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Application of r-vine copula method in Istanbul stock market data: A case study for the construction sector

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Abstract

In the stock market, the relationship between the sectorial changes can be very informative in order to predict the changes in prices of assets from each sector. In order to understand these sectorial relations, various studies have been conducted. In one of the recent studies, the construction sector in Turkey was investigated in terms of its effect in other Turkish sectors since it is one of the leading sectors in Turkey and its assets have a significant impact in stock markets. Hereby, in this study we detect the sectorial relationship of the construction sector via the Regular vine, also known as R-Vine, approach. The R-Vine copula is a specific type of copula which enables us to be flexible in the distributional assumptions of the observations while stating their joint distribution function. By this way, we can investigate the structure of the country's financial path, i.e., network, under a graphical model.

1. Introduction

The detailed analyses of the stock market data have been increasingly noticed since such analyses help us to better understand the customer's demand and priorities and make profits. Indeed, the analyses of this type of time series data are challenging because the data have high dimensions and highly nonlinear structure. In order to represent these complex structures, various methods have been suggested in the literature Some of these methods are based on advance computational approach such as neural networks, machine learning methods (Guresen, et al. 2011; Kara, et al. 2011; Kimoto, et al. 1990; Mizuno, et al. 1998; Qian, et al. 2007 and Yand et al. 2002) and data mining techniques (Enke and Thawornwon, 2005; Ou and Wang, 2009 and Yang, et al. 2002). These methods are distribution-free but there are some approaches such as granger causality test (Ramachandra, et al. 2014) and the co-integration analyses which depend on the transformed version of random variables (Ogunbiyi, et al. 2017) as well as some parametric mathematical methods which are based on some constraints about the interrelationship of the variables based on their probability distributions like independent predictors and normality assumption (Dobra and Lenkoski, 2009). There are also some linear and nonlinear modeling approaches which are specifically designed for the time series data such as autoregressive models (Abegaz and Wit, 2013), moving average models and volatility models (Engle and Patton, 2007). In terms of vine copula, there are several studies in economics that only some of them are going to be mentioned here such as the study of Patton (2012) which is a review of copula models for time series economic data and Sriboonchitta, et al. (2014) in which the D-Vine copula is used in the inference of pairwise and conditional dependence between variables in three different countries. In another study, the vine copula approach is used to perform a new production function in economic data (Constantino, et al. 2019). Furthermore, in the study of Song, et al. (2019), two multivariate copulas (C and D-Vine copulas versus factor copula) are compared by some economic data apart from a comparison between three groups of data.

In this study, we apply a combination of these approaches in order to better understand the structure of a particular sectorial data in Turkey based on R-Vine copula. Herein, we are interested in the detection of the plausible path between the construction sector and other sectors by using the Istanbul Stock Market (ISM) data. Because to the best of our knowledge there is no such analysis for the ISM data. For this purpose, we use all the sectorial data given in ISM and detect the most related sectors with the construction sector via data-mining techniques and then, we generate different dimensional clusters. Later, we apply these results for constructing parametric and nonlinear models via copula method. This modeling approach is used frequently for its flexibility to capture the nonlinear relationship between random variables, i.e., between predictors and response variable under distinct distributions of observations. In the analysis via data mining, we use the findings of Erkuş and Purutçuoğlu (2019) which perform distinct clustering approaches for totally 23 sectors in the ISM data. In this work, due to the consensus of the outcomes in clustering, we merely take the results of the k-means algorithm. We apply the R-vine copula approach in whole sectoral data without clustering in order to have a general view. By this way, we can present a country's financial path among sectors and can interpret the effects of the sectoral changes in a complex nonlinear model by using the findings of the k-means algorithm. In these analyses, we prefer the R-Vine copula method since it is able to describe the high dimensional joint distribution of the sectors via a connectivity term, called pair copula terms. Moreover, we construct some alternative models by using the finding of the k-means algorithm. By those newmodels, we aim to describe the sectorial relationships specifically with the construction sectors in small copula models. In the selection of the connectivity term, the R-Vine copula can assume different structures while presenting the relationship in every pair of sectors. Therefore, it can be adapted in distinct datasets. Furthermore, as it can divide the joint distributions into pair of marginals, it can simplify the parameter estimation of the complex regression models.

On the other hand, due to the large number of parameters and high correlations between sectors, the parameter estimation in such a complex model can be done via Bayesian methods (Dobra and Lenkoski, 2009). Although the Bayesian algorithm can accurately estimate the parameters, they are computationally demanding Whereas in this study, we infer our model parameters via the maximum likelihood method, rather than any Bayesian method, since it is as accurate as Bayesian approach when the number of observations is large and also computationally efficient.

Hereby, in the following part, we initially explain the copula in general, then, vine copula and finally represent the R-Vine copula approach. In Application part, k-means clustering method is explained briefly and the findings of the R-Vine copula under the clusters generated by the k-means algorithm are presented. The clusters from k-means algorithm are created in such a way that the selected cluster includes the construction sector and its close neighbourhoods. Lastly, in conclusion, we give our outputs and summarize the results obtained from the R-vine copula model.

2. Materials and Methods

2.1 Copula

Copula approach (Genest and Favre, 2007) is one of the well-known methods to investigate the joint behaviour between the variables when the data structure or the joint density is problematic or complicated. The base theorem of the Copula is the Sklar's theorem (Trivedi and Zimmer, 2007). This theorem indicates that for every data, there is a unique copula which can fit the best model for the joint density of the variables (Brechmann and Schespsmeier, 2013). The theorem can be simply presented basically as follows:

Theorem: Every joint distribution function of two or more variables can be written by their marginal distributions and a copula as $F(x) = C(F_1(x_1), F_2(x_2), \dots, F_d(x_d))$ where $x = (x_1, x_2, .., x_d)$ and F is the d-dimensional distribution of the random variable x. Accordingly, the multivariate copula formula can be written as $C(u_1, u_2, .., u_d) = F(F_1^{-1}, F_2^{-1}, \dots, F_d^{-1})$ for $u_1 = F_1^{-1}, u_2 = F_2^{-1}, \dots, u_d = F_d^{-1}$ where d denotes the number of variables.

In the application, the inverse of the cumulative distribution functions is used as the copula units u_is, and then, a copula formula applies those units in order to fit the best model as their joint density function.

The copulas are divided into two main parts (Brechmann and Schespsmeier, 2013): These are Archimedean and Elliptical Copulas. The Archimedean copulas consist of the Gaussian and student-t copula with one and two parameters, in order. As the characteristic of this copula branch, they are both symmetric. Furthermore, the Gaussian copula has no tail dependence, whereas, student-t copula with ν degrees of freedom, has the tail dependence. The parameters and the properties of this copula type are summarized in Table 1. In this table, the Kendall's tau (τ) term represents the value of the correlation (ρ) between variables in a non-parametric way and the tail dependence denotes the numerical value of the acceptable tail features of the Archimedean copula.

Table 1. The list and properties of the Archimedean copulas. $T_{\upsilon+1}(.)$ denoted the student-t value under (.) given probability and $\upsilon + 1$ degrees of freedom.

Names	Parameter range	Kendall's τ	Tail dependence
Gaussian	$\rho \in (-1,1)$	$2/\pi$ arcsin (ρ)	0
Student-t	$\rho \in (-1,1), \nu > 2$	$2/\pi$ arcsin (ρ)	$2T_{\upsilon+1}(-\sqrt{\upsilon+1}\sqrt{\frac{1-\rho}{1+\rho}})$

On the other hand, the Elliptical copulas are composed of one parameter families, such as Frank, Joe, Clayton and Gumbel copulas which can be fitted to non-symmetric or one- sided tail dependent distributions and two parameter families, which are made by combining one-parameter elliptical families. Therefore, they are more flexible than the Archimedean copulas. By this way, we are able to model the two-sided tail dependent data where necessary. The properties and the list of this type of copulas are represented in Table 2.

Table 2. The list and properties of the Elliptical copulas. θ and δ are the parameters of the given copulas.

Name	Generator function	Parameter range	Tail dependence
Clayton	$\frac{1}{\theta}(t^{-\theta}-1)$	$\theta > 0$	$(2^{-\frac{1}{\theta}}, 0)$
Gumbel	$(-\log t)^{\theta}$	$\theta \geq 1$	$(0, 2-2^{\frac{1}{\theta}})$
Frank	$-\log[\frac{e^{-\theta t}-1}{e^{-\theta}-1}]$	$\theta \in \mathbb{R}\backslash\{0\}$	(0,0)
Joe	$-\log[1-(1-t)^{\theta}]$	$\theta > 1$	$(0, 2-2^{\frac{1}{\theta}})$
Clayton-Gumbel (BB1)	$(t^{-\theta}-1)^{\delta}$	$\theta > 0, \delta \geq 1$	$(2^{\frac{-1}{\theta\delta}}, 2-2^{\frac{1}{\delta}})$
Joe-Gumbel (BB6)	$(-\log[1-(1-t)^{\theta}])^{\delta}$	$\theta \geq 1, \delta \geq 1$	$(0, 2-2^{\frac{-1}{\theta\delta}})$
Joe-Clayton (BB7)	$(1-(1-t)^{\theta})^{-\delta}-1$	$\theta \geq 1, \delta > 0$	$(2^{\frac{-1}{\delta}}, 2-2^{\frac{1}{\theta}})$
Joe-Frank (BB7)	$-\log[\frac{1-(1-\delta t)^{\theta}}{1-(1-\delta)^{\theta}}]$	$\theta \geq 1, \delta \in (0,1]$	(0,0)

In this table, instead of using the copula function, unlike Gaussian or student-t distribution, we apply their generator functions. By this way, it is possible to write a high-dimensional version of the family by only a simple expression (Brechmann and Schespsmeier, 2013). Accordingly, a bivariate copula term can be written as $C(u_1, u_2) = \varphi^{[-1]}(\varphi(u_1) + \varphi(u_2))$ for the Elliptical family, where φ is the generator function and $\varphi^{[-1]}$ stands for the pseudo inverse that is equal to the inverse of the generator function only for non-negative values and zero for negative values. More mathematical details about the selection of the generator function and the elliptical copula can be found in Brechmann and Schespsmeier, (2013).
Finally, the parameter estimation in the copula model can be done by the maximum likelihood method (MLE). For one-parameter families, the parameters can be also estimated by the Kendall's tau (τ) correlation coefficient for the one-to-one relationship with the τ.

2.2 Vine Copula

It is mentioned earlier about the Sklar's theorem that for each dataset, there is an exclusive way to express the joint density. But catching the best model cannot be easy when the model has a high dimensional and complicated structure. In order to solve these computational challenges, the vine copula approach has been proposed (Brechmann and Schespsmeier, 2013). The vine copula solves these problems in a very simpler way in the sense that it can write the joint density function by conditional expressions in pair of densities without any specific assumption or omitting some terms. We can represent its application in in an example by using the following three-dimensional joint density: We can write the joint distribution as $f(x_1,x_2,x_3) = f_1(x_1)f(x_2|x_1)f(x_3|x_1,x_2)$ among all possible ways of writing as an example. Then, by using the definition for the conditional probability function, $f(x_2|x_1) = \frac{f(x_1,x_2)}{f_1(x_1)}$. On the other hand, according to the Sklar's theorem

$$f(x_1,x_1) = c_{1,2}(F_1(x_1),F_2(x_2))f_1(x_1)f_2(x_2). \quad (1)$$

Thereby, $f(x_2|x_1) = c_{1,2}\big(F_1(x_1),F_2(x_2)\big)f_2(x_2)$ and

$$f(x_3|x_1,x_2) = c_{2,3|1}(F(x_2|x_1),F(x_3|x_1))c_{1,3}(F_1(x_1),F_3(x_3))f_3(x_3). \quad (2)$$

Finally, we can define the joint function as

$$f(x_1,x_2,x_3) = c_{1,2}\big(F_1(x_1),F_2(x_2)\big) \times c_{2,3|1}\big(F(x_2|x_1),F(x_3|x_1)\big) \times c_{1,3}(F_1(x_1),F_3(x_3)) \times f_1(x_1) \times f_2(x_2) \times f_3(x_3). \quad (3)$$

From these derivations, it is obviously seen that the model can be presented in terms of a pair of joint functions (conditional or non-conditional) that can be summarized as 1-2, 1-3 and 2-3|1. Indeed, there are some other ways of writing this joint distribution such as 1-2, 2-3 and 1-3|2. Also, regarding the order of the pair of functions, some other alternative ways for expressing the joint density can be possible.

There are different kinds of copulas. The Regular vine, shortly, R-Vine copula is the general form of vine copula and due to its generality, it is flexible in the construction of the network model. The C and D-Vine copulas are the specific version of R-Vine. They are appropriate models for some data structures mostly corresponding to prior knowledge. In this study, due to its generality, we prefer the R-Vine copula for the model of the construction sector.

2.2.1 R-Vine Copula

In the construction of a model for any kind of datasets, there are various ways to write the order of variables, their combinations and the conditioned variables according to the decomposition method which we prefer to represent their joint densities. To depict the structure of the vine copula, we show an example of a 4-dimensional C and D- Vine structure in Figure 1.

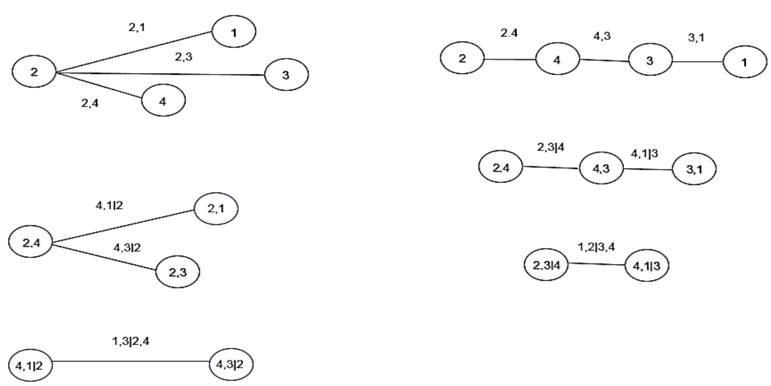

Figure 1: The C-Vine (left side with order {2,4,1,3}.) and D-Vine (right side with order {2,4,3,1}) structure for 4-variable case.

As seen in Figure 1, each row indicates a tree. In the C-Vine copula, the order of variables is determined in each tree while for the D-Vine copula, the order is determined only in the first tree and the remaining trees are written consequently. Whereas, the R-Vine can be expressed as their mixtures. For example, the first tree can be D-Vine for the first two paths and then, C-Vine for the third variable. So, one of the challenging issues in the R-Vine selection can be the order and also the structure of the networks. Because there are $\binom{d}{2}$ amounts of pairs, where d is the number of variables. Accordingly, in order to select the best model description, we need some statistical tests, alternatively in order to compare the proposal models, we need certain model selection criteria such as AIC (Akaike information criteria) or BIC (Bayesian information criteria). In this study, among many of alternatives, we perform the Voung and Clark test which can select the optimal model with the maximum score based on the sum of scores (Brechmann and Schepsmeier, 2013). This test computes two-by-two comparisons of all alternative models and then, computes a chi-square test statistic for its significance.

3. Application

In this part, we describe the data and the clustering results of Erkuş and Purutçuoğlu (2019) before presenting the application of the R-Vine copula model. In their analyses, to investigate the relationship between the construction sector and other factors, the k-means clustering algorithm is applied for the cluster number $k = 3$ and 4. In general, the clustering is an unsupervised method that separates the dataset into k clusters by considering similarities. In the calculation, the k value can be determined by different methods such as the elbow graph or within cluster sum of square scores. In the k-means algorithm, once a k value is found, the algorithm selects k random points which are not necessarily among the observations. Then, it arranges each data point to the nearest k points. Later, it updates the centroid of each cluster and again assigns each data point regarding to the new centroid. The algorithm continues this process until no change is observed in the clusters.

Description of data

The data used in this study are a weekly average time series dataset from all sectors of ISM starting from February 2002 to August 2019.

Table 3. The clusters of sectors for 3 and 4-cluster cases. (Erkuş and Purutçuoğlu, 2019).

	3-cluster case	**4-cluster case**
Cluster 1	Banking Construction Finance Investment Minerals Printing Sports Telecommunication Tourism	Chemical product Food Industry Machines Service Supermarket Technology Textile Transportation
Cluster 2	Electricity	Electricity
Cluster 3	Chemical products Food Industry Information Technology Insurance Leasing Machines Metals Service Supermarket Technology Textile Transportation	Construction Printing Sports Telecommunication Tourism
Cluster 4	-	Banking Finance Information Technology Insurance Investment Leasing Metals Minerals

It is composed of 23 sectors, as the return series. These sectors are listed as the banking, chemical products, construction, electricity, finance, food, industry, information technology, insurance, real estate investment, leasing, machines, metals, minerals, printing, service, sports, supermarkets, technology, telecommunication, textile, tourism and the transportation. In Table 3, we present the clustering results of these 23 sectors based on 3 and 4 clusters respectively, as presented in the study of Erkuş and Purutçuoğlu (2019).
In the application of the R-Vine copula, we describe the model of the construction sector by including all 23 sectors, the sector in Cluster 1 under 3-cluster case and the sectors in Cluster 3 under 4-cluster case as seen in Table 3.

3.1 R-Vine analysis by including all sectors

The data used in this part contain 23 sectors, i.e., variables. In the modeling, the significant relationships between sectors are chosen based on the Kendall's τ value. When the absolute value of Kendall's τ greater than 0.5, we find a meaningful relationship between the corresponding variables. The significant copulas between each sectorial pair with their associated Kendall's τ are also listed in Table 4. From the outputs it is seen that there are high and positive correlations between each pair apart from the relationship between banking and investment, which is negative and relatively low. However, it is found that the sectorial change in the construction sector is mostly correlated to the sectorial change of mineral and this interaction is positive.

Table 4. The list of significantly related variables based on the whole ISM data

Pair of variables	**Copula family**	**Kendall's τ**
Chemical, industry	Gumbel (180^0 rotated)	0.92
Supermarket, services	BB1	0.90
Machines, industry	Frank	0.89
Industry, supermarket	Frank	0.88
Insurance, machines	BB1	0.86
Investment, industry	BB1 (180^0 rotated)	0.866
Metals, industry	BB1 (180^0 rotated)	0.86
Banking, finance	BB7	0.82
Service, transportation	Gumbel	0.82
Insurance, industry	BB7	0.82
Textile, machines	BB7	0.80
Information, technology	BB7	0.80
Food, service	BB7	0.80
Leasing, textile	BB7 (180^0 rotated)	0.79
Finance, investment	BB7 (180^0 rotated)	0.77
Telecommunication, service	BB7	0.76
Minerals, finance	BB7	0.75
Printing, investment	BB7	0.75
Construction, minerals	BB7	0.65
Banking, investment	Gaussian	-0.68

On the other hand, we draw all the estimated interactions between sectors in Figure 2 as an undirected graph for visualization.

Figure 2: The estimated network of the ISM dataset including all sectors by the R-Vine approach

3.2 R-Vine analysis by including the sectors in Cluster 1 under 3-means clustering

This analysis is based on 9 variables, namely, banking, construction, finance, investment, mineral, printing, sports, telecommunication and tourism sector as shown in Table 3. Similar to the previous application, when we investigate the sectorial relationships between the construction and other sectors based on their correlational structures, via the R-Vine copula approach, we obtain the structure as presented in Table 5. From Table 5, it is found that there are correlational structures between the telecommunication-construction sectorial pair and the banking-investment sectorial pair. The remaining 5 variables are found uncorrelated from these sectorial changes.

Table 5. The list of significantly related variables based on Cluster 1 of 3-means clustering results of the ISM data

Pair of variables	Copula family	Kendall's τ
Telecommunication, construction	BB7 (180^0 rotated)	0.69
Banking, investment	Gaussian	-0.68

3.3 R-Vine analysis by including the Cluster 3 in 4-means clustering

Finally, from the R-Vine copula modeling of Cluster in 4-means clustering results, it is seen that based on 5 sectorial model, composed of construction, printing, sports, telecommunication and tourism sectors, the change in the construction sector is related to both telecommunication and printing sector on a positive direction. The list of the significant relations is represented in Table 6.

Table 6. The list of significantly related variables based on Cluster 3 of 4-means clustering results of the ISM data

Pair of variables	Copula family	Kendall's τ
Construction, printing	BB7 (180^0 rotated)	0.59
Construction, telecommunication	Gaussian	0.55

According to the output obtained through the R-Vine copula approach on the whole data, it is seen that the ratio of the significant relations to the total combination is $20/\binom{23}{2} = 0.079$ that contains only 1 pair with the construction sector. This means that among 20 edges, only one of them is related to the relationship of the construction sector while for the first clusters of 3-means clustered data which are done based on the relation with the construction sector, although this ratio decreases to $2/\binom{9}{2} = 0.057$. But one of the two significant relations is related to the construction sector. Furthermore, for the third cluster of the 4-means clustering that contains 5 variables including the construction sector, it is found that the number of significant relationships is 2 and the ratio is $2/\binom{5}{2} = 0.2$ that both edges describe a relationship of the construction sector with other elements in the same cluster. As a result, it is seen that this kind of clustering makes the analysis easier and it can produce an alternative way for the dimension reduction of the complicated model. In the number of variables, for instance in the data used for this study, in order to investigate the important factors for the construction section, instead of working with 23-dimensional data with only one edge related to construction, dealing with 9 or better with 5 variables can give us a better fortune to concentrate on our proposed factor. Research and publication ethics were followed in this study.

4. Conclusions

In this study, we have investigated a network architecture in order to model the Turkish sectorial path by using the Istanbul Stock Market data from 2002 to 2019, specifically, focusing on the sectorial change in the construction. In modeling, we have performed the copula approach and generated alternative forms of copula expressions. Specifically, we have chosen the mixture of the C-Vine and D-Vine copula, called the R-Vine copula, due to its flexibility in the implementation. For the analysis, we have used the k-means clustering results of Erkuş and Purutçuoğlu (2019) for the same data. From the results, we have seen that the proposed method can detect the sectorial relationship between construction and other sectors when the network model is composed of 23, 9 and, 5 sectors. The strength of their plausible interactions can also be estimated successfully via the maximum likelihood estimation method. We consider that the proposed modeling approach can be used to describe the highly connected, nonlinear relationships in finance as an alternative approach of time series models and machine learning methods.

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Conflicts of Interest

The authors declared that there is no conflict of interest.

Contribution of researchers

Dr. Hajar Farnoudkia: She did the statistical analysis and wrote the manuscript and followed the publishing process.
Prof. Dr. Vilda Purutçuoğlu: She suggested and designed the main subject of this study and also provided the data from one of her previously published proceedings. She controlled the manuscript and made lots of change and corrections. Finally, the last version was revised and verified by her.

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