wambua *et al*. prepared natural fibers reinforced polypropylene by compression molding using a film stacking method [1]. Those composites were tested to estimate and compare the mechanical properties of the different natural fiber composites. They also carried out a comparison study with the corresponding properties of glass mat reinforced polypropylene composites from the open literature. Ahankari *et al*. [2] carried out a study wherein green composites were fabricated by incorporating agro-residues as corn straw, soy stalk and wheat straw into the bacterial polyester, poly(3-hydroxybutyrate-co-3-hydroxyvalerate), usually known as PHBV, by melt mixing technique. In their work they have investigated the effects of these biomass fibers on mechanical, thermal, and dynamic mechanical properties of PHBV. A comparative study was also conducted between these green composites and traditional polypropylene composites. Khalil *et al*. [3] prepared epoxy–lignin composites, with varying lignin content (15%, 20%, 25% and 30%), reinforced with empty fruit brunches fibers, and have further investigated the effect of empty fruit brunches-based lignin on the mechanical, thermal and morphology properties of the composites. They found that epoxy composites cured with 25% lignin content proved to be a better matrix.

Valente *et al*. [4] manufactured hybrid thermoplastic composites from wood flour and recycled glass fibers through a two-step process involving a kinetic mixer and a compression molding machine. A study on the evaluation of these mechanical behavior characteristics, namely flexural modulus and strength, hardness as a function of temperature, screw withdrawal resistance and water absorption behavior were carried out. Coutinho *et al*. [5] prepared sawdust (20 wt%) reinforced polypropylene composites with 22.4 wt% maleated polypropylene coated fibers. Processing conditions, as well as the development of interfacial fiber/matrix adhesion, are discussed in their work.

Another work, focused on the development of procedural guidelines taking into account the ecodesign concepts in the selection of materials, was presented by Alves *et al*. [6]. According to the authors, the results pointed out the importance of the introduction of environmental parameters since the beginning of the project. Regarding to composite materials, it was noted that vegetable fibers were a good choice to replace fiberglass. Osarenmwinda and Nwachukwu [7] presented a work focusing on the properties' estimation of agro waste composite materials produced in a previous experimental work investigation. These estimations were carried out for hardness, yield strength, ultimate tensile strength, modulus of elasticity; modulus of rupture, internal bond strength, density, thickness swelling and water absorption.

A research study on the use of untreated and treated jute fiber composites, as candidates to replace glass fibers as reinforcement to produce structural composites with better environmental performance, was carried out by Alves *et al*. [8]. The mechanical characterization of the composites was obtained according to the ASTM standards (D-3039/D-790) and dynamic mechanical analysis. ASTM D-3039 is a tensile test used to measure the force needed to break a composite specimen and the extent to which it stretches to that point. Concerning to the ASTM D-709 test, it is a flexural test meant to measure the force required to bend a specified distance, the composite specimen under three point loading conditions or the force needed to break it. These jute composites were also compared with glass composites and the authors concluded from the increase of the mechanical properties due to jute fiber treatments, without damaging their environmental performances.

Concerning the existence of published reviews, it was possible to find a few works with different scopes and ranges of study, where relevant information is summarized. One of these works is due Pandey *et al*. [9] which presented a review where the advancement in the application of cellulose based materials is analyzed, discussing the fundamental research in these areas. By modifying either the resin system or the natural fiber, biocomposites may be designed for different applications ranging from products of commodity to aerospace, including electroactive papers, fuel cell membranes, controlled drug release mechanisms and biosensors. Another review article on the structure, composition and properties of pineapple leaf fiber and sisal, the chemical modifications of these fibers and pineapple leaf fiber/sisal-reinforced thermosets, thermoplastics, rubber, cement, hybrids and biocomposites, was presented by Mishra *et al*. [10]. In their work it was focused the modification of these fibers as a key area of research at present to obtain optimum fiber-matrix properties. John and Thomas [11] also presented a review work on the various aspects of cellulosic fibers and biocomposites. The classification of composites into green composites, hybrid biocomposites and textile biocomposites is discussed.

New developments dealing with cellulose based nanocomposites and electrospinning of nanofibers were also reported. Saheb and Jog [12] reported the published work on natural fiber reinforced composites with special reference to the type of fibers, matrix polymers, treatment of fibers and fiber-matrix interface, and, La Mantia and Morreale [13] presented a short review on green composites illustrating the main paths and results of research (both academic and industrial) on this emergent area, providing a quick overview as well as appropriate references for further in-depth studies.

Studying on green composites is an important issue and it is possible to see that a growing number of works were published in the recent years, and a great number of them are experimental works. The main motivation to this work is due to the belief that it is important to articulate and to deepen the link between experimental results and analytical and/or numerical solutions in order to predict composites behavior. Therefore, in the present work, it was considered the use of the agro waste composites, whose properties were estimated by Osarenmwinda and Nwachukwu [7]. These composites were made from by-products of agro-industrial processes, namely palm kernel shell and sawdust. The palm oil plant produces an edible fruit, similar to an apricot, which is processed through a steaming process, in order to extract crude palm oil. The remaining nuts are crushed to extract the seeds (kernels). Those crushed shells are called palm kernel shells, also used as a source of biomass fuel. On the other hand, sawdust is produced in lumber industries, and log saw milling. Such biomaterials, known as agro waste, are being increasingly used to improve the strength of polymers.

In this context, the goal of the present study is to predict the mechanical behavior of composite beam structures made of agro waste composite materials. Additionally, we have also carried out to optimization studies, using sequential quadratic programming technique, in order to obtain better structural performances under the corresponding loading and boundary conditions.

2. Agro Waste Composite Properties Modeling

The analyses and the optimization processes considered in this study were carried out on the constituent's percentage composition domain, wherein the properties descriptions obtained by Osarenmwinda and Nwachukwu [7] were determined. This domain is presented in Table 1.

Table 1
Fiber composition of composite panels tested by Osarenmwinda and Nwachukwu [7]

	Fiber constituent's percentage composition (%)					
Sawdust	100	90	80	70	60	50
Palm kernel shell	0	10	20	30	40	50

Osarenmwinda and Nwachukwu [7] tested several panels whose dimensions were 330×110×30 mm, and with different agro waste fibers compositions as it can be seen in Table 1. According to the experimental results the reference authors obtained a set of approximating functions depending on the sawdust and palm kernel shell percentage compositions. The composite properties were modeled by these authors using quadratic functions of these compositions.

For the purposes of this work, some of these properties fitting functions were used. However taking into account the relation between the fiber constituents' percentage, visible in Table 1, the curves can be expressed using a single parameter dependence, which was chosen to be the palm kernel shell percentage composition (P_{pks}). In equations 1-3, we present the quadratic fitting functions used (presented to three decimal places) namely for Young modulus (E), density (ρ) and yield strength, which reproduce the curves originally obtained [7].

$$E = 1.599\times10^{9} + 2.557\times10^{7} P_{pks} - 1.129\times10^{5} P_{pks}{}^{2} \tag{1}$$

$$\sigma_y = 1.088\times10^{6} + 72727.273 P_{pks} - 81.169 P_{pks}{}^{2} \tag{2}$$

All the properties fitting functions are expressed in SI units. In Figs. 1 and 2 it is possible to observe the plots corresponding to the estimation functions for Young's modulus and yield strength respectively.

The estimation function for the composite density is given as:

$$\rho = 811.140 + 0.062 P_{pks} + 0.0713 P_{pks}{}^{2} \tag{3}$$

The plot of this fitting curve can be observed in Fig. 3.

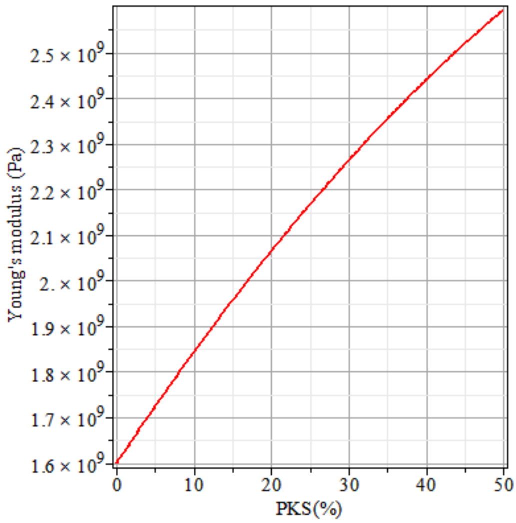

Fig. 1 Young's modulus estimation function

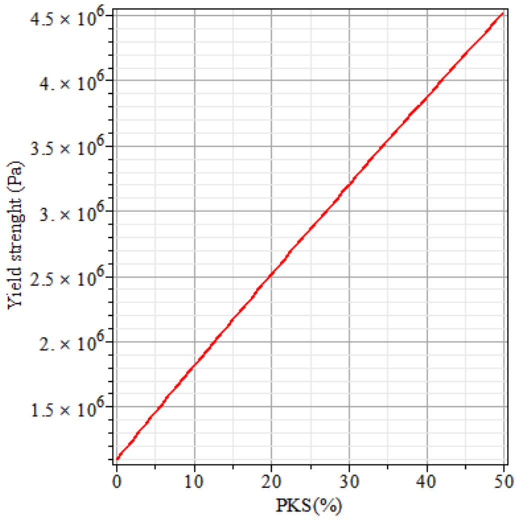

Fig. 2 Yield strength estimation function

Although in the literature, there is no information on Poisson's ratio [7], in this study it was assumed a value of 0.3. One of the reasons for this value is related to the fact that the composites studied here can be considered isotropic composites, as the particle sizes are in the order of 300 μm, and this Poisson ratio value is a typical value for this type of cases. Additionally in other literature works' results, namely on Silva *et al*. [14] where some numerical studies on the effect of the Poisson ratio estimation were carried out for banana and sisal fibers composites and compared to experimental results, it was also found that this value would be an acceptable value.

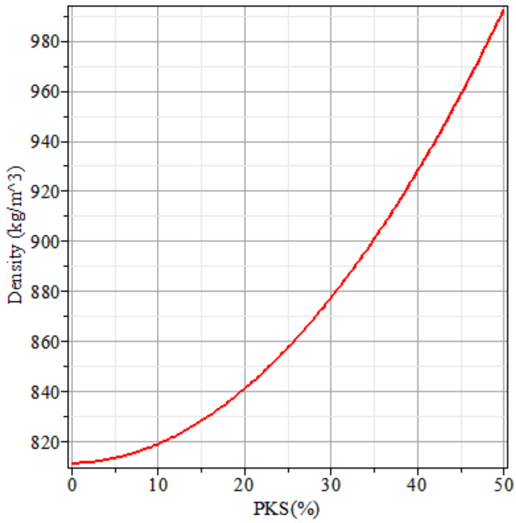

Fig. 3 Composite density estimation function

3. Methodology

3.1. Beam Analysis

To study the mechanical behavior of these agro waste composite beams, we have considered the First Order Shear Deformation Theory [15], also known as FSDT. The displacement field associated to this theory is given as:

$$\begin{cases} u(x,z,t) = u^0(x,t) + z\,\theta_y^0(x,t) \\ w(x,t) = w^0(x,t) \end{cases} \tag{4}$$

where u^0, w^0 and θ_y^0 are respectively the mid-plane displacements associated to the longitudinal direction (x) and transverse normal direction (z), and the rotation around (y) direction (see Fig. 4).

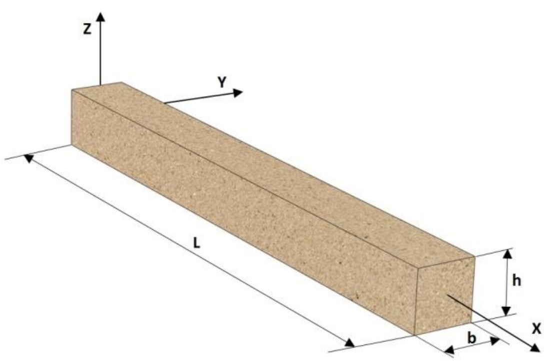

Fig. 4 Coordinate system and geometrical characteristics of agro waste composite beam

The thickness coordinate given by z and t is the time instant. The strain field was then obtained by considering the kinematical relations of elasticity for small deformations. Linear static analysis and free vibration analysis are carried out according to the usual procedures fully described in the literature [16]. It is worth mentioning that the implementation of these procedures is done symbolically and the numerical evaluation occurs only at final calculations. Following the analysis phase, one has proceeded to optimization studies.

3.2. Optimization Technique

The optimization technique applied here was the sequential quadratic programming (SQP). SQP, more than a specific optimization method, is a class of optimization algorithms that has evolved from a common basis, based upon Newton's method. The first SQP method was proposed by Wilson [17] and was applied to solve constrained nonlinear problems.

Although, only after the work published by Han [18], the optimization community started to pay attention to this technique. Further details about the SQP class of algorithms can be found on the review papers of Boggs and Tolle [19] and Gill and Wong [20]. This technique was found to be adequate because of the non-linearity of both the fitness function and the constraints considered. According to the referred literature this technique yields a good performance on these situations. In the present work, the considered SQP method is the one referred by Fishback [21].
A general minimization problem is written as:

$$\begin{aligned} &Minimize\left\{\Omega\left(\boldsymbol{b}\right)\right\} \\ &subject\ to:\ b_i^{low} \le b_i \le b_i^{up},\ i=1..ndv\ ;\ \Psi_j\left(\boldsymbol{q},\boldsymbol{b}\right) \le 0,\ \ j=1..nbc \end{aligned} \tag{5}$$

where $\Omega(\boldsymbol{b})$ is the objective or fitness function, $\boldsymbol{b}$ is the design variables vector, $\Psi_j\left(\boldsymbol{q},\boldsymbol{b}\right)$ are the *nbc* inequality behavioral constraint equations, b_i^{low}, b_i^{up} are the lower and upper limits of the design variables respectively, and *ndv* is the total number of design variables. In this work, some test cases were studied, where the objective functions were the maximum transverse displacement, the fundamental frequency and the weight of the beam. As behavioral constraints, we have selected the weight, the maximum normal stress and the fundamental frequency constraints. As it will be shown, the optimization technique used shows to perform very effectively, presenting good convergence characteristics.

Considering the nature of the optimization technique used, in the cases studied one has considered a maximum number of iterations was established to 1000, and for each optimization case, six different initial points were tested. In all the cases the design variable was chosen to be the palm kernel shell percentage composition (P_{pks}), thus the initial points were the elements belonging to the set {0, 10, 20, 30, 40, and 50}. The side constraints were equal for all the cases studied and correspond to the bounds associated to the domain of study, that is, $0 \le P_{pks} \le 50$, and the feasibility tolerance and the optimality tolerance were 0.1×10^{-7} and 0.302×10^{-11} respectively.

4. Results and Discussion

This section is constituted by two main parts. In the first part we carried out linear static and free vibrations analyses of the composite beam structures for different length to thickness ratios, different boundary conditions and different agro waste fibers percentages compositions. Maximum load designs were also determined for typical cases. In the second part, we considered some constrained and unconstrained optimization cases, considering different objective functions and behavioral constraints. It is important to note here that the acronyms that will be used to mention the boundary conditions are respectively CC, CF, and SS for clamped-clamped, clamped-free and simply supported boundaries. The length to thickness ratio is referenced as L/h.

4.1. Validation Case Study

To study the performance of the present FSDT model, for different length to thickness ratios, it was considered a simply supported straight beam structure subjected to a uniformly distributed loading p=1 N/m, in the z direction. The material and geometric properties were E=200 GPa (Young's modulus), b=0.01 m (width) and h=0.01 m (height). It was used a shear correction factor of 5/6.

In Table 2, we can see the maximum transverse displacements obtained through the Elasticity theory as well as the solutions obtained with another FSDT model implemented numerically using Fortran and double precision (Loja *et al.* [22]).

Table 2
Maximum transverse displacement of simply supported beam (m)

L/h	Owen and Hinton (1980)	FSDT (Loja *et al.*, 2001)	Present FSDT
1	0.256250×10^{-11}	0.190972×10^{-11}	0.228125×10^{-11}
2	0.196250×10^{-10}	0.170139×10^{-10}	0.185000×10^{-10}
4	0.228500×10^{-9}	0.218056×10^{-9}	0.224000×10^{-9}
10	0.799063×10^{-8}	0.792535×10^{-8}	0.796250×10^{-8}
20	0.125713×10^{-6}	0.125451×10^{-6}	0.125600×10^{-6}
50	0.488727×10^{-5}	0.488563×10^{-5}	0.488660×10^{-5}

The present model was implemented symbolically using Maple. It can be observed that even for lower L/h ratios, where FSDT model is known to have a stiffer behavior, the present model presents an overall reasonable performance when compared to the Elasticity solutions. This is the case of the L/h=4 ratio, where it is known that the shear deformations can become significant. A very good agreement between the implemented model and the elasticity solutions can be concluded for the L/h ratios that are within the typical range of applicability of first order theory. This fact, has also supported the option to use an FSDT approach instead of classical theory based one. It is relevant to mention that the model implemented using symbolic computation presents a closer proximity to elasticity solutions (Owen and Hinton, [23]).

4.2. Agro Waste Composite Beam Analysis

The analyzed agro waste composite beam is schematically represented in Fig. 1, having the following geometrical characteristics: height h=0.030 m, width b=0.090 m. In the whole study concerning the agro waste beam, it was also considered a shear correction factor of 5/6.

To study the mechanical behavior of the composite beam we started to consider that it was only submitted to its own weight. The static behavior of the beam was analyzed for different length to thickness ratios and different boundary conditions, and these dependences were characterized. In Tables 3 and 4, it is possible to observe, that for each value of palm kernel shell percentage composition, the maximum transverse displacement follows the expected pattern according to the nature of the boundaries where CC, CF, SS stand for clamped-clamped, clamped-free and simply supported boundaries.

Additionally, as we could expect the thinner beam (higher L/h ratio) presents greater displacements, for any percentage composition value, in any boundary conditions situation,.

From the results obtained within the domain of the study, one can conclude that the palm kernel shell percentage composition plays an important role as a stiffening agent of the structure. It is important to mention that the minimum displacement occurs for the 40% percentage composition value, which seems to be related to the properties estimation function of Young's modulus (see Fig. 1).

Table 3
Maximum displacements for L/h=10, different percentage compositions and boundary conditions

Percentage composition PKS (%)	Maximum transverse displacement (m)		
	Boundary conditions		
	CC	CF	SS
0	0.119×10^{-5}	0.505×10^{-4}	0.535×10^{-5}
10	0.104×10^{-5}	0.442×10^{-4}	0.468×10^{-5}
20	0.952×10^{-6}	0.405×10^{-4}	0.429×10^{-5}
30	0.905×10^{-6}	0.386×10^{-4}	0.408×10^{-5}
40	0.888×10^{-6}	0.378×10^{-4}	0.400×10^{-5}
50	0.894×10^{-6}	0.381×10^{-4}	0.403×10^{-5}

In a second phase within these first characterization studies, we also carried out free vibration analyses, which similarly meant to evaluate the influence of the same parameters on the beam fundamental frequency. The results are shown in Tables 5 and 6.

It is possible to observe the decreasing trend of the fundamental frequency when the boundary conditions go from a more restrictive to a less restrictive character. This decreasing trend is also visible between the fundamental frequency of the thicker beam and the fundamental frequency of the thinner beam. In any case, the behavior patterns are what we could expect as in the static analyses carried out.

It is also worth noting that there is a visible relation between the density fitting function and the fundamental frequency. A maximum value reached around 40% palm kernel shell percentage composition.

Table 4
Maximum displacements for L/h=20, different percentage compositions and boundary conditions

Percentage composition PKS (%)	Maximum transverse displacement (m)		
	Boundary conditions		
	CC	CF	SS
0	0.172×10^{-4}	0.801×10^{-3}	0.838×10^{-4}
10	0.151×10^{-4}	0.701×10^{-3}	0.734×10^{-4}
20	0.138×10^{-4}	0.643×10^{-3}	0.672×10^{-4}
30	0.132×10^{-4}	0.612×10^{-3}	0.640×10^{-4}
40	0.129×10^{-4}	0.600×10^{-3}	0.628×10^{-4}
50	0.130×10^{-4}	0.604×10^{-3}	0.632×10^{-4}

Considering the experimental knowledge about the properties variation of the composite, and its approximating functions, it would be relevant to know the maximum load that it is admissible to consider in addition to the beam's own weight. For this purpose, two situations were also analyzed: the first one was a situation where the additional external load will be a uniformly distributed loading along the whole beam length, and the second one was another situation where the external load will be applied at midspan.

Table 5
Fundamental frequency for L/h=10, different percentage compositions and boundary conditions

Percentage composition PKS (%)	Fundamental frequency (Hz)		
	Boundary conditions		
	CC	CF	SS
0	519.634	86.952	241.790
10	555.298	92.920	258.385
20	580.026	97.058	269.891
30	594.681	99.510	276.710
40	600.401	100.468	279.372
50	598.522	100.153	278.497

Table 6
Fundamental frequency for L/h=20, different percentage compositions and boundary conditions

Percentage composition PKS (%)	Fundamental frequency (Hz)		
	Boundary conditions		
	CC	CF	SS
0	136.920	21.879	61.267
10	146.317	23.380	65.472
20	152.833	24.422	68.387
30	156.694	25.039	70.115
40	158.202	25.279	70.790
50	157.706	25.200	70.568

For this purpose, we considered a clamped-clamped beam, with a length to thickness ratio of 10. Taking into account that the maximum stress value cannot exceed the yield strength, for each value of palm kernel shell percentage composition, we obtained the maximum external uniformly distributed load Q distribution that can be observed in Fig.

5. The least squares fitting function for the admissible uniformly distributed load Q, was determined to be:

$$Q_{adm} = 2102.996 + 135.902 P_{pks} + 0.090\, P_{pks}^{\;2} - 0.0032 P_{pks}^{\;3}$$

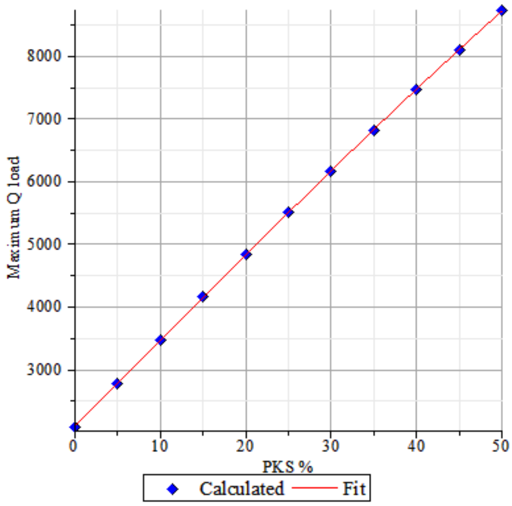

Fig. 5 Admissible distributed load Q

Considering now the second load case, where an external concentrated load W was applied at beam midspan in addition to the beam own weight, one have obtained for the admissible load W for each palm kernel shell percentage composition, the distribution presented in Fig. 6, in the study domain.

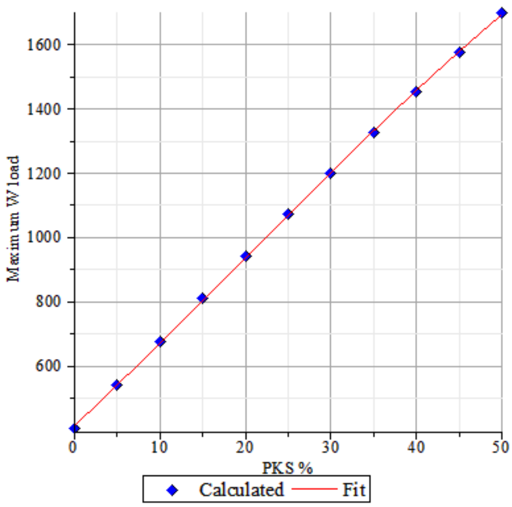

Fig. 6 Admissible concentrated load W

The fitting function associated to the admissible concentrated load W was determined using a least squares methodology which yields:

$$W_{adm} = 414.302 + 24.995\ V_f + 0.090\ V_f^{\ 2} - 0.0016\ V_f^{\ 3}$$

In any of these cases it can be concluded that the percentage composition of palm kernel shell parameter directly influences the maximum external load that it is possible to apply to the structure.

4.3. Agro Waste Composite Beam Optimal Designs

Following the static and free vibration analyses cases presented, some optimization studies were carried out, using the sequential quadratic programming technique. The parameterization of this optimization technique was presented in Methodology section.

As a first optimization case study, it was considered a clamped-clamped beam, with a length to thickness ratio equal to 10, submitted to a combined load of its own weight and an external load W given by $W = \left(P_{pks}/100\right)^3 10^5$ N, applied at beam midspan. In this case it was intended to maximize the maximum transverse displacement w (in terms of its absolute value).In the optimization study carried out, the behavioral constraint selected was expressed by an inequality constraint between the maximum stress value and the yield strength value for the corresponding value of W. The results obtained can be observed in Table 7.

Table 7
Admissible distributed load Q

Percentage composition PKS (%)	Load Q (N/m)
0	2085.61425
5	2786.09062
10	3478.61027
15	4163.17320
20	4839.77941
25	5508.42890
30	6169.12168
35	6821.85774
40	7466.63707
45	8103.45969
50	8732.32560

The initial P_{pks}=10% and P_{pks}=20% lead to a slight slower convergence. However in all those situations we get an optimal value that is equal until the seventh decimal place, except for the first one (P_{pks}=0%). There is also a situation corresponding to considering an initial point P_{pks}=30% which cannot lead to a feasible solution. Because of this, we chose two other near initial points and as expected we achieve a solution as can be observed in the same table. These situations, frequent in this type of optimization techniques, happen due to the convexity characteristics of the problem. The optimal solution can be therefore considered to correspond to a percentage composition of palm kernel shell of 21.385% to obtain a maximum transverse displacement of 0.276×10^{-3} m. Thus the maximum concentrated load that would be possible to apply under the present conditions will be 978 N.

Taking the same beam under similar conditions, it was now carried out an optimization study to minimize the maximum transverse displacement w, electing now as preferential a stiffer composite beam. The results obtained are presented in Table 8.

Table 8
Admissible concentrated load W

Percentage composition PKS (%)	Load Q (N/m)
0	405.86254
5	542.17591
10	676.94090
15	810.15751
20	941.82572
25	1071.94556
30	1200.51701
35	1327.54007
40	1453.01475
45	1576.94104
50	1699.31895

As we can observe, this case was more time consuming when compared to the latter. The best solution is achieved either we consider as starting point, P_{pks} =30%, P_{pks} =40% or P_{pks} =50%, where we get an optimal value that is equal in all cases until the seventh decimal place. However the optimization process converges faster with the last percentage selection. The initial point P_{pks}=0% provides a solution that is very close to the best found in this process. When we take P_{pks} =10% or P_{pks} =20% as initial point we are not able to converge to the mentioned better solution, although the number of iterations reaches its maximum limit. The optimal solution in this case corresponded to a percentage composition of palm kernel shell of 0.411%. This composition leads to an optimal minimum transverse displacement equal to 0.118×10^{-5} m.

Other optimal designs' studies were also considered taking into account the free vibrations behavior of these composite beams. As a first case we wished to maximize the fundamental frequency, neglecting any behavioral constraints. The results can be seen in Table 9.

Table 9
Maximum transverse displacements w

Initial Point PKS (%)	maximization w (m)		
	Optimum w (m)	Number of iterations	PKS (%)
0	0.1185600×10^{-5}	1	0
10	0.2756547×10^{-3}	13	21.385440509
20	0.2756547×10^{-3}	10	21.385440509
30	-	-	-
40	0.2756547×10^{-3}	7	21.385440509
50	0.2756547×10^{-3}	7	21.385440509

The first five initial points in the table perform similarly in terms of convergence characteristics. The last initial point selection, correspondent to P_{pks} =50% requires a higher number of iterations to approach the best solution. In this case the optimum palm kernel percent composition was 42.267% to achieve a minimum fundamental frequency of 600.591 Hz.

Finally we wished to minimize the beam weight submitted to behavioral constraints. The beam characteristics were similar to the previous optimization case studies, as well as the side constraints. The behavioral constraints were related to two inequality relations: the first one, established in a static situation, between the maximum stress value and the yield strength value, and the second established between the fundamental frequency and a minimum value of 600 Hz. The results obtained are summarized in Table 10.

Table 10
Minimum transverse displacements w

Initial Point PKS (%)	minimization w (m)		
	Optimum (m)	Number of iterations	PKS (%)
0	0.1185600×10^{-5}	3	0.000297471
10	0.3286460×10^{-4}	1000	9.990849531
20	0.2277957×10^{-3}	1000	19.968195830
30	0.1180500×10^{-5}	615	0.410924697
40	0.1180500×10^{-5}	274	0.410924735
50	0.1180500×10^{-5}	82	0.410925592

From this table we find that no matter the initial point, we obtain the same optimal solution which is a minimum weight of 7.295 N for a percentage of palm kernel shell of 38.290%. In this case, there is no significant difference in the number of iterations required for convergence. One can also conclude from a very good convergence performance.

5. Conclusions

The use of green composite materials is spreading in application fields where mechanical requisites do not collide to some of these materials shortcomings, namely in terms of durability and strength limits. In those cases they are becoming a realistic alternative to other reinforcement agents traditionally used in composite materials.

This work presents an illustrative set of case studies on the static and free vibration analysis of a beam made of sawdust and palm kernel shell composite, in order to characterize its mechanical behavior.

Within the study domain considered, it can be clearly concluded that the palm kernel shell has an effective contribution to a better mechanical behavior of the agro waste composite. The sawdust is not as efficient as the palm kernel shell.

Optimization studies were also carried out considering a few test cases. These studies can be relevant as they may enable to understand under which conditions we will get a better performance according to a given set of requisites or constraints.

It is thought that as a subsequent study on the evaluation of the performance of these composites, it would be desirable that adequate experimental tests could be carried out, which in turn would also enable to conclude on the accuracy of the numerical results and on their deviations when compared to the experimental ones.

Both analysis and optimization studies were developed on a symbolic computation platform. An aspect that is worth mentioning is related to the advantages of using such type of platform in research work, which enables for an integrated ambient of development and simulation, where the libraries access and the functionalities available, enable a more expedite and perceptive way of achieving results.

Another important factor is the one related to the accuracy of results. It was possible to verify quantitatively that the symbolic manipulation may carry with it a gain in terms of the results accuracy obtained. This was proved through the results comparison with an equal theory based model implemented numerically using double precision, in Fortran90. Concerning the results obtained from the analyses and the optimization cases considered, it can be concluded that they globally respect the trends that would be expected, taking into account the nature of the parameters studied: the length thickness ratio, the boundary conditions and the percentages of the reinforcement agents' mixtures.

As far as it is possible to conclude from the works already published and the one carried out in this study, it is important to articulate experimental and simulation results either obtained analytically or numerically. This type of studies may be relevant for a better prediction of green composite materials' behaviors prone to further developments.

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