JEOFİZİK 6,67-75, 1992

APPLICATION OF KALMAN FILTER TO SYNTHETIC SEISMIC TRACES

Kalman Süzgecinin Sismik Verilere Uygulanması

Ali SAYMAN*

ABSTRACT

In this paper, the application of the Kalman filter for the seismic deconvolution problem is discussed. The behaviors of these feedback filters with different dimensions are examined by applying them to different kind of synthetic seismic traces. The outputs of discrete Kalman filter for deconvolution are noisy as expected. For this reason, optimal fixed interval smoothing filters are applied to the outputs of these filters to attenuate high frequency energy. From the outputs of the filters, it has been observed that the Kalman and the fixed-interval smoothing filters are successful in random noise suppression.

ÖZET

Bu yazıda geri beslemeli bir süzgeç türü olan Kalman süzgecinin sismik tersevrişim işlemlerine uygulanması yapılarak, çeşitli boyutlardaki süzgeçlerin farklı nitelikteki sismik izler karşısındaki davranışı tartışılmıştır. Beklenildiği gibi, tersevrişim için ayrık Kalman süzgecinin çıktıları gürültülüdür. Bu süzgeçlerin çıktılarına yüksek sıklıktaki enerjiyi sönümlemek için, en iyi belirli aralık yuvarlatma süzgeci uygulanmıştır. Kalman ve belirli aralık yuvarlatma süzgeçlerinin çıktıları, bu süzgeçlerin gürültü bastırmada başarılı olduklarını göstermiştir.

INTRODUCTION

One of the major problems of exploration seismology is the devonvolution of seismic traces. Most of the algorithms use the Wiener filtering. In the present paper, we have attempted to use the Kalman filtering. The first attempt to use the Kalman filtering for seismic deconvolution has been made by Bayless and Brigham (1970). Ott and Meder (1972) have used the Kalman filtering and the associated state-space representation to design a prediction error filter. Crump (1974) has used the discrete Kalman filter for deconvolution of seismic traces. Mendel (1976, 1977) has used the Kalman filtering to obtain the optimal smoothing estimation for the reflection coefficient series. Mendel and Kormylo (1977, 1978) and Kormylo and Mendel (1980) have developed a Kalman filtering approach to obtain optimal estimation of the reflection coefficient sequence. Their approach is applicable to the time varying or time invariant processes along with stationary or nonstationary noise processes. Mendel et al. (1981) have demonstrated some examples of the use of the state-space representation models in seismic applications. Aminzadeh and Mendel (1983) have studied the state-space representation of normal incidence waves propagating in a multilayered dissipative medium.

In this paper, the discrete Kalman filters with different dimensuons are applied to synthetic seismic traces for deconvolution purpose. The optimum fixed-interval smoothing filter with two different dimensions are also used to eliminate the high frequency energy. The RMS values of the filter outputs are compared with those of the input signal

DISCRETE KALMAN FILTER

State and measurement equations for a single channel data are given (Medich (1969) and Mendel (1979)) as

$$\underline{x}(k+1) = \Phi\, \underline{x}(k) + \wp\mu(k) \qquad (1)$$

$$z(k)= h'\underline{x}(k) + v(k) \qquad (2)$$

In the state equation : $\underline{x}$ is the nx1 state vector; Φ is the nxn state transition matris; $\wp$ (k) is the input distribution vector (disturbance transition vector); μ (k) is the scalar input

* Valideçeşme Armağan Sokak, No: 31, Beşiktaş, İstanbul.

noise value. In the measurement equation: z(k) is the scalar measurement (observation) value; v(k) is the scalar measurement (observation) noise value; h(k) is nx1 the measurement vector and h′(k) denotes transpose of h(k); and k= 0, 1, 2, 3, ... is the discrete time index.

In this paper, I have made some assumptions about two noise sequences (k) and v(k). These are:

a) $\mu(i)$ and v(i) are two zero mean sequences.

$$E[\mu(i)] = 0 : i > 0 \quad (3)$$
$$E[v(i)] = 0 : i > 0$$

b) $\mu(i)$ and v(i) are stationary white noise sequences.

c) The variance of $\mu(i)$ and v(i) are known and denoted by q and r, respectively:

$$E[\mu(i)\,\mu(j)] = q\,\delta ij \quad (4)$$
$$E[v(i)\,\mu(j)] = r\,\delta ij$$

where δij is the Kronecker delta and defined as unity when i = j, and zero othervise.

d) $\mu(i)$ and v(i) are uncorrelated.

$$E[\mu(i)\,v(j)] = 0 \quad (5)$$

The Kalman filter can be applied to measurement sequence z(1), z(2), ..., z(k). For simplicity, I use "k" to indicate the time index for the present time. Any time t (t_k and t) t_k indicate the past and future times, respectively.

$\hat{\underline{x}}(k|j)$ indicates the optimal filtered estimate of $\underline{x}$ (k) based on all measurements of z(1), z(2), ..., z(k). According to this representation, for j=k, the estimate of $\underline{x}(k)$ is the optimal filtered estimate; for j<k, the estimate of $\underline{x}(k)$ is the optimal predicted estimate; for j>k, the estimate of $\underline{x}(k)$ is the optimal smoothed estimate.

The Kalman filter algorithm for a single channel data can be represented by the following prediction and correction equations.

Prediction equation

$$\hat{\underline{x}}(k+1\,|k) = \Phi\,(k,k)\,\hat{\underline{x}}\,(k\,|\,k) \quad (6)$$
$$P(k+1\,|\,k) = \Phi\,P\,(k\,|\,k)\,\Phi' + \wp\,q\,\wp \quad (7)$$

Correction equations

$$\hat{\underline{x}}\,(k+1\,1\,k+1) = \hat{\underline{x}}\,(k+1\,|\,k) + K(k+1)[z(k+1) - h'\hat{\underline{x}}\,(k+1\,|\,k)] \quad (8)$$
$$K(k+1) = P(k+1\,|\,k+1)\,h\,[h'P\,(k+1\,|\,k)h + r]^{-1} \quad (9)$$
$$P(k+1\,|\,k+1) = [I - K(k+1)\,h']\,P\,(k+1\,|\,k) \quad (10)$$

In these equations: $\hat{\underline{x}}$ (k+1 | k+1) is the nx1 optimal filtered estimate; $\hat{\underline{x}}$ (k+1 | k) is the nx1 optimal predicted estimate; X(k+1 | k+1) is the nxn filtered error covarience matrix; P(k+1 | k) is the nxn predicted error covariance matrix; K(k+1) is the nxn Kalman gain matrix; I is the nxn identity matrix. Additionally, P(010) = P(0) and $\hat{\underline{x}}$ (010) = $\hat{\underline{x}}$ (0) are taken as initial conditions.

In the calculation of Φ, $\wp$, and h, the Laplace transform method which was discussed in detail in Mendel and Kormylo (1978) is used.

OPTIMAL FIXED INTERVAL SMOOTHING FILTER

The Kalman filter with k(j) is used for the smoothing process which was explained in the previous section. The state space representation of the smoothed estimates was given by eq. (1). In that equation $\hat{\underline{x}}$ (k | N), k=0, 1, 2, ..., N-1, N is called the the fixed-interval smoothed estimate; $\hat{\underline{x}}$ k | j), j = k-1, k-2, ..., N is called the fixed-point estimate; $\hat{\underline{x}}$ (k | k+N), k = 0, 1, 2, ...1, N is the fixed-lag smoothed estimate. In this section, I will be interested in the fixed-interval smoothing estimate.

The optimal fixed-interval smoothing estimate for single channel data is given in the following equations:

$$\hat{\underline{x}}(k\,|\,N) = \hat{\underline{x}}\,(k\,|\,k) + A(k)\,[\hat{\underline{x}}(k+1\,|\,N) - \hat{\underline{x}}\,(k+1\,|\,k)] \quad (11)$$
$$A(k) = P(k\,|\,k)\,\Phi'(k+1,k)\,P^{-1}(k+1\,|\,k) \quad (12)$$
$$P(k\,|\,N) = P(k\,|\,k) + A(k)\,[P(k+1\,|\,N) - P(k+1\,|\,k)]A'(k) \quad (13)$$

In these equations: $\hat{\underline{x}}$ (k | N) is the nx1 fixed-interval smoothed estimate vector; A(k) is the nxn smoothing filter gain matrix; P(k | N) is the nxn smoothing error covarience matrix; K = N-1, N-2, ..., 1, 0 is the discrete time index. All values of $\hat{\underline{x}}$ (k | k), $\hat{\underline{x}}$ (k+1 | k), P(k | k), P(k+1 | k) as input for algorithm of optimal fixed-interval smoothed estimates are requrithm for each k in the Kalman filter. Initial conditions or boundary conditions, $\hat{\underline{x}}$ (N | N) and P(N | N) the last value of $\hat{\underline{x}}$ (k | k) and P(k | k) for k = 0, 1, 2, ..., N are taken from Kalman filter. These initial values are the first values of the fixed-interval filtered estimate for k = N-1, N-2, ..., 0.

SEISMIC TRACE

Robinson (1967) showed that a simple synthetic seismogram can be obtained from the deconvolution of the source wavelet with the reflection coefficient sequence. This process can be shown as:

$$y(k) = \sum_{\vartheta=N}^{k} [\omega(k,\vartheta)\,\mu(\vartheta)] + v(k),\ k = 1, 2, 3, \ldots \quad (14)$$

where y(k) is the seismic signal; w(k,ϑ) is the time varying seismic wavelet; $\mu(\vartheta)$ is the reflection coefficient sequence; v (k) is measurement noise; k is the sampling number; N is the index that is defined by N=1 for k<L and N=k-1+1 for k≥L where L is the wavelet length in number of sample periods. The term w(k,ϑ) represents the (k-ϑ+1) th sample of the wavelet model corresponding to the time of k sample periods.

TIME VARYING SINGLE CHANNEL DISCRETE KALMAN FILTER

The state-space representation is obtained incorporating the base (transition) matrix into the discrete Kalman filter in Crumpt (1974). Let us look at again equation (14). A change in k is defined by i = k-ϑ+1. This transforms the equation (14) to:

$$y(k) = \sum_{i=1}^{j} [w(k,k-i+1)\,\mu(k-i+1)] + v(k),\ k = 1, 2, \ldots \quad (15)$$

where J=k for k<L and J=L for k≥L.

h (k) takes the wavelet values as $h'(k) = w(k,k-i+1)$. Then the equation (15) can be written as:-

$$y(k) = \sum_{i=1}^{j} [h'(k)\, \mu(k-i+1)] + v(k) \qquad (16)$$

According to this defination, measurement vector can be written as

$$y(k) = h'(k)\, \underline{x}(k) + v(k) \qquad (17)$$

The state vector can also be defined as

$$\underline{x}(k) = \begin{bmatrix} \mu(k) \\ \mu(k-1) \\ \cdots\cdots \\ \cdots\cdots \\ \mu(k-L+1) \end{bmatrix} \qquad (18)$$

The reflection coefficient sequence is assumed as:

$$\mu(k+1) = \sum_{i=1}^{k} [b_i(k)\, \mu(k)] + u(k) \qquad (19)$$

where u(k) is the white random process and b(k) is a vector in which the reflectivity function is expected to change with depth.

Using these definations, the state equation can be written as

$$\underline{x}(k+1) = \Phi(k+1,k)\, \underline{x}(k) + qu(k) \qquad (20)$$

where

$$\Phi(k+1,k) = \begin{bmatrix} b_1(k) & b_2(k) & b_3(k) & \cdots & b_{L-1}(k) & b_L(k) \\ 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \qquad (21)$$

and

$$q = [1 \quad 0 \quad 0 \quad 0 \quad \ldots \quad 0]' \qquad (22)$$

Transition matrix or base matrix can be given in this states-pace representation as

$$\Phi(k+1,k) = \Phi(k+1,k)^k \qquad (23)$$

Here, transition matrix is time-variant, therefore discrete Kalman filter becomes a time-variant filter (Cazdow and Martens, 1970).

APPLICATIONS:

As I have mentioned before, the Kalman filter has been applied to synthetic seismograms. In calculating these synthetics, I have used the actual sonic log taken by the Turkish Petroleum Company. These data are assumed to represent the reflection coefficient series. Using Robinson's approach, which was explained above, I convolved them with a source wavelet.

Two different source wavelets have been used. These are defined by

$$w(t) = \exp(-100t) \sin(100t) \qquad (24)$$

and

$$w(t) = -1360t \exp(-500t) + 0.5 \exp(-15.3t) \sin(2\pi t/0.06) \qquad (25)$$

The source wavelet calculated using (24), the sonic log and the synthetic seismogram are given in Figure 1. As it is seen in Figure 1 (b), the sonic log contains a certain level of noise. Due to this, the synthetic seismogram given in Figure 1 (c) is slightly noisy. However, in order to see the operation of the Kalman filter on noisy signals, I have added additional Gaussian noise onto the synthetic seismogram. The signal to noise ratio calculated from

$$\frac{S}{N} = \frac{1/M \sum_{k=1}^{M} s^2(k)}{\sigma_M^2} \qquad (26)$$

is taken as unity, where s(k) is the noise free discrete synthetic seismic trace; M is the number of sample, σ_M^2 is the variance of the Gaussian (normal) noise.

The result of the 2x2 dimensional discrete Kalman filter are shown in Figure 2. Figure 2 (a) and (b) are the synthetic and noisy synthetic traces. The output of the Kalman filter is given in Figure 2(c). As it is seen in Figure 2(b), the signals are masked by the noise. Though the noise is suppressed in certain amount by the Kalman filter, as it is observed from Figure 2(c), the S/N ratio is still close to 1. On the other hand the fixed-interval smoothing filter is more successful in eliminating the random noise as is seen in Figure 2(d).

The same data have been filtered using a 26x26 dimensional Kalman filter. The result is given in Figure 2(e). Comparing the output of this filter with that of 2x2 Kalman filter given in Figure 2(c), we can see that the 26x26 dimensional filter is better. However, due to the large matrices, it takes more CPU time in numerical calculations. In addition, much more memory space is needed to run the computer program.

The other data set used in numerical calculations was obtained using the source wavelet given in equation (25). The wavelet along with the reflection coefficient sequence and synthetic seismogram are given in Figure 3. These data have been filtered using 4x4 and 40x40 dimensional Kalman filters. Figure 4 shows the result of the 4x4 dimensional Kalman filter and that of the fixed-interval smoothing filter. The results of the 40x40 Kalman filter are given in Figure 4(e).

ERRORS IN THE OUTPUTS OF THE DISCRETE KALMAN AND SMOOTHING FILTERS

Information about the quality of the filtering can be acquire by showing the integral-square-errors of the outputs of discrete Kalman filter and optimal fixed-interval smoothing filter which have been applied to noisy synthetic seismic trace. The square errors between noiseless and noisy seismogram, output of the Kalman filter and fixed-interval smoothing filter can be defined respectively as:

$$\begin{aligned} e_1^2 &= [z(k) - y(k)]^2 \\ e_2^2 &= [h'\hat{\underline{x}}(k+1 \mid k+1) - y(k)]^2 \\ e_3^2 &= [h'\hat{\underline{x}}(k \mid N) - y(k)]^2 \end{aligned} \qquad (27)$$

where y(k) is the noiseless seismic trace, z(k) is the noisy seismic trace, $h'\underline{x}(k+1 \mid k+1)$ is the output of the discrete Kalman filter, $h'\underline{x}(K \mid N)$ is the output of the optimal fixed-interval smoothing filter.

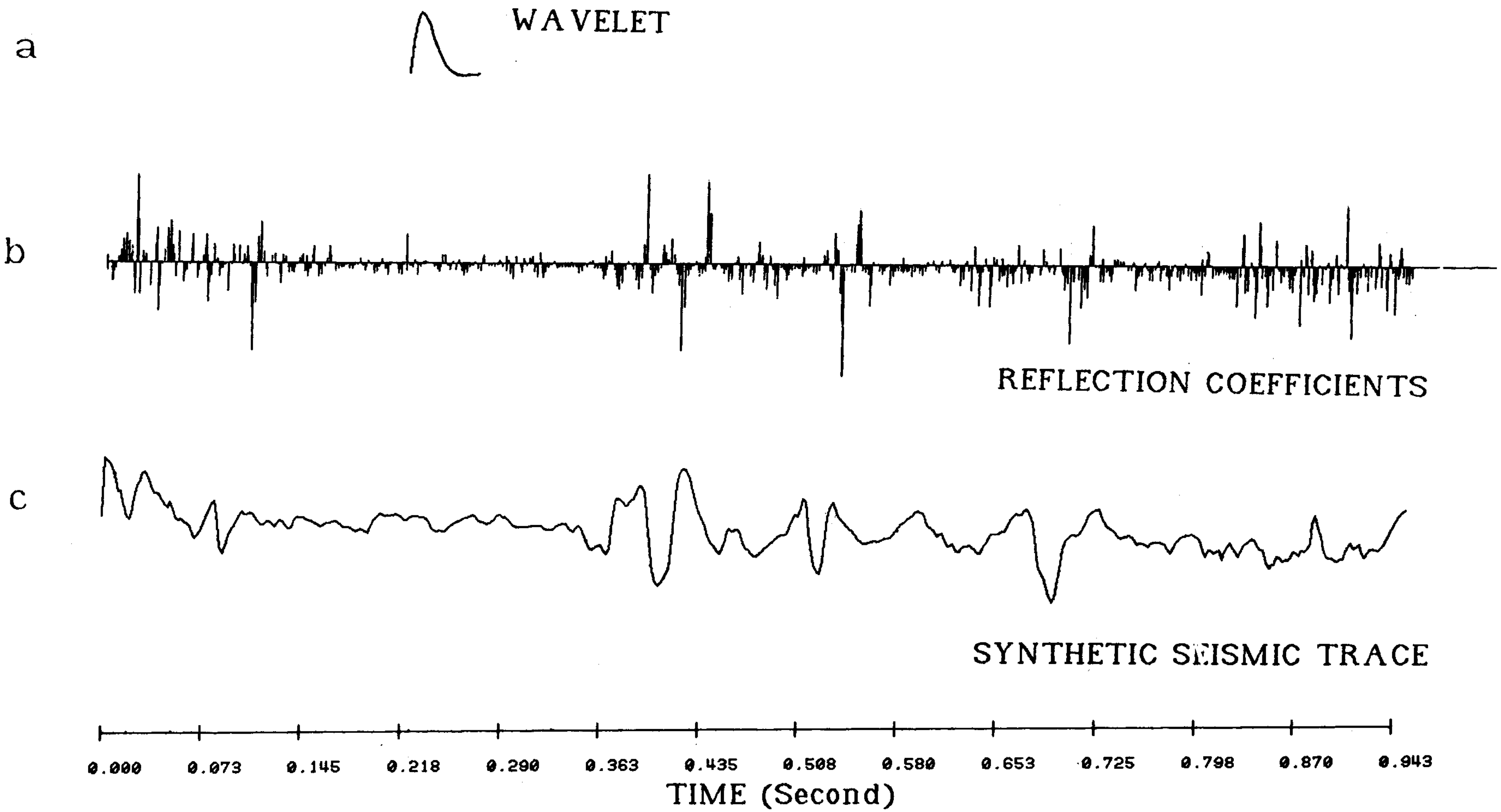

Fig. 1. The source seismic wavelet (a), the reflection coefficient sequence (b),and the noise free synthetic seismic trace (c).

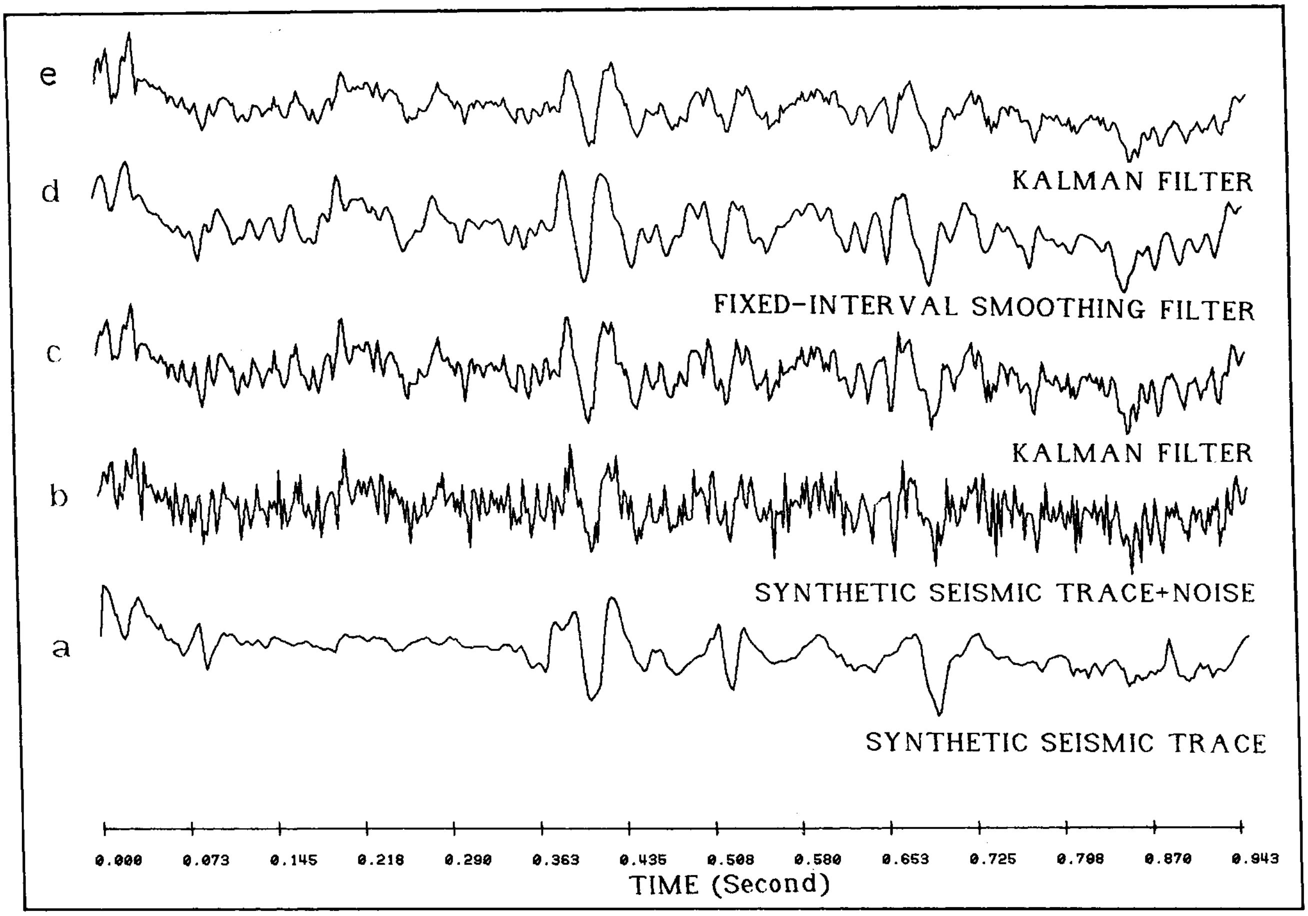

Fig. 2. The application of 2x2 (c) and 26x26 (e) dimensional discrete Kalman filter and 2x2 (d) dimensional fixed interval smoothing filter. Synthetic seismic trace (a, b), (S/N=1).

The square error of 2x2 and 26x26 dimensional Kalman and fixed-interval smoothing filters are shown in Figure 5. Figure 5 (a) shows the square error between the original and noisy seismograms. Figure 5 (b), (c) and (d) display the normalized square errors between original seismogram and the output of 2x2 dimensional Kalman filter, 2x2 dimensional fixed-interval smoothing filter and the output of 26x26 dimensional Kalman filter. Comparing these with the Kalman filter outputs given in Figure 2, we can see that the square errors shown in Figure 5 (b) and (c) are large in those parts of seismogram that the seismic wavelet exist while the square errors are small in the outputs corresponding to random noise. On the other hand, the same feature does not exist in 26x26 dimensional Kalman filter output as it is observed in Figure 5 (d).

The same comparison has been made for 4x4 and 40x40 dimensional filters. The results are given in Figure 6 (a), (b), (c), (d). These are also normalized to their maximum values. If we compare these outputs with those given in Figure 5, we may conclude that the performance of 4x4 dimensional filter is better than that of 2x2 filter. If we compare the large dimensional filters, we may again say that 40x40 dimensional filter is more efficient than 26x26 dimensional filter.

The sequence of the square error is not a good criterion for determination of the filter achievement. Indeed, when I compared only the square erros, I concluded above that 4x4 Kalman filter was better than 2x2 Kalman filter. On the other hand, the performance of the filter is best examined by calculating the integral-square-error of each trace. The total integral square error for 2x2, 4x4, 26x26, 40x40 dimensional Kalman filters calculated are: 0.1001, 0.2576, 0.1513, 0.0277, respectively. Obviously, the best filter is the one with the smallest-integral-square-error. In the present case, 40x40 filter has the smallest-integral-square-error. On the other hand, this filter is too large dimensional filter, consequently it is not economical. The second least-integral-square-error filter is the 2x2 filter with 0.1001 total error. Thus, we may conclude that the most economical and reliable filter is the 2x2 dimensional Kalman filter for the present data set.

CONCLUSION

In this study, the application of the Kalman filter to seismic deconvolution is discussed. The behaviors of these feedback filters with different dimensions are examined by applying them to different kind of seismic traces. It works in the time domain, and one of the most significant features of the Kalman filter is its feedback. This feature makes the computing easy on digital computers. However, large dimensional Kalman filters need too much computing time in application. The experiments showed that the noise reduction is not too sensitive to the filter length.

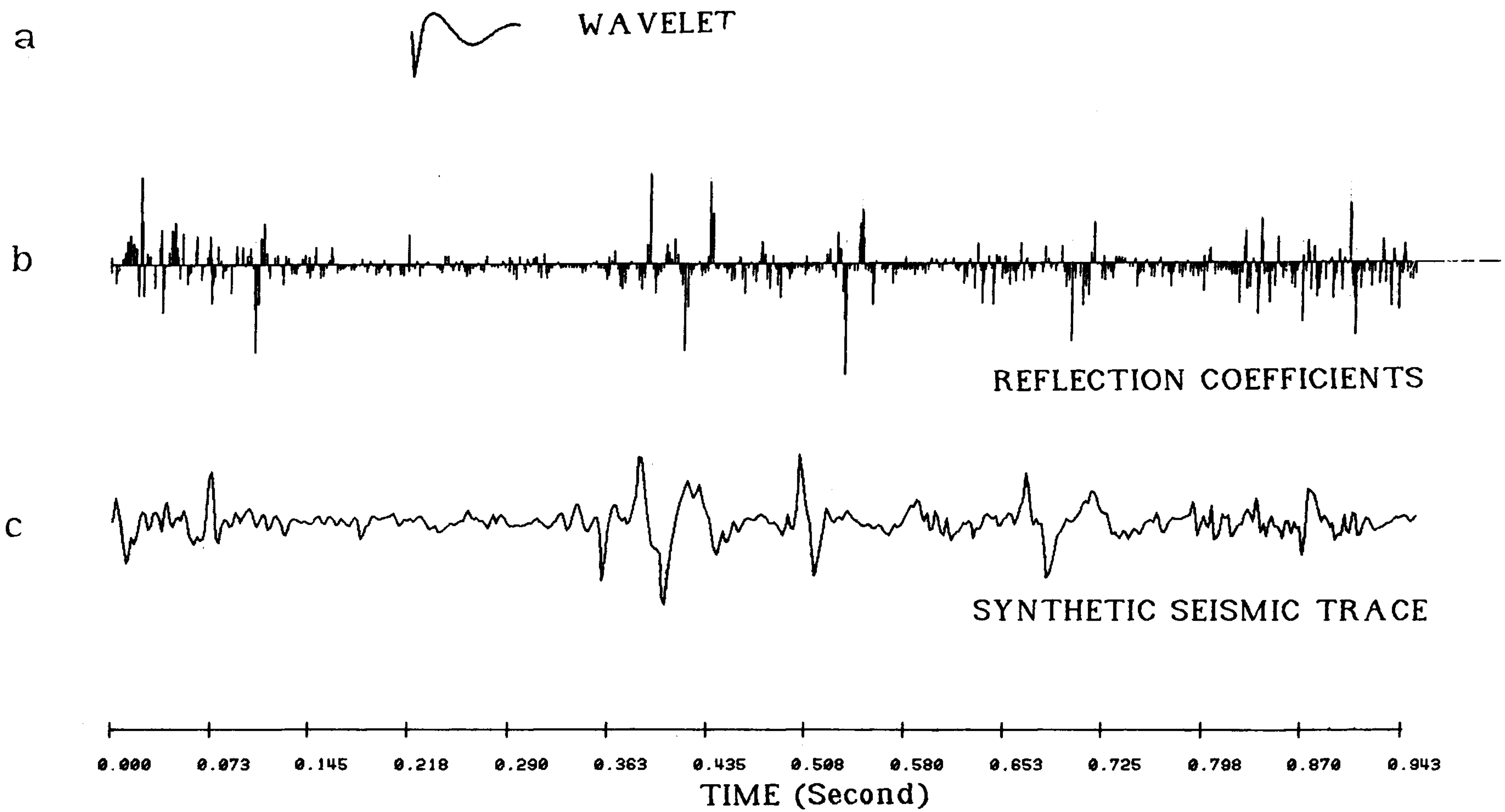

Fig. 3. The source seismic wavelet (a), the reflection coefficient sequence (b), and the noise free synthetic seismic trace (c).

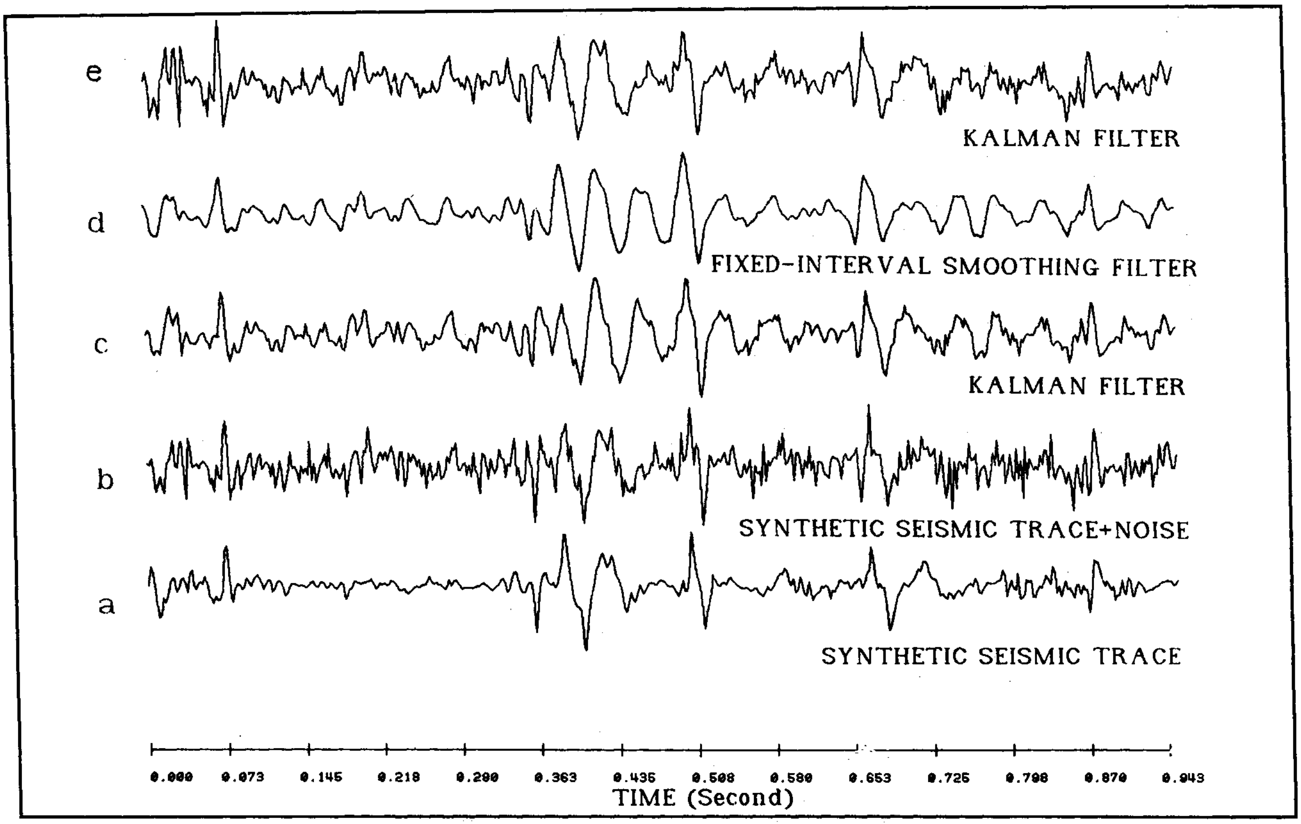

Fig. 4. The application of 4x4 (c) and 40x40 (e) dimensional discrete Kalman filter and 4x4 (d) dimensional fixed interval smoothing filter. Synthetic seismic trace (a,b), (S/N=1).

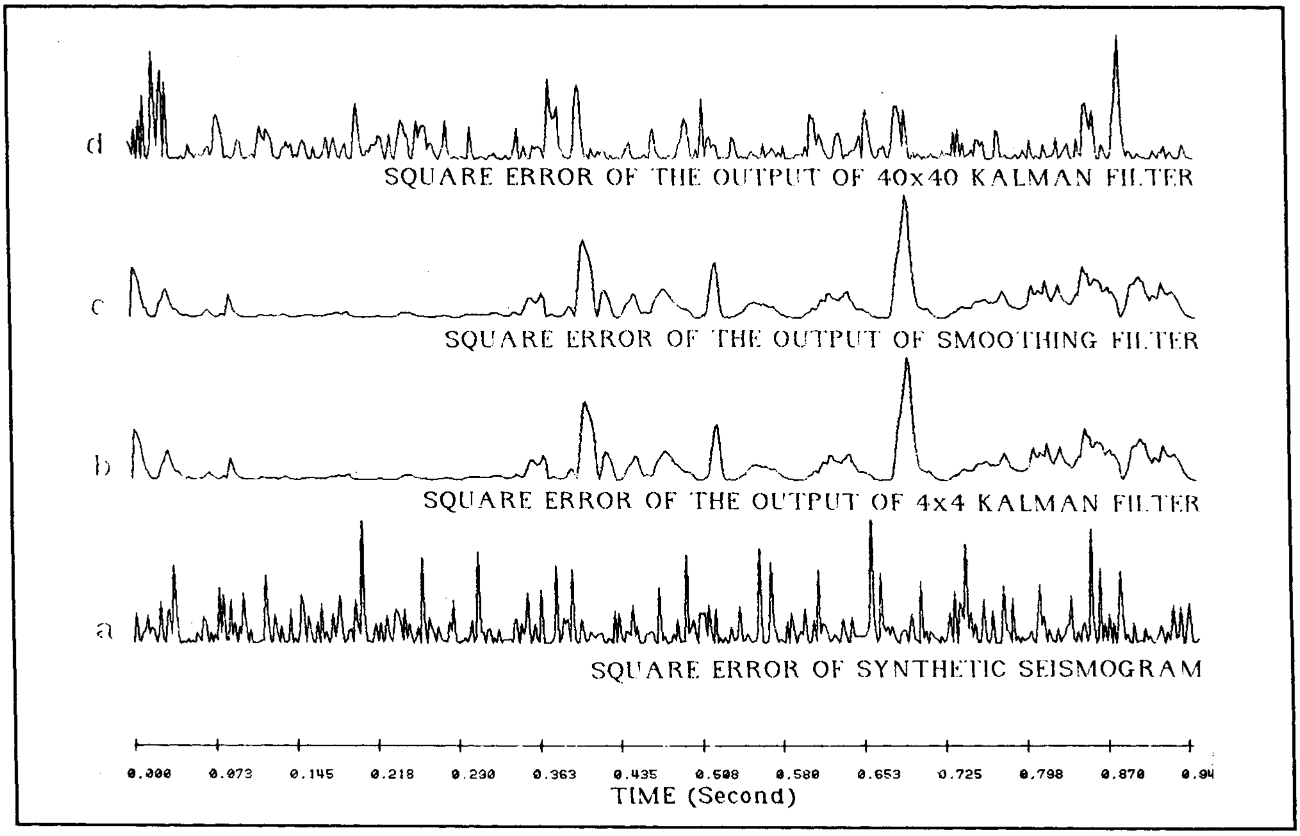

Fig. 5. The value of square error obtained from the synthetic seismogram (a), 2x2 (b) and 26x26 (d) dimensional discrete Kalman filter and 2x2 (c) dimensional optimal fixed interval smoothing filter (S/N=1).

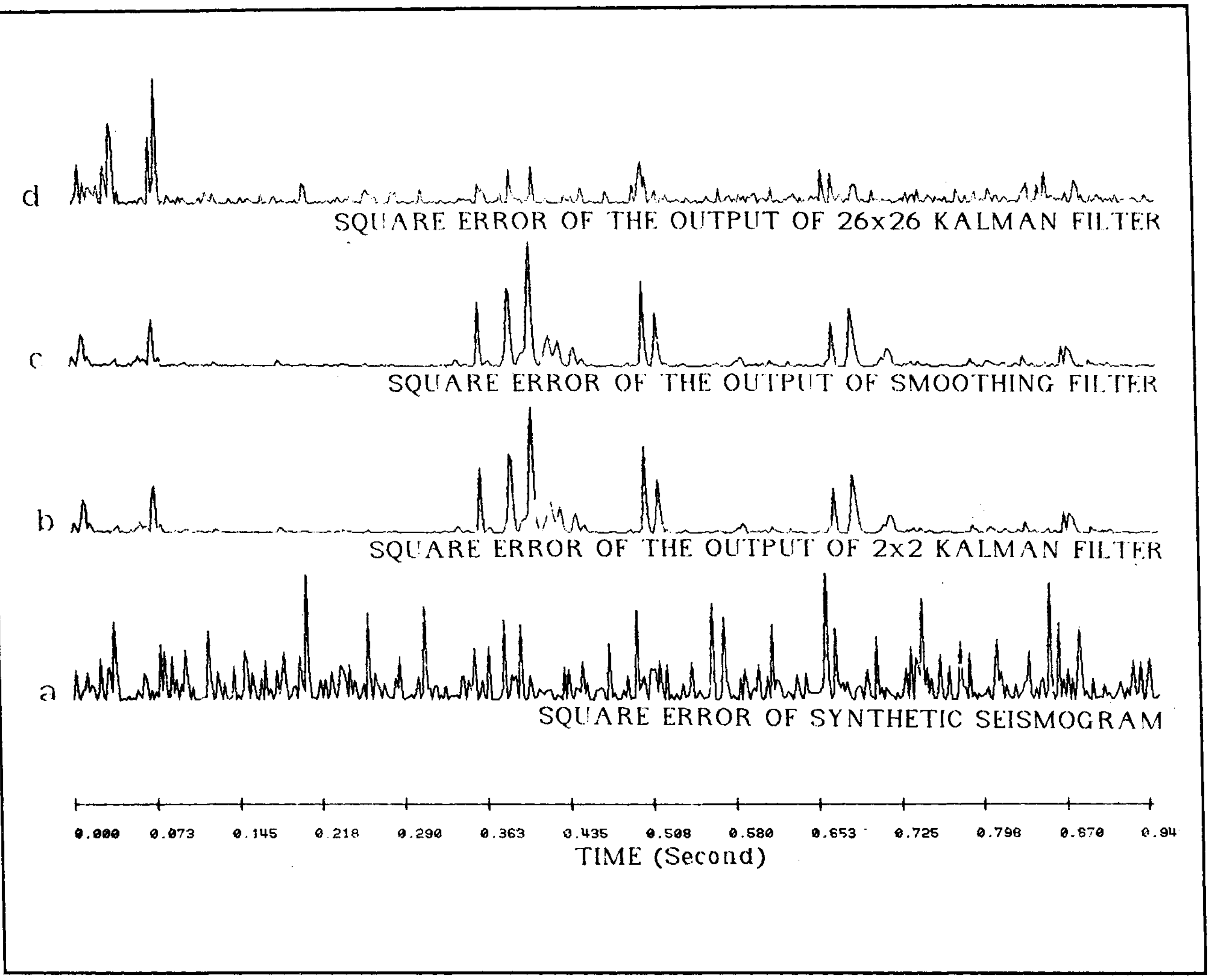

Fig. 6. The value of square error obtained from the synthetic seismogram (a), 4x4 (b) and 40x40 (d) dimensional interval Kalman filter and 4x4 (c) dimensional optimal fixed interval smoothing filter (S/N=1).

When I compare the integral-square-errors of the Kalman filters with different dimension, I obtained that 40x40 dimensional one is the best. On the other hand, due to the large dimension, this filter is not economical. During the same analysis, I have arrived at the point that 2x2 dimensional Kalman filter is the second least-integral-square-error filter. Its total error is even smaller than that of 26x26 dimensional Kalman filter. As a result, 2x2 dimensional Kalman filter is more efficient on seismic records with S/N=1.

ACKNOWLEDGMENT

This work is the part of my Ph. D. thesis while I was on the study of Marine Science and Technology Institute of the Dokuz Eylül University. I thank Turkish Petroleum Company for providing me the sonic log. I appreciate Prof. Bülent Sankur of Boğaziçi University and Prof. Murat Aşkar (METU) for their valuable suggestions. I would like to express my sincere appreciation to Prof. Nezihi Canıtez for critically reading and making many valuable improvements.

REFERENCES

Aminzadeh, F. and Mendel, J.M. 1983, Normal incidence layered system state-space models which include absorption effects. Geophysics 48,259-271.

Bayless, J.M. and Brigham, E.O. 1970, Application of the Kalman filter to continuous signal restoration, Geophysics 35, 2-23.

Cazdow, J.A. and Martens, H.R. 1970, Discrete Time and Computer Control Systems, Prentice Hall.

Crump, N.D. 1974, A Kalman filter approach to the deconvolution of seismic signals, Geophysics 39, 1-13.

Korymlo, J. and Mendel, J.M. 1980, Simultaneous spherical divergence correction and optimal devoncolution, IEEE Trans. Geosci. and Remote Sensing V. GE-18, 273-280.

Meditch, J.S. 1969, Stochastic optimal lineer estimation and control, McGraw Hill, New York.

Mendel, J.M. 1976, Single channel white noise estimators for predictive deconvolution, Presented of the 46th. Annual International SEG Meeting, Houston, Texas.

Mendel, J.M. 1977, White noise estimators for seismic data processsing in oil exploration, İEEE Trans. on Auto. Contro. V.AC-22, 694-706.

Mendel, J.M. 1979, Kalman Filtering for Exploration Geophysicsts In-depth Course, University of Southern California, Dept. of Electrical Engineering.

Mendel, J.M. and Kormylo, J. 1977, New-fast optimal white noise estimators for deconvolution, Spec. Iss., IEEE Trans. on Geosci. Electron V.GE-15, 32-41.

Mendel, J.M. and Kormylo, J. 1978, Single channel white noise estimators for deconvolution, Geophysics 43, 102-124.

Mendel, J.M.,Kormylo, J., Aminzadeh, F., Lee, J.S. and Ashrafi, F.H. 1981, A novel approach to seismic signal processing and modelling, Geophysics 46,1398-1414.

Ott, N. and Meder, H.G. 1972, The Kalman filter as a prediction error filter, Geophysical Probpecting 20, 549-560.

Robinson, E.A. 1967, Predictive Decomposition of Time Series with Application to Seismic Exploration, Geophysics 32,414-484 (1954 Ph.D. Thesis).