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New notions of triple sequences on ideal spaces in metric spaces

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Abstract

In this paper, the concepts of I_3-localized and I_3^*-localized sequences in metric spaces are introduced. Furthermore, some properties related to the I_3-localized and I_3-Cauchy sequences are proved. Otherwise, the notions of uniformly I_3-localized sequences in metric spaces are defined.

Keywords: I_3-localized sequences I_3^*-localized sequence I_3-Cauchy sequences uniformly I_3-localized sequences.
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1. Introduction and preliminaries

The concept of I-convergence of real sequences was defined by Kostyrko et al. [2] as a generalization of statistical convergence, which is based on the structure of the ideal I of subsets of the set of natural numbers.

Otherwise, the notion of localized sequence is defined in [4] as a generalization of a Cauchy sequence in metric spaces. Besides, by using the properties of localized sequences and the locator of a sequence, some results taking into account closure operators in metric spaces were obtained in [4]. If X is a metric space with a metric $d(\cdot,\cdot)$ and (x_n) be a sequence of points in X, we can call the sequence (x_n) to be localized in some subset $M \subset X$ if the number sequence $\alpha_n = d(x_n, x)$ converges for all $x \in X$. The maximal subset on which (x_n) is a localized sequence is said to be the locator of (x_n). Besides, if (x_n) is localized on X, then it becomes localized everywhere. If the locator of a sequence (x_n) contains all elements of this sequence, except for a finite number of elements, then (x_n) is said to be localized in itself. It is important to recall that every Cauchy sequence in X is localized everywhere. In addition, if $B : X \to X$ is a function with the

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condition $d(B_x, B_y) \leq d(x, y)$ for all $x, y \in X$, thus for every $x \in X$ the sequence $(B^n x)$ is localized at every fixed point of the function B. This means that fixed points of the function B are contained in the locator of the sequence $(B^n x)$.

On the other hand, the notion of statistical convergence was introduced for triple sequences by Sahiner and B. C. Tripathy [7]. Recently some works on I-convergence of triple sequences have been studied (see [8], [6], [9]). Additionally, the study of sequences on ideal spaces has reached a big importance on different fields of mathematics in the last decade (see [11], [12], [13], [14]). Besides, some authors have extended these notions on multiple sequences in probabilistic normed spaces [15] and fuzzy real valued [16]. Motivated by the mentioned above, Nabiev et al. [5] introduced the notion of I-localized sequences in metric spaces and they obtained some interesting results.

In this paper, the main idea is to generalize the concept of I-localized sequence by using the notions of ideal I of subset of the set $\mathbb{N}$ ($\mathbb{N}$ denotes the set of natural number) of positive integers and triple sequences I_3-convergent.

Definition 1.1. *([2, 3]) Let X be a non-empty set, the family $I \subset 2^X$ is said to be an ideal if satisfies that if $A \subset I$ and $B \subset A$, then $B \in I$, besides if $A, B \in I$, then $A \cup B \in I$. Additionally, a non-empty family of subsets of $F \subset 2^X$ is a filter on X if satisfies that $\emptyset \in F$, if $A, B \in F$, then $A \cap B \in F$, moreover if $A \in F$ and $A \subset B$, then $B \in F$. In addition, an ideal I is called non-trivial if $I \neq \emptyset$ and $X \notin I$. Thus, $I \subset 2^X$ is a non-trivial ideal if and only if $F = F(I) = \{X - A : A \in I\}$ is a filter on X. Finally, a non-trivial ideal I is said to be admissible if $\{\{x\} : x \in X\} \subset I$.*

Remark 1.2. *Throughout this paper, I_3 is an admissible ideal on $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$.*

Definition 1.3. *([7]) A triple sequence $x = (x_{nmj})$ of elements of X is said to be I_3-convergent to $L \in X$ if for every $\varepsilon > 0$, $\{(n, m) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : d(x_{nmj}, L) \geq \varepsilon\} \in I_3$, we write $I_3\text{-}\lim_{n,m,j \to \infty} x_{nmj} = L$.*

Definition 1.4. *([7]) A triple sequence $x = (x_{nmj})$ of elements of X is said to be I_3-Cauchy sequence if for every $\varepsilon > 0$ there exists $n_0 = n_0(\varepsilon), m_0 = m_0(\varepsilon), j = j_0 \in \mathbb{N}$ such that $\{(n, m, j) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : d(x_{nmj}, x_{n_0 m_0 j_0}) \geq \varepsilon\} \in I_3$.*

Definition 1.5. *([7]) A triple sequence $x = (x_{nmj})$ of elements of X is said to be I_3^*-convergent to $L \in X$ if there exists $M \in F(I_3)$, i.e. $R = \mathbb{N} \times \mathbb{N} \times \mathbb{N} - M \in I_3$ such that $\lim_{k,i,l \to \infty} d(x_{n_k m_i j_l}, L) = 0$ and $M = \{n_1 < ... < n_k; m_1 < ... < m_i; j < ... < j_l\} \subset \mathbb{N} \times \mathbb{N} \times \mathbb{N}$.*

Definition 1.6. *([7]) A triple sequence $x = (x_{nmj})$ of elements of X is said to be I_3^*-Cauchy sequence if there exits a set $M = \{n_1 < ... < n_k; n_1 < ... < n_p; m_1 < ... < m_i; m_1 < ... < m_j; w_1 < ... < w_o; t_1 < ... < t_a\}$ such that $\lim_{k,i,p,j,o,a \to \infty} d(x_{n_k m_i w_o}, x_{n_p m_j t_a}) = 0$.*

We can see that I_3^*-convergent and I_3^*-Cauchy sequences imply I_3-convergent and I_3-Cauchy sequences, respectively. Moreover, if I is an ideal with property (AP3) (see [7]), then I and I^*-convergence coincide, as well as, I_3-Cauchy and I_3^*-Cauchy sequences coincide.

2. I_3 and I_3^*-localized sequences

In this section, we introduced the notions of I_3-localized and I_3^*-localized sequences. Besides, some of their properties and characterizations are proved. Moreover, throughout this paper, (X, d) denotes a metric space and I_3 is a non-trivial ideal of $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$.

Definition 2.1. *A triple sequence (x_{nmj}) of elements of X is said to be I_3-localized in the subset $M \subset X$ if for each $L \in M$, $I_3\text{-}\lim_{n,m,j \to \infty} d(x_{nmj}, L)$ exits, i.e. the triple number sequence $\alpha_{nmj} = d(x_{nmj}, L)$ is I_3-convergent.*

Remark 2.2. *The maximal set on which a sequence (x_{nmj}) is I_3-localized we will call the I_3-locator of (x_{nmj}) and we will denote this set as $loc_{I_3}(x_{nmj})$.*

Definition 2.3. *A triple sequence (x_{nmj}) is said to be I_3-localized everywhere if I_3-locator of (x_{nmj}) coincides with X, i.e. $loc_{I_3}(x_{nmj}) = X$*

Definition 2.4. *A triple sequence (x_{nm}) is said to be I_3-localized in itself if $\{(n,m,j) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : x_{nmj} \notin loc_{I_3}(x_{nmj})\} \subset I_3$.*

Remark 2.5. *The definitions mentioned above imply that if (x_{nmj}) is an I_3-Cauchy sequence, then it is I_3-localized elsewhere. In fact, since $|d(x_{nmj}, L) - d(x_{n_0m_0j_0}, L)| \leq d(x_{nmj}, x_{n_0m_0j_0})$. Then, we have that $\{(n,m,j) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : |d(x_{nmj}, L) - d(x_{n_0m_0j_0}, L)| \geq \varepsilon\} \subset \{(n,m,j) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : d(x_{nmj}, x_{n_0m_0j_0}) \geq \varepsilon\}$. This indicates that the sequence is I_3-localized, indeed it is I_3-Cauchy sequence.*

Remark 2.6. *By the Definitions mentioned above we conclude that every I_3-convergent sequence is I_3-localized.*

Remark 2.7. *If I_3 is an admissible ideal, then it is an open problem if every triple localized sequence in X is I_3-localized sequence in X.*

Remark 2.8. *If X is a vector space, and (x_{nmj}), (y_{nmj}) are two I_3-localized sequences, then $(x_{nmj}y_{nmj})$, $(\frac{x_{nmj}}{y_{nmj}})$, where $y_{nmj} \neq \emptyset$ and $(x_{nmj} + y_{nmj})$ are I_3-localized sequences.*

Definition 2.9. *A triple sequence (x_{nmj}) is said to be I_3^*-localized in a metric space X if the sequence $d(x_{nmj}, L)$ is I_3^*-convergent for each $L \in X$.*

Remark 2.10. *From the above Definitions, it follows that every I_3^*-convergent or I_3^*-Cauchy sequence in a metric space X is I_3^*-localized.*

Now, we show some results which were obtained taking into account the previous notions.

Lemma 2.11. *Let I_3 be an admissible ideal on $\mathbb{N}\times\mathbb{N}\times\mathbb{N}$ and X be a metric space. If a triple sequence $(x_{nmj}) \subset X$ is I_3^*-localized in the set $M \subset X$. Then, (x_{nmj}) is I_3-localized in the set M and $loc_{I_3^*}(x_{nmj}) \subset loc_{I_3}(x_{nmj})$.*

Proof. Let (x_{nmj}) be I_3^*-localized in M. Then, there exists a set $R \in I_3$ such that for $R^c = \mathbb{N}\times\mathbb{N}\times\mathbb{N} - R = \{k_1 < ...k_i; k_1 < ... < k_l; k_1 < ... < k_p\}$, we have that $\lim_{i,l,p\to\infty} d(x_{ilp}, L)$, for each $L \in M$. Then, the sequence $d(x_{nmj}, L)$ is an I_3^*-Cauchy sequence, this implies that $d(x_{nmj}, L)$ is an I_3-Cauchy sequence. Therefore, the triple number sequence $d(x_{nmj}, L)$ is I_3-convergent. This means that (x_{nmj}) is I_3-localized in the set M. $\square$

Remark 2.12. *By Lemma 2.11, we prove that $loc_{I_3^*}(x_{nm}) \subset loc_{I_3}(x_{nm})$, but under which conditions the equality is satisfied. This is an open problem.*

Proposition 2.13. *Every I_3-localized sequence is I_3-bounded.*

Proof. Let (x_{nmj}) be I_3-localized. Then, the triple number sequence $d(x_{nmj}, L)$ is I_3-convergent for some $L \in X$. This implies that $\{(n,m,j) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : d(x_{nmj}, L) > U\} \in I_3$ for some $U > 0$. In consequence, the triple sequence (x_{nmj}) is I_3-bounded. $\square$

Theorem 2.14. *Let I_3 be an admissible ideal with the (AP3) property and $P = loc_{I_3}(x_{nmj})$. Besides, a point $L_1 \in X$ be such that for any $\epsilon > 0$ there exits $L \in P$ which satisfies*

$$\{(n,m,j) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : |d(L, x_{nmj}) - d(L_1, x_{nmj})| \geq \varepsilon\} \in I_3 \tag{1}$$

Then, $L_1 \in P$.

Proof. It will be sufficient if we show that the triple number sequence $\alpha_{nmj} = d(x_{nmj}, L_1)$ is an I_3-Cauchy sequence. Now, let $\varepsilon > 0$ and $L \in P = loc_{I_3}(x_{nmj})$ is a point with the property (1). By the (AP3) property of I_3, we have

$$|d(L, x_{k_n k_m k_j}) - d(L_1, x_{k_n k_m k_j})| \to 0 \text{ as } n, m, j \to \infty,$$

and

$$|d(x_{k_n k_m k_j}, L) - d(x_{k_s k_t k_r}, L) \to 0 \text{ as } n, m, j, s, t, r \to \infty$$

where $R = \{k_1 < ... < k_n < ...; k_1 < .... < k_m < ....; k_1 < ... < k_r\} \in F(I_3)$. Thus, for any $\varepsilon > 0$ there is $n_0, m_0, j_0 \in \mathbb{N}$ such that

$$|d(L, x_{k_n k_m k_j}) - d(L_1, x_{k_n k_m k_j})| < \frac{\varepsilon}{3} \tag{2}$$

and

$$|d(L, x_{k_n k_m k_j}) - d(L, x_{k_s k_t k_r})| < \frac{\varepsilon}{3} \tag{3}$$

for all $n \geq n_0$, $m \geq m_0$, $j \geq j_0$,$s \geq n_0$, $t \geq m_0$ and $r \geq j_0$. Now, combing (2) and (3) with the following estimation

$$|d(L_1, x_{k_n k_m k_j}) - d(L_1, x_{k_s k_t k_r})|$$
$$\leq |d(L_1, x_{k_n k_m k_j}) - d(L, x_{k_n k_m k_j})| + |d(L, x_{k_n k_m k_j}) - d(L, x_{k_s k_t k_r})| + |d(L, x_{k_s k_t k_r}) - d(L_1, x_{k_s k_t k_r})|$$

We have that $|d(L_1, x_{k_n k_m k_j}) - d(L_1, x_{k_s k_t k_r})| < \epsilon$, for all $n \geq n_0$, $m \geq m_0$, $j \geq j_0$,$s \geq n_0$, $t \geq m_0$ and $r \geq j_0$, which give

$$|d(L_1, x_{k_n k_m k_j}) - d(L_1, x_{k_s k_t k_r})| \to 0 \text{ as } n, m, j, s, t, r \to \infty$$

for $R = (k_{nmj}) \subset \mathbb{N} \times \mathbb{N} \times \mathbb{N}$ and $R \in F(I_3)$. This implies that $d(x_{nmj}, L_1)$ is an I_3-Cauchy. □

Definition 2.15. *If X is a metric space. Then,*

1. *The point L_1 is an I_3-limit point of the tripe sequence $(x_{nmj}) \in X$ if there exists a set $R = \{k_1 < ...k_i; w_1 < ...w_i; r_1 < ... < r_i\} \subset \mathbb{N} \times \mathbb{N} \times \mathbb{N}$ such that $R \notin I_3$ and $\lim_{k,w,r\to\infty} x_{n_k m_w j_r} = L_1$.*
2. *A point L_1 is said to be an I_3-cluster point of the triple sequence (x_{nmj}) if for each $\varepsilon > 0$, $\{(n, m, j) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : d(x_{nmj}, L_1) < \varepsilon\} \notin I_3$. Additionally, if $R = \{k_1 < ...; w_1 < ...; r_1 < ...\} \in I_3$, the triple subsequence $(x_{k_n w_m r_j})$ of the sequence (x_{nmj}) is called I_3-thin subsequence of the triple sequence (x_{nmj}). Besides, if $M = \{s_1 < ...; t_1 < ....; u_1 < ...\} \notin I_3$, the triple sequence $x_M = (x_{stu})$ is called I_3-nonthin triple subsequence of (x_{nmj}).*

Proposition 2.16. *All I_3-limit points (I_3-cluster points) of the I_3-localized triple sequence (x_{nmj}) have the same distance from each point L of the locator $loc_{I_3}(x_{nmj})$.*

Proof. If L_1 and L_2 are two I_3-limit points of the triple sequence (x_{nmj}). Then, the triple numbers $d(L_1, L)$ and $d(L_2, L)$ are I_3-limit points of the I_3-convergent sequence $d(L, x_{nmj})$. Therefore, $d(L_1, L) = d(L_2, L)$.
With I_3-cluster point is proved similarly. □

Proposition 2.17. *$loc_{I_3}(x_{nmj})$ does not contain more than I_3-limit (I_3-cluster) point of the triple sequence (x_{nmj}).*

Proof. If $L, L_1 \in loc_{I_3}(x_{nmj})$ are two I_3-limit points of the triple sequence (x_{nmj}), then by the Proposition 2.16, $d(L, L) = d(L, L_1)$. But, $d(L, L) = 0$. This implies that $d(L, L_1) = 0$ for $L \neq L_1$ and this is a contradiction.
With I_3-cluster point is proved similarly by using Proposition 2.16. □

Proposition 2.18. *If the tripe sequence (x_{nmj}) has an I_3-limit point $L_1 \in loc_{I_3}(x_{nmj})$. Then, I_3-$\lim_{n,m,j\to\infty} x_{nmj} = L_1$.*

Proof. The triple sequence $(d(x_{nmj}, L_1))$ is I_3-convergent and some I_3-nonthin subsequence of this triple sequence converges to zero. Then, (x_{nmj}) is I_3-convergent to L_1. □

Definition 2.19. *For the given I_3-localized triple sequence (x_{nmj}), with the I_3-locator $P = loc_{I_3}(x_{nmj})$, the number*

$$\sigma_3 = \inf_{L\in P}(I_3\text{-}\lim_{n,m,j\to\infty} d(L, x_{nmj}))$$

is called the I_3-barrier of (x_{nmj}).

Theorem 2.20. *Let $I \subset 2^{\mathbb{N}\times\mathbb{N}\times\mathbb{N}}$ is an ideal with the property (AP3). Then, and I_3-localized triple sequence is I_3-Cauchy if and only if $\sigma_3 = 0$.*

Proof. Let (x_{nmj}) be an I_3-Cauchy triple sequence in a metric space X. Then, there is a set $R = \{k_1 < .. < k_n; k_1 < ...k_m; k_1 < ... < k_j\} \subset \mathbb{N}\times\mathbb{N}\times\mathbb{N}$ such that $R \in F(I_3)$ and $\lim_{n,m,j,s,t,r\to\infty} d(x_{k_nk_mk_j}, x_{k_sk_tk_r}) = 0$. In consequence, for each $\varepsilon > 0$ there exists $n_0, m_0, j_0 \in \mathbb{N}$ such that

$$d(x_{k_nk_mk_j}, x_{k_{n_0}k_{m_0}k_{j_0}}) < \varepsilon \text{ for all } n \geq n_0,\ m \geq m_0 \text{ and } j \geq j_0.$$

Since (x_{nmj}) is an I_3-localized triple sequence, I_3-$\lim_{n,m,j\to\infty} d(x_{k_nk_mk_j}, x_{k_{n_0}k_{m_0}k_{j_0}})$ exist and we have that I_3-$\lim_{n,m,j\to\infty} d(x_{k_nk_mk_j}, x_{k_{n_0}k_{m_0}k_{j_0}}) \leq \varepsilon$. Therefore, $\sigma_3 \leq \varepsilon$, this is due to $\varepsilon > 0$, then we have that $\sigma_3 = 0$.

Now, let's prove the converse by taking $\sigma_3 = 0$. Then, for each $\varepsilon > 0$ there is a $L \in loc_{I_3}(x_{nmj})$ such that $d(L) = I_3$-$\lim_{n,m,j\to\infty} d(L, x_{nmj}) < \frac{\varepsilon}{2}$. In this case

$$\{(n,m,j) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : |d(L) - d(L, x_{nmj})| \geq \frac{\epsilon}{2} - d(L)\} \in I_3$$

This implies that $\{(n,m,j) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : d(L, x_{nmj}) \geq \frac{\varepsilon}{2}\} \in I_3$. Therefore, I_3-$\lim_{n,m,j\to\infty} d(L, x_{nmj}) = 0$, this means that (x_{nmj}) is an I_3-Cauchy triple sequence. □

Theorem 2.21. *If the triple sequence (x_{nmj}) is I_3-localized in itself and (x_{nmj}) contains an I_3-nonthin Cauchy subsequence, then (x_{nmj}) will be an I_3-Cauchy triple sequence itself.*

Proof. Let (y_{nmj}) be an I_3-nonthin Cauchy subsequence. It might be assumed that all members of (y_{nmj}) belong to the $loc_{I_3}(x_{nmj})$. Since (y_{nmj}) is a triple Cauchy sequence, by the Theorem 2.20, $\inf_{y_{nmj}} \lim_{s,t,r\to\infty} d(y_{str}, y_{nmj}) = 0$. Otherwise, since (x_{nmj}) is I_3-localized in itself, then

$$I_3\text{-}\lim_{s,t,r\to\infty} d(x_{str}, y_{nmj}) = I_3\text{-}\lim_{s,t,r\to\infty} d(y_{str}, y_{nmj}) = 0$$

Therefore, the I_3-barrier of (x_{nmj}) is equal to zero. Then, we have that (x_{nmj}) is and I_3-Cauchy triple sequence. □

Definition 2.22. *A tripe sequence (x_{nmj}) in a metric space X is said to be uniformly I_3-localized on a subset $M \subset X$ if the triple sequence $(d(L, x_{nmj}))$ is uniformly I_3-convergent for all $L \in M$.*

Lemma 2.23. *Let (x_{nmj}) be a triple sequence uniformly I_3-localized on the set $M \subset X$ and $L_1 \in Y$ is such that for every $\varepsilon > 0$ there is $L_2 \in M$ for which*

$$\{(n,m,j) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : |d(L_1, x_{nmj}) - d(L_2, x_{nmj})| \geq \varepsilon\} \in I_3$$

is satisfied. Then, $L_1 \in loc_{I_3}(x_{nmj})$ and (x_{nmj}) are uniformly I_3-localized on the set of such points L_1.

Proof. The prove of this Lemma is analogously to Theorem 2.14. □

3. Conclusion

The main idea of this paper was to extend the notion of I-localized sequences in triple sequences. As we could see, we got some interesting properties and results. Nevertheless, there are some open problems (see Remarks 2.7 and 2.12) which would be interesting whether we study them for future work. Moreover, for future work, it would also be interesting whether we make a deeper study of the uniformly I_3-localized triple sequence.

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