# Statistical Analysis of the Wind Resources at Darling for Energy Production

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**Abstract-**This study presents a statistical analysis of the time varing wind resources at Darling site for wind energy assessment, evaluation and generation. Three statistical distribution functions were considered and fitted to a collection of wind speed data on the 10, 50 and 70m heights above ground level for the year 2010 to identify a suitable distribution function for describing the wind speed variation at these heights. The results show that the Rayleigh function modeled the observed wind speed best at these hub heights as compared to the other statistical functions. Also, the accuracy of these statistical functions were validated to determine the goodness of fit using an independent wind speed dataset, collected on a 40m hub height for the year 2009. The accuracy test results of the predicted wind distribution were compared to the results obtained on the 10, 50 and 70m heights. The Rayleigh function proved to be accurate for modeling of the wind speed at various hub heights. The choice of the Rayleigh function is based on the accuracy of the function in modeling the wind speed at the various heights and the testing criteria. Furthermore, the wind resources were mapped with the wind power density as the annual mean wind power densities were estimated at, 288.9 W/m² and 333.2 W/m², and the annual mean wind speed were estimated at 6.19 m/s and 6.49m/s on 50m and 70m heights, respectively.

**Keywords**Wind Data; Maximum Likelihood Estimator (MLE); Goodness of fit; Wind Power Density

## 1. Introduction

The assessment of the wind resources at a given site is one of the preliminary steps in the sitting of a wind farm project. The assessment of the wind resources involves analyzing in detail the wind profile at a given height such as the wind speed and its prevailing direction, turbulence intensity, the shape and scale parameters, the wind distribution, wind power density and class etc. At a given site, a known wind power class is regarded as one of the approaches for assessing the wind resource of a given site. To determine the suitability of this site for wind energy generation; the mean wind speed, the shape and scale parameters of the site are estimated. The estimated shape and scale parameters are used alongside with the various statistical functions to model the wind speed, and the wind distributions which best describe the variation of the wind at the site are obtained. Once the wind distributions are obtained, evaluation of the wind resources is conducted based on the known wind distribution for accurate sizing of the wind energy systems. The obtained wind distribution, the power curve of the wind generator (WG), and the site parameters are used to develop the site power curve. The developed site power curve is used to analyze the availability of wind energy generation for a known wind speed.

Over few decades, a number of studies have been conducted on the use of probability density function for modeling of the wind speed around the world. Some of these density functions include the Weibull, Rayleigh, Gamma, Lognormal, Exponential, and Gaussian etc [1-4]. The Weibull function is widely used in the wind industry as the preferred approach for modeling of the wind speed for energy assessment due to its wide range of versatility, flexibility, and usefulness for describing the wind speed variation. It applicability can also be found in reliability engineering and life data analysis. Some authors, [5-7] suggested the Weibull function as the best for modeling the wind speed at a given site. However, Hennesessey and Aidan [3,8] reported that for sites having very low/calm wind speed, the Weibull function does not model well the wind speed. The choice of a function for modeling of the wind speed at a given site shouldn't be based on a general rule of thumb but should depend mainly on the wind speed at the area under consideration, the modeling approach, and the accuracy of the model fitting the wind speed

Based on the study carried out at Darling site, the Rayleigh modeling of the wind speed was found to fit a collection of wind speed dataset at 10, 50m and 70m hub heights. The study takes into consideration the varying nature of the wind and the effects of the surface roughness of the site on the wind speed at different heights. The wind parameters such as the shape and scale of the site were determined at these hub heights. The obtained shape and scale parameters were applied to the Weibull, Rayleigh and Gamma distribution functions to determine the wind distribution that best describe the wind speed variation at these hub heights. In addition, accuracy test was conducted to validate the prediction accuracy of the various distribution functions used for modeling of the wind speed at 10m, 50 and 70m heights. An independent wind speed data obtained on 40m hub height are used for the validation of the distribution functions. The Rayleigh function modeled the wind speed best at various heights as compared to other functions. Furthermore, the wind resources were mapped with the wind power density and the class of wind power at different heights were determined. The aim of the paper is to demonstrate that (1) for this site under study, the Rayleigh distribution outperforms the Weibull and Gamma function at various hub heights, and (2) the site can be assessed for small to utility scale energy application.

## 2. Wind Site Description

The wind data between Jan 2010 to Dec 2010 were collected at Geelbek weather station (GWS) on a hub height of 10m above ground level as shown in table 1. This time series wind data were continuously measured by the wind acquisition systems deployed on a measurement mast at this weather station, sampled at every second (1s) and stored as 5-minute mean wind data. The wind data collected include the mean wind speed and direction, gust, temperature, atmospheric pressure and air humidity. The strong wind flow(s) are South-Easterly, and South-Westerly to North-Westerly.

**Table 1.** Location of Geelbek weather station

| Station | Latitude | Longitude | Height (m) |
|---|---|---|---|
| G.W.S. | 33°11'46.6"S | 18°7' 26.25"E | 10.0 |

A total of 105,096 wind data points were collected from the G.W.S on a 10m hub height and 90.45% data points remain valid for this study. The wind speed data on a 10m height are extrapolated to 50m and 70m to obtain the wind speed profile at different hub heights.

### *2.1. Mean Wind Speed (MWS)*

The wind speed is one of the most important parameters in the wind profile of any given site. The mean wind speed (MWS) indicates the suitability of a wind site for small-scale to large scale energy generation. The mean wind speed $\bar{v}$ (m/s) of a given site is defined in Eq. (1).

$$\bar{v} = \frac{1}{N}\sum_{i}^{N} v_i \tag{1}$$

where $V_i$ is the wind speed observation at $i^{th}$ time, and $N$ is the number of wind speed data points.

The summary of the monthly mean wind speed at both 10m and 50m hub heights for the year 2010 are shown in tables 2 and 3. The annual mean wind speed recorded at 10m height shows the suitability of the wind resources at this site for micro and small wind energy systems. Also, the annual mean wind speed at 50m height shows the suitability of this site for utility scale.

Figure 1 shows the comparison of the monthly mean wind speed variation at 10m height. In tables 2 and 3, the Vmin, Vmax, Data and MWS are the monthly minimum wind speed, maximum wind speed, number of wind speed data points, and the estimated mean wind speed. It can be seen that the observed wind speed and the estimated monthly mean wind speed (MWS) increases with heights. Column 5 shows the monthly mean wind speed using Eq. (1) and the highest wind speed values were observed in the month of January and December. This means that these months have potentials of recording the highest amount of wind energy generation at this site.

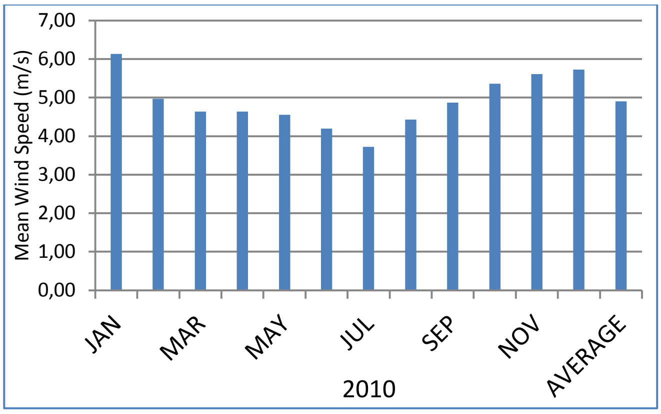


**Fig1.** Comparison of the monthly mean wind speed variation at a 10m hub height for the year 2010

**Table 2.** Monthly mean wind observations at on a 10m hub height for 2010

| Month | Vmin | Vmax | Data | MWS |
|---|---|---|---|---|
| Jan | 1.01 | 15.50 | 8561 | 6.14 |
| Feb | 1.01 | 15.50 | 7631 | 4.97 |
| Mar | 1.01 | 14.95 | 8393 | 4.63 |
| Apr | 1.01 | 14.20 | 7914 | 4.64 |
| May | 1.01 | 16.20 | 8208 | 4.55 |
| Jun | 1.01 | 15.30 | 8024 | 4.19 |
| July | 1.01 | 13.55 | 7898 | 3.72 |
| Aug | 1.01 | 12.85 | 7530 | 4.43 |
| Sept | 1.01 | 13.85 | 7557 | 4.87 |
| Oct | 1.01 | 16.35 | 7931 | 5.36 |
| Nov | 1.01 | 15.35 | 7909 | 5.61 |
| Dec | 1.01 | 16.05 | 8375 | 5.73 |
| **Sum** | **-** | **-** | **95931** | **4.90** |

**Table 3.** Monthly mean wind observations at 50m tower height for 2010

| Month | Vmin | Vmax | Data | MWS |
|---|---|---|---|---|
| Jan | 1.01 | 19.50 | 8766 | 7.56 |
| Feb | 1.01 | 19.50 | 7860 | 6.10 |
| Mar | 1.01 | 18.81 | 8690 | 5.67 |
| Apr | 1.01 | 17.86 | 8313 | 5.61 |
| May | 1.01 | 20.38 | 8482 | 5.58 |
| Jun | 1.01 | 19.25 | 8250 | 5.16 |
| July | 1.01 | 17.05 | 8262 | 4.53 |
| Aug | 1.01 | 16.17 | 7748 | 5.45 |
| Sept | 1.01 | 17.42 | 7711 | 6.03 |
| Oct | 1.01 | 20.57 | 8108 | 6.22 |
| Nov | 1.01 | 19.31 | 8035 | 6.96 |
| Dec | 1.01 | 20.19 | 8532 | 7.09 |
| **Sum** | **-** | **-** | **98757** | **6.00** |

However, for the accurate estimation of the wind power density, Eq. (1) is not used. Using Eq. (1), will underestimate the wind power potential at this site [9-10]. As a result, the roots mean cube $v_{rmc}$ of the wind speed (m/s) is preferred as expressed in Eq. (2):

$$v_{rmc} = \sqrt[3]{\sum_{i}^{N} v_1^3} \tag{2}$$

where $v_{rmc}$ is the roots mean cube wind speed

*2.2. Wind Speed Variation with Height*

The wind speed varies continuously as a function of time and height, and it's another important factor to be considered when assessing the wind resources potential of any location. The wind speed measurements for this study were obtained at a 10m hub height ($\boldsymbol{h_1}$) above sea level. And since changes in the wind speed propagate with the wind, the wind speed above the sea level changes with increasing height. The most common expression for variation in wind speed with height is known as wind shear or power law equation as defined in Eq. (3).

$$\frac{v_2}{v_1} = \left(\frac{h_2}{h_1}\right)^{\alpha} \tag{3}$$

where $v_1$ is the reference wind speed at 10m hub height ($\boldsymbol{h_1}$), is the wind speed at 50m hub height ($\boldsymbol{h_2}$) and is the exponents which depends upon the surface roughness of the site

The variation of wind at different heights is primarily due to the local geographic of the site. Often times, the wind shear exponent value is taken as 1/6$^{th}$ for a flat or smooth terrain. The shear values for any given wind site vary from this value depending on the topography of the site. Several studies have dealt with the wind shear exponent at a given site. The wind shear value is crucial to a site especially for wind site with only one measurement mast. The wind shear value varies with increasing heights, time and season, nature of the terrain, weather effect etc [11].For an accurate assessment of any wind site, two or more masts are installed at an area under consideration. For this study, only a 10m measurement mast were available for this site and the wind shear exponent based on the site description was estimated at 0.143.To obtain the wind speed at 50m and 70m hub heights, Eq. (1) is applied to the wind speed at 10m hub height and the wind shear exponent ($\alpha$) is estimated at 0.143 because of the topography of the site [6,10].

*2.3. Air Density Variation with Altitude*

The air density is another important parameter for estimating both the wind power and its density. The air density has a significant effect on the performance of a wind energy system. The wind power production is proportional to the air density at a height (h), as a function of the atmospheric pressure and air temperature. The density of air "ρ" is defined as the mass per volume of air in the earth's atmosphere. At 15 °C above sea level, the dry air has a density of approximately 1.225kg/m3. Instead of using the value of constant dry air density at sea level, the time varying air density at 10m, 50m and 70m heights are estimated using the 5-minute values of the air temperature and atmospheric pressure in the wind data measurement.

In this study, the time varying varied air density at 10m, 50m and 70m heights were obtained using Eq. (4) [12].

$$\rho(h) = \frac{P}{RT} e^{-\left(\frac{gh}{RT}\right)} \tag{4}$$

where *ρ(h)* is the varied air density as a function of height (kg/m³), *P* is the atmospheric pressure (hPa), *R* is the molar gas constant (287.05J/(kgK)), *T* is the temperature (K), *g* is the gravitational constant (9.81m/s²), and *h* is the height above ground level

The summary of the monthly and annual mean air densities at 10m, 50m and 70m hub heights are shown in tables 4.

**Table 4.**Summary of the monthly mean air densities @ 10m, 50m and 70m hub heights

| **2010** | | *ρ(h)* | |
|---|---|---|---|
| Mon | 10m | 50m | 70m |
| Jan | 1.206 | 1.201 | 1.198 |
| Feb | 1.201 | 1.196 | 1.193 |
| Mar | 1.205 | 1.200 | 1.197 |
| Apr | 1.228 | 1.223 | 1.220 |
| May | 1.232 | 1.232 | 1.224 |
| Jun | 1.243 | 1.237 | 1.234 |
| July | 1.245 | 1.240 | 1.237 |
| Aug | 1.240 | 1.235 | 1.232 |
| Sept | 1.232 | 1.227 | 1.224 |
| Oct | 1.224 | 1.219 | 1.216 |
| Nov | 1.214 | 1.209 | 1.206 |
| Dec | 1.200 | 1.194 | 1.192 |
| **Aver** | **1.222** | **1.218** | **1.214** |

## 3. Modelling of the Wind Speed

The wind speed variation at a given site is usually described using the wind distribution. Around the world, to identify the suitable statistical distribution for describing the wind speed variation, the following functions are been used and they include the Weibull [13], Rayleigh [14], Gamma [8,15], Lognormal [16], Logistic [17-18] etc. However, the Weibull and Rayleigh functions are the widely accepted and extensively used statistical models for wind energy application. In this study, three distribution functions are considered and they include the Weibull function, the Rayleigh function, and the Gamma function.

### *3.1. Weibull Functions*

The Weibull function is the most widely used function among the several distribution functions for modeling of the wind speed at a given site. In Weibull function, the variations in wind speed are described by using the Weibull shape and scale parameters. The Weibull cumulative distribution function (cdf) is defined by Eq. (5).

$$F_w = 1 - \exp\left[-\left(\frac{v}{c}\right)^k\right] \tag{5}$$

where $F_w$ is the Weibull cumulative distribution function, $k$ is the Weibull shape parameter and $C$ is the Weibull scale parameter (m/s).

Eq. (5) is defined as the fraction of time at which an observed wind speed is equal or below a particular speed.

The Weibull probability density function of a 2-parameter continuous distribution is defined as the derivative of the cumulative distribution function (cdf) as expressed in Eq. (6)

$$f_w = \frac{k}{C}\left(\frac{v}{c}\right)^{k-1} \exp\left[-\left(\frac{v}{c}\right)^k\right] \tag{6}$$

where $f_w$ is the Weibull density function (pdf).

Eq. (6) is defined as probability at which the wind speed $v$ prevails at a given site.

The mean wind speed in the terms of Weibull distribution is expressed as

$$\bar{v} = \int_0^\infty v\, f_w(v)dv$$

$$= \int_0^\infty \frac{vk}{C}\left(\frac{v}{c}\right)^{k-1} \exp\left(-\frac{v}{c}\right)^k dv$$

$$= k\int_0^\infty \left(\frac{v}{c}\right)^k \exp\left(-\frac{v}{c}\right)^k dv \tag{7}$$

Putting $x = \left(\frac{v}{c}\right)^k$, $dx = \frac{k}{C}\left(\frac{v}{c}\right)^{k-1} dv$ and $x^{\frac{1}{k}} = \frac{v}{c}$

Equation (7) can be re-expressed as

$$\bar{v} = c\int_0^\infty x^{\frac{1}{k}} \exp(-x)dx \tag{8}$$

Substituting the Euler gamma function

$\Gamma(n) = \int_0^\infty x^{n-1}\exp(-x)dx$ and putting y $= 1 + \frac{1}{k}$ into

Eq. (8), the actual mean wind speed in term of Weibull shape $k$ and scale parameter $C$ is expressed as

$$\bar{v} = C\,\Gamma\{1 + \frac{1}{k}\} \tag{9}$$

The shape parameter as denoted by $k$ represents the nature of the wind (variability or stability of the wind). For most fairly wind site, the value of $k$ ranges between 1.51-1.99. Smaller values of $k \leq 1.5$ correspond to highly variable or gust wind, whereas $k$ =2 corresponds to moderate wind and for $k \geq 3$ , indicates regular, steady wind.

There are several approaches for estimating the shape parameter of a Weibull function. The shape parameter $k$ of the Weibull distribution used for this study were estimated using the maximum likelihood estimator "MLE" as expressed in Eq. (10)

$$k = \left(\frac{\sum_{i=1}^{N}\ln(v_i)v_i^k}{\sum_{i=1}^{N}v_i^k} - \frac{\sum_{i=1}^{N}\ln(v_i)}{N}\right)^{-1} \tag{10}$$

where $k$ is the Weibull shape parameter obtained using the MLE approach as defined in Eq. (10)

Once the value of the Weibull shape parameter is obtained, the Weibull scale parameter $C$ is estimated using the MLE as expressed in Eq. (11)

$$C = \left(\frac{\sum_{i=1}^{N}v_i^k}{N}\right)^{\frac{1}{k}} \tag{11}$$

where $C$ is the scale parameter of the Weibull distribution.

The obtained Weibull shape and scale parameters are set into Eq. (5-6) to obtain the Weibull pdf and cdf.

The standard deviation of the Weibull distribution function in term of its shape and scale parameters is defined as

$$\delta_W = C\left(\Gamma\left(1+\frac{2}{k}\right) - \Gamma^2\left(1+\frac{1}{k}\right)\right)^{\frac{1}{2}} \tag{12}$$

where $\delta_W$, $\Gamma$ (.) are the weibull standard deviation and the gamma function of (.) respectively.

The summary of the monthly mean Weibull shape and scale parameters using the MLE is shown in Table 5. At a

50m height and above, the Weibull shape parameters remain constant and the site has fairly wind moving towards the moderate wind range in the month of November. The Weibull scale parameter increases with the hub height(s) as shown in table below.

**Table 5.** Comparison of the values of the monthly Weibull shape and scale parameters at 50m and 70m hub heights

| Mon | *k* | *C* | | *k* | *C* |
|---|---|---|---|---|---|
| | **50m** | | | **70m** | |
| Jan | 1.833 | 8.520 | | 1.832 | 8.940 |
| Feb | 1.816 | 6.878 | | 1.820 | 7.229 |
| Mar | 1.759 | 6.393 | | 1.759 | 6.708 |
| Apr | 1.580 | 6.281 | | 1.580 | 6.590 |
| May | 1.613 | 6.263 | | 1.613 | 6.571 |
| Jun | 1.869 | 5.843 | | 1.869 | 6.131 |
| July | 1.774 | 5.121 | | 1.774 | 5.374 |
| Aug | 1.876 | 6.163 | | 1.875 | 6.468 |
| Sept | 1.885 | 6.817 | | 1.885 | 7.153 |
| Oct | 1.903 | 7.476 | | 1.903 | 7.845 |
| Nov | 1.911 | 7.862 | | 1.911 | 8.249 |
| Dec | 1.845 | 7.999 | | 1.845 | 8.393 |
| **Aver** | **1.805** | **6.801** | | **1.806** | **7. 137** |

### *3.2. Rayleigh Function*

The second widely accepted distribution function which is extensively used in modeling of the site wind speed is the Rayleigh function. The Rayleigh distribution is known to be a special case of the Weibull distribution. This function is found to typically model the wind speed at some sites where the Weibull function could not accurately model. At a wind site, where the value of *k* is 2, is commonly referred to as the Rayleigh function. At a wind site where the Weibull function is a poor model for fitting the wind speed, it may be appropriate to model the wind speed with a Rayleigh function. This is based on changing the shape parameter of an dependent shape variable "$k\neq 2$" for Weibull function to a independent variable "$k$=2" for a Rayleigh function.

Putting $k$=2 into Eq. (6), the Rayleigh probability density function of a continuous distribution is defined by Eq. (13)

$$f_r = \frac{2v}{c^2}\exp\left[-\left(\frac{v}{c}\right)^2\right] \tag{13}$$

where $C$ is the scale parameter at $k$=2 and $f_r$ is the Rayleigh pdf.

And the cumulative distribution function is defined by Eq. (14)

$$F_r = 1-\exp\left[-\left(\frac{v}{c}\right)^2\right] \tag{14}$$

where $F_r$ is the cdf of the Rayleigh distribution

The Rayleigh scale parameter $C_r$ is obtained using the maximum likelihood estimator as expressed in Eq. (15) [19]

$$C_r = \sqrt{\frac{1}{2N}\sum_{i=1}^{N} v_i{}^2} \tag{15}$$

where $C_r$ is the Rayleigh scale parameter and $v_i$ is the wind speed observations at $i^{th}$ time

The mean of the Rayleigh distribution function is defined by Eq. (19) [19-20]

$$\overline{v_r} = c_r\sqrt{\frac{\pi}{2}} \tag{16}$$

where $\bar{v}_r$ is the mean of the Rayleigh distribution function

### *3.3. Gamma Function*

The probability density function of a Gamma distribution is defined by Eq. (17)

$$f_g = \frac{v^{k-1}}{c^k\Gamma(k)}\exp\left[-\left(\frac{v}{C}\right)\right] \qquad k, C>0 \tag{17}$$

where $C$, $k$ and $f_g$ are the shape, scale parameters and probability density function of a Gamma distribution respectively.

The cumulative distribution function is defined as

$$F_g = \frac{1}{c^k\Gamma(k)}\int_0^v t^{k-1}\exp\left(-\left(\frac{t}{C}\right)\right)dt \tag{18}$$

where $F_g$, $\Gamma(k)$ are the Gamma cumulative distribution and Gamma function of (k) respectively.

The gamma distribution function can be found applicable in the modeling of low wind speed data and modeling errors in multi-level Poisson regression models

### *3.4. Actual Function*

The probability density function of an actual distribution with the mean $\mu$ and standard deviation $\delta$ is defined as

$$f_a = \frac{1}{\delta\sqrt{2\pi}}\exp\left(-\frac{(v-\mu)^2}{2\delta^2}\right) \tag{19}$$

And the cumulative distribution function is defined as

$$F_a = \frac{1}{2}\left(1+erf\,\frac{(v-\mu)}{\delta\sqrt{2}}\right) \tag{20}$$

Using the maximum likelihood estimator (MLE) called the biased estimator, the standard deviation in term of the sampled wind speed data $v_i$, and the mean wind speed $\mu$ can be defined as

$$\delta = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(v_i-\mu)^2} \tag{21}$$

where $\boldsymbol{\mu}$, $\boldsymbol{\delta}$, $\boldsymbol{f_a}$, $\boldsymbol{F_a}$ are the mean, standard deviation, pdf and cdf of the actual distribution respectively.

### *3.5. Goodness of Fit*

There are several tests used for validating the accuracy of the predicted wind distribution obtained from various statistical functions. The wind distributions obtained from these functions indicate whether there is an accurate modeling of the wind speed, or that the functions fail to accurately model the wind speed at a given site.

For the accuracy test, an independent wind speed dataset is used to validate the accuracy of the statistical distrbution functions in modeling the wind speed at this site. The wind speed dataset used for these tests were obtained on a 40m hub height for the year 2009. The various tests which have been used for validating the goodness-of-fit of these distribution functions are explained below.

#### *3.5.1. Root Mean Square Error (RMSE)*

The RMSE has been used for comparison of the actual deviation between the predicted and the actual (measured) values. The root mean square error value is defined by Eq.(22)

$$\text{RMSE} = \left( \frac{\sum_{i=1}^{N} (y_i - x_i)^2}{N} \right)^{\frac{1}{2}} \tag{22}$$

where $x_i$ is the $i^{th}$ actual wind distribution, $y_i$ is the $i^{th}$ predicted wind distribution from the Weibull, Rayleigh, and Gamma functions etc. and $N$ is the number of the wind speed dataset.

The actual wind distribution is obtained from Eq. (19) and the predicted wind distributions are obtained from the Weibull, Rayleigh and Gamma functions.

Comparison of the annual RMSE values from the distribution models at both 40m and 50m heights for the year 2009 are shown in figures 2 and 3. From the figures, it is clearly shown that only the RMSE value of the Rayleigh distribution reduces slightly with hub height. The best wind distribution with the lowest RMSE value is chosen as the accurate function to be used for modeling of the wind speed.

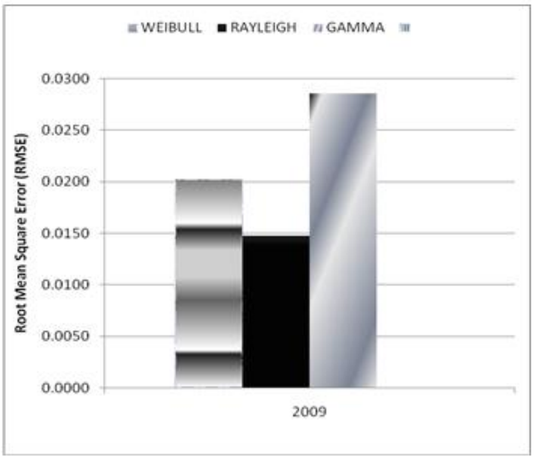


**Fig. 2.** Annual RMSE @ 40m hub height

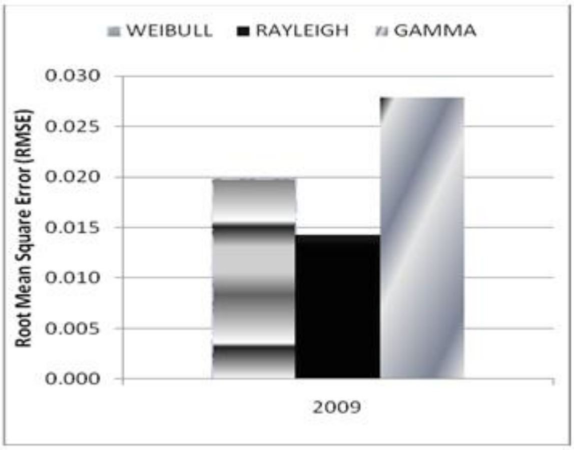


**Fig. 3.** Annual RMSE @ 50m hub height

#### *3.5.2. Chi-Square Test (χ²)*

The Chi-Square method is used for testing the predicted wind distribution with respect to the actual wind distribution. The mathematical expression for the Chi-square test "$\chi^2$" is defined as:

$$\chi^2 = \frac{\sum_{i=1}^{N} (y_i - x_i)^2}{N - n} \tag{23}$$

where $x_i$, $y_i$ and $N$ are defined in Eq. (22). $n$ is the number of constant wind data.

#### *3.5.3. Correlation Coefficient (R)*

The correlation coefficient is a statistical technique that is used to determine the linear relationship between two datasets. The mathematical equation for R is defined as

$$R = \frac{\sum_{i=1}^{N} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{N} (x_i - \bar{x})^2 \sum_{i=1}^{N} (y_i - \bar{y})^2}} \tag{24}$$

where $\bar{x}$ and $\bar{y}$ are the mean of the actual and predicted wind distributions respectively.

The values of R always lies between -1 and 1, and is greater than the values of $R^2$

#### *3.5.4. Coefficient of Determination ( R² / COD)*

Another method of assessing the goodness of fit of the wind distribution is known as the coefficient of determination "$R^2$"or "COD". It is simply defined as

$$R^2 = \frac{N \sum x_i y_i - \sum x_i y_i}{\left[\sqrt{N \sum x_i^2 - (x_i)^2}\right]\left[\sqrt{N \sum y_i^2 - (y_i)^2}\right]} \tag{25}$$

The value of $R^2$ lies between 0 and 1 and is always less than or equal to R. The annual $R^2$ values from the three distribution models at 40m height is depicted in figure 4.

The distribution function that accurately modeled the wind speed data are selected according to the highest value of the $R^2$.

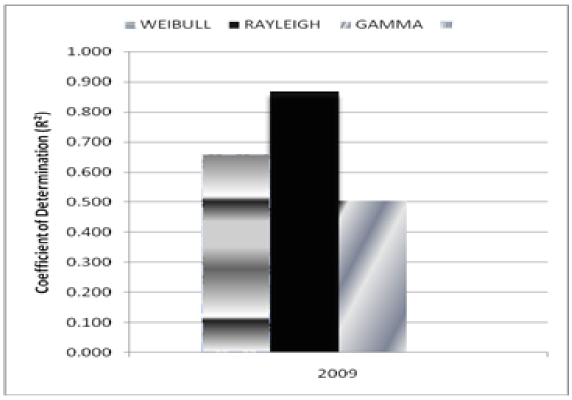


**Fig. 4.** Annual COD @ 40m hub height

*3.6. Estimation of Wind Power Density*

The available wind power per unit swept area known as wind power density {W/m²} is defined as

$$P = \frac{1}{2}\boldsymbol{\rho(h)}V^3 \tag{26}$$

where $v$ is the observed wind speed, $\rho(h)$ is the varied air density sweeping the rotor blades, and P is the wind power density.

The theoretical maximum power {W} of the wind that flows across the rotor swept area (A) at a given speed $V$ is given by

$$P_o = \frac{1}{2}\boldsymbol{\rho(h)}AV^3 \tag{27}$$

where $A$ is the swept area of the rotor blades and $P_o$ is the theoretical wind power.

The mechanical power {W} of the wind turbine is defined as

$$P_m = C_p * \frac{1}{2}\boldsymbol{\rho(h)}AV^3 \tag{28}$$

where $P_m$ and $Cp$ are the mechanical power developed by the rotor blades and the power coefficient of the rotor, respectively. Based on the bertz law, the maximum wind power that can be extracted at any given time is 59% but in the practical design of the rotor blades, the maximum Cp values range from 0.2 to 0.4 [9].

The electrical power outputs {W} of the wind generator is defined as

$$P_e(v) = \eta_{R,G} * P_m \tag{29}$$

where $\eta_g$ is the efficiency of the gearbox and electrical generator which is always estimated or specified, and $P_e(v)$ is the electrical power of the wind generator.

*3.6.1. Actual Power Density $P_A$*

In this study, the actual wind power density used in study is estimated using Eq. (30). From Eq. (26), the speed is obtained using the actual wind distribution. ie. $v^3 = \int_0^\infty v^3 f(v)dv$

Equating $V^3 = \int_0^\infty V^3 f(v)dv$ into (26)

$$P_A = \frac{1}{2}\boldsymbol{\rho(h)}\int_0^\infty V^3 f(v)dv \tag{30}$$

where $f(v)$ is the actual wind distribution and $P_A$ is the actual wind power density.

*3.6.2. Weibull Power Density $P_W$*

The wind power density using the Weibull distribution is estimated by using Eq.(31) [22-23]

$$P_W = \frac{1}{2}\boldsymbol{\rho(h)}C^3\Gamma\left(1+\frac{3}{k}\right) \tag{31}$$

where $k$, and $C$ are the shape and scale parameters of the Weibull distribution, and $P_W$ is the Weibull wind power density.

*3.6.3. Rayleigh Power Density $P_R$*

The Rayleigh wind power density $P_R$ is estimated using Eq. (32) [24]

$$P_R = \frac{3}{\pi} * \boldsymbol{\rho(h)} * (\bar{v})^3 \tag{32}$$

From Eq. (9), putting $k = 2$

$$\bar{v} = C\Gamma\left(1+\frac{1}{2}\right) = C\,\Gamma(1.5)$$

$$\bar{v} = C\sqrt{\frac{\pi}{4}} = 0.8862C \tag{33}$$

where $\bar{v}$ is the Rayleigh mean wind speed for $k$=2, $C$ is the Rayleigh scale parameter at $k$=2 and $P_R$ is the Rayleigh wind power density

And equating (33) into (32), for $k$=2, the Rayleigh wind power density is re-defined as

$$P_R = \frac{3}{\pi} * \boldsymbol{\rho(h)} * c\sqrt{\frac{\pi}{4}} \tag{34}$$

*3.6.4. Gamma Power Density $P_G$*

The wind power density using the Gamma distribution is estimated by [8]

$$P_G = \frac{1}{2}\boldsymbol{\rho(h)}C^3[k(k+1)(k+2)] \quad (35)$$

where $k$, $C$ and $P_G$ are the Gamma shape and scale parameters; wind power density of the Gamma function respectively.

The wind power potential at a given location is usually classified according to its wind power class. The wind power class of any given site ranges between classes 1 to 7 depending on the prevailing wind resources. Each wind power class represents a range of mean wind power density (W/m²) and its equivalent mean wind speed (m/s) at 10m and 50m hub height(s) above sea ground [10,22]. For example, the wind power class 1 is used to denote a very poor wind site while a class 6 is used to denote an excellent wind site.

A typically 10-40meters WPD estimates are suitable for small to medium scale wind energy system while a 50-meter WPD estimates are the industry standard metric used to gauge a the site wind resource for large-scale wind energy systems. The criteria for the selection of the hub height(s) for the energy system depends mainly on the site wind resources and the associated capital cost. The wind resources at this site are mapped with the wind power density and the summary of the monthly mean wind power densities estimated at this site are shown in tables 6-8. In tables 6-8, the wind power densities “WPD” at these hub heights were determined using the mean wind speed, wind distribution and the estimated air densities. The results obtained from the wind power densities estimate denote that the selected wind site falls under 3, 2 and 3 of the international system of wind power classification as the annual mean wind speed recorded in the area are 4.99 m/s, 6.19 m/s and 6.49m/s, and the corresponding annual mean power density are estimated at 151.5 W/m², 290.50 W/m² and 336 W/m² at the considered hub heights, respectively

**Table 6.** Comparison of the monthly mean wind power densities and its error estimate @ 10m height

| **2010** | **Wei** | | **Ray** | | **Gam** | |
|---|---|---|---|---|---|---|
| | $P_W$ | Error | $P_R$ | Error | $P_G$ | Error |
| Jan | 278.5 | 0.30 | 273.6 | 0.27 | 317.8 | 0.48 |
| Feb | 149.6 | 0.23 | 145.9 | 0.20 | 164.1 | 0.35 |
| Mar | 125.9 | 0.19 | 121.9 | 0.15 | 134.9 | 0.27 |
| Apr | 142.1 | 0.16 | 133.9 | 0.09 | 155.5 | 0.27 |
| May | 137.7 | 0.14 | 127.9 | 0.06 | 143.9 | 0.19 |
| Jun | 092.6 | 0.16 | 090.6 | 0.15 | 093.5 | 0.15 |
| July | 067.2 | 0.14 | 065.5 | 0.11 | 067.7 | 0.15 |
| Aug | 149.3 | 0.69 | 105.7 | 0.19 | 113.6 | 0.28 |
| Sept | 140.5 | 0.21 | 139.8 | 0.19 | 152.3 | 0.30 |
| Oct | 184.1 | 0.24 | 182.3 | 0.23 | 203.6 | 0.37 |
| Nov | 212.1 | 0.25 | 207.7 | 0.23 | 233.8 | 0.38 |
| Dec | 223.3 | 0.26 | 222.7 | 0.20 | 253.7 | 0.37 |
| **Aver** | **159.4** | **0.25** | **149.8** | **0.17** | **169.5** | **0.30** |

**Table 7.** Comparison of the monthly mean wind power densities and its error estimate @ 50m

| **2010** | **Wei** | | **Ray** | | **Gam** | |
|---|---|---|---|---|---|---|
| | $P_W$ | Error | $P_R$ | Error | $P_G$ | Error |
| Jan | 546.1 | 0.25 | 523.8 | 0.20 | 635.2 | 0.46 |
| Feb | 290.7 | 0.24 | 277.0 | 0.18 | 334.9 | 0.38 |
| Mar | 243.1 | 0.19 | 229.7 | 0.13 | 265.2 | 0.30 |
| Apr | 276.5 | 0.19 | 247.1 | 0.06 | 299.9 | 0.29 |
| May | 267.0 | 0.15 | 242.8 | 0.05 | 281.9 | 0.21 |
| Jun | 177.0 | 0.16 | 172.5 | 0.06 | 184.0 | 0.21 |
| July | 127.7 | 0.13 | 121.9 | 0.53 | 130.9 | 0.16 |
| Aug | 206.6 | 0.20 | 201.0 | 0.17 | 224.2 | 0.31 |
| Sept | 276.1 | 0.21 | 269.2 | 0.18 | 302.3 | 0.32 |
| Oct | 357.8 | 0.21 | 349.9 | 0.18 | 405.7 | 0.37 |
| Nov | 410.6 | 0.24 | 402.3 | 0.22 | 465.7 | 0.41 |
| Dec | 410.6 | 0.14 | 402.3 | 0.11 | 465.7 | 0.29 |
| **Aver** | **299.2** | **0.19** | **288.9** | **0.16** | **332.1** | **0.31** |

**Table 8.** Comparison of the monthly mean wind power densities and its error estimate @ 70m

| **2010** | **Wei** | | **Ray** | | **Gam** | |
|---|---|---|---|---|---|---|
| | $P_W$ | Error | $P_R$ | Error | $P_G$ | Error |
| Jan | 629.5 | 0.25 | 603.8 | 0.20 | 659.7 | 0.31 |
| Feb | 334.2 | 0.23 | 320.2 | 0.18 | 375.2 | 0.39 |
| Mar | 280.2 | 0.21 | 264.8 | 0.13 | 305.6 | 0.30 |
| Apr | 321.3 | 0.19 | 287.1 | 0.07 | 348.5 | 0.30 |
| May | 306.5 | 0.15 | 278.7 | 0.04 | 323.6 | 0.21 |
| Jun | 204.0 | 0.16 | 198.8 | 0.13 | 212.1 | 0.21 |
| July | 147.2 | 0.13 | 140.5 | 0.08 | 150.8 | 0.16 |
| Aug | 238.9 | 0.21 | 232.6 | 0.18 | 259.4 | 0.31 |
| Sept | 318.2 | 0.21 | 310.2 | 0.18 | 348.4 | 0.32 |
| Oct | 412.4 | 0.24 | 403.4 | 0.21 | 467.3 | 0.41 |
| Nov | 473.3 | 0.24 | 463.7 | 0.22 | 536.6 | 0.41 |
| Dec | 513.7 | 0.23 | 495.2 | 0.19 | 582.1 | 0.40 |
| **Aver** | **348.3** | **0.20** | **333.2** | **0.15** | **380.8** | **0.31** |

where $P_W$ ,$P_R$, $P_G$ error are the monthly mean wind power densities (W/m²) of the Weibull, Rayleigh, Gamma functions and its respective error estimation with respect to the actual wind power density (W/m²).

The performance of the models in estimating the wind power densities were evaluated using Eq. (36) and (37) below. The monthly errors in estimating the wind power densities are obtained using Eq. (36) and results shown in tables 6-8.

The percentage error (%) of the WPD is defined as

$$\text{Error (\%)} = \frac{P_{W,R,G} - P_A}{P_A} * 100\% \quad (36)$$

While the annual mean error (%) in calculating the wind power density is estimated using Eq. (37)

$$\text{Error(\%)} = \frac{1}{12}\sum_{i=1}^{12}\frac{P_{W,R,G} - P_A}{P_A} * 100\% \quad (37)$$

where $P_{W,R,G}$, are the wind power densities of the Weibull, Rayleigh, Gamma distributions respectively, and $P_A$ is the actual wind power density.

From the tables 6-8, the more the error values are closer to 0%, the better the performance of the model. The monthly mean error in predicting the wind power densities are slightly higher at 10m hub height and decreases with hub heights.

## 4. Discussions

The wind resource at darling site has been statistically analyzed at various heights for the wind energy assessment, evaluation and production. The monthly mean wind speed and standard deviation at 10m, 50m and 70m heights have been estimated as shown in the tables 9-10. The mean wind speed increases with the hub height(s) and the actual annual mean wind speeds are estimated at 4.92m/s on 10m, 6.05m/s on 50m and 6.35 m/s on 70m heights respectively. The standard deviation and the mean wind speed are the important site parameters for determining the wind turbulent intensity at different hub heights.

The shape and scale parameters of the Weibull, Rayleigh and Gamma functions have been estimated and used to obtain the wind distributions at different heghts as shown in the tables 11-13. A lower value of k<2 indicates a greater deviation from the mean wind speed while a higher value of k>2 indicates the small variation from the mean wind speed. As the values of k increases, the probability curve becomes peaked indicating small variation from the mean wind speed.

At a 10m hub height, the wind site has k values ranges between 1.68 to 1.98 using the maximum likelihood estimator and 1.65 to 1.98 using the analytical approach. The difference in the values of k was as a result of the discrepancy using the analytical approach which depends mainly on the mean wind speed and its standard deviation values to obtain the values of k. The wind power class of the site turbines suitable on a 10m height ranges between class 1 to 5 based on the estimated wind power density. At 50m height, the wind speed increases with height with decreasing values of k and increasing values of scale parameter C. This means that most of the high wind speed values obtained at 50m height comes during a period of short and strong wind as shown in table 12. The shape parameter k ranges between 1.58 1.91 using the maximum likelihood estimator and 1.57 to 1.92 using the standard deviation approach. This means that the class of wind energy systems suitable for 50m height ranges between classes 1+ to 5-. Also, at 70m height the values of k remain unchanged with the value of C increases with hub heights as shown in table 13. From the wind power density analysis, the wind power class suitable for 70m height at this site range between classes 1+ to 6.

The air density which is an important site parameter for estimating the wind power density has been estimated at different hub heights. The air density varies with atmospheric pressure and air temperature with increasing altitude. The obtained estimated varied air densities at 10m, 50m and 70m heights consist of the densities of mixed dry air and water vapor molecules. The air densities estimate are based on the 5-minute atmospheric pressure and temperature measurement on a 10m hub height. The annual mean air densities are estimated at 1.222kg/m³, 1.218kg/m³ and 1.214kg/m³ on the 10m, 50m and 70m heights at this site as shown in the table 4.

**Table 9.** Summary of the monthly mean wind speed and standard deviation of different distribution functions @ 10m hub height

| **2010** | **WEIB** | | **RAYL** | | **GAM** | |
|---|---|---|---|---|---|---|
| **Mon** | $\bar{v}_a$ | $\delta$ | $\bar{v}_a$ | $\delta$ | $\bar{v}_a$ | $\delta$ |
| Jan | 6.14 | 3.33 | 6.19 | 3.24 | 6.14 | 3.61 |
| Feb | 4.98 | 2.72 | 5.03 | 2.63 | 4.97 | 2.88 |
| Mar | 4.65 | 2.61 | 4.73 | 2.47 | 4.63 | 2.70 |
| Apr | 4.66 | 2.85 | 4.85 | 2.54 | 4.64 | 2.95 |
| May | 4.57 | 2.80 | 4.77 | 2.50 | 4.55 | 2.84 |
| Jun | 4.22 | 2.26 | 4.24 | 2.22 | 4.19 | 2.26 |
| July | 3.75 | 2.08 | 3.81 | 1.99 | 3.72 | 2.06 |
| Aug | 4.45 | 2.37 | 4.47 | 2.34 | 4.43 | 2.45 |
| Sept | 4.89 | 2.62 | 4.92 | 2.57 | 4.87 | 2.73 |
| Oct | 5.37 | 2.83 | 5.38 | 2.81 | 5.36 | 3.03 |
| Nov | 5.62 | 2.98 | 5.64 | 2.95 | 5.61 | 3.20 |
| Dec | 5.74 | 3.13 | 5.79 | 3.03 | 5.72 | 3.34 |
| **Aver** | **4.92** | **2.72** | **4.99** | **2.61** | **4.90** | **2.84** |

**Table 10.** Summary of the monthly mean wind speed and standard deviation of different distribution functions @ 50m and 70m hub heights

| **Mon** | **h** | $\bar{v}_a$ | $\delta$ | $\bar{v}_a$ | $\delta$ | $\bar{v}_a$ | $\delta$ |
|---|---|---|---|---|---|---|---|
| Jan | 50m | 7.57 | 4.28 | 7.70 | 4.03 | 7.56 | 4.68 |
| | 70m | 7.94 | 4.49 | 8.08 | 4.22 | 7.94 | 4.91 |
| Feb | 50m | 6.11 | 3.49 | 6.24 | 3.26 | 6.10 | 3.72 |
| | 70m | 6.42 | 3.66 | 5.85 | 3.42 | 6.41 | 3.91 |
| Mar | 50m | 5.69 | 3.34 | 5.85 | 3.06 | 5.67 | 3.49 |
| | 70m | 5.97 | 3.51 | 6.14 | 3.21 | 5.95 | 3.66 |
| Apr | 50m | 5.64 | 3.65 | 5.96 | 3.11 | 5.61 | 3.78 |
| | 70m | 5.92 | 3.83 | 6.25 | 3.27 | 5.89 | 3.97 |
| May | 50m | 5.61 | 3.56 | 5.91 | 3.09 | 5.58 | 3.64 |
| | 70m | 5.89 | 3.74 | 6.20 | 3.24 | 5.85 | 3.82 |
| Jun | 50m | 5.19 | 2.88 | 5.27 | 2.75 | 5.16 | 2.92 |
| | 70m | 5.44 | 3.02 | 5.53 | 2.89 | 5.42 | 3.06 |
| Jul | 50m | 4.56 | 2.65 | 4.69 | 2.45 | 4.53 | 2.66 |
| | 70m | 4.78 | 2.79 | 4.92 | 2.57 | 4.75 | 2.79 |
| Aug | 50m | 5.47 | 3.03 | 5.54 | 2.90 | 5.45 | 3.17 |
| | 70m | 5.74 | 3.18 | 5.82 | 3.04 | 5.72 | 3.33 |
| Sep | 50m | 6.05 | 3.34 | 6.12 | 3.20 | 6.03 | 3.51 |
| | 70m | 6.35 | 3.50 | 6.43 | 3.36 | 6.33 | 3.69 |
| Oct | 50m | 6.63 | 3.63 | 6.70 | 3.50 | 6.62 | 3.91 |
| | 70m | 6.96 | 3.81 | 7.03 | 3.67 | 6.95 | 4.11 |
| Nov | 50m | 6.97 | 3.80 | 7.04 | 3.68 | 6.96 | 4.10 |
| | 70m | 7.32 | 3.99 | 7.38 | 3.86 | 7.30 | 4.30 |
| Dec | 50m | 7.11 | 3.99 | 7.22 | 3.77 | 7.09 | 4.30 |
| | 70m | 7.46 | 4.19 | 7.58 | 3.96 | 7.44 | 4.51 |
| **Ave** | **50m** | **6.05** | **3.47** | **6.19** | **3.23** | **6.03** | **3.66** |
| | **70m** | **6.35** | **3.64** | **6.49** | **3.39** | **6.33** | **3.84** |

where $\bar{v}_a$, $\delta$ are the monthly mean wind speed of the Weibull, Rayleigh, Gamma functions, and its standard deviation at 50m and 70m hub heights respectively

**Table 11.** Summary of the various monthly mean shape and scale parameters @ 10m hub height

| 2010 | Wei | | Rayl | | Gam | |
|---|---|---|---|---|---|---|
| **Mon** | ***k*** | *C* | ***k*** | *C* | ***k*** | *C* |
| Jan | 1.92 | 6.93 | 2.00 | 6.99 | 2.88 | 2.13 |
| Feb | 1.91 | 5.62 | 2.00 | 5.67 | 2.98 | 1.67 |
| Mar | 1.85 | 5.24 | 2.00 | 5.35 | 2.94 | 1.58 |
| Apr | 1.68 | 5.22 | 2.00 | 5.47 | 2.48 | 1.87 |
| May | 1.68 | 5.13 | 2.00 | 5.39 | 2.56 | 1.78 |
| Jun | 1.94 | 4.75 | 2.00 | 4.78 | 3.45 | 1.22 |
| July | 1.87 | 4.22 | 2.00 | 4.30 | 3.27 | 1.14 |
| Aug | 1.96 | 5.61 | 2.00 | 5.05 | 3.27 | 1.36 |
| Sept | 1.95 | 5.52 | 2.00 | 5.54 | 3.18 | 1.53 |
| Oct | 1.98 | 6.06 | 2.00 | 6.08 | 3.14 | 1.71 |
| Nov | 1.97 | 6.34 | 2.00 | 6.36 | 3.08 | 1.82 |
| Dec | 1.91 | 6.47 | 2.00 | 5.53 | 2.94 | 1.95 |
| **Aver** | **1.89** | **5.59** | **2.00** | **5.63** | **3.01** | **1.65** |

**Table 12.** Summary of the various monthly mean shape and scale parameters @ 50m hub height

| **2010** | **Wei** | | **Rayl** | | **Gam** | |
|---|---|---|---|---|---|---|
| Jan | 1.83 | 8.52 | 2.00 | 8.68 | 2.61 | 2.90 |
| Feb | 1.82 | 6.88 | 2.00 | 7.04 | 2.68 | 2.27 |
| Mar | 1.76 | 6.39 | 2.00 | 6.60 | 2.64 | 2.15 |
| Apr | 1.58 | 6.28 | 2.00 | 6.72 | 2.20 | 2.55 |
| May | 1.61 | 6.26 | 2.00 | 6.68 | 2.36 | 2.37 |
| Jun | 1.87 | 5.84 | 2.00 | 5.94 | 3.13 | 1.65 |
| July | 1.77 | 5.12 | 2.00 | 5.29 | 2.90 | 1.56 |
| Aug | 1.88 | 6.16 | 2.00 | 6.25 | 2.96 | 1.84 |
| Sept | 1.88 | 6.82 | 2.00 | 6.92 | 2.95 | 2.05 |
| Oct | 1.90 | 7.48 | 2.00 | 7.57 | 2.86 | 2.31 |
| Nov | 1.91 | 7.86 | 2.00 | 7.95 | 2.88 | 2.42 |
| Dec | 1.85 | 8.00 | 2.00 | 8.15 | 2.72 | 2.60 |
| **Aver** | **1.81** | **6.80** | **2.00** | **6.98** | **2.74** | **2.22** |

**Table 13.** Summary of the various monthly mean shape and scale parameters @ 70m hub height

| **2010** | **Wei** | | **Rayl** | | **Gam** | |
|---|---|---|---|---|---|---|
| Jan | 1.83 | 8.94 | 2.00 | 9.12 | 3.04 | 2.61 |
| Feb | 1.82 | 7.23 | 2.00 | 7.40 | 2.69 | 2.38 |
| Mar | 1.76 | 6.71 | 2.00 | 6.93 | 2.64 | 2.26 |
| Apr | 1.58 | 6.59 | 2.00 | 7.06 | 2.20 | 2.67 |
| May | 1.61 | 6.57 | 2.00 | 7.00 | 2.36 | 2.49 |
| Jun | 1.87 | 6.13 | 2.00 | 6.24 | 3.13 | 1.73 |
| July | 1.77 | 5.37 | 2.00 | 5.54 | 2.90 | 1.64 |
| Aug | 1.88 | 6.47 | 2.00 | 6.56 | 2.96 | 1.93 |
| Sept | 1.88 | 7.15 | 2.00 | 7.25 | 2.95 | 2.15 |
| Oct | 1.90 | 7.85 | 2.00 | 7.93 | 2.86 | 2.43 |
| Nov | 1.91 | 8.25 | 2.00 | 8.33 | 2.88 | 2.54 |
| Dec | 1.85 | 8.39 | 2.00 | 8.56 | 2.72 | 2.73 |
| **Aver** | **1.81** | **7.14** | **2.00** | **7.33** | **2.78** | **2.30** |

The comparisons of the monthly and annual mean wind power densities were made using the varied and constant air densities. The Weibull model returns an annual value of 24.4%, Rayleigh model returns 17.3%, and Gamma model 30.0% at a 10m hub height for the year 2010. The high error percentage was as a result of the inability of the actual distribution function to accurately model the wind speed which is been used as the reference wind power density. As a result, the actual distibution function is often used as a bench-mark to check how much improvement other distribution functions have made over it. At a 50m hub height, the Weibull model returns an annual error value of 19.98%, Rayleigh model returns a small value of 14.64%, and Gamma, 31.81%. Also, at 70m hub height, the Weibull model returns an annual error value of 20.41%, Rayleigh model returns 14.92%, and Gamma 30.93% as shown in tables 6-8. To determine the accuracy of the models in fitting the wind speed at this site, an accuracy tests were was conducted using an independent wind speed dataset collected on a 40m hub height for the year 2009. The results of the accuracy tests are are summarized in the tables 14-16. The predicted wind distribution that represent the actual wind distribution with the lowest values of $\chi^2$, RMSE; and with the highest values of COD, R describe best the wind speed variation at this site. From the result in the tables, the Weibull COD and R values are 0.660 and 0.811 while the RMSE and Chi-square ($\chi^2$) error values are 4.0E-4 and 230E-4 at a 40m height. The Rayleigh COD, R, RMSE and $\chi^2$ values are 0.868, 0.931, 2E-4, and 147E-4. Also, the Gamma COD and R values are 0.504, 0.708 at a 40m height while the RMSE and $\chi^2$ error values are 8E-4 and 286E-4. Comparing the accuracy test results (COD, R, RMSE and $\chi^2$ values) at a 40m height for the year 2009; the best distribution function are selected according to the the highest value of COD and R; the lowest RMSE and $\chi^2$ values. At the different hub heights, the Rayleigh function proves to be the best function for the modeling of the wind speed data and prediction of the wind power density.

## 5. Conclusion

From several literatures, the Weibull function has been preferred and the most widely used for modeling of the wind speed at a given site. At Darling site, the Rayleigh function proved to be accurate for modeling and describing the wind speed variation at various hub heights as compared to the most accepted Weibull function. From the testing criteria, though the two most acceptable functions for **modeling of** the wind speed at this site are the Weibull and the Rayleigh functions. The Rayleigh shows significant improvement over the actual distribution as compared to the Weibull distribution. Also, from the accuracy test results, it can be seen that filtering the calm wind speed values will avoid underestimation of the wind parameters since most of these wind speed values only introduces error to the prediction model. The choice of Rayleigh function is based on the accuracy of the modeled wind speed and the testing criteria. From the predicted wind distribution, results show that the wind site is ideal for small to utility scale energy application. The wind resources assessment results are needed for evaluation of the wind site and sizing of the wind energy systems. Future study will focus on sizing of wind generators and analysis of wind energy generation at this site based on the predicted wind distributions. Hence, a detailed statistical analysis of the site's wind resources is a crucial tool when performing wind resources assessment for energy evaluation and generation at any given site

**Table 14.** Comparison of the accuracy test results using the Weibull distribution @40m hub height for the year 2009

| Mon | **COD** | **R** | $\chi^2$ | **RMSE** |
|---|---|---|---|---|
| Jan | 0.644 | 0.802 | 003 | 168 |
| Feb | 0.645 | 0.803 | 003 | 171 |
| Mar | 0.608 | 0.780 | 005 | 216 |
| Apr | 0.543 | 0.737 | 008 | 279 |
| May | 0.616 | 0.785 | 005 | 226 |
| Jun | 0.584 | 0.764 | 005 | 226 |
| July | 0.709 | 0.842 | 005 | 228 |
| Aug | 0.676 | 0.822 | 006 | 239 |
| Sept | 0.750 | 0.866 | 003 | 180 |
| Oct | 0.765 | 0.874 | 002 | 157 |
| Nov | 0.796 | 0.892 | 002 | 132 |
| Dec | 0.581 | 0.762 | 004 | 201 |
| **Aver** | **0.660** | **0.811** | **004** | **202** |

**Table 15.** Comparison of the accuracy test results using the Rayleigh distribution @40m hub height for the year 2009

| Mon | **COD** | **R** | $\chi^2$ | **RMSE** |
|---|---|---|---|---|
| Jan | 0.855 | 0.925 | 001 | 118 |
| Feb | 0.871 | 0.933 | 001 | 117 |
| Mar | 0.882 | 0.939 | 002 | 139 |
| Apr | 0.934 | 0.966 | 003 | 166 |
| May | 0.886 | 0.941 | 002 | 155 |
| Jun | 0.931 | 0.965 | 002 | 151 |
| July | 0.830 | 0.911 | 004 | 188 |
| Aug | 0.896 | 0.947 | 003 | 176 |
| Sept | 0.839 | 0.916 | 002 | 154 |
| Oct | 0.815 | 0.903 | 002 | 143 |
| Nov | 0.797 | 0.893 | 002 | 132 |
| Dec | 0.882 | 0.939 | 002 | 126 |

**Table 16.** Comparison of the accuracy test results using the Gamma distribution @40m hub height for the year 2009

| Mon | **COD** | **R** | $\chi^2$ | **RMSE** |
|---|---|---|---|---|
| Jan | 0.438 | 0.662 | 006 | 242 |
| Feb | 0.436 | 0.660 | 006 | 243 |
| Mar | 0.429 | 0.655 | 009 | 297 |
| Apr | 0.458 | 0.677 | 013 | 356 |
| May | 0.502 | 0.709 | 010 | 309 |
| Jun | 0.519 | 0.720 | 009 | 292 |
| July | 0.559 | 0.748 | 012 | 342 |
| Aug | 0.588 | 0.767 | 011 | 337 |
| Sept | 0.587 | 0.766 | 008 | 275 |
| Oct | 0.555 | 0.745 | 006 | 246 |
| Nov | 0.572 | 0.756 | 005 | 215 |
| Dec | 0.401 | 0.633 | 008 | 274 |
| **Aver** | **0.504** | **0.708** | **008** | **286** |

**Note:** The values of $\chi^2$ and RMSE are approximated to 4 significant figures (10E-4)

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