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A New Method to Determine the Optimum Load of a Real Solar Cell using Special Trans Function Theory (STFT)

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Abstract- An analytical expression for optimum load for a real solar cell, having resistive and current leakage losses, has been obtained using special trans function theory (STFT) for solving the transcendental current-voltage relationship. Theoretically calculated value using STFT is compared with the experimental data as well as the values obtained by other methods. Impact of series resistance and shunt resistance on optimum load is also studied. The numerical results indicate that STFT method has the best accuracy over other available methods in literature.

Keywords- STFT, Optimum load, Solar Cell.

1. Introduction

Maximum power from a solar cell can be extracted only when it is operated with an optimum load. Even a small deviation of 1% from the optimum power transfer condition can cause a loss of output power by ~10% in solar cell panels operating under normal conditions [1,2].The effect is even more pronounced at high intensities of illumination [8].

According to maximum power transfer theorem, for a linear circuit network, the maximum power transfer occurs at an optimal load having impedance equal to the complex conjugate of the source impedance [3]. However, the current-voltage characteristic of a solar cell is non-linear and transcendental in nature. To address the problem of determining the optimum load (R_{Lmp}) of a solar cell, researchers adopted different techniques [4-10]. Loferski [4] and Wysocki and Rappaport [5] calculated the load impedance at the maximum power transfer point for an ideal solar cell where the series resistance is zero and the shunt resistance is infinite. Another method of estimating the value of R_{Lmp} from a set of typical current-voltage characteristics was presented by Rao and Padmanabhan [6] for a silicon solar cell used in terrestrial applications. A trial and error method was adopted by Govil [7] to obtain the maximum output condition for a p-n junction solar cell with a series resistance or a shunt resistance; also derived the expressions for the optimum load R_{Lmp}. Kothari et al [8] obtained the expression for optimum load of a real solar cell, having both resistive and current leakage losses (taking into account both R_s and R_{sh}) using Lagrange's method of undetermined multipliers. The study of Wyatt and Chua [9] reports a multiport maximum power theorem for nonlinear resistive networks presenting the optimal nonlinear optimal characteristics as a closed form expression involving the constitutive relation of the source. It is applied to the optimal load-matching problem for a solar cell. The method of Lambert W-function was used by Ding and Radhakrishnan [10] to get an exact explicit expression for optimum load of an illuminated solar cell containing the parasitic resistances.

Of late, Special Trans Function Theory (STFT) [11] has been used to determine the parameters of real solar cells [12, 13]. STFT, originated from the work of Perovich, can analytically examine various problems in the field of applied physics and engineering domains. In this communication, an analytical expression for optimum load of a real solar cell is obtained, based on the solution of the basic current-voltage characteristics of the cell using STFT by writing it in a form that complies with the properties of STFT. The possibility of getting different gradients in STFT makes it possible to analyze analytically any problem rigorously and is applicable for arbitrary non-linear forms [11-13], and so this analysis should hold for various types of solar cells. Comparisons are also made with the calculated results of other analytical methods available in literature.

2. Theory

The current-voltage relation of a real solar cell based on the single diode model under illumination can be written as

$$I = I_0\left(\exp\left(\frac{V-IR_s}{nV_{th}}\right)-1\right)+\frac{V-IR_s}{R_{sh}}-I_{ph} \quad (1)$$

where I and V are the current and voltage output of the cell respectively, V_{th} ($=k_BT/q$)is the thermal voltage, I_{ph} is the light generated photocurrent, I_0 is the reverse saturation current of the diode, n is the diode ideality factor, R_s and R_{sh} are the parasitic series and shunt resistances respectively. The explicit analytical expression for current in terms of V and the newly defined trans function ($trans_+(D)$) and the model's parameters using STFT is [12]

$$I = \left(\frac{\frac{V}{R_{sh}}-\left(I_{ph}+I_0\right)}{1+\frac{R_s}{R_{sh}}}\right)\left(\frac{nV_{th}\left(1+\frac{R_s}{R_{sh}}\right)}{R_s\left(I_0+I_{ph}-\frac{V}{R_{sh}}\right)}trans_+\left(D\right)-1\right) \quad (2)$$

or

$$I = \left(\frac{\left(I_{ph}+I_0\right)-\frac{V}{R_{sh}}}{1+\frac{R_s}{R_{sh}}}\right)+\frac{nV_{th}}{R_s}\left(D\left(\frac{\sum_{m=0}^{[x]}\frac{D^m\left(x-m\right)^m}{m!}}{\sum_{m=0}^{[x+1]}\frac{D^m\left(x+1-m\right)^m}{m!}}\right)\right) \quad (3)$$

The trans function is defined by ($trans_+(D)$)

$Z=(trans_+(D)$ (4)

where $trans_+(D)$ is a new special trans function [11-13] defined as

$$trans_+\left(D\right)=\lim_{x\to\infty}\left[D\left(\frac{\varphi_+\left(D,x\right)}{\varphi_+\left(D,x+1\right)}\right)\right] \quad (5)$$

and

$$\varphi_+\left(D,x\right)=\sum_{m=0}^{[x]}\frac{D^m\left(x-m\right)^m}{m!} \quad (6)$$

so that

$$trans_+\left(D\right)=\lim_{x\to\infty}\left(D\left(\frac{\sum_{m=0}^{[x]}\frac{D^m\left(x-m\right)^m}{m!}}{\sum_{m=0}^{[x+1]}\frac{D^m\left(x+1-m\right)^m}{m!}}\right)\right) \quad (7)$$

and D as given in [12]

$$D=\frac{I_0R_s\exp\left(\frac{V}{nV_{th}}\right)\exp\left(\frac{R_s\left(I_0+I_{ph}-\frac{V}{R_{sh}}\right)}{nV_{th}\left(1+\frac{R_s}{R_{sh}}\right)}\right)}{nV_{th}\left(1+\frac{R_s}{R_{sh}}\right)} \quad (8)$$

Here [x] denotes the greatest integer less than or equal to x. The number of accurate digits in the numerical structure of the parameter will depend on [x] [11-13].

The differentiability of the trans function in STFT [11] make it useful in the analytical analysis of any physical problem. That is

$$\frac{\partial Z}{\partial V}=\frac{trans_+(D)}{D\left(1+trans_+(D)\right)}\frac{\partial D}{\partial V} \quad (9)$$

Using this property of differentiability of the trans function, the gradient of I with respect to V is obtained to be

$$\left(\frac{\partial I}{\partial V}\right)=\frac{1}{R_{sh}\left(1+\frac{R_s}{R_{sh}}\right)}+\frac{1}{R_s\left(1+\frac{R_s}{R_{sh}}\right)}\frac{trans_+\left(D\right)}{\left(1+trans_+\left(D\right)\right)} \quad (10)$$

The power output is

$P= IV$

and the maximum power transfer condition can be written as

$$\left(\frac{\partial P}{\partial V}\right)_{V=V_{mp}}=0 \quad (11)$$

or

$$\left(\frac{\frac{2V}{R_{sh}}-\left(I_{ph}+I_0\right)}{1+\frac{R_s}{R_{sh}}}+\frac{nV_{th}}{R_s}trans_+\left(D\right)+\frac{V}{R_s\left(1+\frac{R_s}{R_{sh}}\right)}\frac{trans_+\left(D\right)}{\left(1+trans_+\left(D\right)\right)}\right)_{V=V_{mp}}=0 \quad (12)$$

Equation (12) is the general condition for maximum power from which V_{mp} can be calculated for given values of other parameters.

The inverse of the slope on the I-V characteristic at $V=V_{mp}$ i.e., $\left(\frac{\partial I}{\partial V}\right)_{V=V_{mp}}=\frac{1}{R_{Lmp}}$ gives the value of the optimum load for the given set of solar cell parameters.

$$\frac{1}{R_{Lmp}} = \left(\frac{1}{R_{sh}\left(1+\frac{R_s}{R_{sh}}\right)} + \frac{1}{R_s\left(1+\frac{R_s}{R_{sh}}\right)} \frac{trans_+\left(D\right)}{\left(1+trans_+\left(D\right)\right)} \right)_{V=V_{mp}} \tag{13}$$

The analytical expression for the optimum load of a real solar cell is represented by equation (13) corresponding to the maximum power point where $V=V_{mp}$.To recall V_{mp} will be obtained from equation (12) using the device model parameters.

3. Results and Discussion

To investigate the STFT method of determining the RLmp, it is applied to three different experimental solar cells (namely grey, blue and plastic solar cells) *I-V* characteristics based on the single diode model of a real solar cell (Table 1).

The theoretical values of RLmp were evaluated using equation (13) with respect to the condition of equation (12). Using the data of Charles et al [14] and Phang et al [15], optimum loads of two silicon solar cells (grey solar cell and blue solar cell) were calculated. Similarly, the optimum load of a plastic solar cell was determined using its recently published experimental data reported in [16]. Calculated results are compared with experimental data and the values determined by other methods available in literature. Relative accuracy is also calculated to show the significance of the STFT method (Table 2).

Table 1. Parameters of experimental solar cells used for calculating optimum load resistance

Parameters	**Experimental Data**		
	Charles et al [14]and Phang et al [15]		Conde et al [16]
	Grey Solar Cell	**Blue Solar Cell**	**Plastic Solar Cell**
R_s	77.69 (mΩ)	68.26 (mΩ)	8.59 (Ωcm^2)
R_{sh}	25.9 (Ω)	1000.0 (Ω)	197.24 (Ωcm^2)
I_0	5.514 (µA)	0.1036 (µA)	13.6 (nAcm^{-2})
I_{ph}	0.5610 (A)	0.1023 (A)	7.94 (mAcm^{-2}
V_{th}	26.479 (mV)	25.875 (mV)	25.875 (mV)
n	1.7168	1.5019	2.31
V_{mp}	0.390 (V)	0.437 (V)	0.548 (V)
I_{mp}	0.481 (A)	0.0925 (A)	4.71 (mAcm^{-2})
T	307 (K)	300 (K)	300 (K)

Table 2. Comparison of R_{Lmp} values obtained by different methods

			Grey Solar Cell	**Blue Solar Cell**	**Plastic Solar Cell**
Experimental		$\boldsymbol{R_{Lmp}}$	0.8108 (Ω)	4.724 (Ω)	116.35 (Ωcm^2)
Present method (STFT method)		$\boldsymbol{R_{Lmp}}$	0.7994 (Ω)	4.6404 (Ω)	116.23 (Ωcm^2)
		Accuracy (%)	1.406	1.769	0.103
Lambert W-function method		$\boldsymbol{R_{Lmp}}$	0.7985 (Ω)	4.638 (Ω)	116.18 (Ωcm^2)
		Accuracy (%)	1.517	1.821	0.146
Loferski's method		$\boldsymbol{R_{Lmp}}$	1.5498 (Ω)	4.9036 (Ω)	458.37 (Ωcm^2)
		Accuracy (%)	91.149	3.801	293.958
Govil's method	Only R_s	$\boldsymbol{R_{Lmp}}$	0.6459 (Ω)	4.0337 (Ω)	27.10 (Ωcm^2)
		Accuracy (%)	20.334	14.612	76.712
	Only R_{sh}	$\boldsymbol{R_{Lmp}}$	0.7951 (Ω)	4.6698 (Ω)	111.22 (Ωcm^2)
		Accuracy (%)	1.940	1.147	4.411
Kothari's method		$\boldsymbol{R_{Lmp}}$	0.7414 (Ω)	4.2192 (Ω)	115.39 (Ωcm^2)
		Accuracy (%)	8.554	10.686	0.821

The calculated results show that the STFT method has the best accuracy over other available methods in literature. In this scheme, substituting the values of the model parameters in equations (12) and (13), yield the value of R_{Lmp}.

It is worth to analyze the dependence of R_{Lmp} on the series resistance and the shunt resistance of the solar cell. Fig. 1 represents the effect of the series resistance R_s on the R_{Lmp} of various cells. The graphs are in consistent with the conclusions of Kothari et al [8] and Ding and Radhakrishnan [10], according to which, higher the series resistance, the greater its effect on the optimum load. Fig. 2 shows the effect of shunt resistance R_{sh} on R_{Lmp}. From the graph, it is observed that smaller the shunt resistance, the greater its effect on optimum load, in consistent with the conclusions of Kothari et al [8] and Ding and Radhakrishnan [10].

4. Conclusion

A new analytical method to determine the optimum load of a real solar cell from its model parameters is described using the properties of STFT. The method involves no approximation. The effects of the series resistance and shunt resistance on the optimum load are studied which are in consistent with the results reported in literature. The advantage of the STFT method is that the calculation time is independent of the response time, and the transcendental calculation scheme for current independent of the time domain. We may conclude that the method presented here is a powerful theoretical tool for the study of real solar cells.

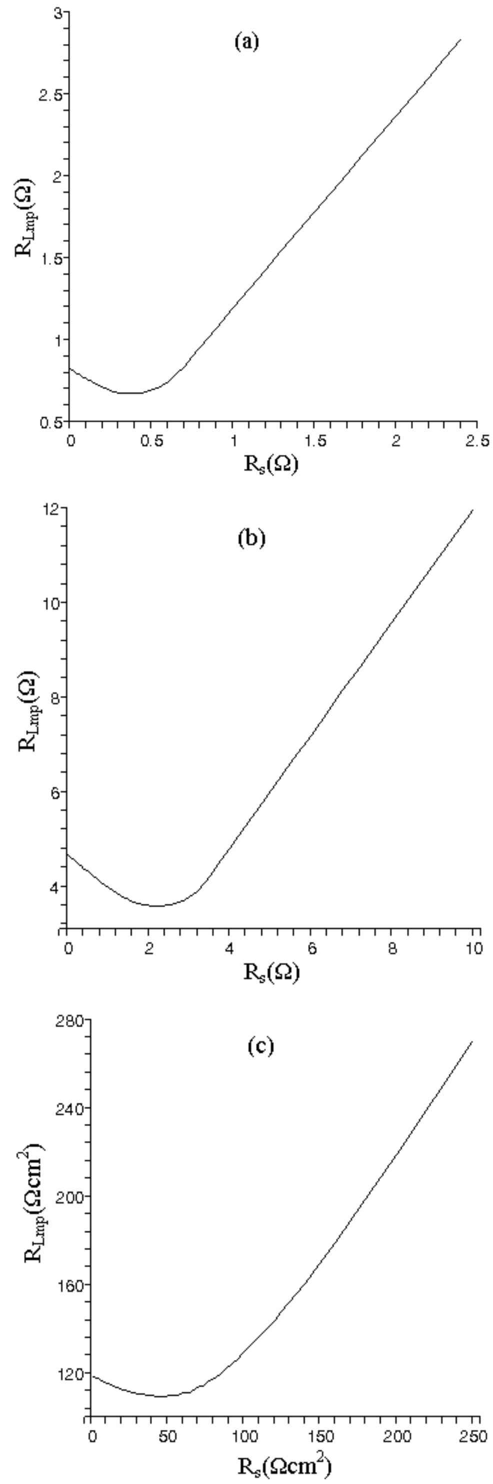

Fig. 1. Effect of the series resistance R_s on the optimum load R_{Lmp} of a (a) grey solar cell, (b) blue solar cell and (c) plastic solar cell

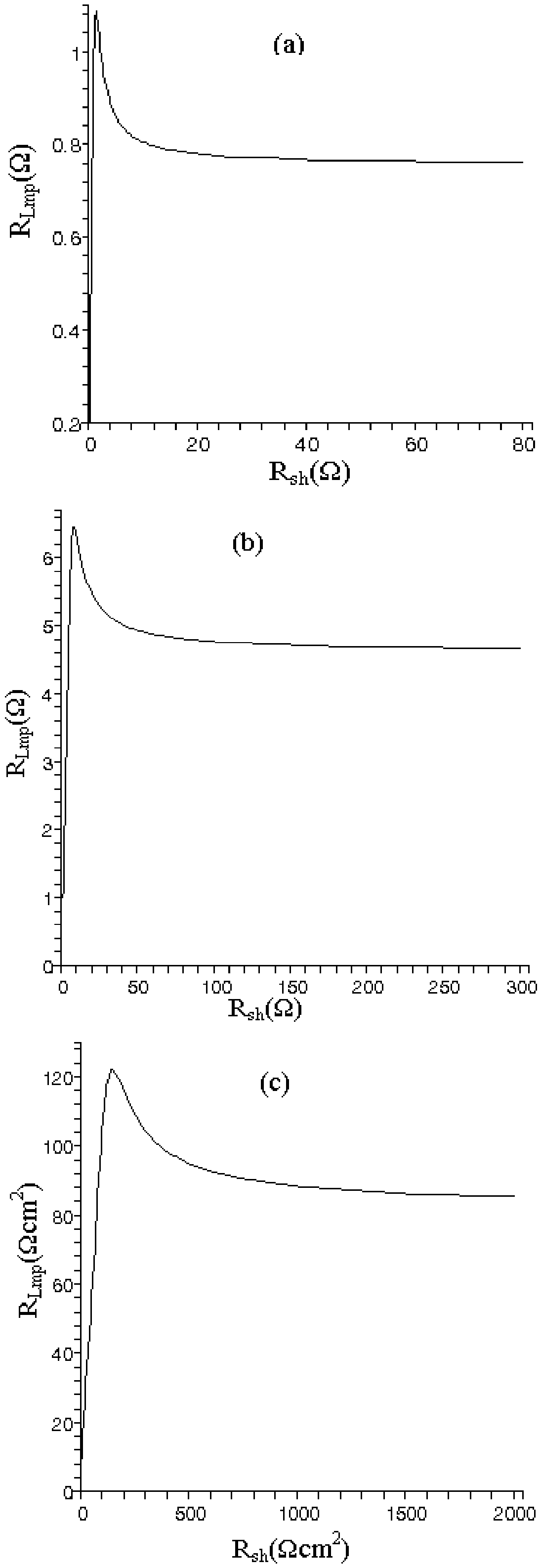

Fig. 2. Effect of the shunt resistance R_{sh} on the optimum load R_{Lmp} of a (a) grey solar cell, (b) blue solar cell and (c) plastic solar cell

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