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Stress concentration analysis between two neighboring circular holes under internal pressure of a non-explosive expansion material

Patlamayan-kabaran malzemenin içsel basıncı altındaki komşu dairesel iki delik arasındaki gerilme konsantrasyonunun analizi

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ABSTRACT

Stress analysis for a rock medium is essential for determination of stress concentration between two neighboring circular holes and prediction of fracture behavior. When two neighboring circular holes in a hard rock medium such as granite are loaded internally by the pressure of a Non-Explosive Expansion Material (NEEM), stress concentration occurs between the holes which then causes the rock to fracture. In this study, Finite Element (FE) analysis using a Phase^2 computer code was employed to study the stress concentration between two neighboring circular holes under internal pressure induced by the NEEM. The effects of different hole diameters and spacings, rock properties and NEEM pressures were analyzed, and the data obtained from numerical analysis and statistical studies were then used to develop two models. These models were then modified by using the FE data and polynomial regression analysis. The developed statistical models were shown to be in a very good agreement with the FE analysis. Validation of the equations is only for the points located on the line passing through the centers of the holes in the elastic state. Hence, the developed models can be used with confidence to determine stress distribution and concentration factors around two neighboring circular holes, which are excavated in a hard -brittle rock and loaded internally by the pressure induced from the NEEM.

Keywords: Finite element method, non-explosive quarry mining, statistical model, stress concentration.

ÖZ

Komşu konumlu iki dairesel delik arasındaki gerilme konsantrasyonunun tayini ve kırık davranışının tahmini için kaya ortamında gerilme analizi yapılması gereklidir. Granit gibi sert kaya ortamlarındaki komşu konumlu dairesel delikler Patlamayan-Kabaran Malzeme (PKB)'nin basıncıyla yüklendiklerinde, kayanın kırılmasına neden olan gerilme konsantrasyonları meydana gelir. Bu çalışmada, PKB'nin neden olduğu içsel basınç altındaki komşu konumlu iki dairesel delik arasındaki gerilme konsantrasyonunun araştırılması için Phase adlı bilgisayar programı kullanılarak Sonlu Elemanlar (SE) analizi yapılmış olup, bu analizlerde farklı delik çapları ve uzaklıkları, kaya özellikleri ve PKB basınçları analiz edilmiştir. Sayısal çözümlemelerden ve istatistiksel analizlerden elde edilen veri iki modelin geliştirilmesi için kullanılmıştır. Daha sonra bu modeller SE ve polinomial regresyon analizleriyle modifiye

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edilmişlerdir. Geliştirilen istatistiksel modeler SE analiziyle çok iyi bir uyum göstermiştir. Eşitlikler, sadece elastik durumda deliklerin merkezinden geçen hattın üzeride yer alan noktalar için geçerlidir. Bu nedenle geliştirilen modeler, PKB'den kaynaklanan basınçla içsel olarak yüklenmiş ve sert kayada kazılmış komşu konumlu dairesel delikler çeveresindeki gerilme konsantrasyonu faktörlerinin ve gerilme dağılımının belirlenmesi için güvenilirlikle kullanılabilirler.

Anahtar Kelimeler: *Sonlu elemanlar yöntemi, patlatmasız taş ocağı işletmeciliği, istatistiksel model, gerilme konsantrasyonu.*

INTRODUCTION

One of the main methods in quarry mining, especially in hard rocks, is the controlled fracture method that is carried out by the introduction of a slowly advancing crack using Non-Explosive Expansion Material (NEEM). The application of NEEM in hard rock quarry mining has recently been increased (Hayashi et al., 1994; Pal Roy, 2005; Arshadnejad, 2007). This method of rock breakage is without noise and vibrations and its operation, compared to the blasting method, is more controllable; it is very safe and easy and does not leave extra undesirable cracks in the rock block.

In this method, some circular holes of equal length, diameter and spacing (centre-to-centre distance) are drilled closely together in a rock block. The holes are then filled with the NEEM, which by its expansion will generate an incremental static load into the holes after about two to four hours (Goto et al., 1988; Zhongzhe et al., 1988; Jana, 1991; Hayashi et al., 1994; Pal Roy, 2005). If the spacing of the holes is suitable, it will create a crack between two neighboring holes, and the rock will fracture along the high-stress concentration path between the holes. If the material of the medium is brittle, such as hard rocks (e. g. granite and quartzite), it will not yield and before failure no plastic behaviour will be observed in the material (Hoek and Bieniawski, 1965; Lajtai, 1972; Lawn and Wilshaw, 1975; Ingraffea and Schmidt, 1978; Fowell, 1995; Eberhardt et al., 1999; Orekhov and Zertsalov, 2001; Yağız, 2009). Thus, the material is considered to behave in a linear elastic mode until the onset of failure.

When there are two neighboring holes in a plate loaded internally, stress concentration will occur. The maximum elastic stresses (stress concentration) have been examined by several methods, such as photoelastic analysis (Hoek and Bieniawski, 1963; Joussineau et al., 2003), direct strain measurement (Nesetova and Lajtai, 1973; Chong et al., 1987) and numerical modeling (Bazant, 1982; Yan, 2007). There are many empirical models for estimating stress concentration in different geometries, such as a circular hole. Stress concentration factors due to uniform and axisymmetrical external pressure around a single circular hole were analyzed by Howland (1929), Forcht (1935), Lipson and Juvinall (1963). One of the first studies of plane elasticity in bipolar coordinates in an infinite plate with two circular holes was carried out by Jeffrey (1920). Howland (1935) investigated the stress distribution around an infinite row of equal sized circular holes spaced equally in an infinite elastic plate. The plate was subjected to a uniaxial stress field. Howland and Knight (1939) presented stress functions for problems involving equal sized circular holes. Ling (1948) developed a solution (in bipolar coordinates) for the stresses in a plate containing two equal circular holes when the distances between them are variable. He considered three stress fields: uniaxial stress parallel, perpendicular to the line of centers and equal stresses in all directions (Gerçek, 2005). Haddon (1967), using the conformal mapping and complex variable techniques, presented a solution for stresses around two unequal circular holes in an infinite plate. The plate was subjected to a uniaxial stress field with a variable inclination to the line of holes' centers (Gerçek, 2005). Obert and Duvall (1967) studied the stress distribution around pillars (rock columns) between two parallel circular excavations subjected to uniaxial compressive external loading, by the photoelasticity method. Two empirical models were developed by Schulz (1942) and Peterson (1974) when the type of external loading is tensile in biaxial, Ling and Tsai (1969) presented an analytical solution for the stresses in a thick plate of infinite size containing a spherical inclusion or cavity eccentrically located between the surfaces. The plate had been subjected to a stress system symmetrical about the axis of revolution of the plate, while the surfaces were stress-free. Gerçek (1988, 1996) presented a solution for boundary stresses for two parallel circular tunnels located in a biaxial in situ stress field. It was obtained by superposing the solutions developed by Ling (1948) (Gerçek, 2005). Zimmerman (1988, 1991) suggested approximate equations for stress concentrations in an infinite elastic plate containing two circular holes.

Unfortunately, almost all existing solutions are only applicable to stress-free conditions at the boundary of the holes, which is not always the case in engineering applications, such as a hole with internal pressure caused by the NEEM. The scope of this study is to develop a model to analyze stress concentration between two neighboring circular holes - the points located on the line passing through the centers of the holes in elastic state - excavated in a hard rock medium and loaded internally by the NEEM. The base of the model is a statistical method, and verification by the finite element method has been upgraded.

STRESS DISTRIBUTION AROUND A CIRCULAR HOLE DUE TO INTERNAL AND EXTERNAL LOADS

Stress distribution around a circular hole depends on the stress field condition. Kirsch (Kirsch, 1898) initially studied this problem for a single circular hole under a biaxial stress field.

The stress distribution within a thick-walled cylinder under uniform external and internal loading is as follows (Timoshenko and Goodier, 1951).

$$\sigma_r = \frac{(a^2P_i - b^2P_o)}{b^2 - a^2} + \frac{a^2b^2(P_i - P_o)}{r^2(b^2 - a^2)} \qquad (1)$$

$$\sigma_\theta = \frac{(a^2P_i - b^2P_o)}{b^2 - a^2} - \frac{a^2b^2(P_i - P_o)}{r^2(b^2 - a^2)} \qquad (2)$$

Where σ_r and σ_θ are the radial and tangential stresses, respectively, and r is the radial distance of the considered point from the hole centre. P_i and P_o are internal and external pressures, respectively, and a and b are the internal and external radius of the thick-walled cylinder, respectively. Because of axisymmetry in the loadings and body geometry, there is no shear stress in the medium. As a convention in rock mechanics, the tensile stress is considered negative and the compressive stress is considered positive (Hoek and Brown, 1980; Goodman, 1989). The constraint for using thick-walled cylinder Equations is as follow (Shigley, 1956; Hertzberg, 1996).

$$\frac{b-a}{a} > \frac{1}{20} \qquad (3)$$

If there is no external pressure ($P_o = 0$) the equations become:

$$\sigma_r = \frac{a^2P_i}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right) \qquad (4)$$

$$\sigma_\theta = \frac{a^2P_i}{b^2 - a^2}\left(1 - \frac{b^2}{r^2}\right) \qquad (5)$$

If the thickness of the cylinder wall increases to infinite ($b \to \infty$), the cylinder will transform to a circular hole in an infinite plate, such as a hole in a rock medium. Then, Equations 4 and 5 convert to:

$$\lim_{b\to\infty} \sigma_r = \frac{a^2P_i}{r^2} = P_i\left(\frac{a}{r}\right)^2 \qquad (6)$$

$$\lim_{b\to\infty} \sigma_\theta = -\frac{a^2P_i}{r^2} = -P_i\left(\frac{a}{r}\right)^2 \qquad (7)$$

STRESS CONCENTRATION BETWEEN NEIGHBORING HOLES UNDER INTERNAL PRESSURE

When two or more circular holes in a plate are loaded by internal pressure, stress concentration will occur among them. When the stress intensity is equal to the rock fracture toughness, cracks may be initiated. Subsequently, the cracks will grow; however, as the length of the crack increases, the stress on the crack tip decreases, due to distancing from the hole, thus decreasing the stress concentration. Nevertheless, by increasing the stress induced from the hole due to application of NEEM, in due time, the stress intensity on the crack tip will again increase up to the level of rock fracture toughness. Thus, again the crack will grow farther, and this circle of events will repeat; hence, a controllable mechanism for crack growth may be achieved. Figure 1 shows a rock fracture between two neighboring holes due to application of the NEEM in a granite quarry mine in Taleghan, Iran.

As stated before, Equations 6 and 7 may be used for determination of stress distribution around a circular hole. However, when there are two neighboring circular holes in a rock medium, these equations have to be modified. The modified equations may be assumed to be as follows.

$$\sigma_r = C_r . P_i \left(\frac{a}{r} \right)^2 \tag{8}$$

Figure 1. Rock fracture between two neighboring holes due to application of NEEM in a granite mine.

Şekil 1. Bir granit işletmesinde PKB'nin uygulanması nedeniyle komşu konumlu iki delik arasında gelişmiş kırık.

$$a = \frac{d}{2} \Rightarrow \quad \sigma_r = C_r . P_i \left(\frac{d}{2r} \right)^2 \tag{9}$$

$$\sigma_\theta = -C_\theta . P_i \left(\frac{d}{2r} \right)^2 \tag{10}$$

Where C_r and C_θ are the stress concentration factors for the radial and tangential elastic stress, respectively, and *d* is the diameter of holes. Other parameters are those explained earlier in Equations 1 and 2.

In this study, the Phase^2 computer code (Rocscience, Inc., 1999 and 2001) based on the finite element method (FEM) was used to determine the radial and tangential stresses around the hole (σ_r and σ_θ) by numerical analysis. In this respect, six nodal triangular elements with nodal averages were utilized. Figure 2 shows the stress trajectories on such nodal points. The model geometry and the parameters were selected based on real conditions of quarry mining operations. Table 1 shows the parameters that were applied in the numerical models. The internal pressures in the holes were due to non-explosive expansion material (NEEM).

While running the program, it was noticed that Young's modulus and internal pressure have no effect on the stress concentration factors. Since the stress concentration factor is essentially related to geometrical characteristics, this finding seems to be justified. However, Poisson's ratio tends to have some effect, as has been confirmed by previous study (Zienkiewicz, 1971). Therefore, around 180 numerical models had to be analyzed. Figure 3 depicts the stress concentration zones between two neighboring circular holes with a typical hole diameter of 44 mm and hole spacing of 50 mm under internal pressure of 15 MPa due to NEEM.

The results from the numerical analysis show that Poisson's ratio, hole diameters and hole spacings are the main parameters affecting the stress concentration between two neighboring circular holes. The data from numerical analysis, along with multiple regression and logarithmical data (Chatfield, 1983; Montgomery et al., 2001), were used to determine C_r and C_θ. Equations 11 and 12 show these primary models.

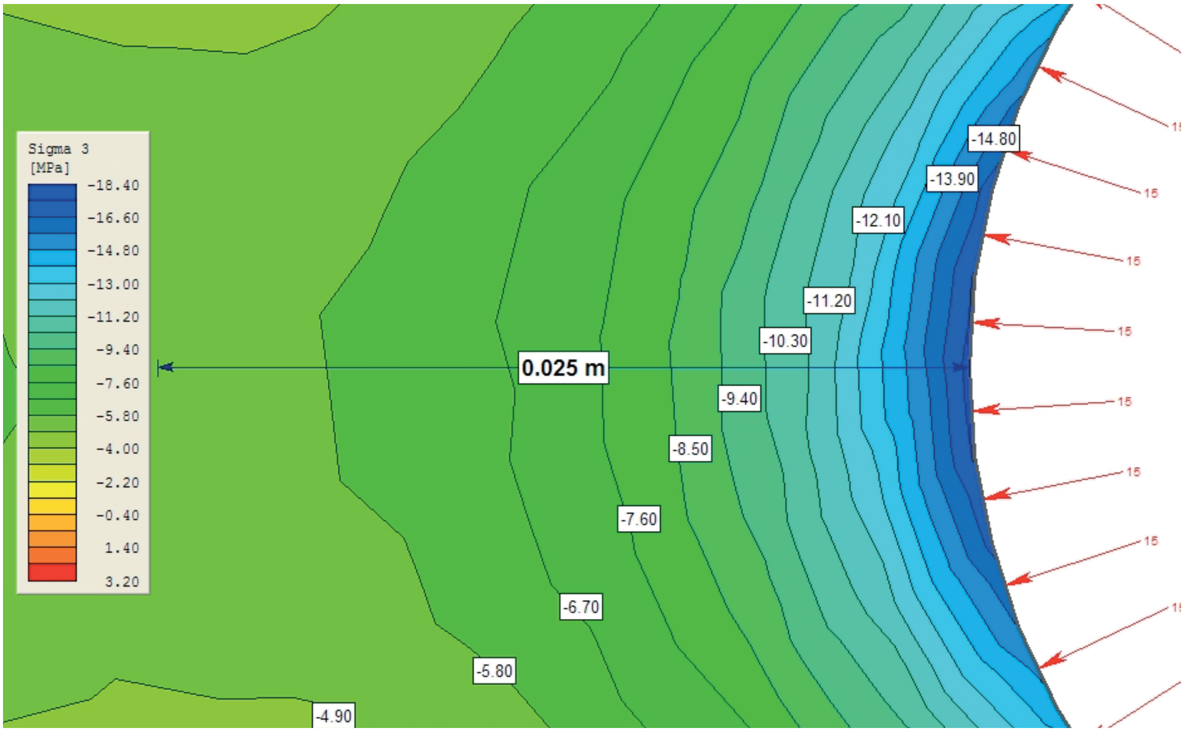

Figure 2. Trajectories of stress at nodal points in the numerical modelling.
Şekil 2. Sayısal modellemede düğüm noktalarındaki gerilme konturları.

Table 1. Applied parameters in the numerical modeling.
Çizelge 1. Sayısal modeled kullanılan parametreler.

Hole diameter (mm)	Hole spacing (mm)	Poisson's ratio	Young's modulus (GPa)	Internal pressure by NEEM (MPa)
28, 32, 38, 44	50, 80, 110, 140, 170, 200, 230, 260, 290	0.1, 0.15, 0.2, 0.25, 0.3	10, 20, 30, 40, 50	10, 20, 30, 40, 60, 80, 100

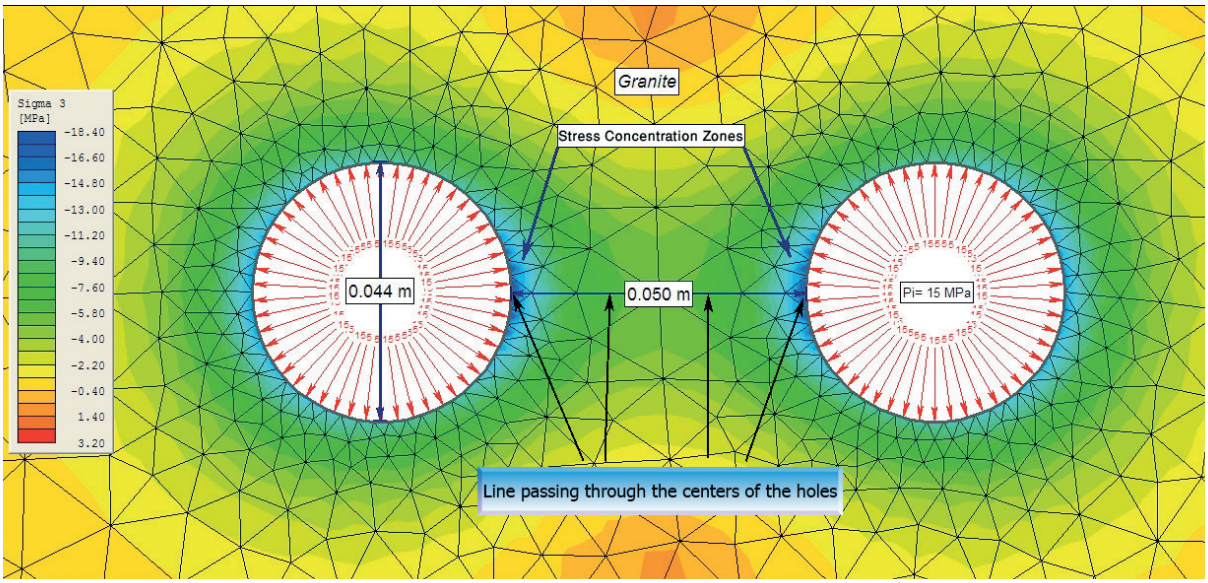

Figure 3. Stress concentration zones between two neighboring holes.
Şekil 3. Komşu iki delik arasındaki gerilme konsantrasyonu zonları.

$$C_r = 1.0352\left(\frac{d}{S}\right)^{0.001} \upsilon^{0.015} \qquad (11)$$

$$C_\theta = 1.1715\left(\frac{d}{S}\right)^{0.124} \upsilon^{-0.025} \qquad (12)$$

Where υ is Poisson's ratio of rock, *d* is the diameter of holes and *S* is the edge-to-edge distance between two neighboring holes (hole spacing). Figures 4 and 5 show a comparison between finite element (FE) data and primary models from Equations 9 and 10 (Primary Models) for the radial and tangential stresses versus distance from hole centre.

MODIFICATION OF THE PRIMARY MODELS

With reference to Figures 4 and 5, it can be observed that stresses determined from FE data and primary models are not quite the same, the-

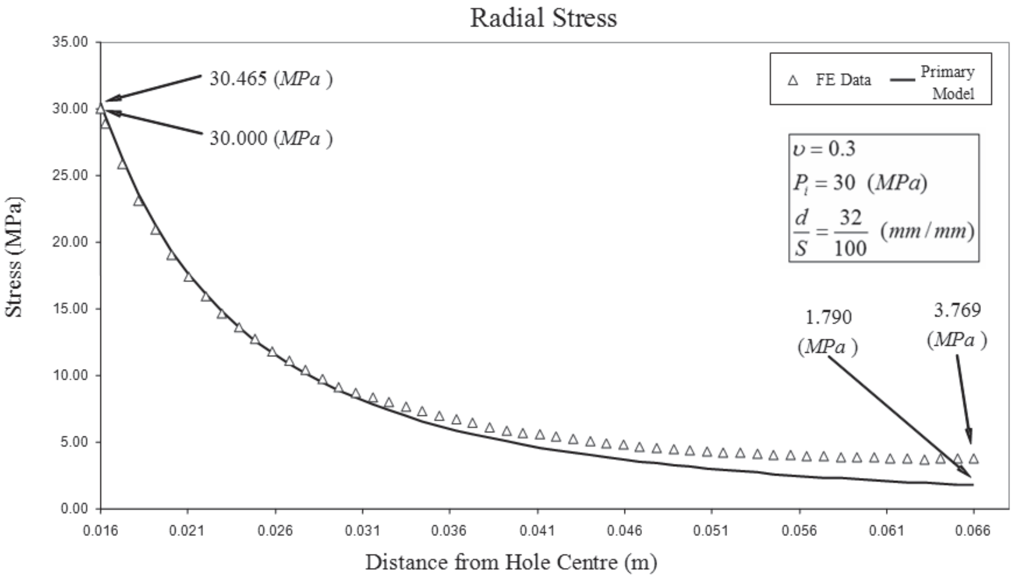

Figure 4. Radial stress distribution at the vicinity of a circular hole from FE data and primary model.
Şekil 4. SE verisi ve ilk modele göre dairesel deliğin çevresindeki radyal gerilme dağılımı.

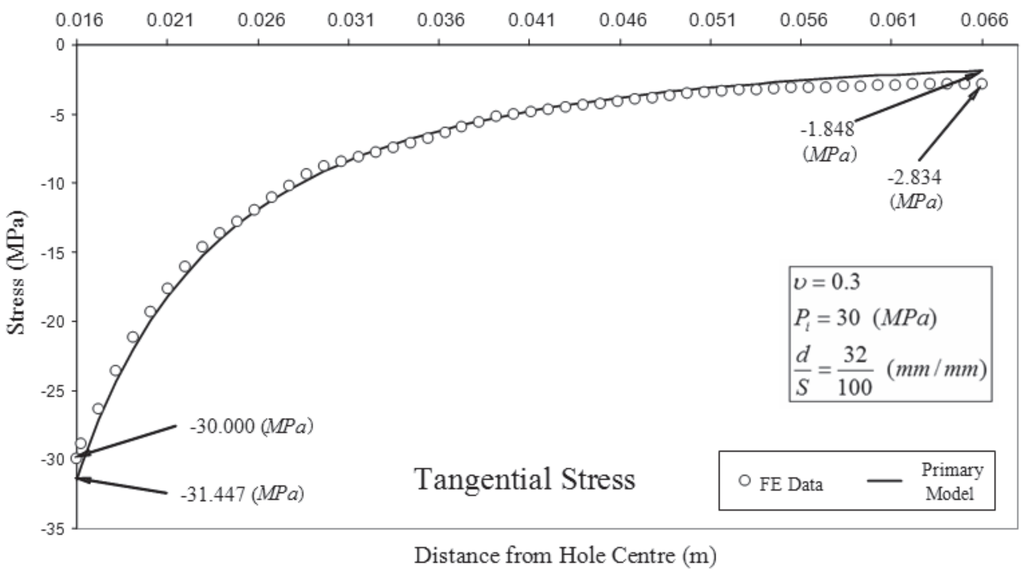

Figure 5. Tangential stress distribution at the vicinity of a circular hole from FE data and primary model.
Şekil 5. SE verisi ve ilk modele göre dairesel deliğin çevresindeki teğetsel gerilme dağılımı.

re are some differences between those. Hence, modifications have to be applied to the primary models so that a closer agreement can be achieved. With that regard, the values of differential stresses (the difference in FE data and primary models) were plotted against the distance from hole centre for both of the radial and tangential stresses. The polynomial regression analysis was then utilized as a modification function and applied to the results given in Figures 6 and 7. The modified models achieved from this analysis are shown in Equations 13 and 15. The corresponding modified functions are also demonstrated by Equations 14 and 16.

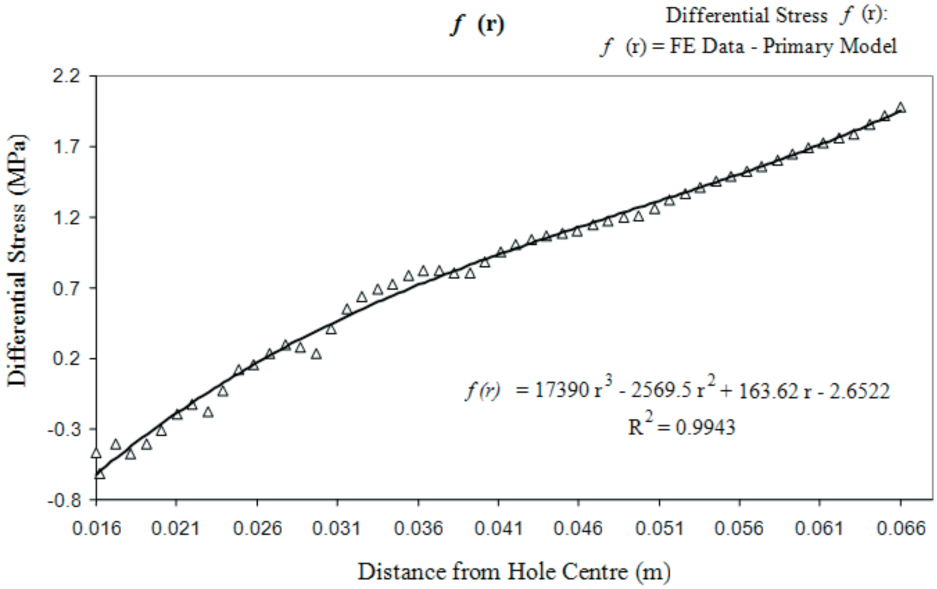

Figure 6. Differential stress (radial stress) against the distance from hole centre.
Şekil 6. Delik merkezinden olan uzaklığa karşı radyal gerilme.

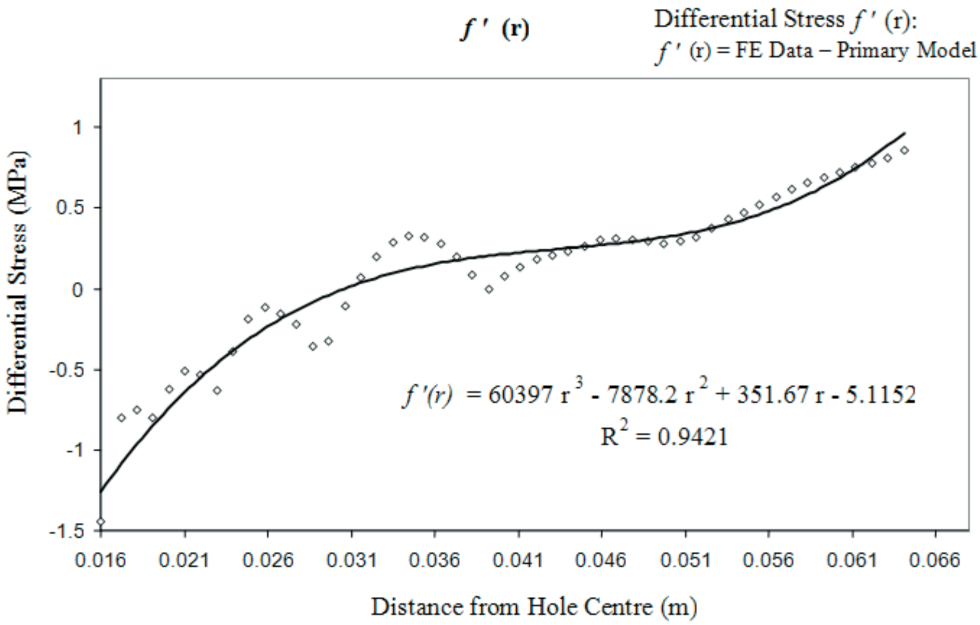

Figure 7. Differential stress (tangential stress) against the distance from hole centre.
Şekil 7. Delik merkezinden olan uzaklığa karşı teğetsel gerilme.

$$\sigma_r = C_r . P_i \left(\frac{d}{2r} \right)^2 + f(r) \qquad (13)$$

$$f(r) = 17390r^3 - 2569.5r^2 + 163.62r - 2.652 \quad (r^2 = 0.9943) \qquad (14)$$

$$\sigma_\theta = -[C_\theta . P_i \left(\frac{d}{2r} \right)^2 + f'(r)] \qquad (15)$$

$$f'(r) = 60397r^3 - 7878.2r^2 + 351.67r - 5.115 \quad (r^2 = 0.9421) \qquad (16)$$

A comparison was then made between the FE data, primary models (Equations 9 and 10) and modified models (Equations 13 and 15) in Figures 8 and 9 by plotting the corresponding radial and tangential stresses against the distance from hole centre. As anticipated, the values ob-

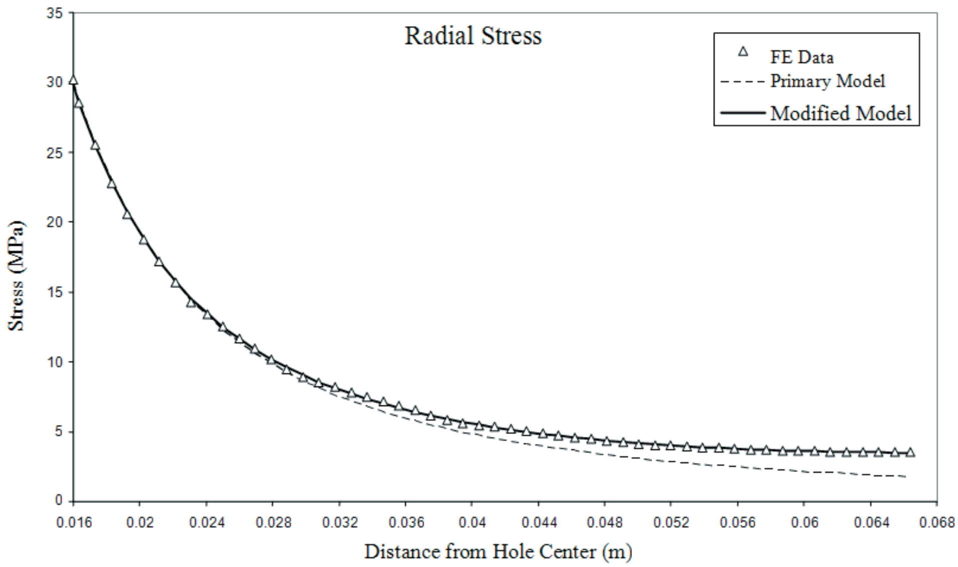

Figure 8. Modified model and FE data for radial stress distribution.
Şekil 8. Radyal gerilme dağılımı için modifiye edilmiş model ve SE *verisi.*

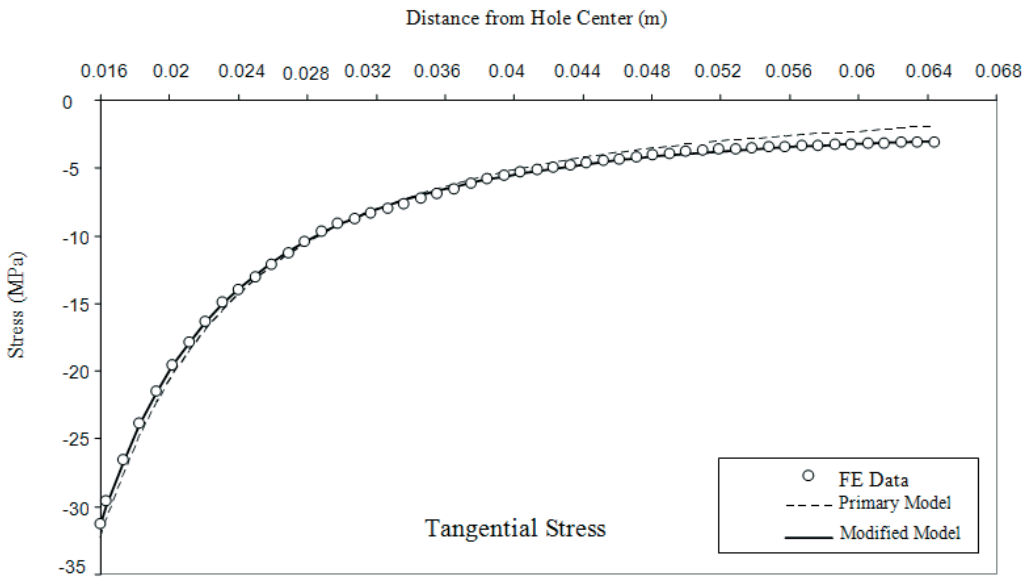

Figure 9. Modified model and FE data for tangential stress distribution.
Şekil 9. Teğetsel gerilme dağılımı için modifiye edilmiş model ve SE *verisi.*

tained from FE data and modified models are almost identical; apparently the values of the primary models show some deviation.

Therefore, the final equations for determining stress distribution including stress concentration factors between two neighboring circular holes due to internal pressure of the NEEM are as follow. Validation of the equations is only for the points located on the line passing through the centers of the holes.

$$\sigma_r = 1.0352\left(\frac{d}{S}\right)^{0.001} \upsilon^{0.015}.P_i\left(\frac{d}{2r}\right)^2 + [17390r^3 - 2569.5r^2 + 163.62r - 2.6522] \quad (17)$$

$$\sigma_\theta = -[1.1715\left(\frac{d}{S}\right)^{0.124} \upsilon^{-0.025}.P_i\left(\frac{d}{2r}\right)^2 + (60397r^3 - 7878.2r^2 + 351.67r - 5.1152)] \quad (18)$$

Where d, r and S are in meters and P_i, σ_r and σ_θ are in MPa. Finally, verification was done by plotting stresses from modified models versus FE data. Figures 10 and 11 show these validation graphs. It is observed that there is a remarkable agreement between the results.

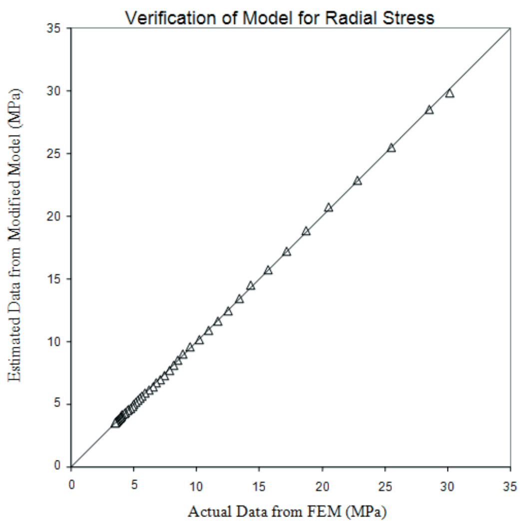

Figure 10. Actual data and estimated data for the radial stress distribution.
Şekil 10. Radyal gerilme dağılımı için belirlenmiş ve tahmin edilmiş veri.

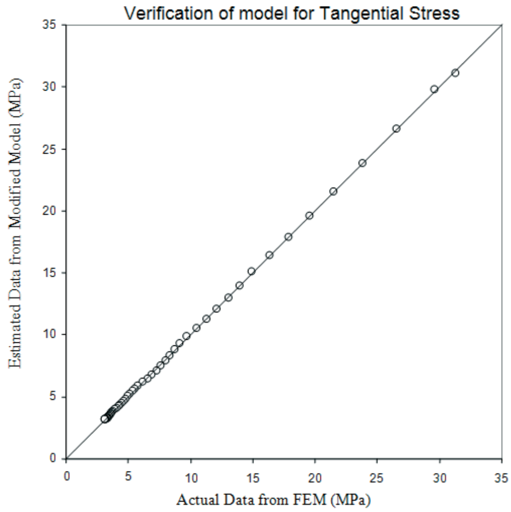

Figure 11. Actual data and estimated data for the tangential stress distribution.
Şekil 11. Teğetsel gerilme dağılımı için belirlenmiş ve tahmin edilmiş veri.

CONCLUSIONS

Based on the solution for thick-walled cylinders and assuming that the wall thickness increases to infinity, an equation for determining stress around a single circular hole in a rock plate maybe obtained. In this study, this equation has been modified to determine the stress concentration between two neighboring circular holes by introducing a coefficient in the equation. This stress concentration coefficient was estimated by numerical modeling based on model geometry and Poisson's ratio, resulting in two equations that are obtained by multiple regression analysis. Due to differences in the stresses determined from FE data and the models, appropriate modifications using polynomial regression analysis were applied in order to achieve a closer agreement between the results. Therefore, the relations obtained can be used confidently to determine stress distribution between two neighboring circular holes internally pressurized by the NEEM in a hard and brittle rock medium such as granite. Validation of the equations is only for the points located on the line passing through the centers of the holes in elastic state.

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