

A New Theoretical Model For Modeling The Wind Speed Frequency Distribution

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Abstract- The probabilistic speed-frequency distribution of the wind is essential for evaluation of the wind potential. Although wind commonly evaluated with weibull distribution, by assigning null value to calm winds it cannot envisage the existence of calm winds. Since for the sites with significant calm wind frequency Weibull distribution is uncertain, in this paper we present a new theoretical approach which employs Maximum Entropy Principle (MEP). The model is improvement of previous proposed MEP which applied to the synoptic sites distributed inside the Tunisian territory with significant calm winds frequency. The obtained results appropriately describe the distribution of measured wind speed data particularly calm speed over the MEP and Weibull models.

Keywords- Wind speed distribution, Calm wind, Meteorological method, Weibull method, Maximum entropy principle, Tunisia..

1. Introduction

The fast growing global need to energy is made wind energy as one alternative source of energy. Wind power stations contributed a significant penetration to the electricity network of the developing countries where certain inhabitants still live without electricity. Not only is the knowledge of the characterization of the wind speed distribution in the establishment sites of the wind projects is indispensable and specially for the evaluation of the wind potential and the selection of the favorable sites, but also in need for the dimensioning of the wind power stations and the suitable choice of the aerogenerators. Most adjustment functions such as Weibull and Rayleigh are used. A wide attention is given to the Weibull law because it has been proved that this method more adapts with majority of the wind pattern in the world [1-7]. The probability density function of Weibull is defined by:

$$f(V) = \begin{cases} \frac{K}{A} \left(\frac{V}{A}\right)^{K-1} \exp\left[-\left(\frac{V}{A}\right)^K\right]; & V \geq 0 \\ 0 & ; V < 0 \end{cases} \quad (1)$$

The function of Weibull distribution cannot predict the existence of the calm wind and it always takes a zero value at the calm speed ($V=0$). Consequently, for the sites where the calm wind is very significant during the measurement period, the Weibull distribution cannot provide an adjustment appropriate to the measured data, particularly for low speeds of the wind [1, 7, 10, 15-18].

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The principal goal of this study is to develop a theoretical model to envisage the speed-frequency distribution of the wind by accommodating of the calm winds. The model is established by the application of the maximum entropy principle (MEP), under constraints of conservation of mass, momentum and energy for the calm wind stream.

This model is an improvement of MEP models used in the wind analysis in previous studies, which intend to accommodate the frequencies of null wind speeds [1, 5, 7, 9-18].

We apply this method for the determination of the wind speed distributions in eight synoptic sites of the Meteorology National Institute (INM) distributed over Tunisian territory with significant frequency of the calm winds (table 1). The used data cover a five-year period (2004-2009) and are measured at height of 11 m above ground level in open country. The statistical approach is performed to calculate the annual values of the mean speed and the wind energy of the selected sites. The obtained results allow us to make a comparative study between the adjustment models: developed MEP with 5 constraints, MEP with 4 constraints cited in the literature and Weibull. Finally, we underline the appropriate distribution for assessment of the wind energy potential.

2. Maximum entropy principle

The entropy can be seen in view of measurement of uncertainty associated to a probability distribution. It is measured by calculating the information quantity associated to such distribution. The entropy concept of information was introduced by Shannon [9-18] in 1940. It showed that the entropy of a finished distribution of probability can be defined by the following expression:

$$S(P) = k \sum_i P_i \ln \left(\frac{1}{P_i} \right) \quad (2)$$

Where P_i is the occurrence probability of state i and $\ln \left(\frac{1}{P_i} \right)$ is the information quantity is received on state i .

In 1957, Jaynes [9-18] introduced the idea of the maximum entropy principle (MEP) under constraints to assign a probability law to a random variable. This optimization problem is written as:

$$\text{Max } S(P) = k \sum_i P_i \ln \left(\frac{1}{P_i} \right) \quad (3)$$

under constraints:

$$\sum_i P_i = 1 \quad (4)$$

$$\sum_i P_i \cdot g_{r,i} = \langle g_r \rangle, \quad r = 1, 2, 3, \dots, m \quad (5)$$

where the equation (3) represents the m physical constraints given by additional information available on the studied system. $g_{r,i}$ is an evaluated function in the state i and $\langle g_r \rangle$ is the expectation of average value of this function g over the entire system. The additional constraint (4) allows us to obtain a standardized probability distribution.

Based on the Lagrange method, the most probable distribution maximizing the entropy function under the constraints of equations (3) and (4) is form [10-18]:

$$P_i = \exp \left(-\alpha_0 - \sum_{r=1}^m \alpha_r \cdot g_{r,i} \right) \quad (6)$$

In the context of the wind energy, this MEP is already applied to determine the wind speed distribution. The MEP Models cited in the literature are limited to four constraints (denoted as PME_4). Furthermore the proposed model in study is formulated with five physical constraints (denoted as PME_5). The imposed constraints are preminialrly expressed from the following conservation laws:

- Mass conservation:

$$\dot{m} = \sum_i \rho P_i V_i S = \rho S V_{10} \quad (7)$$

- Momentum conservation:

$$\dot{M} = \sum_i (\rho P_i V_i S) \cdot V_i = \rho S V_{20}^2 \quad (8)$$

- Energy conservation:

$$\dot{E} = \sum_i (\rho P_i V_i S) \cdot \left(\frac{1}{2} V_i^2 \right) = \frac{1}{2} \rho S V_{30}^3 \quad (9)$$

- Calm wind occurrence probability conservation:

$$P_0 = \exp(-\alpha_0) \quad (10)$$

where V_{10} is the mean wind speed; V_{20} and V_{30} represent the equivalent mean wind speeds, traversing the rotor and producing respectively the

same force and same energy. P_0 is the occurrence probability of the calm wind speed.

Let us consider the air density and swept by the rotor blades are constant, the constraints equations (7)-(9) can be written in the form:

$$\sum_i P_i V_i = V_{10} \quad (11)$$

$$\sum_i P_i V_i^2 = V_{20}^2 \quad (12)$$

$$\sum_i P_i V_i^3 = V_{30}^3 \quad (13)$$

which enable us to calculate three speeds V_{10} , V_{20} and V_{30} starting from the measured wind data. Under the above constraints (10)-(13) and the probability normalization (4), the probabilistic distribution of the wind speed which maximizes the Shannon entropy is an exponential function, similar to the equation (6), of the form:

$$f(V_i) = P_i = \exp(-\alpha_0 - \alpha_1 V_i - \alpha_2 V_i^2 - \alpha_3 V_i^3 - \alpha_4 V_i^4) \quad (14)$$

The multiplier α_0 is computed directly from the measured frequency of the calm wind ($V=0\text{m/s}$), i.e.:

$$\alpha_0 = -\text{Log}[f(0)] \quad (15)$$

The other Lagrangian multipliers α_j ($1 \leq j \leq 4$), satisfy the condition of the distribution normalization (4) and the conservation laws (7)-(9) are determined by the equations system according to:

$$\begin{cases} \sum_i \exp[h(V_i)] - 1 \equiv \xi_1 = 0 \\ \sum_i V_i \cdot \exp[h(V_i)] - V_{10} \equiv \xi_2 = 0 \\ \sum_i V_i^2 \cdot \exp[h(V_i)] - V_{20}^2 \equiv \xi_3 = 0 \\ \sum_i V_i^3 \cdot \exp[h(V_i)] - V_{30}^3 \equiv \xi_4 = 0 \end{cases} \quad (16)$$

with: $h(V_i) = -\alpha_0 - \alpha_1 V_i - \alpha_2 V_i^2 - \alpha_3 V_i^3 - \alpha_4 V_i^4$

By knowing the mean speeds of V_{10} , V_{20} , V_{30} and the value of the multiplier α_0 (15), we can put the system of equations (16) in the following form:

$$\xi(\alpha_j) = 0 \quad (17)$$

where α_j ($1 \leq j \leq 4$) is a unknown vector formed by the four Lagrange multipliers.

To solve the system (17), we use the Newton-Raphson iterative method which requires the computation of Jacobian J matrix of the system, i.e.:

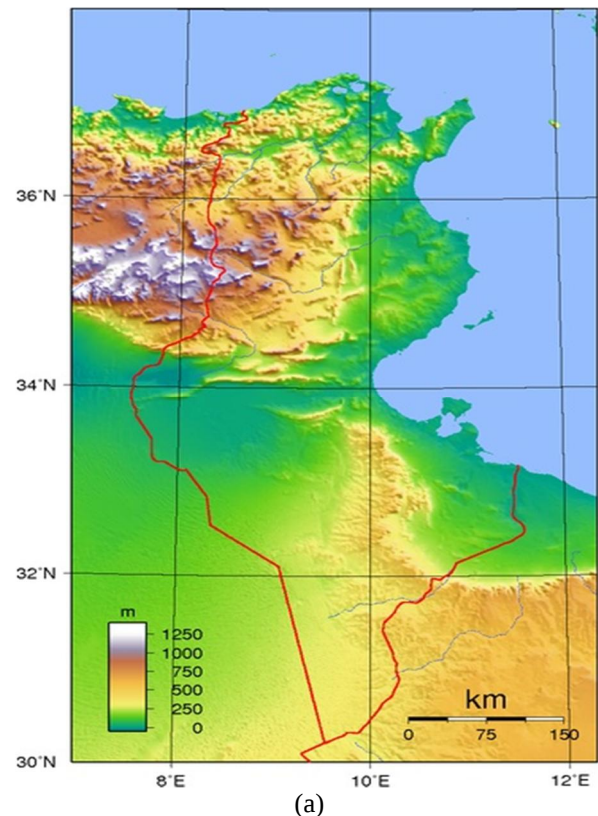
$$J = \left[\frac{\partial \xi}{\partial \alpha} \right] \quad (18)$$

Once α_j is founded, the wind distribution speed-frequency will be described by the model (14).

3. Application to Selected Sites

3.1. Geographical situation and relief of Tunisia

Tunisia, with an area of 164150 km², occupies a geographical zone between 30 and 37°N latitude and between 8 and 12°E longitude. It opens largely on the Mediterranean Sea with 1298 km of coasts, delimited in the West by Algeria and in the south by Libya. Tunisia is an entirely flat country, excluding the North-West and the West which are mountainous areas. It is under the Mediterranean influence as well as the continentalism which appears dices that we go away from the coast (Figure 1). The Maximum altitude is 1544m to the mountain Chaambi, close to Kasserine.



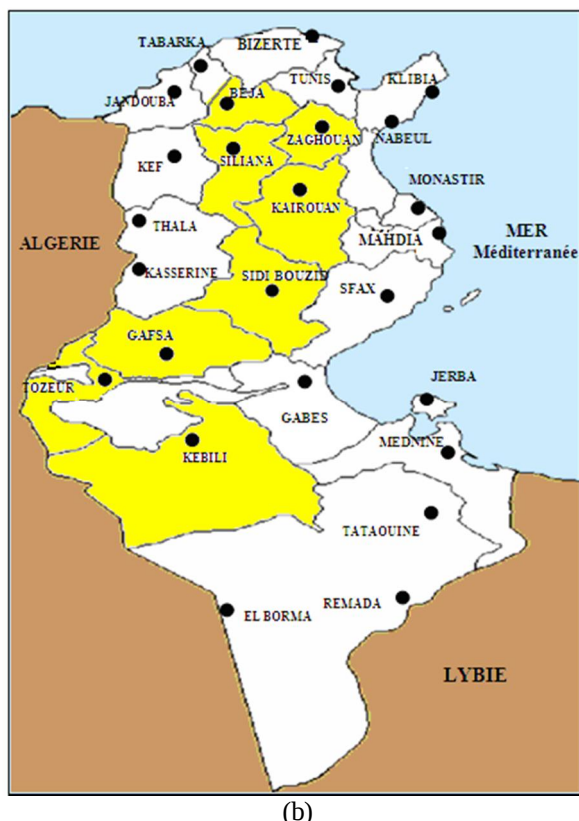


Fig. 1. Relief of Tunisia (a) and geographical position of selected sites (b) yellow zone.

Table 1. Geographical description of the studied meteorological stations.

Stations	Altitude (m)	Latitude (N)	Longitude (E)	Calm wind frequency (%)
Kebili	46	33°70'	8°96'	4.25
Tozeur	87	33°55'	8°06'	3.61
Gafsa	313	34°25'	8°49'	1.31
Zaghuan	156	36°43'	10°76'	5.10
Siliana	443	36°04'	9°22'	4.72
Sidi Bouzid	354	35°00'	9°29'	13.63
Beja	158	36°44'	9°11'	6.70
Kairouan	60	33°40'	10°06'	13.27

The eight sites with significant calm winds frequencies (Table 1) are selected for the study are localized inside the country (Figure 1b).

4. Wind energy potential computation

4.1. Proposed Methodologies

The hourly wind data cover the five-year periods (2004-2009). From the I.N.M. tables are indicating the occurrence frequencies $f(V)$ of the wind speeds, the wind characteristics are given by the relations in table 2.

Table 2. Evaluation methodologies of the wind characteristics.

Wind characteristics	Meteorological method	Weibull method	MEP method
Cumulated frequency	Tableau de l'I.N.M.	$F(V) = \exp \left[- \left(\frac{V}{A} \right)^k \right]$	$F(V) = \int_V^{+\infty} f(V) \cdot dV$
Occurrence frequency	$f(V) = F(V) - F(V+1)$	$f(V) = \frac{K}{A} \left(\frac{V}{A} \right)^{K-1} \exp \left[- \left(\frac{V}{A} \right)^K \right]$	MEP previous with 4 contraintes : $f(V) = \exp(-\alpha_0 - \alpha_1 V - \alpha_2 V^2 - \alpha_3 V^3)$ MEP proposed with 5 contraintes : $f(V) = \exp(-\alpha_0 - \alpha_1 V - \alpha_2 V^2 - \alpha_3 V^3 - \alpha_4 V^4)$
Mean speed	$V_m = \sum_{i=1}^n V_i f(V_i)$	$V_m = A \cdot \Gamma \left(1 + \frac{1}{k} \right)$	$V_m = \int_0^{+\infty} V f(V) \cdot dV$
Power density	$P_d = \frac{1}{2} \frac{16}{27} \rho \sum_{i=1}^n V_i^3 f(V_i)$	$P_d = \frac{1}{2} \frac{16}{27} \rho A^3 \cdot \Gamma \left(1 + \frac{3}{k} \right)$	$P_d = \frac{1}{2} \frac{16}{27} \rho \int_0^{+\infty} V^3 f(V) \cdot dV$
Ecart-type	$\sigma = \left[\sum_{i=1}^n (V_i - V_m)^2 f(V_i) \right]^{1/2}$	$\sigma = A^2 \left[\Gamma \left(1 + \frac{2}{k} \right) - \Gamma^2 \left(1 + \frac{1}{k} \right) \right]$	$\sigma = \left[\int_0^{+\infty} (V - V_m)^2 f(V) \cdot dV \right]^{1/2}$

The distribution of the annual available wind energy, at Betz limit, is expressed as below from:

$$E(V) = 8.76 \frac{1}{2} \frac{16}{27} \rho V^3 f(V) \quad (19)$$

To evaluate the obtained results by the different mentioned models, we calculated the relative error

in the wind power density in the following relation:

$$Err(\%) = 100 \cdot \left| \frac{P_{d_{Weibull/MEP}} - P_{d_{Meteo}}}{P_{d_{Meteo}}} \right| \quad (20)$$

4.2. Wind speed frequency distribution

The probability density functions of Weibull and those based on the maximum entropy principle (MEP_4 and MEP_5) are used for modeling of the

temporal variations of the wind speed frequency. All the Weibull parameters and the Lagrange multipliers for the annual distribution are presented in table 3.

Table 3. Weibull parameters and Lagrange multipliers of various sites.

Sites	Weibull parameters		Lagrange multipliers for MEP_4 previous distribution				Lagrange multipliers for MEP_5 developed distribution				
	A (m/s)	k	α_0	α_0	α_0	α_0	α_0	α_1	α_2	α_3	α_4
Kebili	5.52188	1.52257	2.59129	-0.240133	0.0383947	-0.000722661	3.15769	-1.05462	0.288015	-0.0261804	0.000811422
Tozeur	5.17167	1.79897	3.03259	-0.576811	0.0843701	-0.00196405	3.3221	-1.01229	0.232239	-0.0190761	0.000622267
Gafsa	4.97476	1.95152	3.6497	-1.09546	0.170609	-0.00500188	4.33689	-1.92875	0.437763	-0.0352408	0.00107447
Zaghouan	3.68632	1.46042	2.42122	-0.571665	0.125512	-0.00402487	2.97518	-1.60649	0.543657	-0.0597895	0.00228772
Siliana	3.01456	1.4371	2.29949	-0.725653	0.183691	-0.00653118	3.05313	-2.38902	0.988828	-0.135548	0.00637655
Sidi Bouzid	2.90128	1.35045	1.96527	-0.391775	0.117074	-0.00356359	1.99299	-0.464674	0.150149	-0.00821946	0.000190559
Beja	2.56429	1.30296	1.98767	-0.574412	0.176647	-0.0066883	2.70283	-2.54373	1.26849	-0.202748	0.0108278
Kairouan	2.32972	1.41321	1.86717	0.657386	0.228325	-0.0095228	2.01964	-1.19378	0.549575	-0.0693466	0.0032819

The representative curves of these three distributions are also plotted with the measured data and represented in figure 2. It should be noted that the proposed theoretical model MEP_5 is matching more with the measured data particularly at the calm speeds. It offers more accurate results with respect to Weibull and MEP_4 models. In contrast the theoretical distribution MEP_4 does not provide enough accurate frequency of calm wind speeds. Hence for the sites with significant calm winds during the year, the wind potential evaluation by the empirical distribution of Weibull and MEP_4 distributions are doubtful. On the other hand, the developed MEP_5 model accommodates the calm winds which always starts at not zero frequency and is equal to the measured value. Consequently, the developed distribution MEP_5 is more suitable with respect to Weibull and MEP_4 distributions models provide an appropriate adjustment to the measured data.

4.3. Wind characteristics

Table 4 presents the results of the wind characteristics (speeds and energies) obtained by the meteorological, Weibull, MEP_4 and the MEP_5 distribution models. By comparing the numerical values, can be observed the mean wind speed and the power density calculated by the proposed model MEP_5 perfectly agrees with

findings of the experimental method "meteorological". Indeed, the calculated relative error is approximately null. The results of the Weibull and MEP_4 methods are approximations of the experimental method, and are sometimes unsatisfactory. The obtained relative error can reach 12% with Weibull and 20% with MEP_4.

Similarly, the values of standard deviation σ of the various sites computed by the meteorological experimental distribution and the developed model MEP_5 are identical.

To investigate the different mentioned models, we calculated the following analysis statistical parameters [7-18]:

$$\left\{ \begin{array}{l} R^2 = \frac{\sum_{i=1}^n (y_i - y_m)^2 - \sum_{i=1}^n (y_{ic} - y_i)^2}{\sum_{i=1}^n (y_i - y_m)^2} \\ \chi^2 = \sum_{i=1}^n \frac{(y_i - y_{ic})^2}{y_i} \\ RMSE = \left[\frac{1}{n} \sum_{i=1}^n (y_i - y_{ic})^2 \right]^{1/2} \end{array} \right. \quad (21)$$

where R^2 is the determination coefficient, χ^2 is the chi-square coefficient and $RMSE$ is the root mean square error.

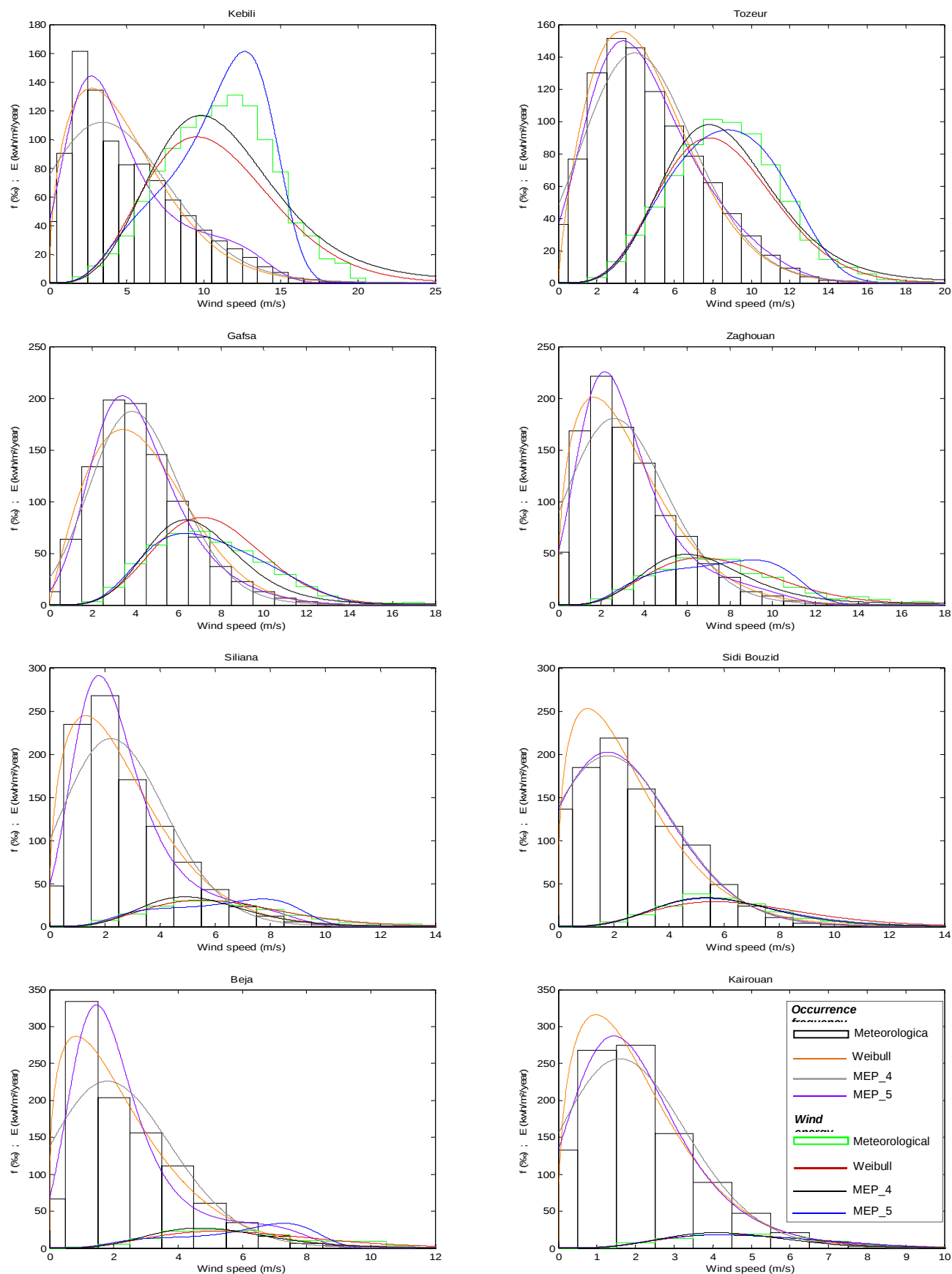


Fig. 2. Annual distributions of the wind speed frequency and the available energy of the various sites.

Table 4. Annual wind characteristics of the various sites.

Sites	V_m (m/s)				P (W/m ²)				σ (m/s)				Err (%)		
	M	W	MEP_4	MEP_5	M	W	MEP_4	MEP_5	M	W	MEP_4	MEP_5	W	MEP_4	MEP_5
Kebili	5.083	4.976	5.090	5.086	135.1	118.9	135.33	135.1	3.622	3.332	3.576	3.622	12	0.16	8.1E-9
Tozeur	4.661	4.599	4.665	4.663	82.22	75.60	82.28	82.22	2.816	2.645	2.805	2.816	8.1	0.08	1.1E-8
Gafsa	4.267	4.411	4.282	4.267	54.45	61.00	57.24	54.44	2.225	2.357	2.175	2.225	12	5.12	2.1E-9
Zaghouan	3.311	3.339	3.330	3.315	37.19	38.25	39.32	37.19	2.299	2.324	2.234	2.299	2.9	5.71	2.1E-8
Siliana	2.739	2.737	2.771	2.741	20.87	21.59	25.07	20.87	1.880	1.933	1.819	1.880	3.4	20.12	4.2E-8
Sidi Bouzid	2.657	2.660	2.669	2.668	20.98	21.95	21.12	20.98	2.008	1.991	2.004	2.008	4.6	0.67	2.2E-7
Beja	2.418	2.367	2.450	2.422	16.59	16.46	19.68	16.59	1.816	1.832	1.759	1.816	0.8	18.67	3.3E-7
Kairouan	2.092	2.120	2.114	2.102	10.24	10.31	11.43	10.24	1.563	1.521	1.540	1.563	0.7	11.59	3.4E-7

A distribution model is said ideal if it is characterized by a zero value for the two parameters χ^2 and RMSE and equal to 1 for the parameter R^2 .

These statistical indicators of precision are applied to the annual wind speed distribution of the

three models (Weibull, MEP_4 and MEP_5) and the obtained results are given in table 5. The comparison between the meteorological distribution and the three models shows that the developed model MEP_5 presents a higher adjustment.

Table 5. Statistical analysis parameters for the annual wind speed distribution.

Sites	Weibull distribution			MEP_4 previous distribution			MEP_5 developed distribution		
	R^2	χ^2	RMSE	R^2	χ^2	RMSE	R^2	χ^2	RMSE
Kebili	0.888921	0.080368	0.01517	0.868501	0.065599	0.0165056	0.948125	0.0291199	0.010366
Tozeur	0.963938	0.046027	0.009913	0.974096	0.0182043	0.0084015	0.996945	0.0030566	0.002885
Gafsa	0.966953	0.037278	0.012122	0.978479	0.0284087	0.0097824	0.999097	0.0012030	0.002004
Zaghouan	0.958728	0.057817	0.014142	0.938300	0.0558117	0.0172914	0.986987	0.0174808	0.007941
Siliana	0.968193	0.053920	0.015056	0.906634	0.1069720	0.0257956	0.985032	0.0233116	0.010328
Sidi- Bouzid	0.763019	0.175140	0.034633	0.990250	0.0067400	0.0070249	0.991459	0.0068026	0.006575
Beja	0.943493	0.081470	0.022037	0.831433	0.1376340	0.0380614	0.917692	0.0922387	0.026596
Kairouan	0.841403	0.144060	0.038119	0.774047	0.2217790	0.0454991	0.997523	0.0038717	0.004763

5. Conclusion

Evaluation of the wind energy potential for the eight sites with significant calm winds by the developed distributions models (Weibull, Rayleigh, MEP_4, etc.) during the two last decennies is doubtful. To alleviate this fault problem, a theoretical function of distribution based on the theory of the maximum entropy principle under five constraints (MEP_5) is developed and proposed. The proposed model MEP_5 is validated by comparison of the predicted data with the measured data with Weibull and MEP_4 distributions.

Several results of comparison between the three models of Weibull, MEP_4 and MEP_5 are applied and the results have been observed on sites at Tunisian territory. The parameters of statistical analysis relating to the wind speed distribution are

presented as fitting criteria. It is evident that the proposed distribution model MEP_5 has a significant adjustment to the measured wind speeds can evaluates the wind potential more accurate than Weibull and MEP_4 distributions models.

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Nomenclature

V	Wind speed (m/s)
$f(V)$	Occurrence frequency (%)
$F(V)$	Cumulated frequency (%)
P_i	Occurrence probability of state i
P_d	Wind power density (W/m^2)
n	Number of wind speed classes
E	Available energy ($kW/m^2/year$)
S	Surface swept by the turbine blades (m^2)
A	Weibull scale factor (m/s)
K	Weibull shape factor
P_d^{Meteo}	Mean power density calculated from the meteorological method (W/m^2)
$P_d^{Weibull/MEP}$	Mean power density calculated from the Weibull and MEP functions (W/m^2)
σ	Standard deviation (m/s)
ρ	Air density (1.225 kg/m^3)
α	Lagrange multipliers
k	Boltzmann constant
α_j	Lagrange multipliers ($j=0,\dots,4$)
Γ	Gamma function
Err	Relative error (%)
R^2	Determination coefficient
χ^2	Chi-square coefficient
$RMSE$	Root mean square error
y_i	i th measured value
y_{ic}	i th calculated value
y_m	Mean value
M	Meteorological method
W	Weibull method
MEP	Maximum Entropy Principle method