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The modified trial equation method to the (2+1)-dimensional Broer-Kaup-Kupershmidt equation and Kolmogorov-Petrovskii-Piskunov equation

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Abstract

Many methods have been developed by scientists to find solutions for nonlinear problems. In this paper, the general structure of the modified trial equation method (MTEM) is introduced, and MTEM is used to find some exact solutions of (2+1)-dimensional Broer-Kaup-Kupershmidt (BKK), Kolmogorov-Petrovskii-Piskunov (KPP) equations. Firstly, an algebraic equation system is obtained by reducing the nonlinear partial differential equation (NLPDE) to the ordinary differential equation under the travelling wave transformation. Travelling wave solutions are found by solving the obtained algebraic equation systems. By using Mathematica 9 program, three and two dimensional graphs for suitable parameters were plotted to analyze the physical behavior of wave solutions. MTEM is of great importance in finding exact solutions of some partial differential equations.

Keywords: *(2+1)-dimensional BKK equation, KPP equation, travelling wave solution.*

(2+1)-boyutlu Broer-Kaup-Kupershmidt denklemi ve Kolmogorov-Petrovskii-Piskunov denklemine modifiye edilmiş deneme denklem metodu

Öz

Lineer olmayan problemlerin çözümünü bulmak için bilim insanları tarafından birçok yöntem geliştirilmiştir. Bu yazıda, modifiye edilmiş deneme denklem metodunun (MEDDM) genel yapısı tanıtılmış ve (2+1)-boyutlu Broer-Kaup-Kupershmidt (BKK), Kolmogorov-Petrovskii-Piskunov (KPP) denklemlerinin bazı tam çözümlerini bulmak

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için MEDDM kullanılmıştır. İlk olarak, hareketli dalga dönüşümü altında lineer olmayan kısmi diferansiyel denklemin (NLPDE) adi diferansiyel denkleme indirgenmesiyle bir cebirsel denklem sistemi elde edilmiştir. Elde edilen cebirsel denklem sistemleri çözülerek hareketli dalga çözümleri bulunur. Mathematica 9 programı kullanılarak, dalga çözümlerinin fiziksel davranışını analiz etmek için uygun parametreler için üç ve iki boyutlu grafikler çizilmiştir. MEDDM, bazı kısmi diferansiyel denklemlerin tam çözümlerini bulmada büyük önem taşımaktadır.

Anahtar Kelimeler: *(2+1)-boyutlu BKK denklemi, KPP denklemi, hareketli dalga çözümü.*

1.Introduction

Partial differential equations are mathematical models of physical events. Finding algebraic calculations of complex nonlinear equations has an important place in soliton theory. Many effective methods have been developed to solve these physical problems [1-5]. Studies on (2+1)-dimensional BKK equation have been carried out in many application areas of nonlinear optics, mathematics and physics. In recent years, the many scientists have obtained the solutions of the Broer-Kaup-Kupershmidt equation by applying different methods. Some of those, Song et al. obtained several new solutions to the (2+1)-dimensional BKK Equation by using the bifurcation method and the qualitative theory of dynamic systems [6]. Gurefe et al. obtained new solutions by using an irrational trial equation method method to solve the (2+1)-dimensional BKK Equations [7]. They have obtained the solutions by applying the nonlinear (2+1)-dimensional BKK Equation to different methods.

Recently, many scientists have used various methods to obtain new solutions of Kolmogorov-Petrovskii-Piskunov (KPP) equation. Rouhparvar has obtained new exact solutions by using the integral method to the (KPP) equation [8]. Feng has found hyperbolic, trigonometric and rational wave solutions using the (G'/G)-expansion method [9]. Since finding the solution of nonlinear problems and obtaining algebraic calculations is complex, many methods have been developed not only with a few methods.

Some of those are Hirota's Bilinear method [11, 12], the Jacobi elliptic function method [13], the Tanh Method [14-16], Simple Equation Method [17], Generalized Kudryashov method [18-21], Exp-Function method [22-23], Homogenous Balance Method [24-26], Modified Trial Equation Method [27]. They are effective methods for finding wave solutions of nonlinear problems. In this study, we have applied MTEM to obtain the exact solution of (2+1)-dimensional BKK Equation and KPP Equation. By reducing NLPDE to nonlinear ordinary differential equation (NLODE), an algebraic equation system was obtained by Mathematica 9. By solving these system, travelling wave solutions have been found. The purpose of this method is to find the traveling wave solutions of NLPDEs. In the light of these data, it is a suitable method to find the solutions of NLPDEs.

We consider the following (2+1)-dimensional BKK equation [6-7];

$$m_{ty}-m_{xxy}+2(mm_x)_y+2s_{xx}=0,$$
$$s_t+s_{xx}+2(ms)_x=0. \tag{1}$$

and the following KPP equation [8-10];

$$u_t-u_{xx}+\mu u+vu^2+\delta u^3=0. \tag{2}$$

Here, in μ, v and δ are real valued constants.

2.The Modified Trial Equation Method

Step 1: Consider the NLPDE,

$$P\left(u,u_t,u_x,u_{xx},...\right)=0, \tag{3}$$

wave transform as,

$$u\left(x,t\right)=u\left(\xi\right)=u(kx-wt)\,, \tag{4}$$

where w is a constant. Applying Eq.(4) to Eq.(3), we can observe the following nonlinear ordinary differential equation (NLODE).

$$O\left(t,x,u,u',u'',\cdots\right)=0, \tag{5}$$

where $u'=\dfrac{du}{d\xi}$.

Step 2. The first order trial equation,

$$u'=\frac{M\left(u\right)}{N\left(u\right)}=\frac{\sum_{i=0}^{n}a_iu^i}{\sum_{j=0}^{l}b_ju^j}=\frac{a_0+a_1u+a_2u^2+\ldots+a_nu^n}{b_0+b_1u+b_2u^2+\ldots+b_lu^l}, \tag{6}$$

and

$$u''=\frac{M\left(u\right)\left[M'\left(u\right)N\left(u\right)-M\left(u\right)N'\left(u\right)\right]}{N^3\left(u\right)}, \tag{7}$$

where $M(u)$ and $N(u)$ are polynomial of u. Substituting Eq.(6) and Eq.(7) into Eq.(5), we get

$$\sigma(u) = \chi_0 + \chi_1 u + \ldots + \chi_r u^r = 0. \tag{8}$$

Step 3. Equating the coefficients of $\sigma(u)$ to zero, we can obtain

$$\chi_p = 0, \quad p = 0, \ldots, r. \tag{9}$$

Solving the system (9), we can find the values of $a_0, \ldots, a_n$ and $b_0, \ldots, b_l$.

Step 4. Consider Eq. (6), the following integral form can be written

$$\xi - \xi_0 = \int \frac{N(u)}{M(u)} du. \tag{10}$$

Using the complete discrimination system with the roots of $M(u)$, we obtain exact solutions of Eq. (3).

3.Application to (2+1)-dimensional BKK equation

Getting transformation as

$$m = \varphi(\eta), \quad s = \phi(\eta), \quad \eta = kx + qy + wt, \tag{11}$$

Eq.(1) converts to

$$\begin{aligned} &qw\varphi'' - k^2 q\varphi''' + 2kq(\varphi\varphi')' + 2k^2\phi'' = 0, \\ &w\phi' + k^2\phi'' + 2k(\varphi\phi)' = 0. \end{aligned} \tag{12}$$

Then, it becomes the following NLODE,

$$qk^4\varphi'' - 2qk^2\varphi^3 - 2qwk\varphi^2 - qw^2\varphi + Z_1 = 0, \tag{13}$$

where Z_1 is constant.

The from the balancing principle in Eq. (13), n and l are determined by as $n = l + 2$.

Case 1:

For $l=0$ and $n=2$ then

$$u'=\frac{a_0+a_1u+a_2u^2}{b_0}, \tag{14}$$

$$u''=\frac{\left(a_0+a_1u+a_2u^2\right)\left(a_1+2a_2u\right)}{{b_0}^2}, \tag{15}$$

where $a_2\neq 0$ and $b_0\neq 0$. Then, an algebraic equation system is obtained. By solving these system, the following solutions have been found:

Case 1.1:

$$w=-3\sqrt[3]{\frac{kZ_1}{5q}},\ \ a_2=-\frac{a_1}{2}\sqrt[3]{\frac{5k^2q}{Z_1}},\ \ a_0=-\frac{a_1}{4}\sqrt[3]{\frac{25Z_1}{k^2q}},\ b_0=\pm\frac{a_1}{2}\sqrt[3]{\frac{5k^5q}{Z_1}}\ \ . \tag{16}$$

When we substitute Eq.(16) into Eq.(10), we get the following trigonometric function solution,

$$u(x,y,t)=\sqrt[3]{\frac{Z_1}{5k^2q}}-\sqrt[6]{\frac{27Z_1^2}{200k^4q^2}}\tan\left[\pm\sqrt[6]{\frac{27k^2Z_1^2}{200q^2}}\left(x+\frac{q}{k}y-3\sqrt[3]{\frac{Z_1}{5k^2q}}t\mp 10\sqrt[3]{k^4q^2}\eta_0\right)\right]. \tag{17}$$

Case 2:

For $l=1$ and $n=3$ then

$$u'=\frac{a_0+a_1u+a_2u^2+a_3u^3}{b_0+b_1u}, \tag{18}$$

and

$$u''=\frac{\left(a_0+a_1u+a_2u^2+a_3u^3\right)\left(\left(b_0+b_1u\right)\left(a_1+2a_2u+3a_3u^2\right)-b_1\left(a_0+a_1u+a_2u^2+a_3u^3\right)\right)}{\left(b_0+b_1u\right)^3}, \tag{19}$$

Where $a_3\neq 0$. Then, an algebraic equation system is obtained. By solving these system, the following solutions have been found:

Case 2.1:

$$a_1 = \frac{\left(-159-82i\sqrt{6}\right)a_3^5 Z_1^2}{200q^2 b_0^4},\ a_2 = \frac{\left(18-11i\sqrt{6}\right)a_3^3 Z_1}{20qb_0^2},\ a_0 = \frac{\left(-394+13i\sqrt{6}\right)a_3^7 Z_1^3}{800q^3 b_0^6},$$
$$b_1 = \frac{2\left(14+3i\sqrt{6}\right)qb_0^3}{25a_3^2 Z_1},\ k = \frac{2\left(-14-3i\sqrt{6}\right)qb_0^3}{25a_3^2 Z_1},\ w = \frac{3i\left(2i+\sqrt{6}\right)b_0}{5a_3}, \tag{20}$$

Substituting Eq.(20) into Eq.(10), we get the following trigonometric function solution,

$$u_1(x,y,t) = \varepsilon \tan\left[\frac{\sqrt{-7923+2046\sqrt{6}}a_3^2 Z_1}{200q^2 b_0^5}\left(\frac{\left(28+6i\sqrt{6}\right)qb_0^3}{25a_3^3 Z_1}x + qy + \frac{6-3i\sqrt{6}}{5a_3}ta_3^3 Z_1 + 800q^3 b_0^7 C[1]\right)\right], \tag{21}$$

where $\varepsilon = \frac{a_3^2 Z_1}{20qb_0^2}\left(2-4i\sqrt{6}+\sqrt{-6(23+4\sqrt{6})}\right).$

4.Application to KPP Equation

Getting transformation as

$$u\left(x,t\right) = u\left(\xi\right) = u(kx-wt)\ , \tag{22}$$

Eq.(2) converts to

$$-wu_\xi - k^2 u_{\xi\xi} + \mu u + vu^2 + \delta u^3 = 0. \tag{23}$$

The from the balancing principle in Eq.(23), n and l are determined by as $n = l+2$.

Case 1:

For $l=0$ and $n=2$ then

$$u' = \frac{a_0 + a_1 u + a_2 u^2}{b_0}, \tag{24}$$

$$u'' = \frac{\left(a_0 + a_1 u + a_2 u^2\right)\left(a_1 + 2a_2 u\right)}{b_0^{\ 2}}, \tag{25}$$

where $a_2 \neq 0$ and $b_0 \neq 0$. Then, an algebraic equation system is obtained. By solving these system, the following solutions have been found:

Case 1.1:

$$a_0=\frac{\mu a_1}{v},\quad w=-\frac{kv}{\sqrt{2\delta}},\ a_2=\frac{\delta a_1}{v},\ b_0=\frac{ka_1\sqrt{2\delta}}{v}\quad . \tag{26}$$

when we substitute Eq. (26) into Eq. (10), we the following trigonometric function solution,

$$u(x,t)=-\frac{v}{2\delta}-\frac{\sqrt{v\left(-4\delta a_0+a_1\right)}}{2\delta\sqrt{a_1}}\tan\left[\frac{\sqrt{a_1\left(-4\delta a_0+a_1\right)}}{2b_0\sqrt{v}}\left(kx-\frac{v}{\sqrt{2\delta}}t+vb_0C[1]\right)\right]. \tag{27}$$

Case 1.2:

$$a_0=0,\ \ a_2=\frac{k^2v^2a_1^2+\sqrt{k^4v^2\left(v^2-4\delta\mu\right)a_1^4}}{2k^2v\mu a_1},\ b_0=\sqrt{\frac{k^2\left(v^2-2\delta\mu\right)a_1^2+\sqrt{k^4v^2\left(v^2-4\delta\mu\right)a_1^4}}{\delta\mu^2}}\ . \tag{28}$$

when we substitute Eq. (28) into Eq. (10), we get the following exp-function solution,

$$u(x,t)=\frac{2\mu}{\exp\left(2\mu+\frac{\mu a_1\sqrt{\delta}}{\sqrt{k^2\left(v^2-2\delta\mu+v\sqrt{v^2-4\delta\mu}\right)a_1^2}}\left(kx-\frac{-\theta\sqrt{\frac{\theta}{\delta\mu^2}}}{4k^2\delta a_1^3}t\right)\right)+v+\sqrt{v^2-4\delta\mu}C[1]}, \tag{29}$$

where $\theta=\left(k^2\left(v^2-6\delta\mu\right)a_1^2+\sqrt{k^4v^2\left(v^2-4\delta\mu\right)a_1^4}\right).$

Case 2:

For $l=1$ and $n=3$ then

$$u'=\frac{a_0+a_1u+a_2u^2+a_3u^3}{b_0+b_1u}, \tag{30}$$

and

$$u''=\frac{\left(a_0+a_1u+a_2u^2+a_3u^3\right)\left(\left(b_0+b_1u\right)\left(a_1+2a_2u+3a_3u^2\right)-b_1\left(a_0+a_1u+a_2u^2+a_3u^3\right)\right)}{\left(b_0+b_1u\right)^3}, \tag{31}$$

where $a_3\neq 0$. Then, an algebraic equation system is obtained. By solving these system, the following solutions have been found:

Case 2.1:

$$a_0 = a_0,\ a_1 = a_0\left(\frac{v}{\mu}+\frac{b_1}{b_0}\right),\ a_2 = \frac{a_0\left(\delta b_0 + v b_1\right)}{\mu b_0},\ k = -\frac{\mu b_0}{\sqrt{2\delta a_0}},\ w = -\frac{v\mu b_0}{2\delta a_0}\ . \tag{32}$$

Substituting Eq. (32) into Eq. (10), we get the following trigonometric function solution,

$$u(x,t) = \frac{-v+\sqrt{-v^2+4\delta\mu}}{2\delta}\tan\left[\frac{\sqrt{-v^2+4\delta\mu}}{4k\sqrt{\delta}}\left(-\frac{\mu b_0}{\sqrt{2\delta a_0}}x+\frac{v\mu b_0}{2\delta a_0}t-2\sqrt{\frac{\delta}{2}}\right)\right]. \tag{33}$$

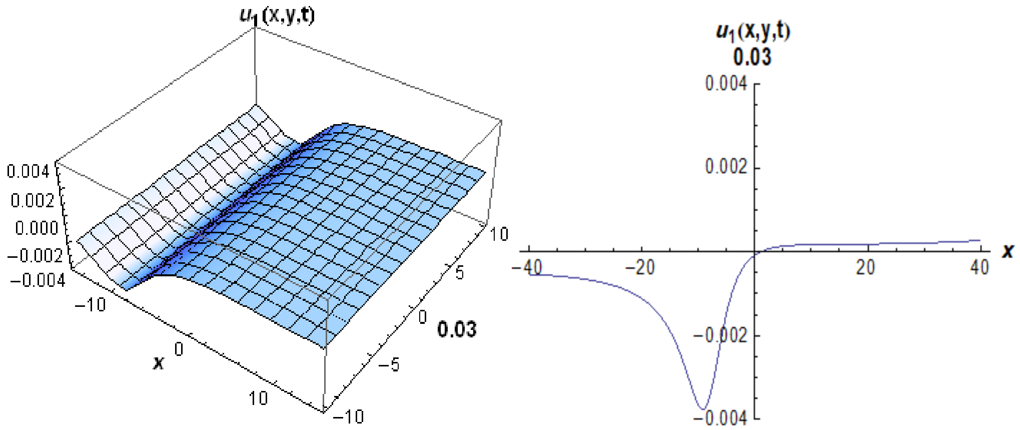

Figure 1. The 3D and 2D surfaces of real values of Eq.(21) for $a_3 = 0.2, q = 1.3, b_0 = 6$, $Z_1 = 1, -15 \le x \le 15, -10 \le t \le 10$ and $y = 0.01$, $t = 0.03$ for 2D.

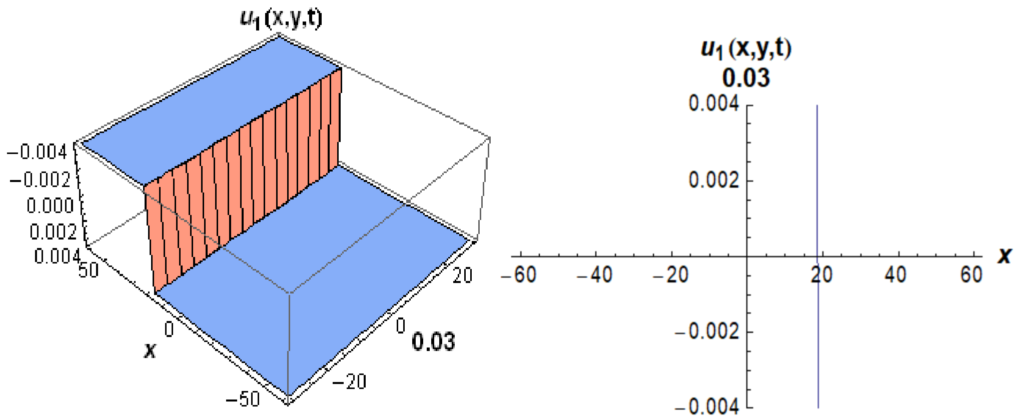

Figure 2. The 3D and 2D surfaces of imaginary values of Eq.(21) for $a_3 = 3, b_0 = 1, q = 7$, $Z_1 = 1$, $-60 \le x \le 60$, $-30 \le t \le 30$, and $y = 0.05, t = 0.03$ for 2D.

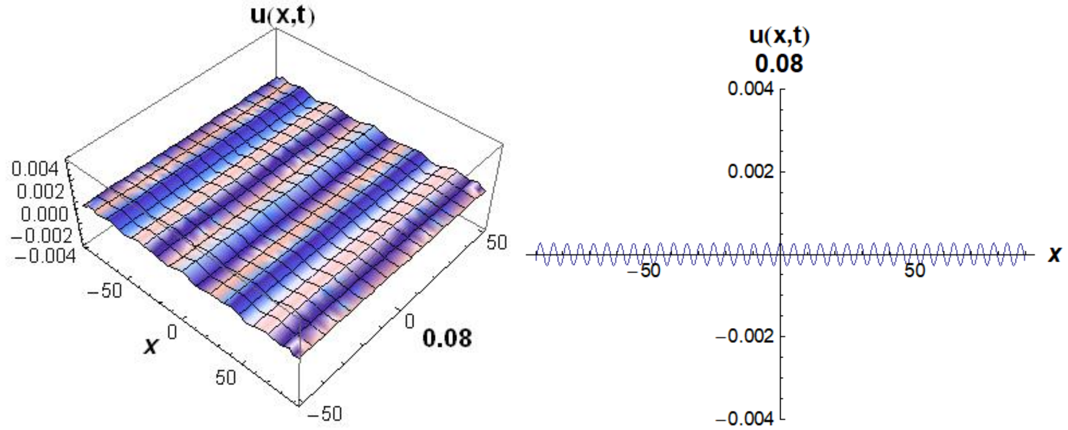

Figure 3. The 3D and 2D surfaces of real values of Eq.(29) for $v=3$, $\delta=-10$, $\mu=5$, $a_1=-7$, $b_0=10$, $k=1, -90 \le x \le 90$, $-50 \le t \le 50$, and $t=0.08$ for 2D.

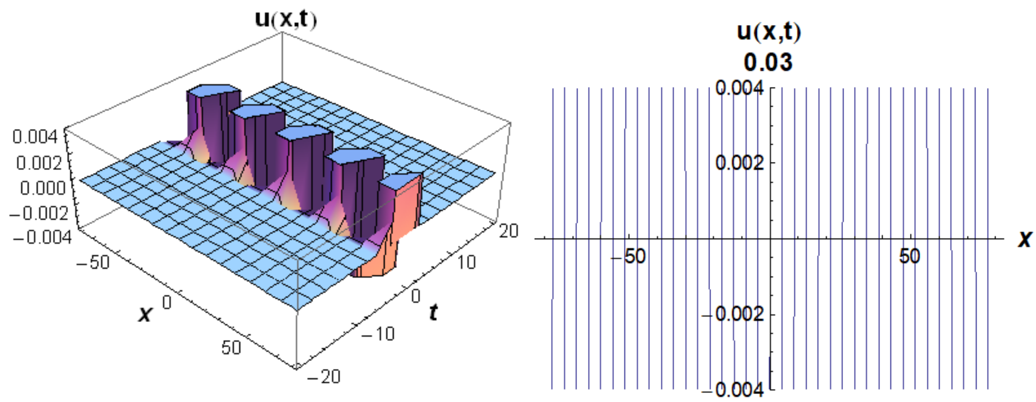

Figure 4. The 3D and 2D surfaces of imaginary values of Eq.(29) for $a_1=-3, b_0=7$, $v=2, \delta=-5$ $\mu=2$, $k=1, -80 \le x \le 80, -20 \le t \le 20$, and $t=0.08$ for 2D.

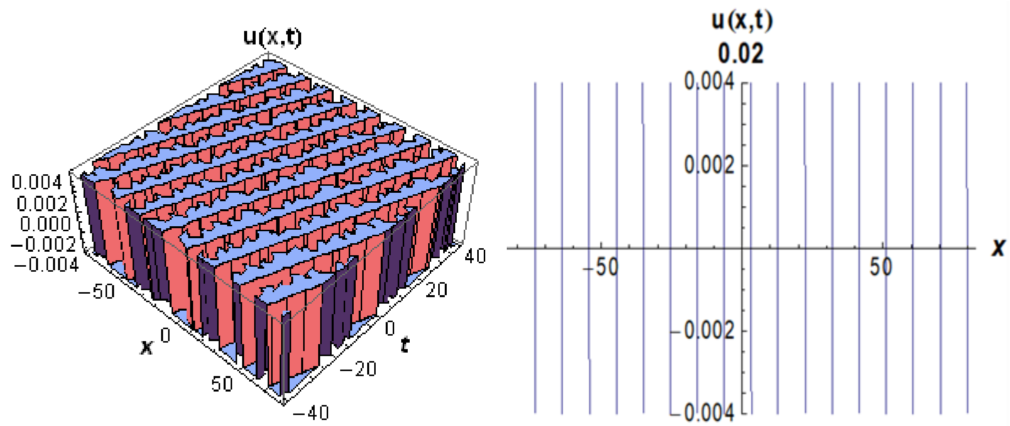

Figure 5. The 3D and 2D surfaces of real values of Eq.(33) for $a_0=-10, b_0=-5, v=3$, $\delta=8, \mu=2, k=1$, $-80 \le x \le 80, -40 \le t \le 40$, and $t=0.02$ for 2D.

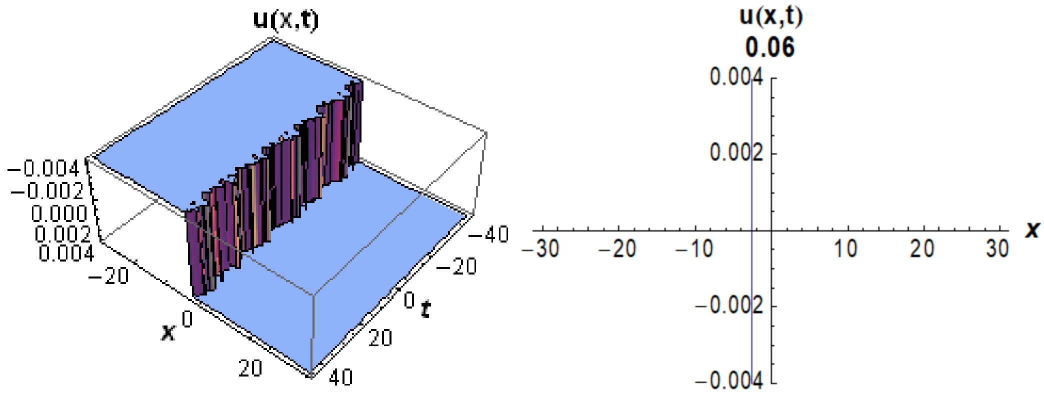

Figure 6. The 3D and 2D surfaces of imaginary values of Eq.(33) for $a_0 = 7, b_0 = -2$, $v = -5, \delta = -3, \mu = 9, k = 3$, $-30 \le x \le 30$, $-45 \le t \le 45$, and $t = 0.06$ for 2D.

In Figs. 1-2, we plot 3D and 2D surfaces of real and imaginary values of $u_1(x, y, t)$ in Eq.(21), which explain the vitality of solutions with appropriate parameters. Also, in Figs. 3-4, we draw 3D and 2D surfaces of real and imaginary values of $u(x,t)$ in Eq.(29), which demonstrate the dynamics of solutions with proper parameters. Finally, in Figs. 5-6, we plot 3D and 2D surfaces of real and imaginary values of $u(x,t)$ in Eq.(33), which show the vitality of solutions with appropriate parameters.

Remark

In this study, travelling wave solutions of Eq. (1) and Eq. (2) are obtained by using MTEM. Also, these solutions were checked in Mathematica 9. The solutions of Eq. (1) are new. Our (33) solution of Eq. (2) is similar to the u_5, u_6, solutions given by Rouhparvar. According to our research, other solutions of Eq. (2) are not given before and are new.

5. Conclusions

In this article, travelling wave solutions of (2+1)-dimensional BKK equation and KPP equation are obtained by using MTEM. Exp-function, trigonometric wave solutions were found by applying this method to the submitted problems. Three and two-dimensional graphs were drawn for appropriate parameters by using Mathematica 9. Physical behaviors were examined by demonstrating three and two dimensional graphics for some values of the parameters. It can be said that MTEM is an effective for finding exact solutions of NLPDEs and it is an important method for obtaining travelling wave solutions. Also, this is a very significiant method for solving nonlinear problems.

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